Complexity matching processes during the coupling of biological systems: Application to rehabilitation in elderly

Presented by Zainy Mshco Hajy Al Murad

The Monday, February 18, 2019

Under the supervision of Didier DELIGNIERES

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Dedication

I dedicate this humble effort to the spirit of my dear father and I will give to him when I meet him. I also dedicate this thesis to all the martyrs of Iraq and to their generous families whose sacrifices, which was their role of preserving Iraq and the Iraqis and I am among them and this effort is nothing in front of their sacrifices. Thank you, gratitude, mercy and forgiveness.
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Introduction

The main motivation of my doctoral project was the prevention of fall in older people. On the basis of this evocation, one could obviously expect a clinically oriented work, testing rehabilitation techniques and their effect on the propensity to fall. However, our approach was mainly theoretical and methodological, based on hypotheses derived from the theories of complexity.

Complexity appears a key concept for the understanding of the perennial functioning of biological systems. By definition, a complex system is composed of a large number of infinitely entangled elements (Didier Delignières & Marmelat, 2012). In such a system, interactions between components are more important than components themselves, a feature that Van Orden et al. (2003) referred to as interaction-dominant dynamics.

Such a system, characterized by a myriad of components and sub-systems, and by a rich connectivity, could lose its complexity in two opposite ways: either by a decrease of the density of interactions between its components, or by the emergence of salient components that tend to dominate the overall dynamics. In the first case the system derives towards randomness and disorder, in the second towards rigidity. From this point of view complexity may be conceived as an optimal compromise between order and disorder (Didier Delignières & Marmelat, 2012; Goldberger, Peng, & Lipsitz, 2002; Lipsitz & Goldberger, 1992). Complexity represents an essential feature for living systems, providing them with both robustness (the capability to maintain a perennial functioning despite environmental perturbations) and adaptability (the capability to adapt to environmental changes)(Whitacre, 2010; Whitacre & Bender, 2010). These relationships between complexity, robustness, adaptability and health were nicely illustrated by Goldberger, Amaral et al. (2002) in the domain of heart diseases.
The experimental approach to complexity has been favored by the hypothesis that links the complexity of systems and the correlation properties of the time series they produce, and the development of related fractal analysis methods, and especially the Detrended Fluctuation Analysis (Peng, Havlin, Stanley, & Goldberger, 1995). A complex system is supposed to produce long-range correlated series (1/f fluctuations), and the assessment of correlation properties in the series produced by a system allows determining the possible alterations of complexity, either towards disorder (in which case correlations tend to extinguish in the series) or towards rigid order (in which case correlations tend to increase).

This interest for complexity was particularly developed in the research on aging. Lipsitz and Goldberger (1992) proposed that aging could be defined by a progressive loss of complexity in the dynamics of all physiologic systems. This hypothesis has been developed in a number of subsequent papers (Goldberger, Amaral, et al., 2002; Goldberger, Peng, et al., 2002; Sleimen-Malkoun, Temprado, & Hong, 2014; Vaillancourt & Newell, 2002). Of special interest for the present work, Hausdorff and collaborators showed that successive step durations during walking presented a typical structure over time, characterized by the presence of long-range dependence (Hausdorff et al., 2001; Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995; Hausdorff et al., 1996). They also showed that these fractal properties were significantly altered in aged participants and in patients suffering from Huntington's disease (Hausdorff et al., 1997). In those cases the fractal organization tended to disappear and step dynamics became more random. Additionally, they showed that the loss of complexity in stride duration series correlated with the propensity to fall.

On these bases, the main question we address in this doctoral project was the following: could it be possible to restore complexity in older people, and especially in the locomotion system?

The working hypothesis that sustains the present work is based on the concept of complexity matching, initially introduced by West, Geneston and Grigolini (2008). The complexity matching effect refers to the maximization of information exchange when interacting systems share similar complexities. This effect has been interpreted as a kind of “1/f resonance” between systems (Aquino, Bologna, West, & Grigolini, 2011). A working conjecture states that interacting systems tend to match their complexities in order to enhance their synchronization (Marmelat & Delignières, 2012). This attunement of complexities has been observed in a number of synchronization experiments (Abney, Paxton, Dale, & Kello, 2014; Coey, Washburn, Hassebrock, & Richardson, 2016; Didier Delignières & Marmelat, 2014; Marmelat & Delignières, 2012; Stephen, Stepp, Dixon, & Turvey, 2008), and interpreted as a transfer of multifractality between systems (Mahmoodi, West, & Grigolini, 2017). Finally, Mahmoodi, West, & Grigolini (2017) showed that when two systems of different complexity levels interact, this transfer of multifractality operates from the most complex system to the less complex (and not the inverse), yielding an increase of complexity in the latter. Then our main hypothesis could be expressed as follows: Is it possible to restore the complexity of locomotion in older people, through a complexity matching effect that could result from the exercise of synchronized walking with young and healthy partners?

This doctoral project cannot be summarized, however, to the test of this essential hypothesis. In a first step a number of theoretical and methodological problems were to be solved, and especially about the identification of relevant methods for measuring (multi)-fractality in experimental series, and for evidencing the presence of genuine complexity matching in synchronized series. This second point was of crucial importance.
When we began this project, a number of papers tended to suggest that complexity matching could represent a very common phenomenon, present in all situations of interaction, cooperation, or synchronization. We thought, in contrast, that complexity matching, in the initial meaning developed by West, Geneston and Grigolini (2008), could be less frequent and maybe quite difficult to discern.

In a second step, it was necessary to show that synchronized walking, between young and healthy participants, gave effectively rise to a complexity matching effect. This first experimental contribution was difficult to organize and perform, but provided satisfying results, allowing the final test of our main hypothesis. We showed that a complexity matching affect was present in synchronized walking, and that this effect was stronger when the two partners were closely coupled.

Then our last experiment aimed at testing the possible restoration of complexity in older participants. The protocol was particularly demanding, requiring from participants a prolonged involvement, and a large amount of work. The recruitment and retention of participants posed many problems and this experiment was very long to achieve. This explains our request for an additional deadline for the defense of this doctoral thesis.

Considering the amount of papers we were able to finalize and publish during this work, we decided to base this doctoral thesis on the presentation of five scientific papers.

1. In the first one, we evaluate an improvement of the Detrended Fluctuation analysis, a widely used method in our domain of research. We show that evenly spacing, in the diffusion plot, significantly reduces the variability of exponent estimates.


2. In a second paper we present a new method for testing for the presence of a genuine complexity matching effect in experimental series. This method is based on the analysis of correlation functions between multifractal spectra, and tries to overcome the pitfalls of more traditional approaches.


3. In a third paper we show that synchronized walking; with young and healthy participants, is governed by a complexity matching effect.


4. A fourth paper is devoted to a formal analysis of a method introduced in the previous article, the Windowed Detrended Fluctuation analysis.


4. Finally a fifth paper shows that complexity matching may restore the complexity of walking in older people.

Chapter 1

Methodological Contribution
Chapter 1

Methodological Contribution: Evenly spacing in Detrended Fluctuation Analysis

Fractal analyses are confronted with important methodological problems. A number of methods have been proposed for assessing the fractal properties of experimental series, either in the time or in the frequency domain. Several studies tried to assess the performances of these methods, to evaluate their intrinsic biases, or to propose methodological improvements (Delignieres et al., 2006; Eke et al., 2000; Eke, Herman, Kocsis, & Kozak, 2002).

The aim of this chapter was to evaluate the effects of evenly spacing on the accuracy and variability of exponent assessment with Detrended Fluctuation Analysis (DFA). As most fractal analyses, DFA seeks at determining the characteristic exponent of a power law. The two sides of the equations are submitted to a logarithmic transformation, and the slope of linear regression between the two logarithmic scales gives the power exponent.

This logarithmic transformation yields a typical increase of the concentration of points, as the abscissa values increase. As a consequence, the higher part the abscissa scale presents a stronger weight in the determination of the exponent. Evenly spacing aims at correcting this bias.

![Figure 1](image)

**Figure 1.** Two example diffusion plots, obtained with the same series. The left panel represents the logarithmically spaced plot, and the right panel an evenly spaced plot.

A number of previous papers using DFA introduced evenly spacing, but this improvement of the original algorithm has never been evaluated.

In this paper we tested two methods: the evenly spaced DFA selects a sample of abscissa value, evenly spaced on the logarithmic scale, and computed the regression on these
selected points. Evenly spaced average DFA determines non-overlapping intervals of equal length on the abscissa logarithmic scale, computes the average values within each interval, and computes the regression on these average values. The first method has been generally used by the authors that applied evenly spacing with DFA. However, the second method was formally expected to provide more satisfying results.

Our results showed that evenly spacing did not improve the accuracy of estimates. However, we observed a significant decrease in variability with the two evenly spaced methods, as compared with the original DFA algorithm. The two evenly spaced methods gave similar results. The average decrease in variability is of about 36% for evenly spaced DFA, and 35% for evenly spaced averaged DFA, as compared with DFA. We showed that evenly spaced DFA methods presented a variability with series of 256 points which was roughly similar to that obtained with DFA, with series of 1024 points.

We used this refinement of DFA in all the papers that are presented in this doctoral thesis. Evenly spacing was also incorporated in the multifractal version of DFA (MF-DFA) we used for determining multifractal signatures of complexity matching.
Evenly spacing in Detrended Fluctuation Analysis:

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Abstract:

Detrended Fluctuation Analysis is a widely used method, which aims at assessing the level of self-similarity in time series. This method analyzes the diffusion properties of the signal, by computing the linear regression slope in the diffusion plot, representing in log–log coordinates the relationship between the variability of the signal and the length of the intervals over which this variability is computed. We compare in this paper the results obtained with logarithmically spaced and evenly spaced diffusion plots. The study shows the substantial benefits of evenly spacing, especially in the reduction of the variability of estimation.

Key-words: Detrended Fluctuation Analysis, power laws, diffusion plot, evenly spacing.

Introduction

The Detrended Fluctuation Analysis (DFA), initially introduced by Peng et al. \cite{1}, is a widely used analysis method which aims at determining the level of self-similarity in a time series. A better understanding of this method supposes a short introduction to the underlying model.

The DFA algorithm is based on the diffusion property of fractional Brownian motion (fBm), a family of stochastic processes introduced by Mandelbrot and Van Ness \cite{2}. Here we focus on the discrete version of fBm, which corresponds to the nature of the series analyzed in experimental research. In such process variance is a power function of the length ($n$) of the interval over which it is computed. Consider a process $x_i$:

\begin{equation}
Var(x_i) \propto n^{2H}
\end{equation}

Where $H$ is the Hurst exponent, which can take any real value within the interval $[0, 1]$. For $H = \frac{1}{2}$, $x_i$ corresponds to ordinary Brownian motion, and variance is just proportional to the elapsed time (normal diffusion). For $H \neq \frac{1}{2}$, $x_i$ is characterized by anomalous diffusion: The process is said subdiffusive for $H < \frac{1}{2}$, and superdiffusive for $H > \frac{1}{2}$.
Fig. 1. Two example diffusion plots, obtained with the same series. The left panel represents the logarithmically spaced plot, and the right panel an evenly spaced plot. The slopes of the regression lines are 0.85 and 0.88, respectively.

A second family of stochastic processes, fractional Gaussian noise (fGn), is defined as the series of increments within a fBm. By definition, a fGn is the series of differences in a fBm, and conversely the summation of a fGn gives a fBm. Each fBm series is then related to a specific fGn, and both are characterized by the same $H$ exponent. $H$ determines the nature of correlations between successive values in the fGn: for $H < 0.5$, successive values are negatively correlated, and the series is said to be anti-persistent. Conversely for $H > 0.5$ successive values are positively correlated, and the series is persistent. For $H = 0.5$, successive values are uncorrelated, and the series corresponds to white noise. An important difference between these two classes of processes is that fBm are non-stationary processes, as suggested by Eq. (1), whereas fGn series present stationary mean and variance over time.

As presented in the first paragraphs of this article, fGn and fBm are defined as two distinct families, which can be considered superimposed, with their relationships of summation/differencing. A number of authors, however, have proposed to consider these two families as a continuum, surrounding the mythical border of “ideal” $1/f$ noise [3–6].

The Detrended Fluctuation Analysis (DFA), Ref. [1] can be applied to both fGn and fBm signals. The details of the DFA algorithm will be detailed later in this paper. Here we just present its general principles, in order to introduce the hypotheses underlying the present work. This method exploits a typical scaling law, which states that the standard deviation of the integrated series is a power function of the interval length over which it is computed, with an exponent $\alpha$. Considering a time series $x_i$:

$$X_j = \sum_{k=0}^{j} x_k$$

$$SD(X_j) \propto n^\alpha$$

This scaling just derives from the original definition of fBm, expressed in Eq. (1). fGn series are characterized by $\alpha$ exponents ranging from 0 to 1, and fBm by exponents ranging from 1 to 2. $\alpha = 1$ corresponds to $1/f$ noise. If the series $x_j$ is a fGn, $X_j$ is the corresponding fBm and $\alpha$ is the Hurst exponent. If $x_j$ is a fBm, $X_j$ belongs to another family of over-diffusive processes, characterized by $\alpha$ exponents ranging from 1 to 2, and in that case $\alpha = H + 1$ [1].

The DFA algorithm works as follows: The series is first integrated, and then this integrated series is divided into non-overlapping intervals of length $n$. Within each interval the series is detrended, and the standard deviation of the residuals is computed. Then one calculates the average (detrended) standard deviation for the intervals of length $n$, noted $F(n)$. This computation is repeatedly performed over all $n$ values. In practice, the shortest interval length is chosen for allowing a valid estimate of standard deviation (for example $n = 10$), and the lengthiest for allowing at least two distinct estimates (e.g., $n = N/2$). Then $F(n)$ is plotted against $n$ in log–log coordinates, forming the so-called diffusion plot, and the exponent $\alpha$ is obtained as the slope of the linear regression (see Fig. 1, left panel).
An important consequence of this logarithmic transformation is that the density of points along the abscissa axis increases as interval length increases (see Fig. 1, left panel). And as regression analysis works on plane geometric principles, the weight of long intervals in the calculation of the slope becomes higher than that of short intervals. Moreover, the long-term region of the diffusion plot often presents irregularities around the linear trend, especially because the number of intervals involved in the computation of $R(n)$ is smaller for long time scales [7]. This results in a greater uncertainty in the determination of the slope in the long-term region of the diffusion plot, where most points are concentrated. A solution for overcoming this problem is to define the diffusion plot over a set of points evenly spaced in the logarithmic scale (Fig. 1, right panel).

Several methods have been proposed for obtaining a series of evenly spaced points in the log–log plot. Some authors have proposed to select a set of interval lengths, evenly spaced on the logarithmic scale. This idea was initially introduced by Peng et al. [8]. For example Jordan, Challis, and Newell [9] used fifty interval lengths distributed between 4 and $N/4$ [10–16]. This method will be designed thereafter as evenly spaced DFA.

However, this arbitrary selection of isolated, evenly spaced points in the original diffusion plot could raise some problems, especially in the long-term region of the diffusion plot, where points often present strong deviations from the regression line. A solution for accounting with this potential problem is to divide the range of $\log(n)$ into a series of intervals of equal length, and then to average the points that fall within each interval [17–19]. This method allows to exploit the whole information contained in the original diffusion plot, while satisfying the evenly spacing principle. This method will be designed thereafter as evenly spaced averaged DFA.

While theoretically convincing, the advantages of evenly spacing have never been rigorously assessed, and a number of recent papers still work with logarithmically spaced points [20–31]. The aim of this paper is to assess the benefits of evenly spacing, in terms of accuracy and consistency of estimates. We hypothesize that evenly spacing should produce a lower variability in the estimation of exponents, and should allow working with shorter series. We also hypothesize that the evenly spaced averaged method should gave better results than simple evenly spacing.

**Methods**

**Series simulation**

In order to assess the performances of DFA over the whole fGn/Bm model, we simulated series with the algorithm proposed by Davies and Harte [32]. This method is known to generate fGn series that preserve the expected correlation structure for a given $H$ value. This algorithm has been used in a number of previous studies aiming at analyzing exact fractal series [4,33–35]. We first generated fGn series for $H$ values ranging from 0.1 to 0.9, by steps of 0.1. A second set of fGn series was generated, for $H$ values ranging from 0.1 to 0.9, by steps of 0.1, and these series were summed for obtaining fBm series for each corresponding $H$ values. In each case we generated 120 series of 1024 data points.

**Detrended Fluctuation Analysis**

Here we present in detail the original algorithm of DFA [1]. Consider a series $x_i$ of length $N$. The series is first integrated, by computing for each $i$ the accumulated departure from the mean of the whole series:

$$X_i = \sum_{j=1}^{i} \left( x_j - \bar{x} \right)$$

This integrated series is divided into $k$ non-overlapping intervals of length $n$. The last $N - kn$ data points are excluded from analysis. Within each interval, a least squares line is fitted to the data (representing the trend in the interval). The series $X_i$ is then locally detrended by
subtracting the theoretical values \( X_i^{th} \) given by the regression. For a given interval length \( n \), the characteristic size of fluctuation for this integrated and detrended series is calculated by:

\[
F(n) = \sqrt{\frac{1}{N - kn} \sum_{i=1}^{N-kn} (X_i - X_i^{th})^2}
\]

This computation is repeated over all possible interval lengths. In the present paper we considered interval lengths ranging from \( n = 10 \) to \( n = N/2 \). Typically, \( F(n) \) increases with interval length \( n \). A power law is expected, as

\[
F(n) \propto n^\alpha
\]

\( \alpha \) is expressed as the slope of the double logarithmic plot of \( F(n) \) as a function of \( n \) (see Fig. 1, left panel).

**Evenly spaced DFA**

The papers using evenly spaced DFA remain often elusive about the selection of the points to introduce in the diffusion plot. Generally the shortest and lengthiest intervals are explicitly defined, and sometimes the number of points included in the diffusion plot. Quite often, however, this information is only accessible through the visual examination of the diffusion plots, when provided. The main idea is that the series of selected interval lengths should present a geometric progression:

\[
n_i = a n_{i-1}
\]

where \( a \) is a constant [8,10,15]. Here we propose a generic solution, considering the number of points to include in the diffusion plot \( k \), the minimum and maximum interval lengths \( n_{min} \) and \( n_{max} \), respectively). The \( k \) interval lengths, noted \( n_i (i = 1,2,\ldots,k) \), are defined as:

\[
\begin{align*}
n_1 &= n_{min} \\
\boxed{n_i} &= \left[n_{i-1} 10^{\frac{\log_{10}(n_{max}/n_{min})}{k-1}}\right]
\end{align*}
\]

The brackets signify that \( n_i \) is rounded to the closest integer value. In the present analysis, for series of 1024 points, we set \( k = 18 \), \( n_{min} = 10 \), and \( n_{max} = 512 \) \((N/2)\). We obtained the following \( n_i \) values: 10, 13, 16, 20, 25, 32, 40, 51, 64, 80, 101, 128, 161, 203, 322, 256, 406, 512. The regression slope was computed over the corresponding points.

**Evenly spaced averaged DFA**

For obtaining the evenly spaced averaged diffusion plots, we divided the \((\log)\) abscissa into \( k \) bins of length \( \frac{\log_{10}(n_{max}/n_{min})}{k} \), starting from \( \log_{10}(n_{min}) \). The \( k \) bins are then defined by:

\[
bin_i = \left[\log_{10}(n_{min}) + \frac{i-1}{k} \left(\log_{10}(n_{max}/n_{min})\right)\right]
\]

With \( i = 1,2,\ldots,k \). The corresponding bins in the natural scale are defined as:

\[
bin_i = \left[n_{min} 10^\frac{(i-1)\log_{10}(n_{max}/n_{min})}{k}, n_{min} 10^\frac{i\log_{10}(n_{max}/n_{min})}{k}\right]
\]

With \( i = 1,2,\ldots,k \). In this natural scale, the length of the successive bins (and hence the
number of points that fall into each successive bin) presents a geometric progression:

$$length_{i+1} = \left(10^{\frac{1}{\log_{10}(\frac{n_{\text{max}}}{n_{\text{min}}})}}\right)length_i$$  \hspace{1cm} (10)

Within each bin \(i\), we computed the average interval length \(\bar{n}_i\) and the average fluctuation size \(F(n)_i\), and determined the diffusion plot with these \(k\) average points. In the present analyses, we set \(n_{\text{min}} = 10\), \(n_{\text{max}} = 512\), and \(k = 18\).

**Statistical analyses**

The results of these analyses were examined in terms of accuracy and consistency. Accuracy refers to the difference between the mean estimate for a set of series and the true exponent that was used for its simulation. This aims at determining systematic biases that could occur in some parts of the fGn/fBm model [33]. Consistency refers to the standard deviation of exponent estimates in a set of series simulated with the same true exponent. In order to assess the effect of series length, we performed additional analyses with shorter series of 512 and 256 data points. In both cases we considered the first segments of our simulated series.

**Results**

We present in Fig. 2 example diffusion plots obtained with the three methods on the same series. As expected, DFA produced a diffusion plot exhibiting large fluctuations around the linear trend in the long-term region. In contrast, the two other methods seem able to allow a more effective determination of the linear trend. Note that in this particular example, evenly spaced averaged DFA seems to present less deviations from the linear slope in the long-term region.

![Example diffusion plots](image)

**Fig. 2**: Example diffusion plots, obtained with a series with true \(\alpha = 0.9\). Left: DFA; middle: evenly spaced DFA; right: evenly spaced averaged DFA.

We present in Fig. 3 the results of the three methods, in terms of accuracy of the mean estimates. All methods gave similar results for fGn series, and mean \(\alpha\) estimates appear very close to the corresponding true \(\alpha\). In contrast, DFA tends to underestimate \(\alpha\) for fBm series. This bias is also present in the two evenly spaced methods, which both present similar results.

The length of series had no noticeable effect of the accuracy of methods for fGn series. Reducing length tended to increase the underestimation bias for fBm close to \(1/f\).
Fig. 3. Mean $\alpha$ estimates, as a function of true $\alpha$ exponents, for DFA (left), evenly spaced DFA (middle), and evenly spaced averaged DFA (right). The solid line represents the theoretical equality between estimated and true exponents.

We present in Fig. 4 (left panel), for the three methods, the standard deviation of the samples of estimates, as a function of true $\alpha$. In all cases variability increases as true $\alpha$ increases, with a kind of ceiling effect for $\alpha > 1.5$.

The most interesting result is the clear decrease of variability with the two evenly spaced methods, as compared with DFA. Note that the two evenly spaced methods gave similar results. The average decrease in variability is of about 36% for evenly spaced DFA, and 35% for evenly spaced averaged DFA, as compared with DFA.

We present in Fig. 4 (right panel) the standard deviation of $\alpha$ estimates as a function of series length, using evenly spaced DFA. As expected, variability increased as series length decreased. However, the examination of the two graphs shows that evenly spaced DFA presents a variability with series of 256 points which is roughly similar to that obtained with DFA, with series of 1024 points. Evenly spaced averaged DFA gave similar results.

Fig. 4. Left: Standard deviation of the samples of estimated exponents, as a function of the true exponent, for the three tested methods. Right: Standard deviation of the samples of estimated exponents, as a function of the true exponent and according to series length (results obtained with evenly spaced DFA, evenly spaced averaged DFA gave similar results).

Discussion

The use of evenly spaced diffusion plots has been recommended for a long time in the literature, but its benefits have never been systematically assessed, and a number of recent papers still exploit logarithmically spaced diffusion plots. We show in this paper that evenly spacing provides substantial benefits, and especially increases the consistency of estimates. Evenly spacing provides reliable results with relatively short series, an important goal in psychological and behavioral sciences in the design of analysis methods [33].

We tested two distinct procedures, evenly spaced DFA and evenly spaced averaged DFA,
hypothesizing that the second one should give better results. Our analyses did not comfort this hypothesis, and both methods gave similar results. One should keep in mind, however, that the present study was performed with synthetic signals. One could suppose that such signals present a more homogeneous correlation behavior over time, and fewer deviations around the regression line in the diffusion plot, as compared with empirical series. Despite the absence of statistically significant results in the present work, the evenly spaced averaged method could in our mind be favored.

Our results confirm the slight underestimation bias of DFA for fBm series, which has been already noticed in several studies [4,33]. It is important to keep in mind that DFA first integrates the series and then analyzes the diffusion properties of the resultant series. In other words the initial series is considered as increments, and DFA works on the diffusion properties of its cumulative sum. However, DFA is mainly sensitive to the correlation structure of the series of increments, and Delignières [36] showed that fGn and fBm presented completely different correlation structures. By definition, the cumulative sum of a fGn is a fBm, which possesses the diffusion property expressed in Eq. (1). In contrast, there is no guaranty that the correlation structure of fBm provides its cumulative sums with such a property. The global underestimation bias with fBm series could be related to this problem. It seems important to notice that when the analyzed series is unambiguously characterized as fBm (e.g., with an $\alpha$ estimate higher than 1.2, see Ref. [33]), it could be considered to re-apply DFA, omitting the summation step. This procedure directly assesses the diffusion properties of the original signal (supposed to be a fBm), and gives an unbiased estimate of the Hurst exponent, which can be converted if necessary in $\alpha$ metrics ($\alpha = H + 1$).

Evenly spacing could also be of interest in the approach of crossovers, a rather common phenomenon in fractal analysis [37–39]. Scale invariance is theoretically revealed by the presence of a linear trend in the bi-logarithmic diffusion plot, over the whole range of time scales. Sometimes, however, correlations do not follow the same scaling law for all time scales, and a crossover can be observed between different scaling regions [37,40]. Such crossover could be related to the presence of a sinusoidal trend in the series, and in that case the timescale where this crossover occurs is inversely related to the oscillatory frequency that dominates the time series [38]. More generally, a crossover appears when series are bounded within upper and lower limits [41].

Because crossovers often appear in the long-term region of the diffusion plots, one could suppose that evenly spacing could allow a more accurate determination of the nature of the crossover [38], and in the case of genuine crossovers separating the diffusion plots in two clearly distinguishable scaling regimes, a better estimate of the crossover point, in order for example to filter out the troublemaking frequency [42].

The present paper focused more on evenly spacing than on DFA itself. However, some concluding remarks about this widely used method could represent an interesting complement. The DFA algorithm is based on the fGn/fBm model, and a number of experiments have proven the suitability of this model for accounting for the long-range correlation properties of a wide diversity of physiological signals. However, a blind application of DFA could yield irrelevant results, and one could wonder whether the fGn/fBm model can be used for any forms of physiological fluctuations [3,43]. Some recent experiments considered signals that clearly fell out of the scope of the fGn/fBm fluctuations [41,44]. Moreover, the fGn/fBm model presents an abrupt breakdown of correlation properties around the $1/f$ boundary, casting some doubts about its relevancy for accounting for $1/f$ behavior [36]. Some alternative models have been proposed for accounting for long-range correlation properties [45,46], which could be considered in order to design more suitable analysis methods.

Despite these reserves, evenly spacing yields substantial benefits, and it seems reasonable to promote the systematic use of this procedure, in order to improve the performances of DFA. Note that this recommendation could be extended to all methods exploiting power laws,
including Power Spectral Density [4], Scaled Windowed Variance Analysis [47], Rescaled Range Analysis [48], or Dispersional Analysis [49,50].

References:


Chapter 2

Multifractal signatures of complexity matching
Chapter 2

Multifractal signatures of complexity matching

*Complexity matching and strong anticipation.*

As indicated in the introduction, complexity matching represents the central concept of this thesis. Complexity matching was introduced by West, Geneston and Grigolini (2008), who showed that information transfer between system is maximized when they present similar complexity. This theory, however, was not directly exploited in the domain of synchronization.

For a long time, synchronization processes were mainly analyzed through the paradigm of synchronization with a regular metronome, and on the basis of representational models (for reviews, see Repp, 2005; Repp & Su, 2013). These models supposed that synchronization was mainly achieved by the correction of the asynchrony perceived by the individual, between the sounds emitted by the metronome and his/her actions (e.g., finger taps). These models received considerable support, but their relevancy was clearly linked to the regularity of the metronome, allowing to anticipate precisely the dates of the next occurrences of the signals.

A decade ago, some authors tried to overcome the limits of this initial paradigm: In real life, synchronization does not occur with regular metronomes, but rather with signals and rhythms that present marked fluctuations. This is especially the case for interpersonal synchronization, in which both partners tend to produce $1/f$ fluctuations. In such cases, precise predictions about the date of occurrence of the next event are difficult, and the traditional models based on error correction seem hardly sustainable.

In a first step authors referred to the model proposed by Dubois (2003), who distinguished the processes of strong and weak anticipation (Vorberg & Wing, 1996). Synchronization with the environment has been often described in terms of anticipation: For example, when participants have to synchronize finger taps with the beats emitted by a metronome, a mean negative asynchrony is consistently reported, suggesting that participants do not react to auditory stimuli, but rather anticipate their occurrence. Such anticipatory behavior can be underlain by the formation of an internal model that allows short-term predictions about the time of occurrence of the next metronome signal. A number of representational models, based on phase correction (Vorberg & Wing, 1996) and/or period correction (Mates, 1994) have been proposed for explaining synchronization in tapping tasks (Repp, 2005). This kind of local, short-term anticipation, based on internal models and corrective processes, is referred by Dubois (2003) to as weak anticipation.

Dubois (2003) evoked a second kind of anticipative behavior he called strong anticipation, which is supposed to occur without reference to any internal model (Stephen et al., 2008; Stepp & Turvey, 2010). According to the authors, strong anticipation is based on the embedding of the organism within its environment, creating a new, organism-environment system, which possesses lawful regularities that allow the emergence of anticipation.

Stephen and Dixon (2011) noted that two divergent approaches to strong anticipation have to be distinguished. The first one suggests that strong anticipation results from an appropriate local coupling between the organism and its environment. For example the synchronization of the rhythmic oscillations of a limb with a periodic metronome has been successfully accounted for by a model of coupled oscillators, including a parametric driving function (Jirsa, Fink, Foo,
More sophisticated models in physics have shown that during the synchronization between a slave and a master systems, the presence of time delays in the master system yields the slave system to synchronize with future states of the master (Voss, 2000). These models of coupled oscillators suggest that anticipation could emerge from the macroscopic properties of the organism-environment system. This conception supposes that anticipation is based on local time scales (Stepp & Turvey, 2010).

A second approach considers that strong anticipation could be based on a more global coordination between the organism and its environment. Stephen, Stepp, Dixon, and Turvey (2008) analyzed synchronization with a chaotic metronome: In that case, local predictions are difficultly conceivable, because of the intrinsically unpredictable nature of the pacing signal. Indeed, the authors showed that tapping behavior in this situation exhibited a mix of reaction, proaction, and synchrony to metronome signals. Importantly, they observed a close matching between the fractal exponents of the chaotic signals and those of the corresponding inter-tap interval series. In this kind of strong anticipation, the organism is not adapted to the states of the environment but to their statistical structure. The presence of 1/f scaling in the environment is essential in this coordination process: the organism exploits the complexity of the environment, and especially the long-range correlated structure of its evolution over time, as a resource for a more adaptive and efficient behavior (Stephen et al., 2008; Stepp & Turvey, 2010).

Note that the concept of strong anticipation has been primarily introduced for accounting for the adaptation of (complex) organisms with their (complex) environment. Anticipation suggests a directional relationship, with a slave system attempting to anticipate the future states of a master system. The principles that underlie strong anticipation, however, can be extended more generally to coordination processes between equivalent systems (Marmelat & Delignières, 2012). In that case systems mutually adapt, with a kind of bi-directional anticipation. Strong anticipation, in this context, suggests that the complexity of both systems is an essential resource for their effective coordination. For example Marmelat and Delignières (2012) analyzed inter-personal coordination in a task where participants had to move pendulums in synchrony. Results revealed a poor local correlation between the series of oscillation periods produced by the two participants of each dyad. The authors analyzed the scaling properties of the series of periods produced by participants, separating short-term and long-term scaling behaviors. They evidenced a close correlation between long-term fractal exponents, but in the short term series behaved more independently.

**The complexity matching effect**

More recently the study of synchronization between complex system was essentially based on the concept of complexity matching (Abney et al., 2014; Coey et al., 2016; Den Hartigh, Marmelat, & Cox, 2018; Fine, Likens, Amazeen, & Amazeen, 2015; Marmelat & Delignières, 2012). Note that this substitution between strong anticipation and complexity matching cannot be considered as a renewal of theoretical foundations, but rather as a switch towards a more insightful perspective (Marmelat & Delignières, 2012). The weak/strong anticipation framework proposed a binary classification between representational and non-representational models of anticipation. The distinction proposed by Stepp and Turvey (2010) between the local and global forms of strong anticipation was just an *ad hoc* tinkering, that tried to distinguish the models of continuous coupling from “something else”, an amazing phenomenon that was observed and which cannot be accounted for neither by representational models, nor by continuous coupling models.

The complexity matching framework provided a theoretical foundation to global strong anticipation. This theoretical step, however, was not so direct. The complexity matching effect was initially evidenced through the analysis of the efficiency of the transfer of information between and within complex systems (West et al., 2008). The authors showed that information
exchange is maximal when systems share the same complexity, and especially $1/f$ scaling. Delignières and Marmelat (2012) derived a simple conjecture for this seminal principle: “In interpersonal coordination tasks, a very efficient information transfer must support the process of coordination between systems: In this case, the best way of exchanging information is to match complexities. This interpretation implies that both systems modified their own dynamics in order to produce a stable pattern of coordination. One could argue that complex systems modify their internal functional organization (Kello, Beltz, Holden, & Van Orden, 2007) when they have to synchronize with another complex system” (Marmelat & Delignières, 2012). This conjecture allowed to deeper the theoretical analysis of processes underlying global strong anticipation.

Complexity matching, from this point of view, is conceived as a multiscale coordination between systems (Den Hartigh et al., 2018). While the synergetics approach considered coordination as the continuous coupling between macroscopic features of both oscillators (Haken, Kelso, & Bunz, 1985), complexity matching suggest a more global form coordination, involving all time scales within the systems.

**Statistical matching and genuine complexity matching**

As previously indicated, the seminal paper in this domain was proposed by Stephen, Stepp, Dixon, and Turvey (2008). In this experiment, the main statistical evidence provided by the authors was the close correlation between the fractal exponents of the chaotic signals and those of the series produced by participants. Marmelat and Delignières (2012) evidenced similar results. Results revealed low local correlations between the series of oscillation periods produced by the two participants of each dyad. The authors analyzed the scaling properties of the series of periods produced by participants, and evidenced a very close correlation between fractal exponents. Similar results were evidenced by Marmelat and Delignières (2012), in an inter-personal coordination task where participants oscillated pendulums in synchrony, and by Abney, Paxton, Dale, and Kello (2014), in the analysis of speech signals during dyadic conversations.

We think, however, that the idea that the matching of scaling exponents could be considered an unambiguous signature of complexity matching remains questionable, and it could be necessary to distinguish between a simple *statistical matching* (i.e., the convergence of scaling exponents) and the genuine complexity matching effect (i.e., the attunement of complexities). Fine et al. (2015), in an experiment on rhythmic interpersonal coordination, observed a typical matching of scaling exponents, but suggested that this statistical matching could just result from local phase adjustments, and not from a global attunement of complexities. As well, Delignières and Marmelat (2014) analyzed series of stride durations produced by participants attempting to walk in synchrony with a fractal metronome. They tried to simulate their empirical results by means of a model based on local corrections of asynchronies, and showed that this model was able to adequately reproduce the statistical matching observed in experimental series. The authors concluded that walking in synchrony with a fractal metronome could essentially involve short-term correction processes, and that the close correlation observed between scaling exponents could in such a case just represent the consequence of local correction processes. Torre et al. (2013) also supported this hypothesis in a tapping task where participants synchronized with different non-isochronous auditory metronomes. They evidenced that inter-tap intervals could be modeled based on the previous inter-beat interval of the metronome and a correction of previous asynchronies...
to the metronome, independently of the level of $1/f$ fluctuations of the metronome (i.e. white noise or pink noise).

**Complexity matching and multifractality**

One of the most appealing hypotheses about the origin of fractal fluctuations in the behavior of complex systems refers to the idea that the interactions between system’s networks are governed by multiplicative cascade dynamics (Ihlen & Vereijken, 2010). Such dynamics is supposed to generate multifractal, rather than monofractal fluctuations, and indeed Ihlen and Vereijken (2010) showed that it was the case in most previous analyzed series in the literature. In the same vein Stephen and Dixon (2011) considered that complexity matching should be conceived as a product of multiplicative cascade dynamics, entailing a coordination of fluctuations among multiple time scales. More recently Mahmoodi, West, and Grigolini (2017) interpreted complexity matching as a transfer of multifractality between systems.

**Statistical signatures of complexity matching**

Our goal in the present paper is to seek for statistical signatures that could unambiguously distinguish between genuine global complexity matching and local corrections or adjustments. As previously explained, most previous papers that tried to evidence complexity matching effects worked on the basis of monofractal analysis. Here we propose to adopt multifractal analyses because they allow for a more detailed picture of the complexity of time series, and also because the tailoring of fluctuations that is typical of complexity matching could be considered as the product of multifractality (Stephen & Dixon, 2011).
Multifractal signatures of complexity matching

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Abstract:
The complexity matching effect supposes that synchronization between complex systems could emerge from multiple interactions across multiple scales, and has been hypothesized to underlie a number of daily-life situations. Complexity matching suggests that coupled systems tend to share similar scaling properties, and this phenomenon is revealed by a statistical matching between the scaling exponents that characterize the respective behaviors of both systems. However, some recent papers suggested that this statistical matching could originate from local adjustments or corrections, rather than from a genuine complexity matching between systems. In the present paper we propose an analysis method based on correlation between multifractal spectra, considering different ranges of time scales. We analyze several datasets collected in various situations (bimanual coordination, interpersonal coordination, walking in synchrony with a fractal metronome). Our results show that this method is able to distinguish between situations underlain by genuine statistical matching, and situations where statistical matching results from local adjustments.

Key-words: Synchronization, coordination, complexity matching, multifractals
Introduction

The concept of *complexity matching* (West et al. 2008) states that the exchange of information between two complex networks is maximized when their complexities are similar. This particular property requires both networks to generate $1/f$ fluctuations, and has been interpreted as a kind of “$1/f$ resonance” between networks (Aquino et al. 2011). In such a situation, a complex network responds to a stimulation by another as a function of the matching of their measures of complexity, i.e. the matching of their $1/f$ fluctuations. In contrast, the response of a complex network to a harmonic stimulus is very weak as compared with that obtained with another network of similar complexity (Aquino et al. 2010; Mafahim et al. 2015).

A direct conjecture exploiting the complexity matching effect is when two complex systems become coupled, they should attune their complexities in order to optimize information exchange. This conjecture has been initially tested by Stephen et al. (2008), in a task where participants had to synchronize finger taps with a chaotic metronome. Results showed that despite the unpredictable nature of the stimuli participants where roughly able to synchronize with the chaotic metronome, with a mix of reaction and proaction. As expected the authors observed a close matching between the scaling properties of the inter-beat interval series of chaotic signals and those of the corresponding inter-tap interval series of participants.

Marmelat and Delignières (2012) evidenced similar results in an inter-personal coordination task where participants oscillated pendulums in synchrony. Results revealed low local correlations between the series of oscillation periods produced by the two participants of each dyad. The authors analyzed the scaling properties of the series of periods produced by participants, and evidenced a very close correlation between fractal exponents. Similar results were evidenced by Abney, Paxton, Dale, and Kello (2014), in the analysis of speech signals during dyadic conversations.

However, the idea that the matching of scaling exponents could be considered an unambiguous signature of complexity matching remains questionable, and it could be necessary to distinguish between the *statistical matching* (i.e., the convergence of scaling exponents) and the genuine complexity matching effect (i.e., the attunement of complexities). For example Fine et al. (2015), in an experiment on rhythmic interpersonal coordination, observed as in previous experiments a typical matching of scaling exponents, but suggested that this statistical matching could just result from local phase adjustments, and not from a global attunement of complexities. Delignières and Marmelat (2014) analyzed series of stride durations produced by participants attempting to walk in synchrony with a fractal metronome. They tried to simulate their empirical results by means of a model based on local corrections of asynchronies, and showed that this model was able to adequately reproduce the statistical matching observed in experimental series. The authors concluded that walking in synchrony with a fractal metronome could essentially involve short-term correction processes, and that the close correlation observed between scaling exponents could in such a case just represent the consequence of local correction processes. Torre et al. (2013) also supported this hypothesis in a tapping task where participants synchronized with different non-isochronous auditory
metronomes. They evidenced that inter-tap intervals could be modeled based on the previous inter-beat interval of the metronome and a correction of previous asynchronies to the metronome, independently of the level of 1/f fluctuations of the metronome (i.e. white noise or pink noise).

Our goal in the present paper is to seek for statistical signatures that could unambiguously distinguish between genuine global complexity matching and local corrections or adjustments. To date, most papers that tried to evidence complexity matching effects worked on the basis of monofractal analysis. Here we propose to adopt multifractal analyses because they allow for a more detailed picture of the complexity of time series, and the tailoring of fluctuations that is typical of complexity matching could be considered as the product of multifractality (Stephen and Dixon 2011). One of the most appealing hypotheses about the origin of fractal fluctuations in the behavior of complex systems refers to the idea that the interactions between system's networks are governed by multiplicative cascade dynamics (Ihlen and Vereijken 2010). Such dynamics is supposed to generate multifractal, rather than monofractal fluctuations, and indeed Ihlen and Vereijken (2010) showed that it was the case in most previous analyzed series in the literature. Stephen and Dixon (2011) consider that complexity matching should be conceived as a product of multiplicative cascade dynamics, entailing a coordination of fluctuations among multiple time scales.

While monofractal processes are characterized by long-term correlations and a single scaling exponent, in multifractal time series subsets with small and large fluctuations scale differently, and their description requires a hierarchy of scaling exponents (Podobnik and Stanley 2008). We propose to assess statistical matching through the point-by-point correlation function between the sets of scaling exponents that characterize the coordinated series.

In the present paper we used the Multifractal Detrended Fluctuation analysis (MF-DFA) introduced by Kantelhardt et al. (2002), which is an extension of the Detrended Fluctuation Analysis (DFA, Peng et al. 1993). Just as DFA, MF-DFA allows to select the range of intervals over which exponents are estimated. Usually authors considers intervals from 8 or 10 data points, in order to allow a proper assessment of statistical moments, up to $N/4$ or $N/2$ ($N$ representing the length of the analyzed series), in order to get at least four or two estimates of these moments. Quite often, however, series present different scaling regimes over the short and the long term, and authors perform separate estimates over different ranges of intervals (Delignières and Marmelat 2014). Here we propose to estimate the set of multifractal exponents in first over the entire range of available intervals (i.e., from 8 to $N/2$), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to $N/2$, from 32 to $N/2$, and then from 64 to $N/2$). We expect to find in both cases (local corrections or global matching), a strong correlation pattern between exponents when considering long length intervals (i.e, 64 to $N/2$). If synchronization is just based on local corrections, we consider that this close statistical matching in long intervals is just the consequence of the short-term, local coupling between the two systems. As local corrections between unpredictable systems remains approximate, we hypothesize that correlations should dramatically
decrease when intervals of shorter durations are taken into consideration. In contrast, in the case of genuine complexity matching, the synchronization between systems is supposed to emerge from interactions across multiple scales. We then hypothesize to find in this case close correlations, even when considering the entire range of intervals, from the shortest to the longest.

**Methods**

In the present paper we re-analyze a set of experimental series which were previously used by Delignières and Marmelat (2014), in a first attempt to derive statistical signature of complexity matching from monofractal analyses. All studies have been approved by the local ethics committee and have therefore been performed in accordance with the ethical standards laid down in the 1964 Declaration of Helsinki. All participants gave their informed consent prior to their inclusion in the study. We first briefly present the three sets of series submitted to analysis.

**Bimanual coordination.**

The first set of series was collected in an experiment where twelve participants performed bimanual oscillations (Torre and Delignières 2008a; Torre and Wagenmakers 2009). This kind of bimanual coordination task has been extensively studied in the dynamical systems approach to coordination, in order to evidence the emergent properties that underlie the macroscopic behavior of complex systems (Haken et al. 1985; Schöner et al. 1986). The two limbs are considered as a system of coupled oscillators, and bimanual coordination represents a nice example of close coordination between complex (sub)systems, embedded to form a global functional system. In the present context, this first set of series represents a limit case, where coordination should be achieved by complexity matching processes.

Participants were instructed to perform smooth and regular forearm oscillations holding two joysticks, synchronizing the reversal points of the motion of the joysticks (in-phase coordination). This experiment used the synchronization-continuation paradigm: during 30 sec participants synchronized their movements with a video model, inducing an initial frequency of 1.5 Hz in a first condition, and 2.0 Hz in a second. Then the model was removed and participants had to continue following the initial tempo during 600 cycles.

For each hand, we computed the series of periods, defined as the time intervals between two successive reversal points in maximal pronation. The mean period of oscillations of the effectors was 665.24 ms (+/- 63.01) in the 1.5 Hz condition, and 524.05 ms (+/- 66.52) in the 2.0 Hz. The standard deviation of asynchronies (i.e., the time intervals between the respective pronation reversal points of the two hands) was 18.65 ms (+/- 5.26) in the 1.5 Hz condition and 14.45 ms (+/- 2.62) in the 2.0 Hz condition, corresponding to a mean relative phase was -6.33° (+/- 3.76), and -5.27° (+/- 6.01), respectively.

**Interpersonal synchronization**

The second set of series were collected in an experiment on interpersonal coordination (Marmelat and Delignieres 2012). In contrast with the previous example, these series
represent coordination between two physically independent systems that interact for achieving a common goal. Twenty-two participants were randomly paired into eleven dyads. Participants in each dyad were instructed to perform synchronized oscillations with pendulums, in the sagittal plane, following an in-phase pattern of coordination. They were instructed to oscillate at the preferred frequency of the dyad, as regularly as possible. The task was performed in three conditions, characterized by increasing levels of coupling between participants. In the weak coupling condition, audition was limited with earplugs, and participants were instructed to visually fix a target in front of them on the wall. In the normal coupling condition, visual and auditory feedbacks were fully available, and participants were invited to visually fix their partner’s pendulum. In the strong coupling condition, participants were instructed to cross their free arms (arm-in-arm), in order to add haptic information to visual and auditory feedbacks. Series of 512 oscillations were collected in each condition. For each participant, we computed the series of periods, defined as the time intervals between two successive reversal points in maximal extension.

Results showed that dyads were able to perform adequately this coordination task, with a mean relative phase of -2.15° (+/- 8.64) in the low coupling condition, -1.67° (+/- 7.50) in the normal coupling condition, and -2.36° (+/- 7.15) in the strong coupling condition. The mean period of oscillations of the effectors was 1035.06 ms (+/- 130.59), 1018.47 ms (+/- 126.70), and 989.35 ms (+/- 51.95), respectively. The standard deviation of asynchronies was 46.51 ms (+/- 7.50), 34.78 ms (+/- 9.10) and 37.71 ms (+/- 6.06), respectively.

**Walking in synchrony with a fractal metronome**

In this experiment participants had to walk in synchrony with a fractal metronome. Eleven participants were involved in this experiment. They walked on a treadmill, and had to synchronize the right heel strikes with metronome signals administered through an earphone. Metronome signals presented fractal fluctuations with a mean α exponent of about 0.9, a mean value of 1135 ms, and their standard deviation was adjusted for obtaining a coefficient of variation of 2%. Series of 512 strides, defined as the time intervals between two successive right heel strikes, were collected. Results showed that participants were able to maintain synchrony with the metronome, with a mean asynchrony of about -52.8 ms (+/- 46.9). The mean stride duration was 1135.53 ms (+/- 51.95), and the standard deviation of asynchronies was 46.94 ms (+/- 11.68).

**Multifractal Detrended Fluctuation analysis (MF-DFA)**

In the present paper we used the MF-DFA method, initially introduced by Kantelhardt et al (2002). Consider the series \( x(i), i = 1, 2, ..., N \). In a first step the series is centered and integrated:

\[
X(k) = \sum_{i=1}^{k} \left[ x(i) - \frac{1}{N} \sum_{i=1}^{N} x(i) \right]
\]

Next, the integrated series \( X(k) \) is divided into \( N_n \) non-overlapping segments of length \( n \) and in each segment \( s = 1, ..., N_n \) the local trend is estimated and subtracted from \( X(k) \). The variance is calculated for each detrended segment:
and then averaged over all segments to obtain $q$th order fluctuation function

$$ F_q(n) = \left\{ \frac{1}{N_n} \sum_{s=1}^{N_n} \left[ F^2(n,s) \right]^{\frac{q}{2}} \right\}^{\frac{1}{q}} $$

where $q$ can take any real value except zero. In the present work we used integer values for $q$, from -15 to +15. Note that Eq. (3) cannot hold for $q = 0$, because of the diverging exponent. A logarithmic averaging procedure is used for this special case:

$$ F_0(n) = \exp \left( \frac{1}{2N_n} \sum_{s=1}^{N_n} \ln \left[ F^2(n,s) \right] \right) $$

Repeating this calculation for all lengths $n$ provides the relationship between fluctuation function $F_q(n)$ and segment length $n$. If long-term correlations are present, $F_q(n)$ increases with $n$ according to a power law:

$$ F_q(n) \propto n^{h(q)} $$

The scaling exponent $h(q)$ is obtained as the slope of the linear regression of $\log F_0(n)$ versus $\log n$. Note that for stationary time series, $h(2)$ is identical to the well-known Hurst exponent $H$, and therefore $h(q)$ is called the generalized Hurst exponent.

For positive values of $q$ the generalized Hurst exponent $h(q)$ describes the scaling behavior of large fluctuations, while for negative values of $q$, $h(q)$ describes the scaling behavior of small fluctuations. For monofractal time series $h(q)$ is independent of $q$, while for multifractal time series small and large fluctuations scale differently and $h(q)$ is a decreasing function of $q$.

The results of the MF-DFA can then be converted into the more classical multifractal formalism by simple transformations (Kantelhardt et al. 2002): first, generalized Hurst exponents $h(q)$ are related to the Renyi exponents $\tau(q)$ defined by the standard partition function-based multifractal formalism:

$$ \tau(q) = qh(q) - 1 $$

For monofractal signals $\tau(q)$ is linear function of $q$, and for multifractal signals $\tau(q)$ is nonlinear function of $q$. Another way to characterize multifractal process is the singularity spectrum $f(\alpha)$ which is related to $\tau(q)$ through the Legendre transform:

$$ \alpha(q) = \frac{d\tau(q)}{dq} $$

$$ f(\alpha) = q\alpha - \tau(q) $$

where $f(\alpha)$ is the fractal dimension of the support of singularities in the measure with Lipschitz-Hölder exponent $\alpha$. The singularity spectrum of monofractal signal is represented by a single point in the $f(\alpha)$ plane, whereas multifractal process yields a single humped function.
The focus-based approach to multifractal analysis

The classical MF-DFA algorithm was shown to perform as well as other multifractal methods (Oswiecimka et al. 2006; Schumann and Kantelhardt 2011). However, especially when applied on empirical series it is known to often produce the so-called “inversed” spectra, exhibiting a zig-zag shapes rather than the expected parabolic shape (Makowiec et al. 2011; Mukli et al. 2015).

Mukli et al. (2015) have recently introduced a focus-based approach, which allows to avoid this pitfall. The standard method assesses the scaling exponents \( h(q) \) independently for each \( q \) value. This procedure can be considered adequate if an assumption on homogeneous structuring holds for the scaling function. This property however may not always be present especially in empirical signals.

Theoretically, the moment-wise scaling functions, for all \( q \) values, should converge toward a common limit value at the coarsest scale. Indeed, substituting signal length \( (N) \) to interval length \( (n) \) in Eq. (3) yields:

\[
F_q(N) = \left\{ \frac{1}{N^q} \sum_{s=1}^{N} \left[ F^2(N,s) \right]^{q/2} \right\}^{1/q} = \left\{ F^2(N,s)^{q/2} \right\}^{1/q} = F(N,s)
\]

\( F(N,s) \) can then be considered the theoretical focus of the scaling functions, and Mukli et al. (2015) proposed to use this focus as a guiding reference when regressing for \( h(q) \). In essence, one can iterate on \( h(q) \) as the ideal multifractal with its given focus and set of associated slopes best fitting to the observed data of the scaling function. Mukli et al. (2015) showed that this method allowed to successfully avoid the occurrence of inversed spectra.

As explained in the introduction, we first applied MF-DFA considering the widest range of intervals, from 8 to 256 \( (N/2) \). We then replicated this analysis by progressively focusing on longer intervals: 16 to 256, 32 to 256, and 64 to 256.

We finally computed for each \( q \) value the correlation between the individual Lipschitz-Hölder exponents characterizing the two coordinated systems, \( \alpha_1(q) \) and \( \alpha_2(q) \), respectively, yielding a correlation function \( r(q) \). As previously explained, we expected to find in all cases a correlation function close to 1, for all \( q \) values, when only the largest intervals are considered. Increasing the range of considered intervals should have a negligible impact on \( r(q) \) when coordination is based on a complexity matching effect. In contrast, if coordination is based on local corrections, a decrease in \( r(q) \) should be observed, as shorter and shorter intervals are considered.

Results

Bimanual coordination.

We present in Figure 1 (upper panels) the averaged multifractal spectra of the period series, considering intervals from 8 to 256 points, in the 1.5 Hz condition (a) and the 2.0 condition (b). The spectra of the right and the left hand are closely superimposed in both conditions. The correlation functions between the multifractal spectra are presented in bottom panels. Correlation coefficients are plotted against their corresponding \( q \) values. Four correlation functions are displayed, according to the shortest interval length.
considered during the analysis (8, 16, 32, or 64). In all cases the correlations functions displayed very high values, close to 1.0 (Figure 1, c and d). Especially in the 1.5 Hz condition the correlations between spectra appeared maximal, over all $q$ values and whatever the considered intervals range. Correlations appeared slightly lower (around 0.90), in the 2.0 Hz condition, for negative values of $q$, and when the entire range of intervals were considered.

**Figure 1:** Bimanual coordination. Upper panels: Multifractal spectra for the right (black circles) and the left (white circle) effectors, for the 1.5 Hz (a) and the 2.0 Hz (b) conditions. Bottom panels: Correlation functions $r(q)$, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$), for the 1.5 Hz (c) and the 2.0 Hz (d) conditions. $q$ represents the set of orders over which the MF-DFA algorithm was applied.

**Interpersonal synchronization.**

We present in Figure 2 (upper panels) the averaged multifractal spectra of the period series in the low coupling (a), normal coupling (b) and strong coupling (c) conditions. As for the previous experiment, we observed a close superimposition of the two averaged spectra, and the superimposition appeared closer as coupling strength increased. The correlation functions between the multifractal spectra are presented in bottom panels (d, e and f). In all cases the correlation function displayed very high values, close to 1.0, when analysis focused to long intervals (i.e. 32 to $N/2$ or 64 to $N/2$).
In contrast with the previous results, the considered range of intervals had a strong impact on the correlation function. When the smallest range was considered (64 to N/2), the correlation function remained close to 1.0. When the range of interval was progressively enlarged, the correlation values decreased, especially for negative q values corresponding to low variance epochs in the series. When all available intervals are considered (i.e. 8 to N/2), r(q) presents non significant values.
regarded with some experiments are consistent with our initial hypotheses and seem sufficiently large to be confirmed by further analyses. However, the differences we observed between the three experiments are consistent with our initial hypotheses and seem sufficiently large to be regarded with some confidence.

In order to provide a clearer picture of the evolution of correlations with the range of considered intervals in the three experiments, we present in Figure 4 a set of scatter plots representing the relationships between the $\alpha(2)$ samples characterizing the two coordinated systems. These graphs show a global narrowing of exponent’s samples, as the considered range of intervals increases. However, the decrease of correlation, especially in the third experiment, clearly arises from a weakening of the relationships between exponents.

**Discussion**

The present results are based on the analysis of behavioral series of relatively short length. 512 data points could be considered insufficient for deriving reliable results, especially with multifractal analyses. However, the application of time series analyses supposes that the system under study remains in stable state during the whole window of observation, and in behavioral experiments the lengthening of the task could raise problems of fatigue or lack of concentration (Delignières et al. 2005; Marmelat and Delignières 2011). On the other hand, a number of improvements have been introduced in fractal analyses, which could allow to consider with a certain confidence the results obtained from relatively short series (Delignières et al. 2006; Almurad and Delignières 2016). Such series lengths are generally considered as an acceptable compromise between the requirements of time series analyses and the limitations of heuristic human experiments (Gilden 1997; Chen et al. 1997; Gilden 2001; Chen et al. 2001).

We are aware that the present results should be considered with caution, and have to be confirmed by further analyses. However, the differences we observed between the three experiments are consistent with our initial hypotheses and seem sufficiently large to be regarded with some confidence.

![Figure 3: Walking in synchrony with a fractal metronome. Panel a: Multifractal spectra for the participant (black circles) and the metronome (white circle). Panel b: Correlation functions $r(q)$, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$).](image-url)
proposed a similar hypothesis in an experiment where participants had to synchronize very large masses compared to the other tasks. Note, however, that Torre et al. (2013) strongly differs from those used in the others, and especially the task used in the third experiment suggested by Marmelat and Delignières (2012). One could argue, however, that this hypothesis should be considered with caution, as the task used in the third experiment strongly differs from those used in the others, and especially the walking task involves very large masses compared to the other tasks. Note, however, that Torre et al. (2013) proposed a similar hypothesis in an experiment where participants had to synchronize finger taps with fluctuating metronomes.

**Complexity matching vs discrete local coupling**

The main result of this paper is the clear distinction between the two first experiments (bimanual coordination and interpersonal coordination) and the third one (walking with a fractal metronome). In the two first cases the correlation function revealed a clear statistical matching between multifractal spectra, whatever the range of intervals considered in the analysis. In contrast, in the third set of data the close statistical matching appeared only when the range was restricted to the lengthiest intervals (i.e., from 64 to 256), and was progressively altered when wider ranges were considered.

These results suggest that in the two first experiments synchronization was governed by a global, multiscale coordination between the two interacting systems, and in the third experiment synchronization was the result of local corrective processes. As in this experiment participants had to synchronize with a series of discrete stimuli, one could hypothesize that these correction processes work on a discrete, step-to-step basis, as suggested by Marmelat and Delignières (2012). One could argue, however, that this hypothesis should be considered with caution, as the task used in the third experiment strongly differs from those used in the others, and especially the walking task involves very large masses compared to the other tasks. Note, however, that Torre et al. (2013) proposed a similar hypothesis in an experiment where participants had to synchronize finger taps with fluctuating metronomes.
It could be interesting, for reinforcing this hypothesis, to check whether a model based on such discrete, local correction processes, could generate series yielding comparable results. We then tried to simulate series that could result from a local coupling with a fractal metronome. In this modeling study we considered that the organism produces intrinsically long-range correlated series. Indeed, a number of previous studies have shown that organisms produced long-range correlated series in self-paced conditions, and that, during synchronization with a regular metronome, this source of long-range correlation was still at work and had to be considered for properly modeling the synchronization process (Torre and Delignières 2008b; Torre and Delignières 2009; Delignières and Marmelat 2014). We then proposed that the organism corrects the interval it intended to intrinsically produce on the basis of the previous asynchronies (Delignières and Marmelat 2014). We worked with a two-order auto-regressive model:

\[
x(i) = y(i) - a[ASYN(i-1)] - b[ASYN(i-2)] + c\epsilon(i),
\]

(10)

\[
ASYN(i) = ASYN(0) + \sum_{k=0}^{i} x(k) - \sum_{k=0}^{i} z(k)
\]

(11)

where \( x(i) \) represents the series of periods effectively produced by the organism, \( y(i) \) the series of virtual periods produced by the organism, and \( z(i) \) the fractal metronome. \( ASYN(i) \) is the series of asynchronies between the events produced by the organism and the signals of the metronome. \( y(i) \) and \( z(i) \) were both modeled as fractional Gaussian noises with \( H = 0.9 \) (mean = 1000 and \( SD = 20 \)). Finally \( \epsilon(i) \) is a white noise process with zero mean and unit variance.

This model suggests that periods are corrected on the basis of the two previous asynchronies, a hypothesis consistent with the cross-correlation functions obtained in this experiment (Delignières and Marmelat 2014).

For the present simulations we used \( a = 0.2, b = 0.4, c = 12 \), and we generated 12 series of 512 data points. We present in Figure 5 (panel a) the averaged multifractal spectra for the ‘participant’ (black circles) and the ‘metronome’ (white circles). Note that these simulated results are characterized by a shift of the first spectrum, similar to that observed in experimental data (see Figure 3). Panel b represents the correlation functions \( r(q) \), for the four ranges of intervals considered. As can be seen we obtained a pattern of results similar to that obtained with experimental results: when focusing on long-term intervals the correlation function remained close to one, and progressively extinguished as more and more shorter-term intervals were taken into consideration.

Figure 5: Simulation of the synchronization to a fractal signal by local corrections of asynchronies. Panel a: Multifractal spectra for the ‘participant’ (black circles) and the ‘metronome’ (white circle). Panel b: Correlation functions \( r(q) \), for the four ranges of intervals considered (8 to \( N/2 \), 16 to \( N/2 \), 32 to \( N/2 \), and 64 to \( N/2 \)).
Another difference between our experiments lies in the coupling between systems: in bimanual coordination and interpersonal synchronization the systems mutually interacted but synchronization with a fractal metronome is characterized by the presence of a 'master' (the metronome) and a 'slave' (the participant). Interestingly, our results revealed an asymmetry in the alteration of correlations when larger ranges of intervals were considered: we especially observed a dramatic decrease of correlations for negative $q$-values, corresponding to low-variance epochs in the signals (Figure 3). This result was particularly obvious in the third experiment, and suggests that high variance epochs in the metronome signals allow a better synchronization, reinforcing the hypothesis of a discrete, perceptual basis of synchronization.

These results question a number of recent experiments dealing with the use of fractal metronomes with the perspective of rehabilitation purposes (Stephen et al. 2008; Hove et al. 2012; Kaipust et al. 2013; Torre et al. 2013; Rhea et al. 2014; Marmelat et al. 2014). In their seminal paper, Stephen et al. (2008) suggested that synchronization with a chaotic metronome was not based on local adjustments but rather on a global, multiscale coordination with the metronome. The present results cast doubt on this conclusion (see also Delignières & Marmelat, 2014; Torre et al., 2013). Fractal metronomes have recently sparked a great interest, suggesting that they could mimic natural variability, and be used for conceiving artificial devices for training and rehabilitation, based on the complexity matching effect. Mimicking natural variability, especially with discrete signal series, seems not sufficient to generate the global and multiscale coordination hypothesized in complexity matching.

**Complexity matching vs continuous local coupling**

Our results, however, do not prove that the strong statistical matching observed in bimanual coordination and interpersonal coordination is due to complexity matching, as defined in the introduction. Recently, Fine et al (2015) questioned the global complexity matching hypothesis, and suggested that a local and continuous coupling between systems could underlain the statistical matching observed in such situations.

The dynamical systems approach to coordination promoted a phenomenological model based on a continuous coupling between oscillators (Haken et al. 1985; Schöner et al. 1986). This so-called HKB model accounts for coordination by non-linear coupling between two hybrid limit-cycle oscillators, based on the two oscillators' state variables (position and velocity):

$$
\begin{align*}
\dot{x}_1 + \delta \dot{x}_1 + \lambda x_1^3 + \gamma x_1^2 \dot{x}_1 + \omega^2 x_1 &= (\dot{x}_1 - \dot{x}_2)[a + b(x_1 - x_2)^2] \\
\dot{x}_2 + \delta \dot{x}_2 + \lambda x_2^3 + \gamma x_2^2 \dot{x}_2 + \omega^2 x_2 &= (\dot{x}_2 - \dot{x}_1)[a + b(x_1 - x_2)^2]
\end{align*}
$$

(12)

where $x_i$ is the position of oscillator $i$, and the dot notation represents derivation with respect to time. The left side of the equations represents the limit cycle dynamics of each oscillator determined by a linear stiffness parameter ($\omega$) and damping parameters ($\delta, \lambda,$ and $\gamma$), and the right side represents the coupling function determined by parameters $a$ and $b$. This model has been proven to adequately account for most empirical features in
bimanual coordination tasks, such as the differential stability of in-phase and anti-phase coordination modes, and the transition from anti-phase to in-phase coordination with the increase of oscillation frequency (Haken et al. 1985; Schöner et al. 1986).

One can indeed suppose that this kind of continuous coupling could be at the origin of the strong statistical matching observed in coordination experiments (Fine et al. 2015), but the multifractal signature proposed in this paper could be unable to distinguish between genuine complexity matching and such local and continuous coupling. A solution for disentangling these two hypotheses is to analyze the series produced by this model, and to compare them to those empirically observed.

However, in order to account for empirical features, the original HKB model should be slightly modified. Especially, it has been proven that in bimanual coordination, the series of periods produced by each effector and the series of relative phase contained long-range correlations, a property that the original HKB model was unable to generate (Torre and Delignières 2008a). The authors proposed to account for this behavior by replacing the fixed linear stiffness parameter $\omega$ in equations (12) by a two independent discrete series $\omega_{1,i}$ and $\omega_{2,i}$, exhibiting long-range correlation properties, and representing the inner frequencies of oscillators 1 and 2, respectively, at cycle $i$.

$$
\begin{align*}
\dot{x}_1 + \delta \dot{x}_1 + \lambda x_1^3 + \gamma x_1^2 \dot{x}_1 + \omega_{1,i}^2 x_1 + \sqrt{Q} \epsilon_{1,i} &= (\dot{x}_1 - \dot{x}_2)(a + b(x_1 - x_2)^2) \\
\dot{x}_2 + \delta \dot{x}_2 + \lambda x_2^3 + \gamma x_2^2 \dot{x}_2 + \omega_{2,i}^2 x_2 + \sqrt{Q} \epsilon_{2,i} &= (\dot{x}_2 - \dot{x}_1)(a + b(x_2 - x_1)^2)
\end{align*}
$$

(13)

Note that they also introduced white noise terms of strength $Q$ in the limit cycle equations, as suggested by Schöner et al. (1986). Torre and Delignières (2008a) showed that a relevant set of parameters allowed to simulate a stable in-phase coordination between the two oscillators, despite the intrinsic long-range correlated fluctuations injected in each oscillator. Their results replicated most empirical results, in terms of mean and standard deviation of relative phase, and also concerning the presence of long-range correlations in the series of relative phase.

We used this model for generating pairs of series of coordinated periods. We attempted to generate series reproducing the main features of the inter-personal coordination experiment, in the strong coupling condition. We used the same set of parameters than Torre and Delignières (2008a): $\delta = 0.5$, $\lambda = 0.02$, $\gamma = 1.0$, $Q = 0.4$. We used the “hopping model” (Delignières et al. 2008; Torre and Delignières 2008a) for simulating the long-range correlated series $\omega_{1,i}$ and $\omega_{2,i}$ around a mean value of $4\pi$. For stabilizing in-phase coordination between the two systems, the coupling parameters were set to $a = 12$ and $b = 6$. These values were much stronger than those commonly reported in the literature (Fink et al. 2000; Assisi et al. 2005; Leise and Cohen 2007), but were necessary for obtaining a stable coordination (Delignières et al. 2008; Torre and Delignières 2008a). We simulated 12 sets of series of 512 data points.

Simulated series broadly reproduced experimental results, with a mean relative phase of $0.74^\circ (+/- 6.93)$. The mean period of oscillations was $1001.14$ ms ($+/- 50.07$). The standard deviation of asynchronies was $19.34$ ms ($+/- 3.38$). We present in Figure 6 (a
and b) the result of the multifractal analysis of these coordinated series. As can be seen, the correlation function $r(q)$ exhibited very high values, even when considering the widest range of intervals. This result appears similar to that obtained with the experimental series (Figure 2), suggesting that this kind of local, continuous coupling, could indeed underlay the observed statistical matching between series.

However, a closer look to the coupled period series casts some doubts about this conclusion. We present in Figure 6 two examples subsets of coupled series (50 points), the first graph (c) corresponding to representative experimental series, and the second (d) to simulated series. These graphs suggest that while providing comparable statistical results, interpersonal coordination and the coupled oscillator model works differently. The simulated series present very close dynamics, resulting from the continuous coupling of positions and velocities in the model. In contrast, experimental series appear more independent, at least on this local scale. In order to quantify these local dependences, we computed the average local cross-correlation coefficient, using a sliding window of 15 points (Marmelat and Delignieres 2012). The average cross-correlation coefficient was 0.87 for simulated series, but close to zero for experimental series.
Figure 6: Panel a: Average multifractal spectra for the two simulated oscillators. Panel b: Correlation functions $r(q)$, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$). Panel c: Representative subsets of coupled experimental series (50 points). Panel d: Representative subsets of coupled simulated series (symmetric model). Panel d: Representative subsets of coupled simulated series, with a 10% detuning.

One could argue, however, that although the stiffness series $\omega_{1,i}$ and $\omega_{2,i}$ included in the model are independent, they still have the same average values, and that more realistic results could be obtained by introducing an asymmetry, i.e. a difference between the natural frequencies of the two systems. In order to check this point, we performed additional simulations introducing a 10% detuning between the two oscillators (mean $\omega_{1,i} = 4\pi$, and mean $\omega_{2,i} = 4.4\pi$). All others parameters were unchanged. This new set of
simulations gave essentially similar results, suggesting that the symmetry of the first model cannot *per se* explain the strong local convergence of the simulated series. We present in Figure 6 (panel e) an example subset of series simulated with these new settings.

These results suggest that a local continuous coupling, as proposed in the HKB model, can indeed mimic the statistical matching supposed to emerge from complexity matching. However, this kind of coupling generates a very close correspondence between the trajectories of oscillators, which seems unrealistic in view of the experimental observations.

**Conclusion**

Complexity matching is a very innovative hypothesis, which has recently motivated a number of theoretical and experimental contributions. In this paper we introduce a method, based on multifractal analysis, for distinguishing between genuine complexity matching and local discrete coupling. Our results show that some situations that were considered prototypic of complexity matching, where participants had to synchronize with a fractal metronome, seem controlled through local adjustments. In contrast, genuine complexity matching seems occurring when two complex systems are mutually coupled.

Complexity matching and local discrete coupling, however, are not necessarily exclusive. One could conceive, for example, that in some tasks synchronization could be achieved through a mix of the two processes. Moreover, the respective contribution of each processes could differ among participants (see, for example, Delignieres and Torre 2011). Further investigations, focusing on individual series and based on cross-correlations should provide some insights about this hypothesis.

**Conflict of interest**

The authors declare that they have no conflict of interest.

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Concluding remarks

In this paper we proposed a method that went beyond the “classical” correlation of mono-fractal exponents and explored more deeply the intimacy of synchronized series. We concluded that multi-fractal correlation function could be able to unambiguously distinguish between asynchrony correction and complexity matching. This conclusion, however, was maybe premature…

Scotti (2017) analyzed series produced interpersonal tapping and forearm oscillation tasks. Applying our multi-fractal correlation test, he found evidence of complexity matching in both situations. However, applying Windowed Detrended Fluctuation analysis (see chapters 3 and 4), he showed that both tasks involved asynchrony correction. Deeper analyses led us to abandon the idea of exclusive or ideal models (i.e., asynchrony correction models vs complexity matching models), and to propose hybrid models, containing both processes but suggesting a possible dominance of one process on the other. This will be developed in the next chapter.

To date, we consider that the multi-fractal correlation function effectively account for the presence of a complexity matching effect, but can not provide clear indication about its strength and its relative dominance with regards of asynchrony correction. Additional tests, and especially the Windowed Detrended Fluctuation analysis (see chapters 3 and 4) seem necessary for definitively concluding about the effective nature of synchronization processes in a given situation.
Chapter 3

*Complexity matching in side-by-side walking*
Chapter 3

Complexity matching in side-by-side walking

At this stage in our project, we have proposed a clear definition of the complexity matching effect: A multi-scale coordination between complex systems. We also highlighted the role of multi-fractality in this process.

We also introduced a first method for distinguishing complexity matching from other kinds of synchronization processes, especially asynchronies corrections. This method exploits the multifractal properties of complexity matching, and is based on the analysis of the correlations between the multifractal spectra produced by the two systems in coordination. We supposed that complexity matching induced strong correlations, over the entire range of the spectra, and even when short intervals were considered. In contrast, we supposed that in the case of short-term coupling, significant correlation functions should appear only when long-term interval were considered, but should extinguish when shorter intervals were introduced in the analysis. This method especially allowed to show that walking in synchrony with a fractal metronome was mainly performed through asynchrony correction, and presented no trace of complexity matching.

Our aim in the present chapter was to show that synchronization, in side-by-side walking, was dominated by a complexity matching effect. This demonstration was a necessary step, in order to pursue our final project, which aimed at testing the hypothesis that complexity matching could allow restoring complexity in deficient systems.

The experiment we designed aimed at analyzing synchronization during synchronized walking, with young and healthy participants. In order to test the effect of coupling strength, we tested two experimental conditions: side-by-side and arm-in-arm walking: coupling is obviously supposed to be stronger in the latter condition. A third condition of independent walking served as control. We analyzed stride duration series with the multi-fractal correlation analysis presented in the previous chapter. We also introduced another method, the Windowed Detrended Cross-correlation analysis. These two methods converged toward the evidence of a clear complexity matching effect in synchronized walking, and that effect was stronger in the strong coupling condition.
Complexity matching in side-by-side walking

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Abstract
Interpersonal coordination represents a very common phenomenon in daily-life activities. Three theoretical frameworks have been proposed to account for synchronization processes in such situations: the information processing approach, the coordination dynamics perspective, and the complexity matching effect. On the basis of a theoretical analysis of these frameworks, we propose three statistical tests that could allow to distinguish between these theoretical hypotheses: the first one is based on multifractal analyses, the second and the third ones on cross-correlation analyses. We applied these tests on series collected in an experiment where participants were instructed to walk in synchrony. We contrasted three conditions: independent walking, side-by-side walking, and arm-in-arm walking. The results are consistent with the complexity matching hypothesis.

Keywords: Synchronized walking Complexity matching Multifractals Cross-correlation

Introduction
Interpersonal synchronization represents a very common phenomenon in daily life activities, for example when people walk together, dance, play music, etc. However, the processes that sustain this kind of coordination are still poorly understood, and several theoretical frameworks are in competition for explaining how interpersonal synchronization occurs.

In the present paper we focus on a very usual activity, side-by-side walking. The final goal of this line of research is concerned by rehabilitation purposes, and this point will be developed in the concluding section. The main aim of the current paper is to enrich the theoretical approach of the alternative frameworks that compete in this domain, and to propose a statistical strategy for disentangling these different points of view. We then apply this theoretical and statistical background in an experimental study on side-by-side walking. In a first step it seems necessary to shortly introduce the theoretical paradigms that that have been proposed in the study of interpersonal synchronization.
**The information-processing approach**

The first framework suggests that interpersonal synchronization is based on cognitive, representational processes of anticipation. This information-processing paradigm originates in the analysis of sensorimotor synchronization (SMS), focusing at the experimental level on the synchronization of simple movements (e.g., finger tapping) with a regular metronome (Repp, 2005; Repp & Su, 2013). A number of studies suggested that in such tasks synchronization is achieved by a systematic correction of the current inter-tap interval, on the basis of the last asynchronies (Pressing & Jolley-Rogers, 1997; Torre & Delignières, 2008; Vorberg & Wing, 1996). This corrective process can be expressed as follows:

\[
ITI_n = \text{ITI}_{th} - \alpha \text{ASYN}_{n-1} + \gamma \varepsilon_n
\]  

(1)

where \(ITI_n\) represents the inter-tap interval produced by the participant at the \(n^{th}\) tap, and \(\text{ASYN}_n\) the asynchrony between the \(n^{th}\) tap and the \(n^{th}\) onset of the metronome. \(\text{ITI}_{th}\) represents the inter-tap interval that should be intrinsically produced. \(\text{ITI}_{th}\) is supposedly produced by an internal timekeeper, and is corrected by a fraction of the preceding asynchrony. Finally \(\varepsilon_n\) is a white noise process.

In order to account for synchronization with more realistic environments, this paradigm has been extended to the study of synchronization with non-isochronous metronomes. The first studies focused on metronomes with regularly modulated deviations around the basic tempo (Madison & Merker, 2005; Thaut, Tian, & Azimi-Sadjadi, 1998). More recently a number of studies analyzed synchronization with metronomes presenting fractal variabilities, which are supposed to represent more closely the kind of fluctuations one encounters with natural situations, and especially with human partners (Delignières & Marmelat, 2014; Hunt, McGrath, & Stergiou, 2014; Kaipus, McGrath, Mukherjee, & Stergiou, 2013; Marmelat, Torre, Beek, & Daffertshofer, 2014; Rankin & Limb, 2014; Torre, Varlet, & Marmelat, 2013). These experiments generally showed that individuals tracked the timing variations of the sequence at a lag of one event (Delignières & Marmelat, 2014; Thaut et al., 1998; Torre et al., 2013). This tracking behavior is essentially similar to that supposed by the basic model proposed in Eq. (1).

This information processing approach to sensorimotor synchronization has been extended to interpersonal synchronization, especially in the study of dyadic finger tapping tasks (Konvalinka, Vuust, Roepstorff, & Frith, 2010; Nowicki, Prinz, Grosjean, Repp, & Keller, 2013; Pecenka & Keller, 2011). These experiments and their results will be presented and discussed latter in this paper.

**The coordination dynamics perspective**

A second theoretical framework has been proposed by the coordination dynamics perspective (Schmidt, Carello, & Turvey, 1990). This approach was initially developed in the analysis of bimanual coordination, and promoted a phenomenological model based on a continuous coupling between oscillators (Haken, Kelso, & Bunz, 1985; Schöner, Haken, & Kelso, 1986):
where $x_i$ is the position of oscillator $i$, and the dot notation represents derivation with respect to time. The left side of the equations represents the limit cycle dynamics of each oscillator determined by a linear stiffness parameter ($\omega$) and damping parameters ($\delta$, $\lambda$, and $\gamma$), and the right side represents the coupling function determined by parameters $a$ and $b$. This model has been proven to adequately account for most empirical features in bimanual coordination tasks, such as the differential stability of in-phase and anti-phase coordination modes, and the transition from anti-phase to in-phase coordination with the increase of oscillation frequency (Haken et al., 1985; Schöner et al., 1986).

Schmidt et al. (1990), in a series of experiments in which two seated participants were asked to visually coordinate their lower legs, showed that interpersonal coordination presents strong similarities with bimanual coordination: anti-phase and in-phase coordination patterns also emerged as intrinsically stable behaviors, with anti-phase being less stable than in-phase coordination, and spontaneous transitions from anti-phase to in-phase coordination were also observed with increasing frequency. Similar results were obtained in diverse interpersonal tasks, such as rocking side-by-side in rocking chairs (Richardson, Marsh, Isenhower, Goodman, & Schmidt, 2007), or swinging pendulums together (Schmidt, Bienvenu, Fitzpatrick, & Amazeen, 1998). Some important predictions of the original model, such as the effect of a difference between the uncoupled eigenfrequencies of the two oscillators, were also evidenced in interpersonal coordination tasks (Schmidt et al. 1998).

**Complexity matching**

Complexity matching represents a third, alternative framework that has been recently proposed for accounting for interpersonal coordination processes (Abney, Paxton, Dale, & Kello, 2014; Delignières & Marmelat, 2014; Marmelat & Delignières, 2012). The concept of complexity matching, introduced by West, Geneston, and Grigolini (2008), states that the exchange of information between two complex networks is maximized when their complexities are similar. The response of a complex network to the stimulation of another network is a function of the matching of their complexities. This property requires that both networks generate 1/f fluctuations, and has been interpreted as a kind of “1/f resonance” (Aquino, Bologna, West, & Grigolini, 2011)

An interesting conjecture exploiting the complexity matching effect supposes that two coupled complex systems tend to attune their complexities in order to optimize information exchange. This conjecture suggests a close matching between the scaling exponents characterizing the series produced by the coupled systems. Such results have been evidenced by Marmelat and Delignières (2012) in an inter-personal coordination task where participants oscillated pendulums in synchrony, and by Abney et al. (2014), in the analysis of speech signals during dyadic conversations.

The processes that underlain this tailoring of fluctuations remain not fully understood. Stephen and Dixon (2011) propose an interesting hypothesis, which explains this
attunement as a case of multifractal cascade dynamics in which perceptual-motor fluctuations are coordinated across multiple time scales. This coordination among multiple time scales could support the apparently predictive aspects of behavior without requiring an internal model.

These three theoretical frameworks have been jointly considered in a series of papers dedicated to the analysis of anticipation processes, and distinguishing several forms of anticipation (Dubois, 2003; Stephen & Dixon, 2011; Stepp & Turvey, 2010). Dubois (2003) considered that synchronization with fluctuating environments was based on a kind of “prediction” of its upcoming behavior (Delignières & Marmelat, 2014; Marmelat & Delignières, 2012; Stephen & Dixon, 2011; Stephen, Stepp, Dixon, & Turvey, 2008). Dubois suggested that a first form of anticipation was based on representational processes, allowing to predict the future of the environment with which the systems has to coordinate. The information-processing approach we previously presented corresponds to this kind of processes. Dubois (2003) proposed to refer this form of anticipation to as “weak” anticipation.

The author proposed a “strong” alternative that does not rely on internal models. Strong anticipation suggests that the organism is embedded within its environment. This embedding asserts lawful constraints upon both the actions of the organism and the environmental effects on those actions, and anticipation emerges as a lawful regularity of the organism–environment system.

Stephen and Dixon (2011) argued that two approaches to strong anticipation have to be distinguished. The first one suggests that strong anticipation results from an appropriate coupling between the organism and its environment. An interesting example was presented by Voss (2000), who showed that during the synchronization between a slave and a master systems, the presence of time delays in the master system yields the slave system to synchronize with future states of the master. The models of coupled oscillators proposed by the coordination dynamics perspective clearly refer to this kind of local strong anticipation processes. This conception supposes that anticipation is based on local time scales, and the quality of anticipation is supposed to be closely related to the strength of coupling between the two systems (Stepp & Turvey, 2010).

A second approach supposes that strong anticipation is based on a more global coordination between the organism and its environment. Stephen et al. (2008) were the first to evidence this kind of strong anticipation in an experiment which analyzed synchronization with a chaotic metronome. In such a situation, local predictions seem difficultly conceivable, because of the intrinsically unpredictable nature of the external pacing signal. Despite this unpredictability, the authors reported a quite acceptable synchronization with the metronome. They also observed a close matching between the fractal exponents of the chaotic signals and those of the corresponding inter-tap interval series. Such global strong anticipation corresponds to the previously presented complexity matching effect.

These three theoretical frameworks have received considerable supports in their respective fields of emergence, including interpersonal coordination tasks. We are not
sure, however, that these frameworks represent alternative hypotheses for accounting for similar phenomena. Depending on the nature and the constraints of the situation, different synchronization processes could be at work, and each framework could offer satisfying accounts in specific tasks. The information processing approach seems particularly relevant for accounting for situations where one has to synchronize discrete movements (e.g., tapping) with series of discrete signals (Konvalinka et al., 2010; Repp, 2005). The coordination dynamics perspective was essentially developed for accounting for the coordination of continuous, oscillatory movements (Schmidt et al., 1990). The scope of complexity matching remains to define, but it has been previously applied to very diverse situations, including non periodic interactions between complex systems (e.g., Abney et al., 2014).

In order to test the relevance of these frameworks in specific situations, we need statistical signatures that could be able to unambiguously identify the processes at work in interpersonal coordination. In the following parts we present three possible tests: the first one is based on multifractal analyses, and has been proposed by Delignières, Almurad, Roume, and Marmelat (2016), the second and the third exploit cross-correlation analyses.

**Multifractal signatures**

Most experiments seeking to evidence a complexity matching effect tried to reveal a close attunement of the (mono)fractal properties of the series produced by the coordinated systems. Typically, the authors showed close correlations between scaling exponents (Delignières & Marmelat, 2014; Marmelat & Delignières, 2012; Marmelat, Delignières, Torre, Beek, & Daffertshofer, 2014; Stephen et al. 2008).

However, Delignières et al. (2016) claimed that the matching of scaling exponents could not be considered an unambiguous signature of complexity matching. They proposed to distinguish between statistical matching (i.e., the convergence of scaling exponents) and genuine complexity matching effect (i.e., the attunement of complexities). Some recent papers showed that the matching of scaling exponents could result from local, short-term adjustments or corrections (Delignières & Marmelat, 2014; Fine, Likens, Amazeen, & Amazeen, 2015; Torre et al., 2013). For example, Delignières and Marmelat (2014) analyzed series of stride durations produced by participants attempting to walk in synchrony with a fractal metronome. They evidenced a close correlation between the scaling exponents of the series of stride durations produced by the participants and those of the series of inter-onset intervals of the corresponding metronomes. The authors tried to simulate their empirical results by means of a model based on local corrections of asynchronies, and showed that this model was able to adequately reproduce the statistical matching observed in experimental series. The authors concluded that walking in synchrony with a fractal metronome could essentially involve short-term correction processes, and that the close correlation observed between scaling exponents could in such a case just represent the consequence of these local corrections.

Delignières et al. (2016) proposed a more binding method for distinguishing genuine complexity matching from local corrective processes. They first suggested to base the
analysis of statistical matching on a multifractal approach, rather than the monofractal analyses previously employed. This choice was motivated by the point developed by Stephen and Dixon (2011), arguing the tailoring of fluctuations that is typical of complexity matching could be considered as the product of multifractality, and also by the fact that multifractals allow for a more detailed picture of the complexity of time series.

Multifractal processes present more complex fluctuations than monofractal series, and cannot be characterized by a single scaling exponent. In multifractal series subsets with small and large fluctuations scale differently, and their description requires a hierarchy of scaling exponents (Podobnik & Stanley, 2008). Delignières et al. (2016) proposed to assess the statistical matching through the point-by-point correlation function between the sets of scaling exponents that characterize the coordinated series.

The authors used the Multifractal Detrended Fluctuation Analysis (MF DFA, see Method section), which is based in its first steps on the analysis of the evolution of average statistical moments with the length of the intervals over which these moments are computed. This method allows to choose the range of interval lengths that is taken into account. The authors proposed to estimate the set of multifractal exponents in first over the entire range of available intervals (i.e., from 8 to \(N/2\), \(N\) representing the length of the series), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to \(N/2\), from 32 to \(N/2\), and then from 64 to \(N/2\)). They then computed the point-by-point correlation functions characterizing the four ranges of interval lengths considered.

The authors supposed that if synchronization is just based on local corrections, the statistical matching in long intervals is just the consequence of the short-term, local coupling between the two systems. As local corrections between unpredictable systems remains approximate, correlations should dramatically decrease when intervals of shorter durations are taken into consideration. In contrast, in the case of genuine complexity matching, the synchronization between systems is supposed to emerge from interactions across multiple scales. The authors hypothesized to find in this case close correlations, even when considering the entire range of intervals, from the shortest to the longest.

We present in Fig. 1 the results obtained by the authors in three experiments. The first one analyzed the series of periods produced by the two hands of participants performing in-phase bimanual coordination. The correlation functions obtained remained close to one, whatever the range of intervals considered (Fig. 1, left panel). The second one was an interpersonal coordination task in which participants were instructed to oscillate pendulums in phase (Fig. 1, central panel). In this experiment the correlation functions remained significant, while a little bit lesser than in the first example. In the third experiment participants had to walk in synchrony with a fractal metronome (Fig. 1, right panel). In that case a close-to-one correlation function was only obtained when the longest intervals were considered (i.e., from 64 to \(N/2\)). When widest ranges of intervals were considered, correlation functions lose statistical significance.
The authors concluded that in bimanual coordination and in interpersonal coordination, the statistical matching resulted from a genuine complexity matching between systems. In contrast during walking in synchrony with a fractal metronome, the apparent statistical matching was just the result of local adjustments.

This multifractal approach allows to clearly distinguishing between weak anticipation processes (i.e. local discrete correction) and strong anticipation processes. However, it seems unable to distinguish between the local and global forms of strong anticipation (Delignières et al., 2016).

**Cross-correlation peaks**

A second kind of signatures can be obtained from cross-correlation analyses. As previously evoked, a number of recent studies analyzed synchronization with non-isochronous metronomes, and especially metronomes presenting fractal fluctuations. These studies showed that synchronization in such situations was sustained by local corrections of the recent asynchronies, as expected from Eq. (1). Such behavior is typically revealed by a positive peak of cross-correlation at lag \(-1\), between the series of asynchronies and the series of periods produced by the participant, or between the series of periods produced by the participant and that produced by the metronome. Note that some more complicated models have been proposed, involving corrective processes taking into account more previous asynchronies (Pressing & Jolley-Rogers, 1997). For example Delignières and Marmelat (2014), in an experiment where participants had to walk in synchrony with a fractal metronome, evidenced positive peaks of cross-correlations at lag \(-2\) and lag \(-1\) between the series of asynchronies and the series of step durations.

The principle of phase correction can also be applied to interpersonal synchronization. When two individuals perform a rhythmic task in synchrony (e.g. tapping), phase...
correction suggests that participant A adapts his/her current inter-tap interval on the basis on the last asynchrony he/she perceived with his/her partner, and conversely for participant B. This mutual phase correction process could be modeled as follows:

\[
\begin{align*}
ITI_{A,n} &= ITI_{th_{A,n}} - \alpha ASYN_{A-B,n-1} + \gamma \epsilon_{A,n} \\
ITI_{B,n} &= ITI_{th_{B,n}} - \alpha ASYN_{B-A,n-1} + \gamma \epsilon_{B,n}
\end{align*}
\]

where \(ITI_{A,n}\) represents the inter-tap interval produced by participant A at the \(n\)th tap, \(ASYN_{A-B,n}\) the asynchrony between the \(n\)th tap of participant A and the \(n\)th tap of participant B (hence, \(ASYN_{A-B,n} = -ASYN_{B-A,n}\)). As in the previous model (Eq. (1)), \(ITI_{th_{A,n}}\) is a long-range correlated series with Hurst exponent H, mean M and variance \(\sigma^2\), representing the series of taps that should be intrinsically produced by participant A, and \(\epsilon_{A,n}\) is a white noise process with zero mean and unit variance. We generated 12 sets of coupled series with this simple model, with the following parameters: \(H = 0.9\), \(\alpha = 0.3\), \(M = 1000\), \(\sigma^2 = 400\), and \(\gamma = 300\). We then computed the cross-correlation function, from lag \(-10\) to lag \(+10\), between the obtained \(ITI\) series (\(ITI_{A,n}\) and \(ITI_{B,n}\)). We present in Fig. 2 the averaged cross-correlation function. This model typically produces positive lag \(-1\) and lag \(+1\) cross-correlations, and a negative lag \(0\) cross-correlation. The positive lag \(-1\) and lag \(+1\) cross-correlations reflect the correction of asynchronies, and the negative correlation at lag \(0\) results from this mutual tendency of each participant to adapt towards the previous \(ITI\) of the other. This typical pattern of cross-correlation was evidenced by Konvalinka et al. (2010), in an experiment where participants had to synchronize their taps, and by Delignières and Marmelat (2014) in an experiment where each participant in a dyad swung a hand-held pendulum, and were instructed to swing in synchrony.

![Fig. 2. Averaged cross-correlation function obtained from series simulated with Eq. (3).](image)

In contrast, both coupled oscillators models and complexity matching are likely to result in a unique, positive peak of cross-correlation, located at lag \(0\). Indeed, coupled oscillators models suggest a local, continuous coupling within the limit cycle, and the oscillators are
clearly expected to synchronize their frequencies. Generally the authors working on coordination dynamics focus on the stability of relative phase, and ignore the possible serial dependencies between the series produced by the two oscillators. However, Delignières and Marmelat (2014) and Coey, Washburn, Hassebrock, and Richardson (2016) clearly evidenced a peak of cross-correlation at lag 0 between the two limbs in bimanual coordination. Complexity matching does not suggest such local coupling, but, rather, a global and multiscale coordination between systems (Stephen & Dixon, 2011). This should induce a close tailoring of fluctuations, which should also be expected to result in a peak at lag 0 in the cross-correlation function.

Then the location of the peak(s) of cross-correlation between the series produced by the two members of the dyad, could allow to distinguish between weak and strong anticipation processes, but not between the local and global forms of strong anticipation. We suggest, however, that the magnitude of the lag 0 cross-correlation peak could represent an interesting test for the respective relevancy of the two last competing models.

**Lag 0 windowed cross-correlation**

Cross-correlations are strongly affected by trends, which could spuriously increase the obtained values. In order to control these biases and to focus on local processes, one could compute the Windowed Detrended Cross-Correlation function (WDCC). In this method the series are divided into non-overlapping intervals of short length (e.g., 15 data points), and detrended within each interval.

The local cross-correlation function is then computed within each interval, and averaged over all intervals (Coey et al. 2016 ; Delignières & Marmelat, 2014; Konvalinka et al, 2010). Fine et al. (2015) suggested that the local and continuous coupling involved in coordination dynamics models could be at the origin of the strong statistical matching observed in interpersonal synchronization experiments. However, several recent studies showed that in such situations, statistical matching occurs despite a lack of substantial short-term cross-correlation, considered as evidence against the local coupling hypothesis (Abney et al., 2014 ; Marmelat & Delignières, 2012; Rhea, Kiefer, D'Andrea, Warren, & Aaron, 2014; Washburn, Kallen, Coey, Shockley, & Richardson, 2015).

Delignières et al. (2016) performed a simulation study based on the HKB model (Eq. (2)). In order to account for the presence of 1/\(f\) fluctuations in limb oscillations, they provided the stiffness parameters (\(\omega^2\)) of both equations with independent fractal properties. They showed that this model required very high coupling parameters (i.e., \(a\) and \(b\) in Eq. (2)) for maintaining the stability of coordination patterns. As a consequence, the local coupling between oscillators was strong and the mean lag 0 WDCC, computed from these simulated series, was of about 0.84. In contrast, Coey et al. (2016) and Delignières and Marmelat (2014) obtained lag 0 WDCCs of about 0.4 in bimanual coordination tasks, and Coey et al. (2016) observed a value of about 0.2 in an interpersonal synchronization tapping task. On the basis of these results, one can consider that the lag 0 WDCC value could allow distinguishing between the alternative models of strong anticipation: Coupled oscillators dynamics should be revealed by a significant peak of WDCC at lag 0, but in the case of complexity matching this peak should remain non-significant.
The aim of the present work was to clarify the nature of synchronization in side-by-side walking. We applied the three previously presented statistical tests to empirical series, and we hypothesized to evidence the typical signatures of complexity matching in this situation.

Methods
Participants
26 participants (16 male and 10 female, mean age: 28.07 yrs ± 8.88, mean weight: 68.65 kg ± 10.5, mean height: 172.92 cm ± 9.67) were involved in the experiment. Participants were paired into 13 dyads. The pairing procedure was performed in order to preserve the homogeneity of weights and heights within each dyad. Participants signed an informed consent approved by the local ethic committee and were not paid for their participation. All work was conducted in accordance with the 1964 Declaration of Helsinki.

Experimental procedure
The experiment was performed around an indoor running track (circumference 200 m), and comprised three experimental conditions:

- Condition 1: Independent walking. Each participant walked individually at his/her preferred velocity
- Condition 2: Side-by-side walking. The two members of the dyad walked together, side-by-side. They were explicitly instructed to Synchronize their steps during the whole trial.
- Condition 3: Arm-in-arm walking. The two members of the dyad walked together, arm-in-arm. They were explicitly instructed to Synchronize their steps during the whole trial. Each trial, in the three conditions, lasted 16 min. Participants had a resting period of at least 10 min between two successive trials. Independent walking was performed at first. The order of the two last conditions was counterbalanced within dyads.

Data collection
Data were recorded with two Mobility Lab systems (APDM, Inc), one for each member of the dyads. Two body-worn inertial sensors were attached on the shanks of each participant. Data were then wirelessly streamed to a laptop. The device performed automated analyses providing a set of raw series (stride duration, stride length, etc., for both limbs). In the present paper we focused on the series of right stride durations.

Statistical analyses
Multifractal detrended fluctuation analysis (MF-DFA). We performed multifractal analyses with the MF-DFA method, initially introduced by Kantelhardt et al. (2002). Consider the series \( x(i), i = 1, 2, ..., N \). In a first step the series is centered and integrated:

\[
F^2(n,s) = \frac{1}{n} \sum_{k=(s-1)n+1}^{sn} \left( X(k) - X_{n}(k) \right)^2
\]  

Next, the integrated series \( X(k) \) is divided into \( N_{\text{N}} \) non-overlapping segments of length \( n \); and in each segment \( s = 1, ..., N_{\text{N}} \). Within each segment the local trend \( X_{n,s}(k) \) is estimated and subtracted from \( X(k) \). The variance is calculated for each detrended segment:
\[ F^2(n,s) = \frac{1}{n} \sum_{k=(s-1)n+1}^{sn} \left[ X(k) - X_{n,s}(k) \right]^2 \]  

(3)

And then averaged over all segments to obtain qth order fluctuation function

\[ F_q(n) = \left\{ \frac{1}{N_n} \sum_{x=1}^{N_n} \left[ F^2(n,s) \right]^{q/2} \right\}^{1/q} \]

(6)

Where \( q \) can take any real value except zero. In the present work we used integer values for \( q \), from \(-15\) to \(+15\). Note that Eq. (6) cannot hold for \( q = 0 \). A logarithmic averaging procedure is used for this special case:

\[ F_0(n) = \exp\left( \frac{1}{2N_n} \sum_{x=1}^{N_n} \ln\left[ F^2(n,s) \right] \right) \]

(4)

This calculation is repeated for all lengths \( n \) (practically, one considers intervals from 8 or 10 data points, in order to allow a proper assessment of statistical moments, up to \( N/4 \) or \( N/2 \)). If long-term correlations are present, \( F_q(n) \) should increase with \( n \) according to a power law:

\[ F_q(n) \propto n^{h(q)} \]

(5)

The scaling exponent \( h(q) \) is obtained as the slope of the linear regression of \( \log F_q(n) \) versus \( \log n \). \( h(q) \) is called the generalized Hurst exponent.

These results are then converted into the more classical multifractal formalism by simple transformations (Kantelhardt et al., 2002): first, generalized Hurst exponents \( h(q) \) are converted into Renyi exponents \( \tau(q) \) by:

\[ \tau(q) = qh(q) - 1 \]

(6)

The singularity spectrum \( f(\alpha) \) is then derived through the Legendre transform:

\[ \alpha(q) = \frac{d\tau(q)}{dq} \]

(7)

\[ f(\alpha) = q\alpha - \tau(q) \]

(8)

Where \( f(\alpha) \) is the fractal dimension of the support of singularities in the measure with Lipschitz-Hölder exponent \( \alpha \).

Note that for avoiding to obtain “inversed” spectra, exhibiting a zig-zag shapes rather than the expected parabolic shape in the singularity spectrum, we applied the focus-based approach introduced by Mukli, Nagy, and Eke (2015). This approach considers that the moment-wise scaling functions, for all \( q \) values, should theoretically converge toward a common limit value at the coarsest scale. Indeed, substituting signal length (\( N \)) to interval length (\( n \)) in Eq. (6) yields:

\[ F_q(N) = \left\{ \frac{1}{N} \sum_{s=1}^{Ns} \left[ F^2(N,s) \right]^{q/2} \right\}^{1/q} = \left\{ F^2(N,s)^{q/2} \right\}^{1/q} = F(N,s) \]

(9)
F(N,s) can then be considered the theoretical focus of the scaling functions, and this focus is used as a guiding reference when regressing for h(q) (Delignières et al., 2016; Mukli et al., 2015).

Correlation functions

Just as DFA, MF-DFA allows to select the range of intervals over which exponents are estimated. As previously indicated, usually authors consider intervals from 8 or 10 data points, up to N/4 or N/2. Quite often, however, series present different scaling regimes over the short and the long term, and authors perform separate estimates over different ranges of intervals (Delignières & Marmelat, 2014). Here we propose to estimate the set of multifractal exponents in first over the entire range of available intervals (i.e., from 8 to N/2), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to N/2, from 32 to N/2, and then from 64 to N/2). We then computed for each q value the correlation between the individual Lipschitz-Hölder exponents characterizing the two coordinated systems, \( \alpha_1(q) \) and \( \alpha_2(q) \), respectively, yielding a correlation function \( r(q) \). As previously explained, we expected to find in all cases a correlation function close to 1, for all q values, when only the largest intervals were considered (i.e. 64 to N/2). Increasing the range of considered intervals should have a negligible impact on \( r(q) \) when coordination is based on a complexity matching effect. In contrast, if coordination is based on local corrections, a decrease in \( r(q) \) should be observed, as shorter and shorter intervals are considered.

Cross-correlation analyses

We first computed the cross-correlation function between the series produced by the two members of each dyad in each condition. Cross-correlations were computed for each dyad from lag −60 to lag +60, and the cross-correlation functions were point-by-point averaged. In a second step we computed for each dyad WDCC functions, from lag −10 to lag 10, between the series produced by the two participants. WDCC were computed over non-overlapping windows of short length (15 data points), and data were linearly detrended within each window before the computation of cross-correlations. WDCC functions were then point-by-point averaged.

Results

The length of the collected stride series obviously depended of the walking speed of each dyad. For the independent walking condition, we deleted for each dyad the last points of the longest series, in order to obtain two series of equal lengths. For the two other conditions, we occasionally deleted some short segments, which presented synchronization errors, either at the beginning of the trial (due to difficulties to enter in synchronization) or at the end of the trial (due to fatigue or boredom). The resulting series lengths ranged from 801 to 990 data points for independent walking, from 716 to 1004 data points for side-by-side walking, and from 650 to 990 data points for arm-in-arm walking.
We present in Fig. 3 (upper panel) two example stride intervals series recorded in a representative dyad in the arm-in-arm condition. This first graph shows how medium- or long-term fluctuations are synchronized within the dyad. The bottom panel of Fig. 3 represents a focus of the previous series (one hundred strides). This graph suggests in contrast a quite poor synchronization on local scales. We analyze these points more deeply in the following section.

![Figure 3](image)

**Fig. 3.** Upper panel: Two example stride intervals series recorded in a representative dyad in the arm-in-arm condition. For a better readability, the series are vertically shifted by 0.15 ms. Bottom panel: A focus on the previous series, between strides #550 and #650.

**Multifractal analysis**

We present in Fig. 4 the correlation functions $r(q)$ between the multifractal spectra, for the three experimental conditions. Correlation coefficients are plotted against their corresponding $q$ values. Four correlation functions are displayed, according to the shortest interval length considered during the analysis (8, 16, 32, or 64). For the independent walking condition (left panel), the correlation functions remained non-significant, whatever the considered range of intervals. In contrast, the correlation functions were systematically above the threshold of significance, whatever the range of interval considered, for side-by-side walking (middle panel) and for arm-in-arm walking (right panel). The correlation functions were close to one in the arm-in-arm condition, when the shortest ranges of intervals were considered (i.e. 32 to $N/2$ and 64 to $N/2$). They appeared a little bit lower in the side-by-side condition, where the correlation functions for the same interval ranges were on average around 0.9 for positive $q$ values, and around 0.82 for negative $q$ values.
Fig. 4. Correlation functions $r(q)$, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$), for independent walking (left), side-by-side walking (middle) and arm-in arm walking (right). $q$ represents the set of orders over which the MF-DFA algorithm was applied.

Cross-correlation analyses

We present in Fig. 5 (left panel) the averaged cross-correlation functions in the three conditions. In the first condition (independent walking), no correlation was observed over the investigated range of lags. In contrast, in the two conditions of synchronized walking cross-correlation functions were organized around a marked peak at lag 0, with an average lag 0 coefficient of about 0.45 in condition 2, and 0.57 in condition 3. Cross-correlations remained significant up to the negative and positive extrema of the investigated range. Finally, cross-correlations were systematically higher in condition 3, showing the effectiveness of the reinforcement of coupling in arm-in-arm walking, as compared with simple side-by-side walking.

The averaged WDCC functions are reported in Fig. 5 (right panel). These functions present a peak at lag 0 for side-by-side and arm-in-arm walking. However in both cases these peaks did not present significant values (0.16 and 0.24, respectively). Note that in contrast with the previous analysis, the decay of cross-correlations was very fast, in both negative and positive directions.
Fig. 5. Left panel: Averaged cross-correlation functions, from lag $-60$ to lag $60$, for independent walking (light grey), side-by-side walking (dark grey), and arm-in-arm walking (black). The horizontal dashed line indicates the significance threshold ($p = 0.05$). Right panel: Averaged windowed detrended cross-correlation functions, from lag $-10$ to lag $10$, for independent walking (light grey circles), side-by-side walking (dark grey circles), and arm-in-arm walking (black circles).

Discussion

These results present strong evidence for the presence of a complexity matching effect in synchronized walking. The first analysis focused on multifractal correlation functions, and the results gave strong support for strong anticipation processes in both side-by-side and arm-in-arm walking. Whatever the range of intervals considered, correlation functions remained above the threshold of significance in both conditions. Note that we expected to find stronger correlations in arm-in-arm walking, whatever the considered range on intervals considered. This was observed for the shortest ranges, focusing on long-term intervals (i.e. $32$ to $N/2$ and $64$ to $N/2$): the correlation functions were in both cases consistently close to one, while they were between $0.8$ and $0.9$ for side-by-side walking (see Fig. 4). When the widest range of intervals was considered ($8$ to $N/2$), however, the correlation function presents somewhat lesser values in arm-in-arm walking, especially for negative $q$ values. We confess that we have no satisfying explanation for this result. Obviously, we did not obtain any significant correlation in the independent walking condition.

Cross-correlation analyses confirmed these first results. The averaged cross-correlation functions in the first condition presented non-significant values over the whole range of investigated lags, a result which was obviously expected from independent series. In contrast, a unique and sharp peak was observed at lag $0$ for both side-by-side and arm-in-arm walking, clearly showing the absence of local cycle-to-cycle adjustments. The second important observation is the persistence of cross-correlations, at least over the considered range, from lag $-60$ to lag $60$. This kind of long-range cross-correlations could be interpreted as an evidence for complexity matching. Short-term adjustments are likely
to produce a quicker, exponential-like decay in cross-correlations. However, this persistence of cross-correlations could also be due to the presence of common local trends in the synchronized series. Finally, we observed systematically higher cross-correlation coefficients in arm-in-arm walking, as compared to side-by-side. This shows that the experimental manipulation (side-by-side vs arm-in-arm) induced an effective difference in coupling strength between the two members of the dyads.

As evoked in the introduction of this paper, such cross-correlation analyses have also been applied in studies about synchronization in music, and especially for synchronization with expressively interpreted musical sequences (Dixon, Goebl, & Cambouropoulos, 2006; Rankin, Fink, & Large, 2014; Rankin, Large, & Fink, 2009; Repp, 1999, 2002, 2006). Repp (2002, 2006) showed that when participants were required to tap along with recordings of such expressively performed music, one observed a lag 0 peak of cross-correlation between the series of inter-tap intervals and the inter-onset intervals of the corresponding tones in the musical excerpt. In contrast, when participants were asked to tap along with a sequence of simple clicks reproducing the expressive timing pattern of a complex piece of music, the peak in the cross-correlation function was shifted by one lag. Repp (2002) considered this latter result as evidence that participants tracked the timing variations of the sequence, at a lag of one event. In contrast, the author considered that the lag 0 peak of cross-correlation in the first condition showed that participants adjusted their inter-tap intervals on the basis of upcoming rather than preceding inter-onset intervals in the music. In other words, they seem able to anticipate or predict ongoing timing fluctuations.

Analyzing the perfect synchronization to musical sequences in terms of prediction is obviously consistent with the representa- tional point of view of the author. However, considering that musical sequences, when expressively interpreted by expert musicians, present fractal fluctuations (Rankin et al., 2009), one could consider such synchronization as a typical case of strong, non representational anticipation. Note also that these results confirm that artificial signals mimicking natural variability do not allow strong anticipation to occur (Delignières & Marmelat, 2014; Delignières et al., 2016).

The two first analyses clearly discarded the hypothesis of local error corrections (or weak anticipation). The last problem was to distinguish between the two remaining theoretical accounts: coordination dynamics and complexity matching. The windowed detrended cross- correlation analysis confirmed the presence of a unique peak of cross-correlation at lag 0, but showed that local synchronization remained weak, and on average non significant. This result is consistent with the graphical example we presented in Fig. 2. This represents in our mind, in conjunction with previous results, a strong argument for complexity matching. The weakness of short-term cross-correlation has been considered in several previous studies as evidence discarding the local coordination account and favoring the complexity matching hypothesis (Abney et al., 2014; Marmelat & Delignières, 2012; Rhea et al., 2014; Washburn et al., 2015).
Conclusion

The complexity matching effect could appear a quite strange phenomenon, and it could certainly hurts common conceptions and models. However, this framework clearly proposes innovative and fruitful ways of thinking about coordination between living systems. The information-processing and the coordination dynamics approaches have been supported by a number of (strongly controlled) experimental protocols, but their relevancy could be limited to these restricted and artificial contexts. The analysis of more complex, daily-life like situations, suggests that coordination between living systems relies on other kinds of processes, which could be accounted for by the complexity matching effect. We propose in the present paper a set of statistical tests that aim to distinguish genuine complexity matching from other kinds of synchronization processes that could mimic some aspects of the complexity matching effect.

Evidencing the presence of a complexity matching effect in side-by-side or arm-in-arm synchronized walking could have important implication, especially for rehabilitation purposes. The presence of fractal fluctuations in stride duration series have been evidenced for a long time, suggesting the complexity of the locomotor system (Hausdorff, Peng, Ladin, Wei, & Goldberger, 1995). However, Hausdorff et al. (1997) evidenced a typical extinguishing of fractal scaling in elderly and patients suffering from neurodegenerative diseases. Additionally, they showed that the level of fractality in stride duration series was predictive of fall propension. These results were consistent with the hypothesis of the loss of complexity with age and disease (Goldberger et al., 2002). This raises a central question, from a rehabilitation perspective: could it be possible to restore complexity in a deficient system?

The complexity matching effect could offer some interesting perspectives in this regard. If a deficient (simplified) system is entrained by a healthy (complex) system, one could suppose that the complexity matching effect should result in a momentary attunement of complexities among systems, and especially an increase of the complexity of the former. In other words, if an elderly person is invited to walk in synchrony with a young and healthy companion, one could expect to observe (at least temporarily) a restoration of complexity. We currently try to test this hypothesis, and future work will aim to analyze the long-term effects of a prolonged training in such situation.

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Concluding remarks

This main result of this experiment was, as expected, to show that synchronized walking was essentially governed by a complexity matching effect. As explained in the introduction, this demonstration was necessary for engaging our final experiment about the possible restoration of complexity through complexity matching.

We also showed that the complexity matching effect was stronger when participants were mechanically coupled (arm-in-arm walking) than when they were just informationally coupled. This should be taken into account in the design of our final protocol.

Finally this experiment convinced us of the heuristic power of cross-correlation analyses, and especially the windowed detrended analysis, for determining the exact nature of the synchronization processes involved in such tasks. This motivated that next paper, in which we engaged a formal analysis of this method.
Chapter 4

Windowed detrended cross-correlation analysis of synchronization processes
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In the paper presented in Chapter 2 (Delignières et al., 2016), we proposed a multifractal signature of complexity matching. In Chapter 3 (Almurad et al., 2017) we completed our approach with a new method, the Windowed Detrended Cross-Correlation analysis (WDCC).

This method has been previously used, with some methodological variants, by some authors (Coey et al. 2015, 2016; Delignières & Marmelat, 2014; Konvalinka et al., 2010). However, the premises and the expectation of the method remained intuitive, and rather rudimentary. In the present paper we propose a formal analysis of the WDCC algorithm, in order to provide a satisfactory support to the obtained results.

WDCC is designed for identifying the processes that underlie intra- or interpersonal synchronization. The principle of windowed cross-correlation was initially introduced by Boker, et al. (2002) for analyzing the association between behavioral series in longitudinal studies. The authors considered that in such situations the assumption of stationarity of the association over the whole time series might not be warranted. The nature and the strength of the association could change over time, and cross-correlations computed over the whole series may only provide a poor picture of the true nature of the relationships between the two series. The authors propose to compute the cross-correlation function within a short sliding window, in order to analyze the possible evolution of the association over time.

This method was used by Konvalinka et al (2010), for analyzing synchronization in a task of interpersonal synchronized tapping. However, the authors considered that in such controlled experiment the association was sufficiently consistent over time for allowing the consideration of the average windowed cross-correlation function. Delignières and Marmelat (2014) proposed to add a detrending procedure within each window before the computation of the cross-correlation function.

WDCC aims at assessing the average cross-correlation function, over intervals of short (fixed) length. It explicitly focuses on local processes of synchronization. This method has been recently used in several publications, sometimes with some minor methodological variants, note that in the previous study and the others, the series are divided into non-overlapping intervals of short length (e.g., 15 data points), and detrended within each interval. The local cross-correlation function is then computed within each interval, and averaged over all intervals (Coey et al. 2015, 2016; Delignières & Marmelat, 2014; Konvalinka et al., 2010). In contrast, in the Windowed Detrended Cross-Correlation
analysis (WDCC), which is presented and discussed in the following work we used a sliding window, as initially proposed by Boker et al. (2002).

We consider that this method provides a more complete picture of cross-correlations between the two series. We formally derive the WDCC results that could be expected from three theoretical frameworks that has been proposed for accounting for inter-personal synchronization: (1) the information-processing approach, (2) the coupled oscillators model and (3) the complexity matching effect. These theoretical accounts differ in their basic assumptions, but also in the kind of tasks or activities that they consider. We show by simulation that each model allows generating series that fit the expected results. We also analyze experimental data sets collected in situations that were supposed to selectively elicit the synchronization processes depicted in the three theoretical frameworks. Our results show that the information-processing and the complexity matching processes are both present in each situation, but with a clear dominance of one of these processes on the other. Finally our results lead us to cast some doubts about the relevance of the coupled oscillators model in interpersonal synchronization.
Windowed detrended cross-correlation analysis of synchronization processes

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Abstract
The aim of this paper was to propose a formal approach of the Windowed Detrended Cross-Correlation (WDCC) analysis, a method designed for identifying the processes that underlie intra- and interpersonal synchronization. We present the three main theoretical frameworks that have been proposed for accounting for synchronization processes, (1) the information-processing approach, (2) the coupled oscillators model and (3) the complexity matching effect. We formally derive the WDCC results that could be expected from each model. We show by simulation that each model allows generating series that fit the expected results. We also analyze experimental data sets collected in situations that were supposed to selectively elicit the synchronization processes depicted in the three theoretical frameworks. Our results show that the information-processing and the complexity matching processes are both present in each situation, but with a clear dominance of one of these processes on the other. Finally our results lead us to cast some doubts about the relevance of the coupled oscillators model in interpersonal synchronization.

Key-words: Synchronization, asynchronies correction, coupled oscillators model, complexity matching
1. Introduction: Synchronization processes

Interpersonal synchronization represents a very common phenomenon in daily life activities, for example when people walk together, dance, play music, etc. However, the processes that sustain this kind of behavior remain still poorly understood, and several theoretical frameworks are in competition for explaining how interpersonal synchronization occurs. According to Almurad, Roume, & Delignières [1], three main theoretical paradigms have been proposed for accounting for synchronization processes: (1) the information-processing approach, (2) the coupled oscillators model and (3) the complexity matching effect. These theoretical accounts differ in their basic assumptions, but also in the kind of tasks or activities that they consider.

1.1. The information-processing approach.

This approach suggests that interpersonal synchronization is based on cognitive, representational processes of anticipation. This paradigm originates in the analysis of sensorimotor synchronization, focusing at the experimental level on the synchronization of simple movements (e.g., finger tapping) with a regular metronome [2,3]. A number of studies suggested that in such tasks synchronization is achieved by a systematic correction of the current inter-tap interval, on the basis of the last asynchronies [4–6].

In order to account for synchronization with more realistic environments, this paradigm has been extended to the study of synchronization with non-isochronous metronomes, and some recent studies focused on synchronization with metronomes that presented fractal variabilities, which are supposed to represent the kind of fluctuations one encounters with natural situations, and especially with human partners [7–12]. These experiments generally showed that individuals tracked the timing variations of the metronome sequence by a discrete correction of the last asynchrony [8,9,13]. This tracking behavior is essentially similar to that supposed by the classical work on synchronization with regular metronomes.

This information processing approach has also been extended to interpersonal synchronization, especially in the study of dyadic finger tapping tasks [14–16]. As previously, the results suggested that interpersonal synchronization was achieved by a mutual correction of the last asynchrony.

1.2. The coupled oscillators model.

A second theoretical framework has been proposed by the coordination dynamics perspective, which was originally developed for accounting for bimanual coordination [17,18]. This approach was based on the hypothesis of a continuous coupling between the two effectors, considered as self-sustained oscillators. Schmidt, Carello, and Turvey [19] suggested to apply this model to interpersonal synchronization. They showed, in a series of experiments in which two seated participants were asked to visually coordinate their lower legs, that interpersonal coordination presented strong similarities with bimanual coordination: anti-phase and in-phase coordination patterns emerged as intrinsically stable behaviors, with anti-phase being less stable than in-phase coordination, and spontaneous transitions from anti-phase to in-phase coordination were also observed with increasing frequency. Similar results were obtained in diverse interpersonal tasks,
such as rocking side-by-side in rocking chairs [20], or swinging pendulums together [21]. Some important predictions of the original model, such as the effect of a difference between the uncoupled eigenfrequencies of the two oscillators, were also evidenced in interpersonal coordination tasks [21]. In contrast to the previous approach, the coordination dynamics perspective does not suggest any form of discrete, cycle-to-cycle correction of asynchronies.

1.3. The complexity matching effect.

Complexity matching represents a third framework that has been recently proposed for accounting for interpersonal coordination processes [1,9,22–24]. The concept of complexity matching was introduced by West et al. [25], and states that the exchange of information between two complex networks is maximized when their complexities are similar. The response of a complex network to the stimulation of another network is a function of the matching of their complexities. This property requires that both networks generate $1/f$ fluctuations, and has been interpreted as a kind of “$1/f$ resonance” [26].

An interesting conjecture exploiting the complexity matching effect supposes that two coupled complex systems tend to attune their complexities in order to optimize information exchange. This conjecture suggests a close matching between the scaling exponents characterizing the series produced by each system. The processes that underlie this tailoring of fluctuations remain not fully understood. Stephen and Dixon [27] proposed an interesting hypothesis, which explained this attunement as a case of multifractal cascade dynamics in which perceptual-motor fluctuations are coordinated across multiple time scales. This coordination among multiple time scales could support the apparently predictive aspects of behavior without requiring an internal model.

These three theoretical frameworks have received considerable supports in their respective fields of emergence, including interpersonal coordination tasks. We are not sure, however, that these frameworks represent alternative hypotheses for accounting for similar phenomena. Depending on the nature and the constraints of the situation, different synchronization processes could be at work, and each framework could offer satisfying accounts in specific tasks. The information processing approach seems particularly relevant for accounting for situations where one has to synchronize discrete movements (e.g., tapping) with series of discrete signals [2,14]. The coordination dynamics perspective was essentially developed for accounting for the coordination of continuous, oscillatory movements [19]. The scope of complexity matching remains to define, but it has been previously applied to very diverse situations, including non periodic interactions between complex systems [22].

Almurad, Roume, and Delignières [1] proposed a set of statistical signatures, in order to test the respective relevance of these frameworks in specific situations. Their goal was to determine statistical tests that could be able to unambiguously identify the processes at work in interpersonal coordination. They proposed three possible tests: the first one was based on multifractal analyses, and has been initially proposed by Delignières et al. [23], the second and the third exploited cross-correlation analyses. In the present paper we focus on the *Windowed Detrended Cross-Correlation* analysis. This method has been applied in some recent papers, but its formal properties have never been explicitly analyzed.
2. Windowed Detrended Cross-Correlation Analysis

The principle of windowed cross-correlation was initially introduced by Boker, Hu, Rotondo and King [28], for analyzing the association between behavioral series in longitudinal studies. The authors considered that in such situations the assumption of stationarity of the association over the whole time series might not be warranted. The nature and the strength of the association could change over time, and cross-correlations computed over the whole series may only provide a poor picture of the true nature of the relationships between the two series. They proposed to analyze cross-correlations over a short sliding window, in order to account for the evolution of the association over time.

This method was used by [14], in a laboratory experiment where paired participants had to tap in synchrony with each other. Auditory feedback was manipulated in order to induce specific coupling mode between the two participants (i.e., no coupling, uni-directional coupling or bi-directional coupling). However, the authors considered that in such controlled experiment the association was sufficiently consistent over time for allowing the consideration of the average windowed cross-correlation function.

Delignières and Marmelat [9] proposed to add a detrending procedure within each window before the computation of the cross-correlation function. Note that this windowing-detrending combination was initially introduced by Lemoine and Delignières [29], in order to improve the performance of auto-correlation analyses for distinguishing between the event-based and the emergent modes of timing. The introduction of detrending by Delignières and Marmelat [9] was motivated by the recurrent observation that behavioral time series typically exhibited 1/f-like fluctuations, and then presented various interpenetrated trends, over diverse time scales. Such trends could strongly affect cross-correlations, and spuriously increase the obtained values. The so-called Windowed Detrended Cross-Correlation analysis (WDCC) explicitly aims at focusing on local processes of synchronization, and it has been recently used in several publications [1,9,30,31].

We now present in details the WDCC algorithm, as discussed and used in the present paper. Consider two series $I_1(n)$ and $I_2(n)$ with length $N$. The main principle is to compute the cross-correlation function, from lag $-k_{\text{max}}$ to lag $k_{\text{max}}$, considering windows of length $L$. The first considered window is the interval $[I_1(k_{\text{max}}+1), I_1(k_{\text{max}}+1+L)]$. The cross-correlation of lag $k$, $k = -k_{\text{max}}, ..., 0, ..., k_{\text{max}}$, is the correlation $r(k)$ between this first interval and the interval $[I_2(k_{\text{max}}+1+k), I_2(k_{\text{max}}+1+L+k)]$.

The first interval is then lagged by one point, and a second cross-correlation function is computed. This process is repeated up to the last interval $[I_1(N-k_{\text{max}}-L-1), I_1(N-k_{\text{max}}-1)]$. Note that in most previous papers WDCC used non-overlapping windows [1,9,30,31]. In the present work we used a sliding window, as initially proposed by Boker et al. [28]. We consider that this method provides a more complete picture of cross-correlations between the two series.

Before the computation of each cross-correlation functions, the data within the window in $I_1(n)$ and the lagged windows in $I_2(n)$ are linearly detrended. Then the cross-correlation
functions are point-by-point averaged. Note that before averaging, the cross-correlation coefficients $r(k)$ are transformed in $z$-Fisher scores $Zr(k)$:

$$Zr(k) = \frac{\log((1 + r(k)) / (1 - r(k)))}{2}$$  \hspace{1cm} (1)

Then the $Zr(k)$ coefficients are averaged over all windows and backward transformed in correlation metrics:

$$\tau(k) = \frac{\exp(2Zr(k)) - 1}{\exp(2Zr(k)) + 1}$$  \hspace{1cm} (2)

Because WDDC uses very narrow windows ($L = 15$ data points in the present paper), and excludes linear trends, one can difficultly expecting to find significant correlations, in the classical sense (i.e., on the basis of the Bravais-Pearson’s correlation test). WDCC provides local traces of the original correlations, and we are more interested in the sign of the average WDCC coefficients, than in their statistical significance. Therefore we test the signs of averaged coefficients with two-tailed location $t$-tests, comparing the obtained values to zero.

Note also that we conduct in the following parts of this paper a formal analysis at the level of covariance, which is more easily decomposable than correlations, but we present graphical results in terms of correlations in order to allow a better readability and a direct comparison between data sets. By definition, the signs of covariance and correlation are identical.

3. Experimental data sets

In the present article we used four sets of experimental data, in order to illustrate the main steps of our argumentation. These data were previously exploited in dissertations and published papers, and the details of the respective protocols are presented in the Appendix. The first data set represents series of inter-tap intervals produced in an experiment where two participants were instructed to perform finger tapping in synchrony [32]. This kind of task was supposed to specifically elicit synchronization processes based on discrete asynchrony correction. The second data set was recorded in an experiment during which participants performed bimanual forearm oscillations [33]. In this situation synchronization was supposed to be governed by a continuous coupling between effectors. The third one was collected in an experiment where dyads had to oscillate pendulums in synchrony [24]. This experimental situation was also expected to be sustained by continuous coupling [21]. Finally the fourth data set was collected in an experiment where dyads walked in synchrony, arm-in-arm, around an indoor running track [1]. The authors presented this task as sustained by a complexity matching effect.

4. Basic properties of asynchronies in synchronization data.

In a first step we highlight some basic properties that should be present in all synchronized series, from the moment where the two systems are effectively synchronized. Let $I_1(n)$ and $I_2(n)$ be the time intervals produced by the first and the
second system, respectively. $A_1(n)$ represents the asynchrony of $I_1(n)$ with respect to $I_2(n)$, and is defined as:

$$A_1(n) = A_1(0) + \sum_{k=1}^{n} I_1(k) - \sum_{k=1}^{n} I_2(k)$$  \hspace{1cm} (3)$$

where $A_1(0)$ represents the initial asynchrony. By definition, $A_1(n)<0$ signifies that the first system leads the second. Note that the reverse asynchrony $A_2(n)$ could also be considered, with $A_2(n) = -A_1(n)$. Considering that the two systems are (closely) synchronized, $A_1(n)$ should be a stationary process, with stable mean and variance over time.

Whichever way synchronization is produced, an increase in $I_1(n)$ should induce an increase of the concomitant asynchrony $A_1(n)$. Then,

$$cov[I_1(n), A_1(n)] > 0$$  \hspace{1cm} (4)$$

As well, in order to maintain synchronization, an increase in $A_1(n)$ should be followed by a decrease in the next interval.

$$cov[A_1(n), I_1(n+1)] < 0$$  \hspace{1cm} (5)$$

We checked these assumptions by computing the cross-correlation function between $I_1(n)$ and $A_1(n)$ in the four previously presented data sets. In order to get useful estimates for the next sections we applied the WDCC algorithm. We present in Figure 1 the averaged WDCC functions, from lag-10 to lag 10. In all cases, the WDCC function presented, as expected, a positive peak at lag 0, and negative peaks at lag -1 and lag 1. As previously explained, we tested the signs of averaged cross-correlation coefficients by means of a two-tailed location $t$-test. In all cases, the lag 0 cross-correlation was positive (data set #1: $t_9 = 27.36$, $p<.01$; data set #2: $t_{11} = 14.85$, $p<.01$; data set #3: $t_{10} = 39.12$, $p<.01$; data set #4: $t_{10} = 8.33$, $p<.01$). The lag -1 cross-correlation was negative (data set #1: $t_9 = -11.76$, $p<.01$; data set #2: $t_{11} = -5.44$, $p<.01$; data set #3: $t_{10} = -7.10$, $p<.01$; data set #4: $t_{10} = -3.47$, $p<.01$), as well as the lag 1 cross-correlation (data set #1: $t_9 = -11.05$, $p<.01$; data set #2: $t_{11} = -7.38$, $p<.01$; data set #3: $t_{10} = -4.42$, $p<.01$; data set #4: $t_{10} = -3.22$, $p<.01$).
Figure 1: Windowed detrended cross-correlation functions, from lag-10 to lag 10, between intervals and asynchronies. a: data set #1, interpersonal tapping task; b: data set #2, bimanual oscillations; c: data set #3, interpersonal pendulum task; d: data set #4, walking in synchrony (*: p < .01).

The properties described by Eq. (4) and Eq. (5) suggest that the lag 1 auto-covariance of $A_1(n)$ should be negative:

$$\text{cov}[A_1(n), A_1(n+1)] < 0$$ (6)

We checked this assumption by computing the auto-correlation function of asynchronies in our four data sets. As previously, we used a windowed detrended auto-correlation algorithm, based on the same principles of WDCC. We computed auto-correlations from lag 1 to lag 20. We present in Figure 2 the average auto-correlation functions, for the four data sets. As expected, the average lag 1 auto-correlation was negative in all cases. We tested the signs of the lag 1 auto-correlation coefficients by means of a two-tailed location $t$-test, which revealed significant differences to zero in all data sets (data set #1: $t_0 = -9.60$, p < .01; data set #2: $t_{11} = -38.62$, p < .01; data set #3: $t_{10} = -7.63$, p < .01; data set #4: $t_{10} = -4.30$, p < .01).

5. Detrending and windowing

Detrending supposes that a series can be decomposed as the sum of the linear trend and the residuals (i.e., the difference between the original value and the trend):

$$x(n) = x_{\text{trend}}(n) + x_{\text{res}}(n)$$ (7)

The WDCC algorithm uses linear detrending. This choice was motivated by the assumption that with a narrow windowing (15 points), the linear trend should be relevant in most intervals. Note that if the series is stationary with zero mean, $x_{\text{res}}(n) = x(n)$. 

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Behavioral series are often modeled as the linear combination of component series [4–6,34,35]. Some of these components are stationary: this is the case for asynchronies, as previously stated, and also for the error series that are modeled as uncorrelated noises. Some others components, in contrast, exhibit 1/f fluctuations [34,36,37]. These series have been characterized as fractional Gaussian noises (fGn), and as such are considered stationary on the long term. However, such series, as previously noticed, present various interpenetrated trends, over diverse time scales (see Figure 3, graph a). In such a case, windowing is supposed to isolate narrow segments in the series that could be effectively stationarized by linear detrending, within each window.

Figure 3 (graphs b and c) illustrates the effect of detrending within a window of 15 data points. The original points (graph b) present a positive trend, and the detrended series (graph c) is stationary, with zero mean. 1/f series are considered stationary around the local trend. On this local scale, this graph suggests a kind of alternation of successive points around the trend.

We present in graph d (black circles) the average autocorrelation function, computed from lag 1 to lag 30, over 12 simulated series of 1024 data points, with Hurst exponent \( H = 0.9 \). As expected, the autocorrelation function shows long-range persistence: auto-
correlation remains significant over 30 lags. Graph d represents also in white circles the average windowed detrended autocorrelation function, using a sliding window of 15 points, over the same set of series. As can be seen, persistence is extinguished by the windowed detrending procedure. This suggests that persistence, in fGn series, is mainly sustained by trends. More interestingly, a location t-test shows that autocorrelation coefficients are negative, from lag 2 to lag 9. This reinforces the idea that within each window, the detrended series presents a kind of anti-persistence around its local trend. This observation will have some importance in the following parts of this paper.

Figure 3: a: fGn simulated series with $H = 0.9$. b: an interval of 15 point extracted from the series of graph a. The dashed line represents the linear trend within the interval. c: The same interval, after detrending. d, black circles: average autocorrelation function, from lag 1 to lag 30, over 12 simulated series of 1024 data points, with $H = 0.9$. d, white circles: averaged windowed detrending autocorrelation function, from lag 1 to lag 30, with a sliding window of 15 points, over the set of 12 simulated series.

In the following parts of this paper, we analyze the models that have been proposed in the three theoretical frameworks we presented in the introduction, and we try to formally derive the results that could be expected in each case with the application of WDCC.
6. Mutual correction of asynchronies

This first model is supposed to account for the synchronization of two participants in event-based timing tasks (e.g., in synchronized tapping). It represents an extension of the model proposed by Vorberg and Wing [4] or Pressing and Jolley-Rogers [5] for accounting for tapping in synchronization with a regular metronome. This initial model could be expressed as follows:

\[
I(n) = I'(n) - \alpha A(n-1) + \gamma \left[ B(n) - B(n-1) \right], \tag{8}
\]

where \( I(n) \) represents the inter-tap intervals produced by the participant, \( I'(n) \) the interval provided by an internal timekeeper, \( A(n) \) the asynchrony between the \( n \)th tap and the \( n \)th metronome signal, and \( B(n) \) a white noise process corresponding to the error produced by the motor component at the \( n \)th tap. The presence of a differenced white noise term \( [B(n)-B(n-1)] \) is related to the event-based nature of the task: \( I(n) \) is defined by the production of two successive taps, and then is affected by the two successive motor errors [35]. Initially \( I'(n) \) was considered a white noise source [4,35], but the analysis of prolonged trials showed that the series of intervals produced by the timekeeper presented fractal properties, and should be modeled as a \( 1/f \) source [36,37].

This model can be extended as follows for synchronized tapping:

\[
\begin{align*}
I_1(n) &= I'_1(n) - \alpha_1 A_1(n-1) + \gamma_1 \left[ B_1(n) - B_1(n-1) \right] \\
I_2(n) &= I'_2(n) - \alpha_2 A_2(n-1) + \gamma_2 \left[ B_2(n) - B_2(n-1) \right]
\end{align*}
\tag{9}
\]

where \( I_1(n) \) and \( I_2(n) \) represent the inter-tap intervals produced by participant 1 and 2, respectively, \( I'_1(n) \) and \( I'_2(n) \) the intervals provided by their respective timekeepers, \( A_1(n) \) and \( A_2(n) \) their mutual asynchronies, and \( B_1(n) \) and \( B_2(n) \) their respective error terms. At this stage, we have no specific assumption about a possible relationship between \( I'_1(n) \) and \( I'_2(n) \), which could be considered either independent or cross-correlated.

Now consider the effect of detrending. Each inter-tap interval can be decomposed as the sum of the theoretical interval given by the linear regression and the associated residual. For participant 1:

\[
I_1(n) = I_{\text{trend}}(n) + I_{\text{res}}(n) \tag{10}
\]

Combining Eq. (9) and (10) yields:

\[
I_1(n) = I'_{\text{trend}}(n) + I'_{\text{res}}(n) - \alpha_1 A_{\text{trend}}(n-1) - \alpha_1 A_{\text{res}}(n-1) \\
+ \gamma_1 B_{\text{trend}}(n) + \gamma_1 B_{\text{res}}(n) - \gamma_1 B_{\text{trend}}(n-1) - \gamma_1 B_{\text{res}}(n-1) \tag{11}
\]

As previously stated \( A_1(n) \) should be stationary and for simplicity we suppose that the asynchronies are centered around zero (note that this assumption supposes that corrections are reciprocal, without systematic leader/follower relationship). On the other
hand, \( B_1(n) \) is by definition a zero mean and stationary process. Then \( A_{1\text{res}}(n) = A_1(n) \) and \( B_{1\text{res}}(n) = B_1(n) \). On the basis on these assumptions Eq. (11) could be simplified as:

\[
I_1(n) = I_{1\text{trend}}^*(n) + I_{1\text{res}}^*(n) - \alpha_1 A_1(n-1) + \gamma_1 B_1(n) - \gamma_1 B_1(n-1)
\]  

(12)

Combining Eq. (10) and (12) yields:

\[
I_{1\text{res}}(n) = I_{1\text{trend}}^*(n) - I_{1\text{trend}}(n) + I_{1\text{res}}^*(n) - \alpha_1 A_1(n-1) + \gamma_1 [B_1(n) - B_1(n-1)]
\]  

(13)

Finally, and considering again \( A_i(n) \) and \( B_i(n) \) as stationary processes, one could suppose that the essential contribution to trends in \( I_1(n) \) comes from \( I_1^*(n) \). Then,

\[
I_{1\text{trend}}^*(n) = I_{1\text{trend}}(n)
\]  

(14)

The whole system could then be rewritten as:

\[
\begin{align*}
I_{1\text{res}}(n) &= I_{1\text{res}}^*(n) - \alpha_1 A_1(n-1) + \gamma_1 [B_1(n) - B_1(n-1)] \\
I_{2\text{res}}(n) &= I_{2\text{res}}^*(n) - \alpha_2 A_2(n-1) + \gamma_2 [B_2(n) - B_2(n-1)]
\end{align*}
\]  

(15)

The distributivity of covariance [4] allows to derive an expression of the lag \( k \) covariance between the residuals of the inter-tap intervals series produced by the two participants:

\[
cov[I_{1\text{res}}(n), I_{2\text{res}}(n+k)] = cov[I_{1\text{res}}^*(n), I_{2\text{res}}^*(n+k)]
\]

\[- \alpha_2 \text{cov}[I_{1\text{res}}^*(n), A_2(n+k-1)]
\]

\[+ \gamma_2 \text{cov}[I_{1\text{res}}^*(n), B_2(n+k)]
\]

\[- \gamma_2 \text{cov}[I_{1\text{res}}^*(n), B_2(n+k-1)]
\]

\[- \alpha_1 \text{cov}[A_1(n-1), I_{2\text{res}}^*(n+k)]
\]

\[+ \alpha_1 \alpha_2 \text{cov}[A_1(n-1), A_2(n+k-1)]
\]

\[- \alpha_1 \gamma_2 \text{cov}[A_1(n-1), B_2(n+k)]
\]

\[+ \alpha_1 \gamma_2 \text{cov}[A_1(n-1), B_2(n+k-1)]
\]

\[+ \gamma \text{cov}[B_1(n), I_{2\text{res}}^*(n+k)]
\]

\[- \alpha_2 \gamma \text{cov}[B_1(n), A_2(n+k-1)]
\]

\[+ \gamma \gamma_2 \text{cov}[B_1(n), B_2(n+k)]
\]

\[- \gamma \gamma_2 \text{cov}[B_1(n), B_2(n+k-1)]
\]
\[-\gamma_{1} \text{cov}\left[ B_{1}(n-1), I_{2\text{res}}^{*}(n+k), \right] \]
\[+ \alpha_{2} \gamma_{1} \text{cov}\left[ B_{1}(n-1), A_{2}(n+k-1) \right] \]
\[+ \gamma_{1} \gamma_{2} \text{cov}\left[ B_{1}(n-1), B_{2}(n+k) \right] \]
\[+ \gamma_{2} \text{cov}\left[ B_{1}(n-1), B_{2}(n+k-1) \right] \] (16)

This expression can be simplified, considering that \( A_{i}(n) = -A_{2}(n) \), and that all covariances involving white noise are zero, except covariances between simultaneous noises and asynchronies. Indeed at the \( n \)th tap, \( B_{1}(n) \) and \( B_{1}(n-1) \) should directly affect \( A_{i}(n) \), in opposite directions, \( \text{cov}[A_{1}(n), B_{1}(n)] \) being positive, and \( \text{cov}[A_{1}(n), B_{1}(n-1)] \) negative. Then we can derive the following expression:

\[\text{cov}\left[ I_{\text{res}}^{*}(n), I_{2\text{res}}^{*}(n+k) \right] = \text{cov}\left[ I_{\text{res}}^{*}(n), I_{2\text{res}}^{*}(n+k) \right] \]
\[+ \alpha_{2} \text{cov}\left[ I_{\text{res}}^{*}(n), A_{1}(n+k-1) \right] \]
\[+ \alpha_{2} \text{cov}\left[ A_{2}(n-1), I_{2\text{res}}^{*}(n+k) \right] \]
\[- \alpha_{2} \alpha_{2} \text{cov}\left[ A_{1}(n-1), A_{2}(n+k-1) \right] \]
\[+ \alpha_{2} \gamma_{2} \text{cov}\left[ A_{2}(n-1), B_{2}(n+k) \right] \]
\[- \alpha_{2} \gamma_{2} \text{cov}\left[ A_{2}(n-1), B_{2}(n+k-1) \right] \]
\[+ \alpha_{2} \gamma_{2} \text{cov}\left[ B_{1}(n), A_{2}(n+k-1) \right] \]
\[- \alpha_{2} \gamma_{2} \text{cov}\left[ B_{1}(n-1), A_{2}(n+k-1) \right] \] (17)

For the lag 1 covariance \( (k = 1) \)

\[\text{cov}\left[ I_{\text{res}}^{*}(n), I_{2\text{res}}^{*}(n+1) \right] = \text{cov}\left[ I_{\text{res}}^{*}(n), I_{2\text{res}}^{*}(n+1) \right] \]
\[+ \alpha_{2} \text{cov}\left[ I_{\text{res}}^{*}(n), A_{1}(n) \right] \]
\[+ \alpha_{1} \text{cov}\left[ A_{2}(n-1), I_{2\text{res}}^{*}(n+1) \right] \]
\[- \alpha_{1} \alpha_{2} \text{cov}\left[ A_{1}(n-1), A_{1}(n) \right] \]
\[+ \alpha_{1} \gamma_{2} \text{cov}\left[ A_{2}(n-1), B_{2}(n+1) \right] \]
\[- \alpha_{1} \gamma_{2} \text{cov}\left[ A_{2}(n-1), B_{2}(n) \right] \]
\[+ \alpha_{1} \gamma_{2} \text{cov}\left[ B_{1}(n), A_{1}(n) \right] \]
\[- \alpha_{2} \gamma_{2} \text{cov}\left[ B_{1}(n-1), A_{1}(n) \right] \] (18)
In the right part of Eq. (18), the first term should be negligible, even if the timekeepers are positively cross-correlated: As previously shown (see Figure 3), the windowed detrending procedure tends to extinguish correlations in fGn series. According to Eq. (4), the second term should be positive. In contrast, the third term should be negligible, considering that the two terms are separated by two lags. According to Eq. (6) the fourth term should be positive, its strength depending of the level of correction in the model ($\alpha_1$ and $\alpha_2$). Our assumptions concerning the relationships between noise and asynchronies suggest that the fifth term should be negative. The sixth term should be negligible, considering that the two terms are separated by two lags. In contrast the seventh and the eighth terms should be positive. On the whole, the lag 1 covariance between the residuals of the interval produced by the two participants should be positive. Considering the symmetry of Eq. (15), the same reasoning holds for the lag -1 covariance, which should also be positive.

Now consider the lag 0 covariance:

$$\text{cov}[I_{1_{\text{res}}}(n),I_{2_{\text{res}}}(n)] = \text{cov}[I_{1_{\text{res}}}(n),I_{2_{\text{res}}}(n)]$$

$$+ \alpha_2 \text{cov}[I_{1_{\text{res}}}(n), A_1(n-1)]$$

$$+ \alpha_1 \text{cov}[A_2(n-1), I_{2_{\text{res}}}(n)]$$

$$- \alpha_2 \alpha_1 \text{cov}[A_1(n-1), A_1(n-1)]$$

$$+ \alpha_1 \gamma_2 \text{cov}[A_2(n-1), B_2(n)]$$

$$- \alpha_2 \gamma_2 \text{cov}[A_2(n-1), B_2(n-1)]$$

$$+ \alpha_2 \gamma_1 \text{cov}[B_1(n), A_1(n-1)]$$

$$- \alpha_1 \gamma_2 \text{cov}[B_1(n-1), A_1(n-1)]$$

$$= \alpha_1 \gamma_2 \text{cov}[B_1(n), A_1(n-1)]$$

$$- \alpha_2 \gamma_1 \text{cov}[B_1(n-1), A_1(n-1)]$$

(19)

The first term of the right side of Eq. (19) should be positive, its strength depending of the level of cross-correlation between the two timekeepers. In contrast, Eq. (5) suggests that the second and third terms are negative, and obviously the fourth term is negative. The fifth and the seventh terms should be negligible, but the sixth and the eighth terms should both be negative. Then the sign of covariance depends on the opposite influences of the level of cross-correlation between the two timekeepers and the strength of the error components.

We tried to simulate the system depicted in Eq. (9), in order to analyze the effect of the correlation between $I_1^*$ and $I_2^*$ on the WDCC function. For simulating $I_1^*$ and $I_2^*$, we used two long-range correlated series, obtained by means of the method described in Zebende [38] and Balocchi et al. [39]. In this method, two long-range cross-correlated series, $x(n)$ and $y(n)$, are obtained as:
\[ x(n) = WX(n) + (1-W)Y(n) + \delta_1 \epsilon_1 \]
\[ y(n) = (1-W)X(n) + (W)Y(n) + \delta_2 \epsilon_2 \]

where \( \epsilon_1 \) and \( \epsilon_2 \) denote two independent white noise processes with zero mean and unit variance, \( \delta_1 \) and \( \delta_2 \) represent the relative strengths of these noise components, with respect to \( X(n) \) and \( Y(n) \), which are two independent auto-regressive fractionally integrated moving-average (ARFIMA) processes, defined as:

\[ X(n) = \sum_{k=1}^{\infty} a_k(d_1)x_{n-k} \]
\[ Y(n) = \sum_{k=1}^{\infty} a_k(d_2)y_{n-k} \]

where \( a_n(d) \) are statistical weights defined by:

\[ a_k(d) = \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(1+k)} \]

In this equation \( \Gamma \) denotes the Gamma function, and \( d \) are parameters ranging from -0.5 to 0.5. \( d \) is related to the Hurst exponent by \( H = d + 0.5 \). In Eq. (22), \( W \) is a free parameter ranging from 0.5 to 1.0 and controlling the strength of cross-correlations between \( x(n) \) and \( y(n) \). \( W = 0.5 \) gives the highest cross-correlation, while the total absence of correlation is obtained for \( W = 1 \).

In the present simulation we set for simplicity \( \delta_1 = \delta_2 = 0 \), and we generated 12 pairs of series of 1024 points, for 6 values of \( W \) (0.5, 0.6, 0.7, 0.8, 0.9, and 1.0). In all cases we used \( d_1 = d_2 = 0.4 \) (\( H = 0.9 \)). The average lag 0 cross-correlation, computed over these simulated series, was 1.0, 0.92, 0.71, 0.45, 0.20, and -0.02, respectively. Then we generated the series of inter-tap intervals, \( I_1(n) \) and \( I_2(n) \), according to Eq. (9), with \( I_1^*(n) = x(n) \) and \( I_2^*(n) = y(n) \), and setting \( \alpha_1 = \alpha_2 = 0.4 \), and \( \gamma_1 = \gamma_2 = 0.5 \). The average WDCC functions between the obtained \( I_1(n) \) and \( I_2(n) \) series, for the six \( W \) values, is reported in Figure 4.
This hypothesis requires additional investigations, as asynchrony correction and suggest ones, because the authors did not use detrending in their approach. Our average lag 0 correlation at lag 0 of about 0.1. We applied WDCC to the series of the data set #1, which was collected in interpersonal synchronized tapping [32]. We present in Figure 5 the obtained average WDCC function. Results reveal positive peaks at lag -1 and lag 1, and a negative peak at lag 0. Location t-tests showed that the mean cross-correlation was positive at lag -1 and lag 1 ($t_0 = 5.36$, $p<0.01$ and $t_0 = 6.32$, $p<0.01$, respectively), and conversely negative at lag 0 ($t_0 = -4.40$, $p<0.01$). A similar result has been evidenced by Konvalinka et al. [14], with a mean cross-correlation at lag 0 of about -0.35, and mean cross-correlations at lag -1 and lag 1 of about 0.3. However, the obtained values should be compared with caution with the present ones, because the authors did not use detrending in their approach. Our average lag 0 WDCC roughly corresponds to those obtained by simulation for $W = 0.8$ and $W = 0.9$, suggesting that the two timekeepers are moderately cross-correlated, which could be interpreted as the presence of a complexity matching effect between the two timekeepers. This hypothesis requires additional investigations, as asynchrony correction and complexity matching were previously considered as mutually exclusive [1].

![Figure 4: Average windowed detrended cross-correlation function for a set of 12 series, simulated from Eq. (9), for $W$ values ranging from 0.5 to 1.0. *: $p<.01$.](image-url)
In conclusion, WDCC seems able to clearly identify trial-to-trial discrete correction processes, essentially though the presence of positive peaks at lag -1 and lag 1, and the lag 0 WDCC provides information about the level of cross-correlation between the two timekeepers. Note also that the WDCC function could exhibit an asymmetry between the lag -1 and the lag 1 coefficients, revealing a leader/follower relationship between participants. The leader is supposed to present a lower correction parameter than the follower (e.g., $\alpha_1 < \alpha_2$), and in such a case the sum of the four last terms of Eq. (18) is different for lag -1 and lag 1.

Finally, the correction process could be more complex, taking into account a wider range of previous asynchronies. For example Pressing and Jolley-Rogers [5] or Vorberg and Wing [4] proposed models based on the correction of the two previous asynchronies. Such models could be expressed as follows:

$$
\begin{align*}
I_1(n) &= I_1^*(n) - \alpha_1 A_1(n-1) - \beta_1 A_1(n-2) + \gamma_1 [B_1(n) - B_1(n-1)] \\
I_2(n) &= I_2^*(n) - \alpha_2 A_2(n-1) - \beta_2 A_2(n-2) + \gamma_2 [B_2(n) - B_2(n-1)]
\end{align*}
$$

This kind of correction process should result in the presence of positive peaks at lag -2, lag -1, lag 1 and lag 2 in the WDCC function.

7. **Coupled oscillators model**

As previously explained, this model was initially developed in the analysis of bimanual coordination, and was based on the hypothesis of a continuous coupling between the two effectors, considered as self-sustained oscillators [17,18]. This model could be written as follows:

$$
\begin{align*}
\ddot{x}_1 + \delta \dot{x}_1 + \lambda \dot{x}_1^2 + \gamma x_1^2 \dot{x}_1 + \omega^2 x_1 &= (\dot{x}_1 - \dot{x}_2)(a + b(x_1 - x_2)^2) \\
\ddot{x}_2 + \delta \dot{x}_2 + \lambda \dot{x}_2^2 + \gamma x_2^2 \dot{x}_2 + \omega^2 x_2 &= (\dot{x}_2 - \dot{x}_1)(a + b(x_2 - x_1)^2)
\end{align*}
$$

Figure 5: Averaged windowed detrended cross-correlation function for data set #1 (interpersonal tapping task). *: $p<.01$. 

![Figure 5](image.png)
where $x_i$ is the position of oscillator $i$, and the dot notation represents derivation with respect to time. The left side of the equations represents the limit cycle dynamics of each oscillator, determined by a linear stiffness parameter ($\omega$) and damping parameters ($\delta, \lambda$, and $\gamma$), and the right side represents the coupling function determined by parameters $a$ and $b$. This model has been proven to adequately account for most empirical features in bimanual coordination tasks, such as the differential stability of in-phase and anti-phase coordination modes, and the transition from anti-phase to in-phase coordination with the increase of oscillation frequency [17,18].

Here we consider a modified version of this model, where the fixed linear stiffness $\omega$ is replaced by a variable parameter $\omega_{i'\prime}$ representing discrete, cycle-to-cycle changes in stiffness [33]. This modification aimed to account for the presence of $1/f$ fluctuations in the series of periods produced by an oscillating effector [34], and in the series of relative phases during bimanual coordination [33].

$$\begin{align*}
\dot{x}_1 + \delta \dot{x}_1 + \lambda x_1^2 \dot{x}_1 + \omega_n^2 x_1 &= (x_1 - \hat{x}_1)(a + b(x_1 - x_2)^2) + q_1 \epsilon_1 \\
\dot{x}_2 + \delta \dot{x}_2 + \lambda x_2^2 \dot{x}_2 + \omega_n^2 x_2 &= (x_2 - \hat{x}_1)(a + b(x_2 - x_1)^2) + q_2 \epsilon_2
\end{align*}$$

(25)

where $\omega_i$ is a fractal process with Hurst exponent $H$, mean $\omega_n$ and standard deviation $\sigma$. $\epsilon_1$ and $\epsilon_2$ are white noise processes with zero mean and unit variance, representing a continuous perturbation independently affecting each oscillator, with respective strengths $q_1$ and $q_2$.

This model suggests that the two oscillators share the same (variable) stiffness, and that perturbations are counterbalanced by the coupling function. Considering that the period of an oscillator is essentially determined by stiffness, this continuous model could be translated at the cycle level as follows, using the preceding notation:

$$\begin{align*}
I_1(n) &= I_1^*(n) + \gamma B_1(n) \\
I_2(n) &= I_2^*(n) + \gamma_2 B_2(n)
\end{align*}$$

(26)

$I^*(n)$ representing a common “timekeeper”, corresponding to the series of stiffness $\omega_n$ in Eq. (25), and the noise terms summarizing perturbations at the cycle level. Note that in contrast with the model of Eq. (9), synchronization is not obtained by means of a cycle-to-cycle correction of asynchronies, but simply by the presence of a common “timekeeper”.

This system predicts that the lag 0 covariance between the two inter-tap intervals series is equal to the variance of the timekeeper, and then positive:

$$\text{cov}[I_{1\text{res}}(n), I_{2\text{res}}(n)] = \text{var}[I^{*\text{res}}(n)]$$

(27)

Considering the other lags, our previous assumptions about the influence of windowed detrending on the auto-correlation of fGn could be applied (see Figure 3), and one could predict to observe slightly negative covariances, especially below lag -1 and above lag 1.
We simulated Eq. (25), setting $\delta = 0.5$, $\lambda = 0.02$, and $\gamma = 1$, $a = 1$ and $b = 0.25$. $\omega_i$ was accounted for by fGn series with $H = 0.9$, $\omega_0 = 4\pi$ and $\sigma = 0.04$, and we set $q_1 = q_2 = 0.03$. Simulations were performed using a four-stage Runge–Kutta algorithm, following the scheme described by Burrage, Lenane, and Lythe [40], for second-order stochastic differential equations with additive noise. We used a fixed step size of 0.001 s, and we generated 12 pairs of period series of 1024 data points. The average WDCC function for these simulated series is displayed in Figure 6 (left panel). As expected, we obtained a positive peak at lag 0 ($t_{11} = 47.86$, $p<0.01$), and negative cross-correlations at lags -5, -4, and -3, and from lag 3 to lag 6.

We finally applied WDCC to the experimental bimanual coordination series of data set #2. The average WDCC function is displayed in Figure 6 (right panel). The results are similar to those obtained by the simulation, with a positive peak at lag 0 ($t_{11} = 10.40$, $p<0.01$), and negative values below lag -2 and above lag 1. This confirms that during bimanual coordination, the two effectors share the same stiffness fluctuations.

**Figure 6:** Left: Average windowed detrended cross-correlation function for a set of 12 series, simulated from Eq. (25). Right: Averaged windowed detrended cross-correlation function for data set #2 (bimanual oscillations). *: $p<0.01$.

As previously indicated, the coupling oscillator model has been extended for accounting for interpersonal coordination, especially in tasks involving continuous movements [19–21]. Our third data set, collected in an experiment were dyads had to oscillate pendulums in synchrony [24], clearly corresponds to this kind of situations.

We present in Figure 7 the average WDCC function obtained with this data set. Clearly the results are different than those expected from the coupled oscillators hypothesis. We obtained positive peaks at lag -1 and lag 1, and a negative peak at lag 0. Location $t$-tests showed that the mean cross-correlation at lag 0 was negative ($t_{10} = -5.63$, $p<0.01$), and conversely positive at lag -1 and lag 1 ($t_{10} = 3.90$, $p<0.01$ and $t_{10} = 4.38$, $p<0.01$, respectively). Note that the average cross-correlation was also positive at lag -2 and lag 2 ($t_{10} = 3.78$, $p<0.01$ and $t_{10} = 4.94$, $p<0.01$, respectively), suggesting that a more complete model, including a correction of the two last asynchronies should be more relevant:
Note that in Eq. (28) included a single error term, and not differenced noise as in Eq. (15). This corresponds to the hypothesis that such continuous task should elicit emergent timing processes [36]. It can be easily shown that this should not affect the signs of the expected windowed detrended covariances.

Quite surprisingly, these results show that synchronization, in this experiment, was governed by discrete, cycle-to-cycle corrective processes, and clearly contradicts the relevance of coupled oscillators models in such interpersonal coordination tasks. Further investigations are currently in progress in our lab for testing the reliability of these results and understanding its determinants.

\[
\begin{align*}
I_1(n) &= I_1^*(n) - \alpha_1 A_1(n-1) - \beta_1 A_1(n-2) + \gamma_1 B_1(n) \\
I_2(n) &= I_2^*(n) - \alpha_2 A_2(n-1) - \beta_2 A_2(n-2) + \gamma_2 B_2(n)
\end{align*}
\] (28)

Figure 7: Averaged windowed detrended cross-correlation function for data set #3 (interpersonal pendulum task). *: \(p<.01\).

8. Complexity matching

Finally we turn to the third theoretical framework we evoked in the introduction, the complexity matching hypothesis. Complexity matching supposes a model quite similar to that advocated for the continuous coupling model, except that the two systems are not driven by a common "timekeeper", but tend to attune their complexity. This model could be expressed as follows:

\[
\begin{align*}
I_1(n) &= I_1^*(n) + \gamma_1 B_1(n) \\
I_2(n) &= I_2^*(n) + \gamma_2 B_2(n)
\end{align*}
\] (29)

\(I_1^*(n)\) and \(I_2^*(n)\) being considered as long-range cross-correlated fGn series. Importantly, the complexity matching hypothesis supposes that synchronization is achieved without any process of asynchronies correction. On the basis of this model, one can obviously expect to observe a positive covariance peak at lag 0.
We generated 12 pairs of series $I_1(n)$ and $I_2(n)$. $\Gamma_1(n)$ and $\Gamma_2(n)$ were long-range cross-correlated fGn series, simulated by the ARFIMA procedure presented in Eqs. (20), (21), and (22), with $d_1 = d_2 = 0.4$ (i.e., $H = 0.9$) and $W = 0.7$. Eq. (9) was implemented setting $\gamma_1 = \gamma_2 = 0.5$. The average WDCC function, for these simulated series, is presented in Figure 8 (left panel). As expected, the WDCC function presented a positive peak at lag 0. Location $t$-tests showed that the mean cross-correlation at lag 0 was positive ($t_{11} = 23.46, p < .01$), and was also positive at lag -1 and lag 1 ($t_{11} = 3.01, p < .05$ and $t_{11} = 4.62, p < .01$, respectively). We also observed negative correlations at lag -4, -5 and -6 ($t_{11} = -3.82, p < .01, t_{11} = -2.94, p < .05$ and $t_{11} = -3.85, p < .01$, respectively), as well as lags 3, 4, 5, 6, 7 and 8 ($t_{11} = -2.76, p < .05, t_{11} = -2.42, p < .05, t_{11} = -2.33, p < .05, t_{11} = -2.74, p < .05, t_{11} = -2.48, p < .05,$ and $t_{11} = -3.44, p < .01$, respectively).

We finally applied WDCC to data set #4. The average WDCC function, for this data set, is presented in Figure 8 (right panel). This function presents a similar shape than that obtained in the simulation study. Location $t$-test showed that the mean cross-correlation at lag 0 was positive ($t_{10} = 5.56, p < .01$), and was also positive at lag -1 and lag 1 ($t_{10} = 5.71, p < .01$ and $t_{10} = 4.60, p < .01$, respectively). We also observe negative correlations at lag -6 and lag -4 ($t_{10} = -4.39, p < .01$ and $t_{10} = -3.57, p < .01$, respectively), as well as lags 4 and 6 ($t_{10} = -6.67, p < .01$ and $t_{10} = -4.51, p < .01$, respectively).

![Figure 8](image-url)

**Figure 8**: Left: Averaged windowed detrended cross-correlation function for 12 simulations of Eq. (29). Right: Averaged windowed detrended cross-correlation function for data set #4 (walking in synchrony). *: $p < .05$; #: $p < .01$.

However, one can notice that the cross-correlation coefficients at lag -1 and lag 1 seem higher in the experimental WDCC function than in that obtained by simulation. This could be explained by a slight process of stride-to-stride correction that could be superimposed to the complexity matching effect. This could be expressed by the following model:

$$
\begin{align*}
I_1(n) &= I_1^*(n) - \alpha_1 A_1(n-1) + \gamma_1 B_1(n) \\
I_2(n) &= I_2^*(n) - \alpha_2 A_2(n-1) + \gamma_2 B_2(n)
\end{align*}
$$

(30)
This model is close to that of Eq. (9), except that the differenced white noise term was replaced by a single noise term for accounting for the continuous nature of the task (see also Eq. (28)). As in the previous simulation, we generated 12 pairs of series \( I_1(n) \) and \( I_2(n) \). \( I_1^*(n) \) and \( I_2^*(n) \) were long-range cross-correlated fGn series, simulated by the ARFIMA procedure with \( d_1 = d_2 = 0.4 \), \( W = 0.7 \) and \( \gamma_1 = \gamma_2 = 0.5 \). We used low values for the correction parameters: \( \alpha_1 = \alpha_2 = 0.2 \). The average WDCC function, for these simulated series, is presented in Figure 9. As can be seen, the introduction of slight corrections did not affect the global shape of the WDCC function, but selectively enhance the average level of correlation at lag -1 and lag 1.

This simulation shows that complexity matching, which tends to dominate synchronization in this situation of synchronized walking, should be completed by a slight, discrete stride-to-stride correction process. We currently try to test this hypothesis, in an experiment where participants walk in synchrony with a clear leader/follower relationship. Preliminary results tend to confirm that the lag -1 and lag 1 WDCC coefficients reflect this asymmetry, confirming that a discrete correction process is at work during synchronization.

Figure 9: Averaged windowed detrended cross-correlation function for 12 simulations of Eq. (30). *: \( p<.01 \).

9. Discussion

In this paper we proposed a formal approach of WDCC, completed with simulation studies and empirical data analysis. We show that the WDCC function contains information revealing the various processes that underlie synchronization. The WDCC function is especially affected by (1) the strength of discrete corrective processes, (2) the level of cross-correlation between ‘timekeepers’, and (3) the level of noise in the system.

Trial-to-trial or cycle-to-cycle discrete corrective processes are revealed by the presence of positive WDCC at lag -1 and lag 1. Correction could be distributed over more successive trials or cycles, and in that case cross-correlation appears positive over more lags (for
example at lags -2, -1, and 1, 2, see data set #3). Finally, an asymmetry between positive and negative lags is likely to reveal leader/follower relationships in synchronization.

The lag 0 WDCC reflects the level of cross-correlation between the two timekeepers. In the limit case of bimanual tasks, in which both hands are governed by the same timekeeper (as in the bimanual oscillation task of data set #2), WDCC should exhibit a strong positive peak at lag 0. When the two synchronized systems are governed by distinct, but long-range cross-correlated fractal processes, as expected in situations where synchronization is sustained by a complexity matching effect, a positive peak is also expected at lag 0, but with rather moderate values, as compared with the previous case (see data set #4).

We show, however, that the lag 0 WDCC is also affected by noise and corrective processes. Obtaining a close-to-zero or even negative lag 0 WDCC does not indicate that timekeepers are independent or anti-correlated. This kind of result simply provides information about the respective importance of the different factors affecting lag 0 WDCC.

This is maybe the main message of the present work, beyond the theoretical and methodological presentation of WDCC. In our previous papers we presented the different frameworks for the analysis of interpersonal synchronization as alternative, and exclusive hypotheses, referring to processes selectively elicited by specific task constraints [1,23]. The present analyses suggest a different point of view. We show in the analysis of our first data set (interpersonal tapping) that beyond a clear discrete process of trial-to-trial correction of asynchronies, a matching of complexities between the timekeepers of the two partners should be considered for a complete account of empirical correlations. As well, we show in the analysis of our last data set (walking in synchrony) that beyond complexity matching, a slight but effective discrete stride-to-stride correction process is at work. In each situation, synchronization seems dominated either by asynchrony correction or by complexity matching, but the discreet influence of the other process cannot be ignored. All situations could be accounted for by a single model, which could simply differ through the relative strength of its essential components.

Another unexpected result in the present paper was the evidence that inter-personal held pendulum task was dominated by corrective processes. Because of its oscillatory character, this task was a good candidate for revealing the essential features of coupled oscillator models, and especially a strong cross-correlation between “timekeepers”, and the absence of cycle-to-cycle correction. This was clearly not the case, and recently Scotti [32] obtained similar results in a task where two participants had to perform synchronized forearm oscillations. This quite deceptive result merits further investigations, considering the amount of theoretical and experimental work that has been devoted during the last decades to the application of the coupled oscillators models to interpersonal coordination.

Finally one might ask what are the advantages of WDCC over the conventional cross-correlational approach? We present in Figure 10 the average cross-correlation functions, computed over the entire series, from lag -60 to lag 60, for our four data sets.
Figure 10: Average cross-correlation functions, computed from lag -60 to lag 60. a: data set #1, interpersonal tapping task; b: data set #2, bimanual oscillations; c: data set #3, interpersonal pendulum task; d: data set #4, walking in synchrony. The dashed line represents the probability threshold $p = .05$.

The essential information provided by WDCC is obviously discernible in these cross-correlation functions (especially the peaks at lag-1 and lag 1 for data sets #1 and #3, the stronger lag 0 peak for data set #2, as compared with data set #4). WDCC has the advantage to focus on short-term processes, and to normalize results over a standardized window size, allowing comparisons among experimental situations. As shown in the previous sections of this paper, WDCC allows to finely disentangling the multiple processes that could underlie synchronization. Cross-correlation functions, however, provide additional information about the persistence and the rate of decay of cross-correlations, which are hidden by the WDCC algorithm, but remain essential for accounting for the long-range nature of cross-correlations in the complexity matching framework.

Finally we would like to insist on the fact that the WDCC function should be considered as a pattern, and not as a sample of correlation coefficients. Comparing Figure 10 with the previous figures of this paper allows understanding why we consider that WDCC functions only contain traces of the original cross-correlations. Each experimental situation seems characterized by a specific WDCC pattern, and the proper way for analyzing WDCC functions is to test the consistency of the obtained pattern over dyads performing in similar conditions (which was done with our location $t$-tests), and not to look for the statistical significance of each WDCC coefficients.
Appendix

Data set #1: Inter-individual coordination in tapping.

This data set was presented in Scotti [32]. 20 participants were involved in this experiment (11 male and 9 female, mean age 22.5 ± 4.3). They were randomly paired into 10 dyads. The series exploited in the present paper was collected in a part of the experiment where the two participants of each dyad were invited to perform finger tapping in synchrony. Participants were sitting face-to-face and tapped with their index finger on a flat pressure sensor fixed on a table. The tempo was initially given by a metronome that emitted 10 successive beeps at a frequency of 1 Hz, and participants were instructed to synchronize their taps with the metronome. Then the metronome was removed and the participants were invited to maintain their synchronization for 10 minutes. The data set exploited in the present paper is the series of inter-taps intervals produced by each participant in each dyad.

Data set #2: Bimanual forearm oscillations

This data set was initially presented in Torre and Delignières [33]. Twelve participants (8 male and 4 female, mean age 29 ± 7) took part in the experiment. They individually performed two bimanual tasks: bimanual forearm oscillations, and bimanual tapping. In the present paper we used the series collected in the first task.

Participants were seated comfortably, with the elbows slightly flexed and the forearms supported in a horizontal position. The task consisted in performing simultaneous in-phase forearm oscillations (by definition in this mode of coordination the two hands move in mirror symmetry). They held two wooden joysticks, 15 cm in length, with a single degree of freedom in the frontal plane. Participants were instructed to perform the oscillations as smoothly and regularly as possible, with an amplitude of approximately 45° on either side of the vertical axis. The angular displacement of the joysticks was captured by two potentiometers.

Each trial was introduced by a 30-s video showing in close-up two hands performing the task at the required frequency (2 Hz). Then participants immediately began the task, following the required frequency as accurately as possible, up to the production of 600 oscillation cycles. The data set exploited in the present paper is the series of periods (computed as the time between two successive maximal pronations) produced by each hand.

Data set #3: inter-individual coordination in pendulum swinging

This data set was initially presented in Marmelat and Delignières [24]. 22 volunteers (16 male and 6 female, mean age 24.5 ± 2.9) were involved in the experiment, and were randomly paired into 11 dyads. Participants were seated side-by-side between the two pendulums. Participant A held his/her pendulum with the right hand, and participant B with the left hand. In each dyad, participants were randomly assigned to the A or B position. Pendulums oscillated in the sagittal plane. The distance between the two
pendulums was 1.10 m, their length was 0.48 m (from the bottom of the handle to the bottom of the pendulum). A mass of 0.150 kg was fixed at the bottom of each pendulum. A potentiometer located at the rotation axis of each pendulum allowed recording the angle position. Participants were instructed to firmly sustain the handle with the entire hand, and to manipulate the pendulum with the wrist joint, in an abduction-adduction movement. The forearm was kept parallel to the floor, without any support.

The data set exploited in the present paper was collected in a condition where participants had to perform synchronized oscillations with the two pendulums, following an in-phase pattern of coordination. They were instructed to oscillate at the preferred frequency of the dyad, as regularly as possible. Visual and auditory feedbacks were fully available. The trial lasted 12 minutes. The data set is composed by the series of periods produced by each participant.

Data set #4: Synchronized walking

This data set was initially presented in Almurad et al. [1]. 26 participants (16 male and 10 female, mean age 28.1 ± 8.9) were involved in the experiment. Participants were paired into 13 dyads. The pairing procedure was performed in order to preserve the homogeneity of weights and heights within each dyad. The experiment was performed around an indoor running track (circumference 200m). The data set exploited in the present paper was collected in a condition where the two members of the dyad walked together, arm-by-arm. They were explicitly instructed to synchronize their steps during the whole trial. The trial lasted 16 minutes.

Data were recorded with two Mobility Lab systems (APDM, Inc), one for each member of the dyad. Two body-worn inertial sensors were attached on the shanks of each participant. Data were then wirelessly streamed to a laptop. The device performed automated analyses providing a set of raw series (stride duration, stride length, etc., for both limbs). In the present paper we focused on the series of right stride durations.

Declarations of interest: none

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References


[33] K. Torre, D. Delignières, Distinct ways of timing movements in bimanual


Concluding remarks

The first aim of this paper was to propose a formal demonstration of the properties of the Windowed Detrended Cross-Correlation analysis. In most previous presentation of Windowed Cross-correlation analysis, authors remained quite allusive concerning the exact meaning of the sign and the magnitude of the correlation coefficients within the obtained function (Coey, 2015; Coey et al., 2016; Didier Delignières & Marmelat, 2014; Konvalinka, Vuust, Roepstorff, & Frith, 2010). Our approach was inspired by that adopted by Wing and Kristofferson (1973), in their analysis of the auto-covariance function of inter-tap intervals in self-paced tapping. We used a systematic decomposition of covariance expressions, for determining the signs of cross-correlations at diverse lags.

Hybrid models

Our approach was initially based on the properties of “ideal” models, each referring to the three theoretical frameworks we previously identified (asynchronies correction, continuous coupling, and complexity matching). In all cases we identified the expected signatures of each model and we showed that the corresponding experimental data, in most cases, produced those signatures.

Our results, however, revealed a more complex picture than that expected by ideal models. We evidenced traces of complexity matching in interpersonal tapping, even if the global WDCC pattern was clearly dominated by asynchrony correction. As well, the WDCC function obtained for synchronized walking was that expected from a complexity matching effect, but we found clear traces of asynchrony correction. These results lead us to propose hybrid models, such as that depicted in equation 30, containing both asynchrony correction and complexity matching terms:

\[
\begin{align*}
I_1(n) &= I_1^*(n) - \alpha_1 A_1(n - 1) + \gamma_1 B_1(n) \\
I_2(n) &= I_2^*(n) - \alpha_2 A_2(n - 1) + \gamma_2 B_2(n)
\end{align*}
\]

In such model, the dominance of one process on the other just depends on the parameters that regulate the strength of their respective terms in the model.

Asynchrony correction in synchronized oscillations

An other surprising result was a discovery of the dominance of asynchrony correction in the pendulum swinging task. We obviously expected in that case a WDCC function similar to that obtained in bimanual coordination, with a clear dominance of complexity matching. This was not the case, and this results leads to cast some doubt on the relevancy of the coupled oscillator model for interpersonal coordination. Interestingly, this result was previously presented by Delignières and Marmelat (2014), who exploited the same data set (see Figure 5, p. 10). The authors did not notice nor discuss this result, which clearly contradicted their expectations. Obviously, this paper focused on another method, the Detrended Cross-Correlation analysis (DCCA), an extention of Detrended Fluctuation analysis, and the presentation of WDCC results remained anecdotic. However, the fact that the authors ignored this result represents an intriguing example of scientific negligence, which should be analyzed from an epistemological perspective.
Recently Scotti (2017) confirmed this result in a task were participants had to perform synchronized forearm oscillations with joysticks. The obtained WDCC function is reported in the following figure:

![Figure 1: Average WDCC function in synchronized forearm oscillations performed with joysticks (Scotti, 2017).](image)

Again, the WDCC function revealed a clear dominance of asynchrony correction, with characteristic positive peak at lag -1 and lag 1. However, one could note that the cross-correlation at lag 0 also positive, suggesting a non-marginal complexity matching effect. This effect was clearly stronger that in the pendulum task. Further investigations are needed for a deeper understanding for the factors that influence the balance between asynchrony correction and complexity matching in such tasks.

**Mechanistic vs nomothetic models**

This article is a nice illustration of the research strategy of our group, which has sometimes been referred to as a “mechanistic” approach (Diniz et al., 2011; Torre & Wagenmakers, 2009). This approach is characterized by the construction of models that incorporate “1/f sources”, in combination with other short-term processes, such as asynchrony correction. In the present paper we went a step beyond by the use of long-range correlated series with the aim of accounting for complexity matching. This approach has been criticized by the proponents of a “nomothetic” approach, considering that this kind of model suggest a “localization” of fractal processes within the systems, whereas this type of fluctuation must be considered as a global product of complexity. These mechanistic models, however, do not postulate a structural localization of fractality, but rather a statistical localization, allowing to disentangle the respective influences of long-term and short-term sources of fluctuations (Diniz et al., 2011).

Our strategy could be summarized as follows: (1) determining, on the basis of previous theories, the models that could account for the production of performance series in a given situation, (2) simulating series with the obtained models, and (3) checking whether the simulated series consistently reproduce the properties of experimental series. This approach, directly inspired by the seminal papers by Wing and Kristofferson (1973),
allowed our group to overcome some erroneous interpretations in previous papers (D. Delignières & Torre, 2011; Torre et al., 2010; Torre, Delignières, & Lemoine, 2007; Torre & Delignières, 2008a, 2008b). The present paper, and especially the results concerning the hybridization of models and the presence of asynchrony correction most inter-individual coordination tasks, provide additional evidence for the relevancy of this mechanistic approach.
Chapter 5

Restoring the complexity of locomotion in older people through arm-in-arm walking
This final chapter is devoted to the test of the hypothesis that initially motivated this doctoral project: the restoration of complexity through complexity matching.

At the beginning of our work, we supposed that the most complex system, being intrinsically more stable than the less complex one, should not be affected by synchronization. In contrast, the less complex system, unstable by definition, should be “attracted” by the most complex one. This initial hypothesis was just sustained by the basic postulate relating complexity and stability: complexity confers systems with robustness and stability, allowing them to resist to external perturbations and to maintain their functioning. This hypothesis was fortunately reinforced by the formal demonstration of Mahmoodi, West, and Grigolini (2017), showing that when two systems of different complexity levels interact, the transfer of multifractality operates from the most complex system to the less complex.

This experiment was essentially exploratory. Our previous results showed that synchronized walking was dominated by a complexity matching, and that this effect was stronger when systems were closely coupled. We supposed, however, that a short experience of complexity matching should not be sufficient for obtaining the expected restoration, and that a prolonged practice should be necessary. In the first steps of the experiment, we decided to pursue the training in synchronized walking up to the obtaining of a statistically significant effect on stride series complexity. Fortunately, we observed with our first participants an increase of complexity after three weeks of training. This observation allowed us to finally fix the protocol to four complete weeks of practice.

As indicated in the paper, this protocol was very challenging, for the participants as well as for the experimenter. It required a large amount of practice, a constant availability during the four weeks of the protocol. It took up to 14 months for achieving the whole protocol for the 24 participants. Some participants, initially involved in the protocol, have been excluded because they were unable, for personal or familial reasons, to strictly follow the training schedule, which was conceived to be performed without interruption nor delay during four weeks, three days a week. It seems important to keep these constraints in mind during the reading of paper that follows...
Abstract

The complexity matching effect refers to a maximization of information exchange, when interacting systems share similar complexities. A working conjecture states that interacting systems tend to match their complexities in order to enhance their synchronization. This effect has been observed in a number of synchronization experiments, and interpreted as a transfer of multifractality between systems. Finally, it has been shown that when two systems of different complexity levels interact, this transfer of multifractality operates from the most complex system to the less complex, yielding an increase of complexity in the latter. This theoretical framework inspired the present experiment that tested the possible restoration of complexity in older people. In young and healthy participants, walking is known to present $1/f$ fluctuations, reflecting the complexity of the locomotion system, providing walkers with both stability and adaptability. In contrast walking tends to present a more disordered dynamics in older people, and this whitening was shown to correlate with fall propensity. We hypothesized that if an aged participant walked in close synchrony with a young companion, the complexity matching effect should result in the restoration of complexity in the former. Older participants were involved in a prolonged training program of synchronized walking, with a young experimenter. Synchronization within the dyads was dominated by complexity matching. We observed a restoration of complexity in participants after three weeks, and this effect was persistent two weeks after the end of the training session. This work presents the first demonstration of a restoration of complexity in deficient systems.

Key words: Complexity matching, restoration of complexity, interpersonal coordination, arm-in-arm walking, rehabilitation.
1. Introduction

Complexity appears a key concept for the understanding of the perennial functioning of biological systems. By definition, a complex system is composed of a large number of infinitely entangled elements (Delignières and Marmelat, 2012). In such a system, interactions between components are more important than components themselves, a feature that Van Orden et al. (2003) referred to as interaction-dominant dynamics.

Such a system, characterized by a myriad of components and sub-systems, and by a rich connectivity, could lose its complexity in two opposite ways: either by a decrease of the density of interactions between its components, or by the emergence of salient components that tend to dominate the overall dynamics. In the first case the system derives towards randomness and disorder, in the second towards rigidity. From this point of view complexity may be conceived as an optimal compromise between order and disorder (Delignières and Marmelat, 2012). Complexity represents an essential feature for living systems, providing them with both robustness (the capability to maintain a perennial functioning despite environmental perturbations) and adaptability (the capability to adapt to environmental changes). These relationships between complexity, robustness, adaptability and health were nicely illustrated by Goldberger, Amaral et al. (2002a) in the domain of heart diseases.

The experimental approach to complexity has been favored by the hypothesis that links the complexity of systems and the correlation properties of the time series they produce, and the development of related fractal analysis methods, and especially the Detrended Fluctuation Analysis (Peng et al., 1995). A complex system is supposed to produce long-range correlated series (1/f fluctuations), and the assessment of correlation properties in the series produced by a system allows determining the possible alterations of complexity, either towards disorder (in which case correlations tend to extinguish in the series) or towards rigid order (in which case correlations tend to increase).

This interest for complexity was particularly developed in the research on aging. Lipsitz and Goldberger (1992) proposed that aging could be defined by a progressive loss of complexity in the dynamics of all physiologic systems. This hypothesis has been developed in a number of subsequent papers (Goldberger et al., 2002a, 2002b; Sleimen-Malkoun et al., 2014; Vaillancourt and Newell, 2002). Of special interest for the present work, Hausdorff and collaborators showed that successive step durations during walking presented a typical structure over time, characterized by the presence of long-range dependence (Hausdorff et al., 1995, 1996, 2001). They also showed that these fractal properties were significantly altered in aged participants and in patients suffering from Huntington's disease (Hausdorff et al., 1997). In those cases the fractal organization tended to disappear and step dynamics became more random. Additionally, they showed that the loss of complexity in stride duration series correlated with the propensity to fall. The main question we address in the present paper is the following: could it be possible to restore complexity in older people, and especially in the locomotion system?
The working hypothesis that sustains the present work is based on the concept of **complexity matching**, initially introduced by West, Geneston and Grigolini (2008). The complexity matching effect refers to the maximization of information exchange when interacting systems share similar complexities. This effect has been interpreted as a kind of “$1/f$ resonance” between systems (Aquino et al., 2011). A working conjecture states that interacting systems tend to match their complexities in order to enhance their synchronization (Marmelat and Delignières, 2012). This attunement of complexities has been observed in a number of synchronization experiments (Abney et al., 2014; Almurad et al., 2017; Coey et al., 2016; Delignières and Marmelat, 2014; Marmelat and Delignières, 2012; Stephen et al., 2008), and interpreted as a transfer of multifractality between systems (Mahmoodi et al., 2017). Finally, it has been shown that when two systems of different complexity levels interact, this transfer of multifractality operates from the most complex system to the less complex (and not the inverse), yielding an increase of complexity in the latter (Mahmoodi et al., 2017).

The very first experimental approaches to complexity matching considered that a close correlation between the mono-fractal exponents characterizing the two synchronized systems could represent a satisfactory evidence for complexity matching (Delignières and Marmelat, 2014; Marmelat et al., 2014; Marmelat and Delignières, 2012; Stephen et al., 2008). However, Delignières et al. (2016) claimed that the matching of scaling exponents should not be considered an unambiguous signature of complexity matching. They proposed to distinguish between a simple statistical matching (i.e., the convergence of scaling exponents) and a genuine complexity matching effect (i.e., the attunement of complexities). Indeed, some studies showed that the matching of scaling exponents could result just from local, short-term adjustments or corrections (Delignières and Marmelat, 2014; Fine et al., 2015; Torre et al., 2013). For example, Delignières and Marmelat (2014) analyzed series of stride durations produced by participants attempting to walk in synchrony with a fractal metronome. They evidenced a close correlation between the scaling exponents of the series of stride durations produced by the participants and those of the series of inter-onset intervals of the corresponding metronomes. The authors tried to simulate their empirical results by means of a model based on local corrections of asynchronies, and showed that this model was able to adequately reproduce the statistical matching observed in experimental series. They concluded that walking in synchrony with a fractal metronome could essentially involve short-term correction processes, and that the close correlation observed between scaling exponents could in such a case just represent the consequence of these local corrections.

Delignières et al. (2016) proposed a more binding method for distinguishing genuine complexity matching from local corrective processes. They suggested to base the analysis of statistical matching on a multifractal approach, rather than on the monofractal analyses previously employed. This choice was motivated by the point developed by Stephen and Dixon (2011), arguing that the tailoring of fluctuations that is typical of complexity matching could be considered as the product of multifractality, and also by the fact that multifractals allow for a more detailed picture of the complexity of time series. In the same vein, Mahmoodi et al. (2017) proposed to consider complexity matching as a transfer of multifractality between system.
Multifractal processes present more complex fluctuations than monofractal series, and cannot be characterized by a single scaling exponent. In multifractal series subsets with small and large fluctuations scale differently, and their description requires a hierarchy of scaling exponents (Podobnik and Stanley, 2008). Delignières et al. (2016) proposed to assess the statistical matching through the point-by-point correlation function between the sets of scaling exponents that characterize the coordinated series. More precisely, they proposed to assess this correlation function over different ranges of scales in the series, in first over the entire range of available intervals (e.g., from 8 to $N/2$, $N$ representing the length of the series), and then over more restricted ranges, progressively excluding the shortest intervals (i.e., from 16 to $N/2$, from 32 to $N/2$, and then from 64 to $N/2$). The authors supposed that if synchronization is based on local corrections, the statistical matching in long intervals is just the consequence of the short-term, local coupling between the two systems. As local corrections between unpredictable systems remains approximate, correlations should dramatically decrease when intervals of shorter durations are taken into consideration. In contrast, in the case of genuine complexity matching, the synchronization between systems is supposed to emerge from interactions across multiple scales. The authors hypothesized to find in this case close correlations, even when considering the entire range of intervals, from the shortest to the longest (Delignières et al., 2016).

Almurad et al. (2017) and Roume et al. (2018) proposed another method, the Windowed Detrended Cross-Correlation analysis (WDCC), based on the analysis of cross-correlations between the series produced by the two systems. In this method, the series is divided into intervals of short length (e.g., 15 data points), and detrended within each interval. The local cross-correlation function is then computed within each interval (e.g. from lag -10 to lag 10), and averaged over all intervals, yielding an averaged windowed detrended cross-correlation function (WDCC). Windowing allows focusing on local synchronization processes, and detrending controls the effect of local trends, which tends to spuriously inflate cross-correlations. Similar approaches were already employed in other papers, albeit differing in some methodological settings (Coey et al., 2016; Delignières and Marmelat, 2014; Den Hartigh et al., 2018; Konvalinka et al., 2010).

WDCC allows distinguishing complexity matching from synchronization processes based on discrete asynchronies corrections: in the first case the cross-correlation function presents a positive peak at lag 0, whereas in the second case one obtains positive peaks at lags -1 and 1, and a negative peak at lag 0 (Almurad et al., 2017; Konvalinka et al., 2010; Roume et al., 2018). Additionally, complexity matching seems characterized by quite moderate levels of lag 0 cross-correlation, in contrast with those expected in continuous coupling models (Coey et al., 2016; Delignières and Marmelat, 2014).

Almurad et al. (2017) used these two methods for clarifying the nature of synchronization in side-by-side walking. In this experiment the authors applied these tests on series collected in three conditions: independent walking, side-by-side walking, and arm-in-arm walking. They evidenced clear signatures of complexity matching in the two last conditions: In both cases the correlation functions between multi-fractal spectra remained significant, whatever the range of intervals considered, and the WDCC functions showed a positive peak at lag 0. Additionally, this experiment showed that complexity matching was more intense in arm-in-arm than in side-by-side walking.
The hypotheses that are tested in the present work derive from the preceding considerations. Our goal was to investigate the possible restoration of complexity in deficient systems. We supposed, as indicated by Hausdorff et al. (1997), that aging should result in a decrease of the complexity of locomotion, as compared with young and healthy persons.

1. If an older person is invited to walk in synchrony, arm-in-arm with a healthy partner, we should observe a complexity matching effect within the dyad.

2. Considering the asymmetry of complexities, complexity matching should result in an increase of complexity in the older person.

3. A prolonged training of walking in synchrony with healthy partners should induce a perennial restoration of complexity in older persons.

2. Materials and Methods

2.1. Participants

24 participants (7 male and 17 female, mean age: 72.46 yrs ± 4.96) were involved in the experiment. They were recruited in local retiree associations, and could be considered as presenting a normal aging. They were free from disease that could affect gait, including any neurological, musculoskeletal, cardiovascular, or respiratory disorders, and had no history of falls. They were randomly assigned to two groups: an experimental group (N = 12, 2 male and 10 female, mean age: 72.83 yrs ± 6.01, mean weight: 64.25 kg ± 10.89, mean height: 162.92 cm ± 6.02), and a control group (N = 12, 5 male and 7 female, mean age: 72.08 yrs ± 3.87, mean weight: 69.91 kg ± 8.63, mean height: 166.50 cm ± 10.39). All work was conducted in accordance with the 1964 Declaration of Helsinki, and was approved by the Euromov International Review Board (n°1610B). Participants signed an informed consent and were not paid for their participation.

2.2. Experimental procedure

The experiment was performed around an indoor running track (circumference 200m). Participants were submitted to a walking training during four consecutive weeks, herein noted as week 1, week 2, week 3, and week 4. Each week comprised three training sessions, performed on Monday, Wednesday, and Friday.

Each week, the Monday session began with a solo sequence, during which the participant was instructed to walk individually around the track, as regularly as possible, at his/her preferred speed, for 15 minutes. The aim of this solo sequence was to assess the complexity of the stride duration series produced by the participant. This solo sequence was performed at the beginning of each week, in order to avoid any effect of fatigue.

Then participants performed during each week three duo sequences in the Monday session and four in the Wednesday and Friday sessions. During these sequences, they were invited to walk with the experimenter, for 15 minutes. All participants walked with the same experimenter (female, 46 yrs). This methodological choice was motivated by the aim of standardizing experimental conditions among participants. In the experimental group, the participant walked...
arm-in-arm with the experimenter, and was explicitly instructed to synchronize its steps with those of the experimenter during the whole trial. In the control group, the participant and the experimenter walked together, without any instruction of synchronization. Note that this control condition cannot be assimilated to the side-by-side condition used by Almurad et al. (2017), in which participants were explicitly instructed to synchronize heel strikes. In both groups, the experimenter was instructed to adapt her velocity to that spontaneously adopted by the participant.

Participants had a resting period of at least 10 minutes between two successive sequences. Each participant performed 44 duo sequences during the whole training program (i.e., 11 walking hours, approximately 67 km). Note that all participants performed approximately the same amount of walk (in terms of duration). The experimental and control groups differed only by the imposed synchronization with the experimenter.

Finally, a solo sequence (post-test) was performed two weeks after the end of the training program (i.e. in week 7).

2.3. Data collection

Data were recorded with two force sensitive resistors (FSR), integrated in soles at heel level. These sensors where wired connected to a Schmitt trigger (LM 393AN), a signal conditioning device that digitally shape the analogic signal of FSR sensors. This device removes noise from the original signal and turns the FSR sensors in on/off switches. The output of the Schmitt trigger was connected to the GPIO interface of a Raspberry Pi model A+. Then, a Wi-Fi dongle (EDIMAX EW7811Un) was plugged in the USB port of the Raspberry and configured as a Hotspot, allowing to launch and remote the device with another. The Schmitt trigger, the Raspberry Pi and a battery (2000 mAh) where packed in a small box entering in a waist bag that was wear on the belt by the participants.

On the software side, the Raspberry Pi was powered by the 2016 February 9th version of the Raspbian distribution. To retrieve the data we wrote a script in Python 3 language, using the internal clock of the Raspberry to time each heel strike, and then to compute stride durations series.

2.4. Statistical analyses

In the present paper we focused on the series of right stride durations. The raw series comprised 700 to 1300 data points. Fractal analyses are known to be highly sensitive to the presence of local trends in the series, which tend to spuriously increase the assessed level of long-range correlation. In the present experiment, such local trends are related to transient periods of acceleration or deceleration. These local trends are essentially present in the first part of the series, where participants seek for their most comfortable velocity, and at the end of the sequences, essentially due to fatigue, or boredom. The corresponding segments were deleted before analysis.

For solo sequences the resulting series had an average length of 924 points (+/- 148, max = 1257, min = 448), and for duo sequences 963 points (+/-64, max= 1198, min = 397). One could consider that most series satisfied the minimal length required for a valid application of fractal analyses (Delignieres et al., 2006).
We assessed the complexity of each series with the Detrended Fluctuation Analysis (Peng et al., 1994). In the application of DFA, we used intervals ranging from 10 to N/2 (N representing the length of the series). We applied the evenly spaced algorithm proposed by Almurad and Delignières (2016), which was shown to significantly enhance the accuracy of the original method.

In order to assess the effect of training on the complexity of series in solo sequences, we used a two-way ANOVA 2 (group) X 5 (week), with repeated measurement on the second factor (including the 4 training weeks and the post-test). Probabilities were adjusted by the Huyn-Feld procedure.

The analysis of synchronization during duo sequences was performed using the methods proposed by Delignières et al. (2016), Almurad et al. (2017) and Roume et al. (Roume et al., 2018). We first analyzed the multifractal signature proposed by Delignières et al. (2016). In this method the series are first analyzed by means of the Multifractal Detrended Fluctuation analysis (Kantelhardt et al., 2002). MF-DFA was successively applied considering four different ranges of intervals: from 8 to N/2, 16 to N/2, 32 to N/2, and 64 to N/2. We used q values ranging from -15 to 15, by steps of 1. The obtained generalized Hurst exponents were then converted into the more classical multifractal formalism by simple transformations (Kantelhardt et al., 2002). We finally obtained singularity spectra, relating the fractal dimension of the support of singularities in the measure f(α) to the Lipschitz-Hölder exponents α(q). We then computed for each q value the correlation between the individual Lipschitz-Hölder exponents characterizing the two coordinated systems, α₁(q) and α₂(q), respectively, yielding a correlation function r(q). As previously explained, we expected to find in all cases a correlation function close to 1, for all q values, when only the largest intervals were considered (i.e. 64 to N/2). Increasing the range of considered intervals should have a negligible impact on r(q) when coordination was based on a complexity matching effect. In contrast, if coordination was based on local corrections, a decrease in r(q) should be observed, as shorter and shorter intervals were considered.

Then we computed for each dyad WDCC functions, from lag -10 to lag 10. We used the sliding version of the method, proposed by Roume et al. (2018). Consider two series I₁(n) and I₂(n) with length N. The main principle is to compute the cross-correlation function, from lag -k_{max} to lag k_{max}, considering windows of length L. The first considered window is the interval [I₁(k_{max}+1), I₁(k_{max}+1+L)]. The cross-correlation of lag k, k = -k_{max},..., 0, ..., k_{max}, is the correlation r(k) between this first interval and the interval [I₂(k_{max}+1+k), I₂(k_{max}+1+L+k)]. The first interval is then lagged by one point, and a second cross-correlation function is computed. This process is repeated up to the last interval [I₁(N-k_{max}-L-1), I₁(N-k_{max}-1)]. Before the computation of each cross-correlation functions, the data within the window in I₁(n) and the lagged windows in I₂(n) are linearly detrended. Then the cross-correlation functions are point-by-point averaged. Before averaging, the cross-correlation coefficients r(k) are transformed in z-Fisher scores. Then the z-coefficients are averaged over all windows and backward transformed in correlation metrics.

Because WDDC uses very narrow windows and excludes linear trends, one can difficulties expect to find significant correlations, in the classical sense (i.e., on the basis of the Bravais-Pearson’s correlation test). WDCC provides local traces of the original correlations, and we are
more interested in the sign of the average WDCC coefficients, than in their statistical significance. Therefore we tested the signs of averaged coefficients with two-tailed location $t$-tests, comparing the obtained values to zero (Roume et al., 2018).

3. Results

We present in Figure 1 the evolution of the average $\alpha$-DFA exponents computed for participants in solo sequences, for the two groups, over the 4 training weeks and the post-test. The ANOVA revealed a significant interaction effect between Group and Week ($F(4,88) = 5.084$, $p = .001$, partial $\eta^2 = 0.19$). The main effect of Week was also significant ($F(4,88) = 6.44$, $p = .00014$, partial $\eta^2 = 0.23$). A Fisher LSD post-hoc test showed a significant difference between, on the one hand, the average $\alpha$-DFA obtained in the experimental group during the fourth week and the post-test, and on the other hand the entire set of other average results.

![Figure 1: Average $\alpha$-DFA exponents computed for participants in solo sequences (red: experimental group, blue: control group), over the 4 training weeks and the post-test. Error bars represent standard deviation. ** : $p<.01$.](image)

We report in Figure 2 the evolution of individual $\alpha$-DFA exponents, obtained in solo sequences over the 4 training weeks and the post-test, for the participants of the experimental group. A detailed examination of this graph reveals some inter-individual differences in the evolution of the scaling parameter. The increase of $\alpha$ exponent at the beginning of the fourth week appeared clearly for 6 participants (# 5, 2, 3, 5, 6, 9, 12), but it occurred early (at the beginning of the third week) for participants 7 and 8. Participant 4 presented at the beginning of the experiment an $\alpha$ exponent close to 1.0, and in that case the protocol had no noticeable effect. One could also note the contrasted evolutions of the exponents between the fourth week and the post-test, with a mix of increases and decreases among participants.
Figure 2: Individual $\alpha$-DFA exponents in solo sequences, over the 4 training weeks and the post-test, for the 12 participants of the experimental group.

Figure 3 presents the evolution of the average $\alpha$-DFA exponents during the four weeks of the experiment, in the solo and the duo sequences. Because the average $\alpha$-DFA exponents for the experimenter were obtained from a single individual, the analysis of variance cannot be applied in the present case. These figures, however, suggest a close convergence of the mean exponents of the experimenter and those of the participants in the experimental group, over the four weeks. Note also that this convergence appears very early during the experiment, in the first week. This convergence appears less obvious in the control group. We present in Table 1 the average correlations between the $\alpha$-DFA exponents of the participants and the corresponding exponents for the experimenter, computed over the four weeks, in the two groups. High correlations were observed in the experimental group, revealing a close statistical matching between the series simultaneously produced by the participants and the experimenter. The following analyses will check whether this statistical matching corresponds to a genuine complexity matching effect, or rather to a more local mode of synchronization. In contrast, correlations appeared moderate and extremely variable in the control group, suggesting a poor statistical matching between series.
Interestingly, one could observe in Figure 3 that in the experimental condition, the experimenter seems poorly affected by synchronization. In contrast, participants appear strongly attracted towards the experimenter, as predicted by the complexity matching framework. In contrast, in the control condition the experimenter and the participants seem converging towards a median level of complexity, halfway between their solo levels.

Table 1: Average correlation between the $\alpha$-DFA exponents of the participants and the corresponding exponents for the experimenter (standard deviations in brackets), computed over the four weeks of the experimental protocol.

<table>
<thead>
<tr>
<th></th>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental group</td>
<td>0.95</td>
<td>0.97</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Control group</td>
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<td>0.51</td>
<td>0.33</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.44)</td>
<td>(0.30)</td>
<td>(0.39)</td>
<td>(0.33)</td>
</tr>
</tbody>
</table>

We present in Figure 4 the average correlation functions $r(q)$ between the multifractal spectra, for the experimental group (top row) and the control group (bottom row), and for the four weeks. Correlation coefficients are plotted against their corresponding $q$ values. Four correlation functions are displayed, according to the shortest interval length considered during the analysis (i.e.: 8, 16, 32, or 64). For the experimental group, the correlation functions are significant, whatever the considered range of intervals. This result suggests the presence of a complexity matching effect within the dyads (Delignières et al., 2016). Note that the complexity matching effect appears from the first week of the experiment, and tends to become stronger over weeks. In contrast, in the control group, the correlation functions exhibit lower, and often non-significant values, especially when the largest ranges of intervals are considered (i.e. 8 to $N/2$ and 16 to $N/2$).
The averaged WDCC functions are reported in Figure 5, for the experimental group (top row) and the control group (bottom row), and for the four weeks. These functions systematically present a peak at lag 0, which appears higher for the experimental group (about 0.3) than for the control group (about 0.15). In both groups and all weeks, however, the location $t$-tests, comparing the obtained values to zero, are significant. The rather moderate values obtained in the experimental group are conformable to that previously obtained in similar experiments (Coey et al., 2016; Delignières and Marmelat, 2014; Den Hartigh et al., 2018; Konvalinka et al., 2010), and to that expected from a complexity matching synchronization (Almurad et al., 2017; Roume et al., 2018). These results provide evidence that synchronization, in this condition, is dominated by a complexity matching effect. In contrast, the values observed in the control group are very low, and suggest a quite poor, or just intermittent synchronization within dyads.

**Figure 4:** Correlation functions $r(q)$, for the four ranges of intervals considered (8 to $N/2$, 16 to $N/2$, 32 to $N/2$, and 64 to $N/2$), for the experimental group (top row) and the control group (bottom row), and over the four weeks. $q$ represents the set of orders over which the MF-DFA algorithm was applied.
Another interesting indication is provided by the cross-correlation values at lag -1 and lag 1, which appear positive and significantly different from zero in the experimental group. This shows that synchronization, while clearly dominated by a complexity matching effect, also involves cycle-to-cycle discrete correction processes: both partners tend to (moderately) correct their current step duration on the basis of the asynchrony they perceived at the preceding heel-strike (see Roume et al., 2018, for a deeper analysis of WDCC properties). One could note a dissymmetry in these correction processes, the lag 1 values being higher than the lag -1 value: According to our conventions, this indicates that participants corrected his/her step duration to a greater extent than the experimenter did. Additionally this dissymmetry, negligible during the first week, becomes more and more salient over weeks. In contrast, we found no trace of correction processes in the control condition.

4. Discussion

The three hypotheses that motivated this experimental work are validated:

1. When an older person is invited to walk in synchrony, arm-in-arm, with a healthy partner, synchronization is mainly achieved through complexity matching. This hypothesis was validated by the two analysis methods we applied to the collected series: The correlation functions between multi-fractal spectra remained significant, whatever the range of exponents considered, revealing a global, multi-scale synchronization between series, and the WDCC functions exhibited a typical positive peak at lag 0, suggesting an immediate synchronization between systems. WDCC results showed that synchronization was clearly dominated by a complexity matching effect, even if slight cycle-to-cycle correction processes were also present, especially for participants, which tended to correct their steps on the basis of previous asynchronies. The main important result, at this level, was to show that forced synchronization, between systems of different levels of complexity, is based on similar processes than forced synchronization between systems of similar complexities (Almurad et al., 2017). Interestingly, the complexity matching effect appeared immediately, from the very first duo sequences, from the moment that the instruction of close synchrony with the experimenter was provided and respected.
2. Considering two systems of different levels of complexity, complexity matching results in an attraction of the less complex system towards the more complex one. This result is one of the most interesting of this experiment, clearly in line with the complexity matching theory (Mahmoodi et al., 2017). One could also argue that a complex system being intrinsically more stable, this attraction results from the relative instability of the less complex one. However, the results in the control group (both systems being equally attracted by each other) seems contradicting this alternative explanation.

3. A prolonged experience of complexity matching, between two systems of different levels of complexity, allows enhancing the complexity of the less complex system, this effect being persistent over time. In the context of our experiment this result suggests a possible restoration of complexity in older people. Note that we tested the persistence of this restoration through a unique post-test, performed two weeks after the end of the training sessions. Further investigations are necessary for analyzing the persistence of this effect, its probable decay over time, and the effects of an additional training session when a significant decay is observed (one could hypothesize that restoration could occur more quickly during a second administration of the rehabilitation protocol).

As far as we know, this is the first evidence for a possible restoration of complexity in deficient systems. Recently Warlop et al. (2017) evoked the effects of Nordic Walking for restoring complexity in patients suffering from Parkinson’s disease, but their experiment focused essentially on the immediate effects of the adoption of a specific locomotion pattern, rather than on the long-term effects of a rehabilitation protocol.

In this experiment a statistical effect was obtained at the beginning of the fourth week. During a pre-testing period, we tried to pursue training up to the obtaining of an increase of long-range correlations. We systematically obtained this effect at the beginning of the fourth week, and decided to limit the protocol to four successive weeks. However, the analysis of individual results shows that this restoration could occur early, at the beginning of the third week. The most important observation is that complexity matching does not spontaneously induce a restoration of complexity in solo sequences, and a repeated and prolonged experience of complexity matching seems necessary. The results of the control group show that an intense training in walking is not sufficient. Walking in close synchrony with a healthy partner appears a key factor in the restoration process, and our analyses about the duo sequences suggest that complexity matching may be the essential ingredient.

Some limitations of the present study have to be pointed out. First, it should be noticed that we evidence in this experiment the possibility of a restoration of complexity, and we just suppose, on the basis on previous assumptions, that this should result in a more adaptable and stable locomotion, and a decrease of fall propensity. Longitudinal studies, using clinical tests and systematic follow-up survey, should be necessary for confirming this hypothesis. However, this was clearly beyond the scope of the present work.

Second, this experiment was extremely difficult to organize (due to the availability of the indoor track), and was very challenging for both the participants and the experimenter. It took up to 14 months for performing the whole protocol for the 24 participants. For practical
reasons, we decided to design the protocol with a single experimenter, who performed all sequences with all participants. This had the advantage of standardizing the experimental conditions, but introduced a possible bias, as our results could be related to some hidden and unsuspected qualities of this specific person. It seems obviously necessary to replicate these results with other accompanying persons. A second experiment is currently engaged in our laboratory for clarifying this point.

Finally, considering the intrinsic difficulty of the experimental protocol, we recruited participants that presented a normal, non-pathological aging, and consequently a rather moderate loss of complexity. The average DFA exponent characterizing the step duration series of our participants was of about 0.83, clearly higher than the mean value reported by Hausdorff et al. (1997) in their group of elderly participants (0.68). Further investigations are required for adapting and testing this kind of protocol with patients suffering of more pronounced locomotion diseases and greater losses of complexity.

In conclusion, this experiment should not be considered a clinical study, aiming at validating and promoting a rehabilitation strategy, but rather a fundamental work testing a theoretical hypothesis (the restoration of complexity in living organisms through complexity matching). We hope, obviously, that it could inspire clinicians for developing, validating and diffusing effective rehabilitation protocols. Currently most research in locomotion rehabilitation focuses on sophisticated devices, involving virtual reality, metronomic guidance, robotic assistance, etc. We are not sure, however, that genuine complexity matching could occur in the interaction with an artificial device (Delignières and Marmelat, 2014). Our experiment suggests that rehabilitation could be achieved with simpler, less expensive and also more humane means. We think especially to countries and situations where the access to sophisticated medical care remains difficult, and often unconceivable. We would be proud that our work can give scientific support to this simple prescription: “Take your eldest's arm and walk together”.

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**Conflict of Interest**

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

**Author Contributions**

ZA and DD contributed conception and design of the study; CR developed the measuring device; ZA conducted the experiment and performed the statistical analysis; DD and HB supervised the whole project; ZA and DD wrote the first draft of the manuscript; All authors contributed to manuscript revision, read and approved the submitted version.

**References**


Concluding remarks

The results of the present experiment were obviously expected. They show that a restoration of complexity in deficient systems could be conceivable. However, we evoke in the conclusion of the paper the limitations of our study. Our goal was not to propose and test an effective protocol of rehabilitation, but rather to test a more fundamental hypothesis, related to the effects of the prolonged experience of complexity matching. This experiment was more driven by theoretical issues than by clinical purposes.

The proposed protocol was very challenging, and as such we decided to recruit participants from a population presenting a “normal aging”, without any gait disorder. The expected complexity deficit was present, but rather moderate, and we cannot provide any information about the effect of this kind of protocol of actually frail patients.

A second problematic point is obviously the fact that this result was obtained using a unique “healthy companion”. The generalization of this result remains difficult, as it was obtained in very specific and personalized conditions. As mentioned in the paper, it could be related to some “hidden” qualities of the experimenter, at the relational or motivational levels. A second experiment is currently in progress, in order to replicate this results with others experimenters.

Despite these limitations, this result remains essential at a more theoretical level: the loss of complexity cannot be considered an ineluctable phenomenon, and a restoration of complexity could be conceivable. A more detailed analysis of the series produced by participants is necessary for a better understanding of this restoration process. Our results focus at the behavioral level, and we evidenced a significant increase of the complexity of stride interval series. The next step should be to infer the causes of this evolution, in terms of interactions within the system, degeneracy properties, etc.
General conclusion

This thesis combines theoretical, methodological, and experimental contributions. On the theoretical side, we tried to deepen the analysis of the complexity matching effect. We especially tried to establish the distinction between statistical matching and genuine complexity matching. For a long time the matching of scaling exponents was considered a satisfying signature for the presence of a complexity matching effect in synchronization. Delignières and Marmelat (2014), however, showed that the matching of mono-fractal exponents could just represent the consequence of local correction processes. Genuine complexity matching seems related to a more global, multi-scale synchronization, in which multi-fractal synchronization a likely to play a central role.

At the methodological level, we introduced two novel methods, the Multifractal Correlation function, and the Windowed Detrended Cross-correlation analysis. These two methods exploit the previous theoretical analysis, and allow disentangling the respective influences of short-term correction processes and genuine complexity matching. The first method is based on the theoretical link between multi-fractals and complexity matching. This method computes correlation functions between the multifractal spectra of the two synchronized series, considering different ranges of scales. We hypothesized that the cross-correlation function should be significant in all cases when only long-term scale were investigated. However, when shorter and shorter intervals were introduced in the analysis, we supposed that correlations should lose significance in the case of short-term corrective processes and conversely maintain a significance level in the complexity matching case. This method allows exploring the intimacy of synchronization, far beyond the global level of mono-fractal exponents. The Multifractal Correlation function, however, presents the disadvantage to be only applicable at the group level.

In contrast, the second method could be considered very simple, exploiting a simple cross-correlation function, between the series produced by the two synchronized systems. We enriched this approach by windowing and detrending procedures, in the aim to focus on local processes and to avoid biases related to non-stationarities in the series. The formal analysis we conducted on this method allowed to clearly establish the signatures that could be expected from local correction processes, on the one hand, and from complexity matching on the other. Note also that this method is applicable at the dyad level.

Quite surprisingly, these methods allowed to evidence that in most situations, synchronization was achieved through a mix of these processes. Even if it seems possible to determine a dominance of one process on the other, especially for asynchrony correction in synchronized tapping, and complexity matching in synchronized walking, the second process still appears at work, albeit to a lesser extend. Further research is necessary for determining the factors that explain the emergence of these processes, and their mutual balance.
At the experimental level, our most spectacular result is the restoration of complexity we observed in the last work. We are obviously aware of the clinical interest of this result, in terms of frailty and fall prevention, but we are also aware of the limits of this experiment, as expressed in the conclusion of the previous chapter. Our main goal was to test a hypothesis directly linked to the complexity matching framework, and to that end it was necessary (1) to check for the presence of a complexity matching effect in synchronized walking between young and healthy participants, and (2) to test the effects of the prolonged experience of complexity matching, between two systems characterized by contrasted levels of complexity. Currently, a number of papers investigate the effects of the entrainment of walking with metronomes, and especially with variable, fractal-like metronomes. The hypothesis that underlies these experiments is that such metronomes mimic the “natural” variability and should help to reinforce walking dynamics. None of those papers, however, evidenced a restoration of complexity, nor a retention effect as that observed in our experiment. As previously explained, we think that artificial devices, mimicking a “natural” variability, are unable to generate the complexity matching effect which seems necessary for restoring complexity in deficient systems.
References


**Substantial summary**

This thesis work focuses on interpersonal coordination. Several theoretical frameworks have attempted to account for the processes underlying inter-individual synchronization. Cognitivist theories, derived from works on the synchronization of discrete movements with a regular metronome (Repp, 2005) suggest that interpersonal synchronization is achieved through a discrete and mutual correction of asynchronies between the two partners (see for example Konvalinka et al., 2010). Dynamic theories, mainly supported by bimanual coordination studies (Haken et al., 1985), have been extended to interpersonal coordination and are based on the assumption of a continuous coupling between the two systems, designed as self-sustaining oscillators (Schmidt et al., 1990). Finally, the complexity matching model, is based on the assumption that the transfer of information is optimized when two interacting systems have similar complexities (West et al., 1999). In this context inter-system synchronization is supposed to emerge from a multi-scale coordination between the two interacting systems (Marmelat & Delignières, 2012).

These three theoretical frameworks have received convincing empirical validations, most often in very specific experimental tasks. Cognitive models of mutual correction of asynchrony thus seem to adequately account for synchronization in discrete tasks such as tapping, and the model of oscillators coupling in continuous tasks such as pendulum oscillations. The essential question that we asked ourselves was to determine whether these three theoretical frameworks represent alternative interpretative frameworks of a single reality, or if we could conceive a multiplicity of synchronization processes, depending in particular on the nature of the implemented tasks. The first objective of this thesis was to develop statistical tests to identify in the experimental data the typical signatures of these three modes of coordination.

We propose two procedures: the first one is based on the analysis of the correlations between the multifractal spectra characterizing the series produced by the two interacting systems, and the second on a windowed cross-correlation analysis, which makes it possible to reveal the local processes of synchronization.

The development of the first method was prepared by validating an improvement of Detrended Fluctuation Analysis (DFA). The DFA, which has been in operation for almost 30 years, calculates scaling invariants by estimating the regression slope of a bi-logarithmic graph plotting the interval lengths in the series on the abscissa and the mean standard deviations calculated for each interval length on the ordinate. The logarithmic distribution of the points logically induces a more massive contribution of the values recorded for the longest intervals. We propose to rectify this bias by calculating the regression on the basis of regularly spaced points on the logarithmic scale. This adjustment of the DFA was previously exploited by a number of authors, but its interest in the traditional method had never been evaluated. We therefore proposed a precise formalization of this method (known as evenly-spaced DFA), and we were able to show that it made it possible to reduce by 36% the variability of the estimates, compared to the original method. We also show that the variability of estimates produced by the evenly-spaced DFA with 256-point series is equivalent to that produced by the DFA with 1024-point series [Almurad, Z.M.H. & Delignières, D. (2016). Evenly spacing in Detrended Fluctuation Analysis. Physica A, 451, 63-69.].

The first interpersonal coordination analysis method we have developed exploits Multifractal Detected Fluctuation Analysis, a variant of the DFA, to which we have obviously added the previously validated evenly spacing. In the last ten years or so, complexity matching was mainly characterized by the presence of a strong correlation between the characteristic mono-fractal
exponents of the series produced by the two interacting systems (Stephen & Dixon, 2008). It has however been shown that this signature was insufficient, being in fact observed in all cases, from the moment when two systems entered in synchronization (Delignières & Marmelat, 2014). The method we propose aims at calculating the correlation functions between the multifractal spectra produced by the two interacting systems. These correlation functions are calculated first of all over the available intervals, then progressively over smaller ranges, focusing on the longest intervals. We assume that in all cases the correlation function should be significant when only long intervals are taken into consideration. If coordination is based on mutual correction of asynchronies, correlations should decrease as shorter intervals are considered. On the other hand, in the case of a continuous coupling or of complexity matching, these correlations should remain significant. The results confirm these hypotheses, and show in particular that the synchronization of walking with a fractal metronome is performed on the basis of a correction of asynchrony [Delignières, D., Almurad, ZMH, Roume, C. & Marmelat, V. (2016). Multifractal signatures of complexity matching. Experimental Brain Research, 243 (10), 2773-2785].

The second method consists in calculating the local cross-correlation function between the two series produced by the interacting systems. This correlation function is calculated over intervals of 15 points, on lags of -10/+10. The intervals considered are systematically detrended before calculating the cross-correlation coefficients. The calculation is carried out in a sliding manner over all the series, and an average function is then calculated. This method, called Windowed Detrended Cross-Correlation Analysis (WDCC), was introduced in a first article [Almurad, Z.M.H., Roume, C. & Delignières, D. (2017). Complexity matching in side-by-side walking. Human Movement Science, 54, 125-136.], and then formally analyzed in a second [Roume, C., Almurad, Z.M.H., Scotti, M., Ezzina, S., Blain, H. and Delignieres, D. (2018). Windowed detrended cross-correlation analysis of synchronization processes. Physica A, 503, 1131-1150.]. We show that the three processes assumed by the three theoretical frameworks previously exposed are revealed by separate signatures. The mutual correction of asynchronies is revealed by the presence of positive cross-correlations at lags -1 and 1. This signature is notably observed in an interpersonal tapping situation. Continuous coupling must produce a positive and significant lag 0 cross-correlation (around 0.5). This signature is obtained on bimanual coordination data, but never on interpersonal coordination data.

Finally, the complexity matching is supposed to produce a positive cross-correlation at lag 0, but more modest than in the case of a continuous coupling. This result is highlighted for synchronized walking. Finally, our analyses show that the different processes are not exclusive, but can be exploited simultaneously. Thus in an interpersonal tapping task the synchronization is dominated by the correction of the asynchronies, but a cross-correlation of the internal clocks also seems present, supposing a certain degree of complexity matching. On the other hand, in synchronized walking, if the coordination is dominated by the complexity matching, asynchronous correction processes also seem to be present. Synchronization can thus reveal hybrid mechanisms mixing, in particular, correction of asynchronies and complexity matching.

These studies have also allowed us to revisit a number of previous works. We show that if the synchronization of discrete tasks such as tapping relies on discrete correction processes of asynchronies, the synchronization of continuous tasks such as pendulum oscillations is essentially based on the same principles of discrete correction, and not on a continuous coupling of the effectors. An important result of these studies is the demonstration that synchronized walking implements a dominant effect of complexity matching, all the more significant as the two partners are closely linked. We show that complexity matching is more intense in arm-to-arm-down than in walking side-by-side.
The previous results allowed us to initiate a last experiment, intended to test the hypothesis of a possible restoration of the complexity of locomotion in the elderly [Almurad, ZMH, Roume, C., Blain, H. and Delignières, D. (2018). Complexity matching: Restoring the complexity of locomotion in older people through arm-in-arm walking. Frontiers in Physiology - Fractal Physiology, 9, 1766]. If in young subjects the analysis of the series of step duration during walking reveals $1/f$ fluctuations, suggesting the optimal complexity of the locomotor system, aging is characterized by a loss of complexity of the series of steps, and this loss of complexity correlated with falling propensity (Hausdorff et al., 1997). The more general hypothesis of a loss of complexity related to age and illness is a fruitful line of research.

As mentioned earlier, the theory of complexity matching assumes that two interacting systems tend to align their complexity levels. It also assumes that when two systems of different levels of complexity interact, the more complex system tends to attract the least complex, causing an increase in complexity in the second (Mahmoodi et al., 2918). We proposed a protocol in which older people were invited to walk arm-in-arm with a young companion. Participants were exposed to extended training for 4 weeks, with 3 sessions per week, each session consisting of 3 or 4 16-minute sessions. A control group performed the same training, but without physical contact with the accompanying person (walking side by side) and without explicit synchronization instructions.

The results show that synchronization between the two partners is achieved through a Complexity matching effect. This effect is more intense in the experimental group than in the control group. During synchronized walking sequences, there is a clear attraction of the complexity of the elderly participants to that of their companion. This effect does not appear in the control group. Lastly, prolonged training in a synchronized way makes it possible to restore the complexity of locomotion in the elderly. This effect appears only in the experimental group, and persists during a post-test performed two weeks after the end of training. This result, in addition to reinforcing one of the essential aspects of the theory of complexity matching, opens new avenues of research for the design of rehabilitation and fall prevention strategies.
Résumé : Plusieurs cadres théoriques rendent compte des processus de synchronisation interpersonnelle. Les théories cognitivistes suggèrent que la synchronisation est réalisée par le biais d’une correction des asynchronies. Les théories dynamiques supposent un couplage continu des deux systèmes, conçus comme oscillateurs auto-entretenus. Enfin le complexity matching repose sur l’hypothèse d’une coordination multi-échelle entre les deux systèmes en interaction. Dans un premier temps, nous développons des tests statistiques permettant de repérer les signatures typiques de ces trois modes de coordination. Nous proposons notamment une signature multifractale, basée sur l’analyse des corrélations entre les spectres multifractals caractérisant les séries produites par les systèmes en interaction. Nous développons également une analyse de cross-corrélation fenêtrée, qui permet de dévoiler les processus locaux de synchronisation mis en œuvre. Nous montrons que si la synchronisation de tâches discrètes telles que le tapping repose en effet sur des processus de correction discrète des asynchronies, la marche synchronisée met en œuvre un effet dominant de complexity matching. Nous proposons dans un second temps d’exploiter ce résultat pour tester la possibilité d’une restauration de la complexité chez les personnes âgées. Le vieillissement a été caractérisé comme un processus de perte graduelle de complexité, et dans le domaine de la marche cette perte de complexité corrèle avec la propension à la chute. La théorie du complexity matching suppose que deux systèmes en interaction tendent à aligner leurs niveaux de complexité, et que lorsque deux systèmes de niveaux différents de complexité interagissent, le système le plus complexe tend à attirer le moins complexe, engendrant un accroissement de la complexité chez le second. Nous montrons, dans un protocole au cours duquel des personnes âgées sont invitées à marcher bras-dessus-bras-dessous avec un accompagnant jeune, que la synchronisation entre les deux partenaires est réalisée au travers d’un effet d’appariement des complexités, et que l’entraînement prolongé en marche synchronisée permet une restauration de la complexité de la locomotion, qui perdure lors d’un post-test réalisé après deux semaines. Ce résultat ouvre de nouvelles voies pour la conception de stratégies de réhabilitation.

Mots-clés : Coordination interpersonnelles, appariement des complexités, vieillissement, restauration de la complexité

Abstract: Several theoretical frameworks could account for interpersonal synchronization. Cognitive theories suggest that synchronization is achieved through discrete corrections of asynchronies. The dynamic theories suppose a continuous coupling between the two systems, conceived as self-sustained oscillators. Finally, complexity matching is based on the assumption of a multi-scale coordination between the two interacting systems. As a first step, we develop statistical tests in order to identify the typical signatures of these three modes of coordination. In particular, we propose a multifractal signature, based on the analysis of correlations between the multifractal spectra characterizing the series produced by the interacting systems. We also develop a windowed cross-correlation analysis, which aims at revealing the nature of the local synchronization processes. We show that if the synchronization of discrete tasks such as tapping relies on discrete correction processes of asynchronies, synchronized walking is based on a dominant effect of complexity matching. We propose in a second step to exploit this result to test the possibility of a restoration of complexity in the elderly. Aging has been characterized as a process of gradual loss of complexity, and considering walking this loss of complexity correlates in older people with the propensity to fall. Complex matching theory assumes that two interacting systems tend to align their complexity levels, and that when two systems of different levels of complexity interact, the more complex system tends to attract the less complex, causing an increase in complexity in the second. We show, in a protocol in which older people are invited to walk arm-in-arm with a younger companion, that synchronization between the two partners is achieved through a complexity matching effect, and that prolonged training in such synchronized walking allows a restoration of the complexity of locomotion, which persists during a post-test conducted after two weeks. This result opens new avenues of research for the design of rehabilitation strategies.

Key-words: Interpersonal coordination, complexity matching, aging, complexity restoration
Appendix: 1

Mobility Lab used for the first experience (chapter3)

**Movement Monitors:** Movement monitors combine a number of sensors within a single package, including a 3 axis accelerometer, a 3 axis gyro, a 3 axis magnetometer, and a temperature sensor. The accelerometers can be configured in a high 6G mode, or a low 2G mode depending on the testing requirements. There are a number of options for securing the monitors on subjects using a selection of straps.

**Docking Station:** The docking station is used to charge and configure the movement monitors. Depending on your configuration, you may have up to 6 docking stations chained together into a single unit.

**Access Point:** The wireless access control point (access point for short) allows for wireless communication between the host computer and Opal movement monitors. A single access point can support up to 6 Opals.

**Instrumenting the Subject:** Testing requires 6 different sensors to be placed on the subject:
- Left and right wrists (monitors with velcro strap attachments)
• Left and right ankles (monitors with velcro strap attachments)

![Velcro strap](image1.jpg)

• Trunk (monitor with the chest holster)

![Trunk monitor](image2.jpg)

• Lumbar (monitor with the elastic belt attachment)

![Lumbar monitor](image3.jpg)

Use the following visual guide and written tips for instrumenting the subject with the monitors:

**Gait**: Gyroscopes attached on the shanks are used to detect the basic gait events, i.e. time feet hit the ground and leave the ground (initial and terminal contacts). Temporal gait measures are then calculated based on the time of gait events. In the next step, ranges of motions (RoM) of the shank segments are estimated by integrating the gyroscopes signals. Finally, spatial gait measures are estimated using a biomechanical model [6].

Gait measures are reported for individual gait cycles in the CSV export. A gait cycle is defined as the period between two consecutive initial contacts (heel-strike) of the right foot.

**Spatial gait measures**:  

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### Measures

<table>
<thead>
<tr>
<th>Measures</th>
<th>Unit</th>
<th>Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stride Length</strong></td>
<td>% of Subjects body height*</td>
<td>Shanks</td>
<td>Distance between two consecutive foot falls at the moments of initial contacts. Averaged for left and right side. The value is normalized for height.</td>
</tr>
<tr>
<td><strong>Stride Length R</strong></td>
<td>% of Subjects height*</td>
<td>Shanks</td>
<td>Distance between two consecutive right foot falls at the moments of initial contacts</td>
</tr>
<tr>
<td><strong>Stride Length L</strong></td>
<td>% of Subjects height*</td>
<td>Shanks</td>
<td>Distance between two consecutive left foot falls at the moments of initial contacts</td>
</tr>
<tr>
<td><strong>Stride Velocity</strong></td>
<td>% of Subjects height / second*</td>
<td>Shanks</td>
<td>Walking speed. Average of the right and left sides.</td>
</tr>
<tr>
<td><strong>Stride Velocity R</strong></td>
<td>% of Subjects height / second*</td>
<td>Shanks</td>
<td>Walking speed of right leg</td>
</tr>
<tr>
<td><strong>Stride Velocity L</strong></td>
<td>% of Subjects height / second*</td>
<td>Shanks</td>
<td>Walking speed of left leg</td>
</tr>
</tbody>
</table>

*Values are given relative to the subject’s body height to enable comparison to normative data. If the subject’s height is entered through Mobility Lab, then values will additionally be provided in units of meters.*

### Temporal gait measures

<table>
<thead>
<tr>
<th>Measures</th>
<th>Unit</th>
<th>Sensors</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cadence</td>
<td>Steps / minute</td>
<td>Shanks</td>
<td>Stepping rate</td>
</tr>
<tr>
<td><strong>Gait Cycle Time</strong></td>
<td>Time (seconds)</td>
<td>Shanks</td>
<td>Duration of a complete gait cycle</td>
</tr>
<tr>
<td><strong>Double Support</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Percentage of a gait cycle that both feet are on the ground</td>
</tr>
<tr>
<td><strong>Swing</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Average percentage of a gait cycle that either foot is off the ground</td>
</tr>
<tr>
<td><strong>Swing R</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Percentage of a gait cycle that right foot is off the ground</td>
</tr>
<tr>
<td><strong>Swing L</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Percentage of a gait cycle that left foot is off the ground</td>
</tr>
<tr>
<td><strong>Stance</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Average percentage of a gait cycle that either foot is on the ground</td>
</tr>
<tr>
<td><strong>Stance R</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Percentage of a gait cycle that right foot is on the ground</td>
</tr>
<tr>
<td><strong>Stance L</strong></td>
<td>% of GCT</td>
<td>Shanks</td>
<td>Percentage of a gait cycle that left foot is on the ground</td>
</tr>
<tr>
<td><strong>Initial Contact R</strong></td>
<td>Time (seconds)</td>
<td>Shanks</td>
<td>The time (relative to the beginning of the trial) that right foot hits the ground. Sometimes referred to as heel-strike.</td>
</tr>
<tr>
<td><strong>Terminal Contact L</strong></td>
<td>Time (seconds)</td>
<td>Shanks</td>
<td>The time (relative to the beginning of the trial) that left foot leaves the ground. Sometimes referred to as toe-off.</td>
</tr>
<tr>
<td><strong>Initial Contact L</strong></td>
<td>Time (seconds)</td>
<td>Shanks</td>
<td>The time (relative to the beginning of the trial) that left foot hits the ground.</td>
</tr>
<tr>
<td><strong>Terminal Contact L</strong></td>
<td>Time (seconds)</td>
<td>Shanks</td>
<td>The time (relative to the beginning of the trial) that left foot leaves the ground.</td>
</tr>
</tbody>
</table>

### References


### Appendix: 2

Raspberry Pi and force sensitive resistors (FSR) for Second experience:
Walk

Apparatus:

Footsteps connected to a Raspberry Pi

Method:
Measure: inter-step interval

References:
Hausdorff et al. (1997):
• Comparison of the complexity of the intervals between the stages of walking between 10 healthy elderly people (age > 70 years) and 22 young adults (age = 24.6 ± 1.9)
• Walking at a preferential rate for 15 minutes on a circular passage of 200 m

<table>
<thead>
<tr>
<th></th>
<th>old</th>
<th>youth</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.68 ± 0.14</td>
<td>0.87 ± 0.15</td>
</tr>
<tr>
<td>IIP (s)</td>
<td>1.05 ± 0.10</td>
<td>1.05 ± 0.07</td>
</tr>
<tr>
<td>IIP CV (%)</td>
<td>2.0 ± 0.7</td>
<td>1.9 ± 0.4</td>
</tr>
<tr>
<td>Speed (m/s)</td>
<td>1.24 ± 0.18</td>
<td>1.42 ± 0.21</td>
</tr>
<tr>
<td>up and go test (s)</td>
<td>7.6 ± 1.0</td>
<td>7.4 ± 1.3</td>
</tr>
</tbody>
</table>