Modeling and hedging strategies for agricultural commodities

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Modeling and Hedging Strategies for Agricultural Commodities

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"Nous ne pouvons réduire le monde à des choses simples si le monde lui-même est compliqué". Michel DESHONS

To dad and Deshons

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Abstract: In agricultural markets, producers incur price and production risks as well as other risks related to production contingencies. These risks impact the producer activity and could decrease his income. The globalization of markets, particularly those of agricultural commodities, provides hedging instruments including futures contracts which will serve to develop a hedging strategy. However, the situation whereby single futures contract-based positions could offset many risks leads to incomplete market. Especially, an producer looking for better hedging strategy could also include insurance, option contract or mutual funds to further guarantee his income, especially when crop yields are lower than expected.

We investigate the hedging strategies in static framework as well as in continuous time framework. Prior, we analyze the behavior of agricultural prices using various statistical approaches and suggest appropriate price modeling for data at hands. The static hedging strategy also accounts for rollover process which gives raise to additional risks due to spread between new futures and nearby futures and inter-crop hedging. We particularly address hedging strategy that combines futures and insurance contracts. Since decisions making in static framework does not include price changes along the hedging horizon, optimal hedging strategy in continuous time framework will take into account jumps and seasonality by combining futures and option contracts.

Résumé : Sur les marchés agricoles, les producteurs encourent les risques de prix et de production ainsi que d'autres types de risques liés aux aléas de production. Ces risques impactent l'activité du producteur et pourraient diminuer ses revenus. La mondialisation des marchés, en particulier ceux des matières premières agricoles, permet de développer une stratégie de couverture en utilisant des instruments comme les contrats à terme. Cependant, la situation selon laquelle une position basée seulement sur un contrat futures devrait couvrir tous les risques entraîne un marché incomplet. Le producteur en recherche de meilleure stratégie de couverture pour ajouter un contrat d'assurance ou d'option pour garantir davantage ses revenus, surtout lorsque les rendements des cultures prévus diminuent.

Nous étudions, ici les stratégies de couverture dans le cadre statique, ainsi que dans le cadre de temps continu. Avant, nous analysons le comportement des prix des matières premières agricoles en utilisant diverses approches statistiques afin de suggérer la modélisation des prix adéquate aux données. La stratégie de couverture statique comprend également le processus de retournement de positions qui pourrait entraîner d'autres risques supplémentaires en raison de l'écart entre les nouveaux contrats à terme et des contrats à terme à proximité ainsi que la couverture interculture. Nous proposons une stratégie de couverture qui combine des contrats futures et d'assurance. Comme la prise de décisions dans le cadre statique ne tient compte des mouvements quotidiens de prix le long de l'horizon de couverture, la stratégie de couverture optimale en temps continu combine des positions en contrat à terme et options tout en prenant en compte les sauts et la saisonnalité dans la dynamique des prix.

Contents

List of Tables

List of Figures

Chapter 1

General Introduction

Commodity prices are subjected to variations of production levels of their underlying assets, as well as to factors related to their economic rationale such as calendar seasons or crop years, consumption and policies, supply and demand balance, inventories ... Commodities trading incurs various risks in both market and in production process. However, the major part of producers' revenue is composed of their crops and any adverse price move will affect their activities. On one hand, the globalization of commodity markets provides financial derivatives like futures, forwards or options to hedge against these risks by shifting them to investors that are looking for speculative opportunities. On the other hand, commodities can also be stored in order to avoid disruptions due to shortage that generates cost of carry stemming from quality deterioration along with storage period. In agricultural markets, a way to avoid these carry costs is to enter the derivatives markets where prices are agreed at inception for a future date, in some way, as contingencies of physical goods. Derivatives instruments postpone the delivery at future date while avoiding real risks. Thus, the holding of commodities in inventories in order to face up to eventual scarcity episodes in the future plays a key role in the relationship between spot price and the futures price.

Arguably, the impact of price variability on the real economy is of greatest in the commodity markets. Price variations in these markets relate to every economic entity; from individuals, to organizations and economies. Therefore, the risk management in the commodity industry is of great importance. Individuals need to manage these risks in order to preserve their revenue, firms to protect their bottom lines and competitiveness, and the countries to protect their macroeconomic stability. Specifically, agricultural commodities are of concern since they are natural resources that are consumed as basic necessities for human diet. They are also used in a number of applications as well. For instance, corn is used in everything from artificial sweeteners to fuel sources and also papers and containers.

Futures markets are the institutions of both risk management and price discovery.[1] In futures markets, the competing expectations of market participants interact to form the *"price discovery mechanism"* that will reflect a broad range of information about upcoming market conditions. Specifically, futures are mainly used as hedging instruments against the exposition in cash positions. But, as they do not equate to direct exposure of actual commodity prices; they will be bets on the expected future spot prices. For instance, a wheat producer who plants a crop is betting that the price of wheat will not drop so low that he would have been better off not to have planted the crop at all. This bet is inherent to the farming business, but the producer may prefer not to make it. Hence, he can hedge this bet by selling a wheat futures contract.

Aside the price risk management, there are a lot of positive externalities associated with hedging. Recall that futures contracts in commodity trading takes place with standardization in sizes as well as in qualities in order to improve efficiency for their extractions, distributions and consumption processes. Hence, the hedging and price discovery functions of futures mar-

[1]Blau [Blau 1944] (p. 1) had stated that *"commodity futures exchanges are market organizations specially developed for facilitating the shifting of risks due to unknown future changes in commodity prices; i.e., risks which are of such a nature that they cannot be covered by means of ordinary insurance."* However, insurance contracts have been improved to become complements of futures markets.

kets will enhance the efficiency of production, storage and marketing operations. Hedging also ensures continuity of cash flows in that it will protect the producer from volatile price movements, and will thereby guarantee uninterrupted and stable revenue streams by bringing some certainty in the production process. That boils down to certainty in production planning at a guaranteed minimum price by using commodity futures to hedge.

Furthermore, agricultural futures markets also serve as an effective hedge against inflation, since prices of foods are often among the first to rise when prices begin to climb. Gorton and Rouwenhorst [Gorton 2006] found that commodity futures returns and inflation are positively correlated to larger-scale at longer horizons. Indeed, commodity futures returns are volatile relatively to inflation, their longer-term correlations better capture the inflation properties of a commodity investment. Agricultural futures can also perform well when global population is growing, or when a growing middle class leads to increased demand. Besides, agricultural futures sometimes function as a hedge against volatility in equity markets when geopolitical tensions in emerging and frontier markets will arise as a result of food shortages.

In agricultural markets, futures contracts allow to hedge against both market and production risks and their price discovery function is embedded in the price change processes and production contingencies. Production risk leads to additional uncertainties that can further lower crop yields and will affect expected prices. Indeed, futures contracts are settled daily and like any other assets, commodity prices can suddenly change with substantial variations; generally, due to news announcements, reforms, political unrest or weather vagaries. A striking example is the political unrest in Ivory Coast, where cocoa prices had peaked in 2002 and 2003. Another illustration of such price moves comes from weather vagaries (coffee production during the Brazilian frost) or prompt extra export demand that (in case of China) can impact final production. Therefore, using futures contracts against all these risks together may not be as effective, if not worse.

The sudden and significant price variation within a very short period is referred as to price jump. Hence, the situation whereby a unique position in a futures contract is taken to hedge multiple sources of risk altogether leads to market incompleteness. Even if there may be an optimal hedge strategy for all the risks, this will be difficult to achieve. The financial literature refers such situation to a non-unique martingale measure. Indeed, all risk processes should be martingales under an equivalent measure. In a such strategy, the risk measure is not unique since there is more than one risk source to hedge with only one state variable.

The issue of hedging such goods against adverse market moves is prominent with the main advantage for the hedge being to significantly lower risk in a portfolio context. The most traditional theory justifying the merit of hedging is Price Risk Insurance Theory in which hedging provides insurance against risks arising from price fluctuations. Keynes [Keynes 1930], Hicks [Hicks 1939] and Kaldor [Kaldor 1940] had supported hedging as a risk mitigation tool. Then, portfolio theory, initiated by Markowitz, had stated hedging as insurance tool against risks by reducing them in tandem with expectation maximization. This theoretical framework has been applied by Johnson [Johnson 1960] and Stein [Stein 1961] to explain hedging as a tool to mitigate risk and earn returns.

There is no effective hedging strategy that would completely eliminate all risks. Rather, it attempts to transform unacceptable risks into an acceptable form; like shifting commodity price risk from hedger to speculators. However, when a hedge becomes ineffective, it bring about losses that may result in bankruptcy. An example was the Metallgesellschaft AG case; the German largest conglomerate that went nearly bankrupt after suffering US$1.5 billion losses from its energy derivative trading activities in December 1993. At the time, the price of oil had dropped while the futures market turned from backwardation to contango and the combination of these market moves led to serious losses on futures position for Metallgesellschaft AG.

Furthermore, a hedge strongly relies on the situation it is applied to, as well as the costs associated to the strategies being implemented. In futures markets, the mismatch between the position in the underlying asset and the futures contract makes the hedging strategy less effective and the risks will not be sufficiently offset. This difference of positions between the underlying and hedging instrument is the basis risk generated by the hedged position. Particularly in commodity markets, when futures contracts mature, they do not exist anymore. An agricultural producer with commitment over a multiple crop years need long maturity hedge instrument. To keep long term hedge with agricultural futures, one has to initiate a rollover strategy process. This consists in rolling over from a position in nearby futures contract to a longer term futures contract. The switch of positions of the two futures contracts incurs additional basis risk due to adverse price spread. This is known as rollover risk and will come from market situation and production contingencies. The production risk will also matter if output is lower than expected. For instance, in rolling short futures positions, the most ideal situation would be a contango market with a decreasing price environment since it offers two opportunities to generate gains. A decreasing price that will obviously generate gains for the short futures position and a contango market that allows the hedger to sell futures at a discount to the spot price, thus, allowing more gains when the spot price decreases. The inverse situation will generate basis risk.

Basis risk and rollover risk are really worrisome as outcomes of inefficient hedging strategy. Paroush and Wolf [Paroush 1989, Paroush 1992] had introduced basis risk in futures market literature by showing how it influences the optimal output and hedge together with other parameters. But earlier, Holthausen [Holthausen 1979] had dealt with hedging and production models in the absence of basis risk to show under which circumstances an agricultural firm, either over-hedges or under-hedges or even full-hedges his output and the effect of increasing risk aversion on the hedge. More generally, the issue of hedging with futures contracts has been investigated using various approaches which deeply rely on the optimization technique. Particularly in static framework, many papers in the literature on hedging issues with futures contract have focused on the modeling aspect and this has gone in tandem with effectiveness measures. However, the existing measures of hedging effectiveness are not consistent because none of the optimization technique to derive hedge ratios turns out to be superior based on them.

In static framework, optimization techniques include minimum-variance, mean-variance, semi-variance, mean-Gini and generalized semi-variance. The minimum-variance is the simplest approach but it does not include portfolio aspects like expected returns or more generally the risk psychology that relates the aversion, prudence and temperance. The other methods come in order to enhance it on taking into account such relevant features. The mean-variance incorporates expected return and risk aversion. For example, Rolfo [Rolfo 1980] had applied mean-variance technique to derive optimal hedging strategy under price risk and output risk for countries that export agricultural products. The semi-variance concept captures only the downside risk, instead of both profits and losses the considered by variance for hedge strategy to reduce only average losses. Plus, the generalized semi-variance, also referred to as lower partial moments (LPM) approach, has been applied in futures hedging literature (Chen et al. [Chen 2001]; for instance in Lien and Tse [Lien 2000]) to include risk psychology with stochastic dominance approach. The mean-Gini approach is also consistent with stochastic dominance as well as with expected utility maximization; especially this could be applied when the mean-variance is bound to fail because of non-normality of returns or biased estimators of ordinary least square; see Shalit and Yitzhaki [Shalit 1984]. Other contributions rely on the unconditional aspect of volatility as risk measure and will be based on conditional distribution approaches like Autoregressive Conditional Heteroskedasticity (ARCH), Generalized ARCH (Baillie and Myers [Baillie 1991]) or multi-period model (Cecchetti et al. [Cecchetti 1988], Chen et al. [Chen 2013] and Lien and Luo [Lien 1993]). Furthermore, Fernandez [Fernandez 2008] and Conlon and Cotter [Conlon 2012] have applied a wavelet[2] decomposition technique to show time horizon effects on the hedge ratio.

In continuous framework, Ho [Ho 1984] and Adler Detemple [Adler 1988a] are pioneers to investigate hedging with futures contract in commodity market. These papers have derived optimal hedging strategies via dynamic programming method as applied in Merton [Merton 1971] in portfolio context. Specifically, the hedging portfolio mainly includes the cash position as non-traded and trading assets consisting in futures contract and other assets like option, stocks or bonds. The authors had considered Brownian motion to represent the risk sources in their approaches and have still stressed about the ineffective of optimal hedge strategy.

The purpose of this thesis is to develop hedging strategies in a portfolio context, for storable commodities. Particularly, the issue of hedging with financial market is considered for agricultural commodities. Hedging strategies are to reduce, as much as possible, losses due to prices fluctuations, and production decisions. Hence, the basic task of hedging instruments is to provide a counter position that will provide a guarantee against losses in part or in full, depending on the nature of the hedge. Before planting, a producer shall decide how he is guaranteeing his income at harvesting. The hedging strategy is analyzed in in portfolio in both static and dynamic frameworks. In the static framework, the hedging problem is stated and some approaches as well as their empirical applications are presented. Mainly, we derive optimal strategies that combines futures and insurance contracts for market risk as well as production risk in rollover process. Since the decisions making in a static framework do not account for feedback patterns along the

[2]The wavelet method is a refinement of Fourier analysis that decomposes time series into its high- and low-frequency components which is short- and long-term variation, respectively.

hedging period, the same issue is investigated in dynamic framework. Indeed, price dynamics substantially influence portfolio strategies along the hedging period.

In continuous time, we analyze hedging shortcomings that may come from either the modeling aspect (risk representation through only Brownian motion) or hedging instruments. On one hand, dynamic hedging strategies include market moves and particularly those relating to price jumps. This provides sharper analysis of commodity futures price behavior as highlighted in Chapter 2. The jump detection tests Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006], Aït-Sahalia and Jacod [Aït-Sahalia 2009] are applied to real data and jump component is significant to be included in price dynamics of commodity futures at hands. However, hedging with only futures contract has to be improved. On another hand, an alternative hedging strategy is to include an option written on non-traded asset. The hedge portfolio with option enhance the hedging strategy by further reducing uncertainties.

The thesis is organized in four main chapters. The first chapter applies recent statistical tests on agricultural futures prices to highlight stylized facts. The empirical study suggests futures prices to follow "mean-reverting jump-diffusion" with seasonal long-run and volatility. Then, the second chapter addresses the model estimation in a two-stage procedure. First, the estimation of mean-reversion speed and periodic long-run parameter is handled using least square. Second, the residuals of the first step allow to estimate the remaining parameters with particle Monte Carlo Markov Chains method. The third and fourth chapters investigate the hedging strategies, respectively, in static and dynamic frameworks. Mainly, static hedging includes market and production risks with applications, while the dynamic hedging strategy will add to the futures contract, option written on the non-traded.

Chapter 2

Agricultural Futures: Economics and Stylized Facts

Abstract: Fundamentals of agricultural commodities are driven by production, inventory and spot price which, all together, contribute to futures price behavior. Following related literature, we describe their futures prices behavior relating to their economic rationales with emphasis on the stylized facts. Particularly, mean-reversion, seasonality and jumps are observed on the futures prices of grains and soft commodities. We highlight these stylized facts by conducting various econometric tests on futures prices in such a way that the features can be taken into account all together. Furthermore, we show that, in the market, the term structure of the selected commodities tends to be in contango.

Contents

2.1 Introduction

The history of agricultural markets has tremendous experience regarding its stylized facts due to various market regulations and their production fashions. The investigation of stylized facts is an usual way to address prior analysis of financial assets for any portfolio strategy that includes. Hence, appropriate empirical analysis helps to highlight the features that would be taken into account for the behavior of asset prices. A massive literature has dealt with stylized facts of asset prices. For instance, Cont [Cont 2001] has compiled various statistical methods to conduct empirical analysis of asset price data; Carr et Al. [Carr 2002] has analyzed the fine structure of asset returns, and had show the departure from normal distributed returns due to non-zero skewness and high kurtosis, what characterizes the density function like a spike. Although agricultural commodities prices are of specific in that they on production of physical goods, they will still remain compatibles with such investigation. Indeed, Mandelbrot [Mandelbrot 1963] had applied similar analysis long ago to highlight stylized facts of cotton prices.

Specifically, the literature on the economic rationale for commodity price behavior is based on two mainstreams as distinguished by Fama and French [Fama 1987]: theory of normal backwardation and theory of storage. The theory of normal backwardation (Keynes [Keynes 1930]) emphasizes the risk reallocation role of futures markets in term of risk premium paid by hedgers to speculators while the theory of storage (Kaldor [Kaldor 1939]) deals with the importance of the convenience yield in storage decisions. This literature also includes empirical analysis to evidence such theories of prices behavior. For instance, Houthakker [Houthakker 1957] and Cootner [Cootner 1960] had showed the existence of risk premia to support the theory of backwardation while Dusak [Dusak 1973] and Carter, Rauser and Schmitz [Carter 1983] examined the risk premium within the context of Capital Asset Price Model, and had come to opposite conclusions. Brennan [Brennan 1958] had found no significant evidence of risk premia and had further formalized the theory of supply of storage. Fama and French [Fama 1987], Bessembinder [Bessembinder 1992] had come to mixed evidence of a risk premium. Roon, Nijman, and Veld [De Roon 2000] revealed strong evidence for "cross-hedging" pressure and risk premia. A recent approach by Gorton et al. [Gorton 2013] tries to combine the two theories to endogenously derive basis and the risk premium.

Apart from the stylized facts known for classical financial assets, commodity prices depict mean-reversion and seasonal pattern (Bessembinder et al. [Bessembinder 1996], Andersson [Andersson 2007], Geman and Nguyen [Geman 2005], Sørensen [Sørensen 2002]). In commodity markets, mean-reversion behavior is often viewed as the long term equilibrium; seasonality simply relates to seasonal fashion of production in tandem with supply and demand and either stochastic volatility or jumps may characterizes the non normality of prices returns. Mandelbrot [Mandelbrot 1963] in stressing the spiky shape of returns density function in agricultural markets, he had suggested a modeling approach based on the class of Paretian distributions. Recent literature in quantitative finance represents asset prices with Lévy processes with jump components. Such analyzes on agricultural prices include Hilliard and Reis [Hilliard 1999] and recently Schmitz et al. [Schmitz 2014].

However, the existing literature on empirical studies for commodity markets mainly deals with non normality, mean-reversion and seasonality. To our knowledge, empirical analysis for jumps evidence in commodity prices is lacking. In this chapter, we apply recent studies of Aït-Sahalia and Jacod [Aït-Sahalia 2009, Aït-Sahalia 2011] on jump and their activities using daily futures prices to evidence such consideration in agricultural markets. In practice, using observed futures prices, we test for inter-temporal relationship, mean-reversion, seasonality of both long run average return and monthly volatility.

The chapter is organized as follows. The first section deals with preliminaries of commodity markets with focus on real market variables as well as efficiency of commodity futures markets. The next section describes the stylized facts and the economic rationale. The third section investigates empirical analysis of selected agricultural futures and the last section concludes.

2.2 Fundamentals of commodity futures prices

This section presents how determinants of real market evolve and how they relate to their futures[1] prices behavior. We base on the two traditional theoretical approaches (theory of normal backwardation and theory of storage) to hedging, speculation, and commodity inventories to derive the futures price behavior. The theories are complementary in their explanation of commodity price formation and are foundations of modern commodity pricing theories as formulated by Brennan [Brennan 1991], Schwartz [Schwartz 1997], and Routledge et al. [Routledge 2000] for example.

2.2.1 Market determinants: production, inventory and price

Prices, rates of production and inventory levels are interrelated in commodity markets. These real market variables are determined at equilibrium in two interconnected markets: a cash market for immediate purchases and sales of the produced good, and a market for inventories held by both producers and consumers of the commodity. In these markets, equilibrium affects and will be affected by changes in price volatility. Specially, for storable commodities, like agricultural goods, inventories play a key role in price formation process. In a competitive markets, the stochastic fluctuations in production and/or consumption for such commodities are absorbed by inventories and to reduce costs[2]. Inventories are also used to reduce the fluctuations in demand and to avoid stock-outs. Then, production is deter-

[1]Futures and forward contracts are considered as same if interest rate is deterministic.

[2]Inventories serve to reduce costs of adjusting production to avoid stock-outs for both producers and consumers (when commodity is used as production input), Pindyck [Pindyck 2001].

mined in light of spot price and price for storage.

In such markets, spot prices do not come from the intersection of supply and demand directly. Instead, the difference between production and consumption characterizes cash market which is function of spot price, S_t, and other demand shifting variables, $Z_{1,t}$, (aggregate income, population growth...) and random shocks in market. Similarly, supply is function of spot price, also supply shifting variables $Z_{2,t}$ (salary, capital stocks,etc...) and random shocks from drought, frost, or thunderstorms, political risks, changes in taste and consumption patterns etc. Let denote by u_t all random shocks in cash market. The changes in inventories, ΔI_t, correspond to the net demand as the difference between supply and demand. Then the spot price is the inverse of net demand function of inventory variations, demand and supply shifting variables and shocks in the markets,

$$S_t = f(\Delta I_t; Z_{1,t}, Z_{2,t}, u_t). \tag{2.1}$$

Therefore, market-clearing in the cash market implies a relationship between the spot price and the changes in inventories. For instance, a higher price is associated to a more supply and less demand, and thus to a important net demand. So, the variations of inventory levels will affect the price at which the market clears making commodity market behavior to depend on equilibrium in both cash market and storage market.

To see this, consider the market for storage. The supply of storage is the total quantity held in inventories I_t. In equilibrium, this quantity must equal the demanded quantity which is a function of the price. The price of storage is the "payment" for the privilege of holding a unit of inventory. For agricultural goods, it is equal to the marginal value corresponding to the value of the flow of services accruing from holding the marginal unit in inventory: the convenience yield, CY_t. Convenience yield is small when the total stock of inventories is large (because one more unit of inventory will be of little extra benefit), but it can rise sharply when the stock becomes very small.

Subsequently, the demand for storage function is downward sloping and convex in convenience yield and this is same for its inverse function.

Then, demand for storage is function of convenience yield and other variables $Z_{3,t}$ such as consumption (or production), volatility of price σ_t, spot price and any other variables that affect demand. Thus market-clearing in the storage market implies a relationship between marginal convenience yield (the price of storage) and the demand for storage. Finally, equilibrium in storage market is described using inverse function of this demand.

$$CY_t = g(I_t; Z_{3,t}, \sigma_t, u'_t) \tag{2.2}$$

Market-clearing in the storage market implies a relationship between convenience yield (the price it takes to store) and the demand for storage. Equations (2.1) and (2.2) describe their dynamic equilibrium in both the cash and storage markets. Note that if there are no exogenous shocks ($Z_{1,t}, Z_{2,t}, Z_{3,t}$ and σ_t), the system will reach a steady-state equilibrium at $\Delta I_t = 0$ (see Pindyck [Pindyck 2001] for how changes in exogenous intervene in equilibrium.).

On the other side, commodity prices are risky and consumers and producers often seek ways of hedging and trading risk. For this, futures and forward contracts, options, swaps, and other derivatives are traded as financial instruments to reduce these risks. Price volatility[3] drives the demand for hedging, whether it is done via financial derivatives (futures contracts or options) or via physical instruments (inventories).

Price volatility is one of main causes of fluctuations in the net demand function, which, in turn, results from fluctuations in consumption, demand and/or production and vice-versa (speculative buying and selling). An in-

[3]Volatility is a key determinant of the values of commodity-based contingent claims, such as futures contracts, options on futures, and commodity production facilities. Indeed, such production facilities can usefully be viewed as call options on the commodity itself, Pindyck [Pindyck 2001].

crease in price volatility is followed by more variations of production and consumption which implies an increase in the demand for storage in order to buffer these fluctuations in production and consumption. But this will also result in an upward shift in the net demand curve. Thus, inventories have effect of reducing the magnitude of short-run market price fluctuations in that they are low when volatility is high. Equilibrium in the two markets affects and will be affected by changes in the level of price volatility which is relevant in driving short-run commodity cash and storage market behaviors.

These changes in the spot price, inventory levels, and convenience yield will be accompanied by changes in futures prices and thus in the futures-spot spread that is referred to as basis. Basis is also a function of marginal value of storage for a commodity inducing a fundamental relationship between spot price, futures price, and inventory behavior that is captured by futures markets. Futures contracts provide relevant information about the two markets because they are jointly used as vehicle for hedging risk.

2.2.2 Futures price behavior

Futures prices relate to spot prices as well as their term structure behavior that evolves with changes in strategies, particularly those including storage of commodity. A major determinant for this relation is convenience yield which induces relevant features for hedging and speculation based on physical markets.

Recall that a futures prices represents the expected spot price under the risk neutral probability measure. A position in a futures contract then relies on anticipations of the corresponding future spot price along with inventory decisions, storage costs, and interest rate. This leads a futures prices to be the discounted expected of its future spot price under no-arbitrage and complete market conditions.[4] If $F_{t,T}$ is the price at t of the futures contract that will mature at time $T > t$, the discount factor is the evolution over time of continuously compounded risk-less interest rate, $r_{t,T}$, plus the cash value of storage costs, $w_{t,T}$, adjusted by convenience yield, $CY_{t,T}$.

$$F_{t,T} = S_t \exp\left\{\left[r_{t,T} + w_{t,T} - CY_{t,T}\right](T-t)\right\}. \tag{2.3}$$

The interest rate, $r_{t,T}$, is the cost of financing the purchase of the physical commodity while, $w_{t,T}$, is the cost that is associated with its ownership. The convenience yield, $CY_{t,T}$, is the utility of holding the physical commodity, in contrast to a pure contractual agreement for the delivery of specific commodity to meet scarcity in market. Convenience yield is subtracted from risk-free rate and storage cost because it reduces the cost of ownership of the asset.

What matters the most, when trading in commodity futures markets is risk premium[5] as sustainable return (Gorton et al. [Gorton 2013]). Averse producers are willing shift future spot prices risk on planned output to speculators by selling futures and speculators will receive risk premia in exchange for long-term financial exposure to future spot prices risk. Then, the risk premium is a function of term structure pattern (downward and upward) and thus will constitute an incentive and a reward to undertaking risk above the risk-free rate (but not for ability to predict market moves).

The two theories describe how inventory level of a commodity is fundamental determinant for risk premium and basis; each focusing on different aspects of futures markets. The theory of normal backwardation, formalized mainly by Keynes [Keynes 1930] and Hicks [Hicks 1939], explains

[4] Lautier [Lautier 2009] points out some sources of imperfections that influence the no-arbitrage conditions.

[5] The risk premium is the expected excess return and the excess return being the difference between the current futures price and the spot price at expiry of futures contract. The difference between a contemporaneous futures price and a current spot price is the basis, Gorton et al. [Gorton 2013].

futures price formation from the net short hedging pressure while the theory of storage of Kaldor [Kaldor 1939], Brennan [Brennan 1958] and Telser [Telser 1958] focuses on spot price formation in relation to futures based on cost-of-carry and the convenience yield, that gives rise to a premium on commodity inventories in periods of scarcity.

Recalling that basis is the return coming from purchasing the commodity at t and selling it at t for delivery at T,

$$S_t - F_{t,T}, \quad \text{or} \quad \big(S_t - F_{t,T}\big)/S_t. \tag{2.4}$$

The theory of normal backwardation focuses the expected variation of the difference between spot price at current time and at futures contract maturity T, and the expected risk premium, $pr_{t,T}$, to explain the basis.

$$S_t - F_{t,T} = \mathbb{E}_t[S_t - S_T] + \mathbb{E}_t[pr_{t,T}]. \tag{2.5}$$

Hence, futures prices is a downward biased estimator of future spot prices. The hedging imbalance caused by producers and the uncertainty in price will jointly determine the size of risk premium, $pr_{t,T}$. Then, the forecast bias of spot price for maturity T is the expected premium $pr_{t,T}$ that naturally induces the backwardation,

$$\mathbb{E}_t[pr_{t,T}] = \mathbb{E}_t[S_T] - F_{t,T}. \tag{2.6}$$

Note that when surplus stocks exist, futures price can vary above the current spot price. This situation, known as contango, is adjusted to backwardation situation by at least the amount due to cost-of-carry.[6] However, this also leads to another risk premium on excess stocks, $S_t\, r_{t,T}$.

The theory of backwardation as natural downward slope of term structure

[6]Keynes [Keynes 1930] defined cost-of-carry as an allowance for deterioration of quality, warehouse and insurance charges, interest charges, and a "remuneration against the changes in the money-value of the commodity during which it has to be carried".

is controversial, as the role of inventories in futures price formation is not sufficiently explained, Telser [Telser 1958]. Cootner [Cootner 1960] generalized the idea in the sense that futures prices may carry either a positive or a negative risk premium. An alternative explanation of futures backwardation comes from theory of storage.

According to the theory of storage, the inventory behavior over time horizon relates to the intertemporal price differences between futures and spot prices. That is the negative of basis as the interest forgone, $S_t \, r_{t,T}$, plus the marginal storage cost, $w_{t,T}$, minus the marginal convenience yield from an additional unit of inventory, $CY_{t,T}$:

$$F_{t,T} - S_t = S_t \, r_{t,T} + w_{t,T} - CY_{t,T}. \tag{2.7}$$

The role of futures markets in storage decisions comes from arbitrage that ensures that the amount of contango in the futures price curve will be limited by the marginal cost of storing one additional unit of the commodity. If inventories are large, hedging the commodity on the futures market ensures a return to storage that covers storage costs and storage will be encouraged to support what would otherwise be a very low price. However, in case of shortage, the convenience yield, $CY_{t,T}$, increases, the marginal return to storage becomes negative, and backwardation arises. Indeed, more stock will be carried if the price is expected to rise and vice versa the risk premium, $S_t \, r_{t,T}$, increases with the amount of stocks held. Stocks will not be held a normal threshold and this risk on spot market for storable commodities can partly or entirely be transferred to speculators on the futures market in short position.

The theory of normal backwardation and the theory of storage to explain futures basis are interrelated through the phenomenon of carrying charge or arbitrage hedging. Seasonal changes in inventories as well as changes in supply and demand will be the determinants for the appropriate approach. Indeed, high inventories and low spot price will lead to futures prices that match cost-of-carry based calculations including a risk premium for the commodity stocks. However, the risk premium on stocks will be close to zero for high spot price and quasi-null inventories. In this case, futures price will come from market expectations including a risk premium depending on market imperfections as e.g. short-selling constraints.

In nutshell, the theory of storage assumes stable supply and demand conditions though the absence of price expectations while the theory of normal backwardation incorporates investor anticipation of the changes in market conditions but excludes the embedded timing option in the spot commodity's price. In all cases, price formation in futures markets results from both current available information and expectations on supply and demand. Then what matters the most is how prices reflect supply and demand and facilitates the carrying of inventories through hedging to make the market viable. This issue relates to market efficiency and is addressed in the following.

2.2.3 Efficiency of agricultural futures markets

In financial markets, price changes[7] come from new information arrival (Karpoff [Karpoff 1987]) and will provide clues to infer about any agents' attitudes. Specifically, in agriculture dominated economies, prices are the major concern of the producers, investors, traders and policy makers. Hence, understanding the underlying core behavior of commodity futures is vital for better decision making. The issue relates to the commodity markets efficiency.

A market is efficient when prices always fully reflect available information (Fama [Fama 1970]). This means that no profit can be made with monopolistically controlled information. Fama [Fama 1970] states three forms of

[7] A measure of price changes is volatility but it is equivalent to squared price variations. Clark [Clark 1973] used squared price changes of futures prices of cotton to represent new information arrival.

efficiency test: weak, semi-strong and strong. The weak form of efficiency examines whether current prices fully reflect the information contained in historical prices. The concept of unbiasedness is a more restrictive version of this form. The semi-strong form test examines how quickly prices reflect the announcement of public information while the strong form investigates whether investors have private information that is not fully reflected in the market prices.

The usefulness of futures markets lies in their ability to forecast spot prices at a specified future date. Thus, they provide agents a way of managing the risks associated with trading in a given commodity, Kellard et al. [Kellard 1999]. That is an efficient futures market provides a mechanism for managing the risk associated with the uncertainty of future events in cash market. In an efficient commodity market, the futures price will be the optimal forecast, in term of information, of future spot price at futures maturity and any random unpredictable error will have zero-mean. The efficient market hypothesis, according the no-arbitrage condition, can then be reduced to the joint hypothesis that agents are, in an aggregate sense, endowed with rational expectations and are risk neutral such that the futures price is an unbiased estimator of the future spot price (Taylor [Taylor 1995]). Hence, the hypothesis that futures price is unbiased forecast of spot price boils down a joint hypothesis of market efficiency and risk neutrality.

Efficient market implies that the futures price, $F_{t,T}$, for a contract expiring at time T, is the unbiased predictor of the future spot price,

$$F_{t,T} = \mathbb{E}\left[S_T|\mathcal{F}_t\right] \quad \text{with} \quad t \in [0, T], \tag{2.8}$$

where the $\mathbb{E}\left[\cdot|\mathcal{F}_t\right]$ is the expectation formed at time $t \in [0, T]$ on information set available, $\mathcal{F}_t$, at that instant. To test for efficiency in futures market, a natural model specification is to assume rational expectations, that is

$$S_T = \mathbb{E}\left[S_T|\mathcal{F}_t\right] + err_T \tag{2.9}$$

where err_t is rational expectation error term orthogonal to all sets in $\mathcal{F}_t$ including the lagged forecasts errors. Then, testing for both unbiasedness and market efficiency together is carried out by specifying the linear regression model

$$S_T = \bar{b}_0 + \bar{b}_1 F_{t,T} + err_T \tag{2.10}$$

with the null hypothesis is $\bar{b}_0 = 0$ and $\bar{b}_1 = 1$. As result, the hypothesis that a futures price is an unbiased estimator of spot price is joint hypothesis that markets are efficient and that there is no risk premium. The test for weak form of efficient market hypothesis is carried out through the unpredictability of current returns from past returns. When returns come to be non-stationary, serial autocorrelation test that includes error correction models are performed. However, this test[8] only focus on linear correlations of price changes in short horizon. Indeed, returns series can be linearly uncorrelated and but non-linearly dependent; Granger [Granger 2001]. An alternative approach is to apply test techniques that take into account the nonlinear serial dependency structures.

The tests for efficiency market hypothesis constitute a useful first step in the evaluation of the social utility of futures markets (Kellard et al. [Kellard 1999]). However, despite a large body of research on efficiency, there is no consensus on whether markets are efficient or not; see Ramírez et al. [Ramírez 2015]. The main reason comes from the fact that markets switch between efficiency and inefficiency at different periods and measuring the degree of efficiency will give more insight than testing it. Besides, such tests do not provide information about the degree of efficiency in any specific market to allow for a quantitative comparison of the functioning of different futures markets. Adaptive market hypothesis theory have been proposed in which the degree of efficiency is based on factors like number of competitors, the magnitude of profit opportunities available and the

[8]Conventional tests for efficient market hypothesis include serial autocorrelation test using the Ljung-Box portmanteau Q-statistic, the runs test of Shiller and Perron, and the variance ratio test amongst others.

adaptability of market participants. Plus, it allows to detect non-linearity in certain period of time.

To analyze the efficiency of futures markets for each commodity selected, we first perform two unit root tests that check for the stationarity of returns series: the traditional augmented Dickey Fuller test with the assumption of Gaussian errors and the residual augmented least squares (RALS) technique (Im et al. [Im 2014]), which does not require knowledge of a specific density function or functional form. Besides processing the efficiency test in (2.10) on returns series, we use Hinich portmanteau bicorrelation test (in Appendix A.3), which is a third order extension of the standard correlation tests for white noise and detects nonlinear serial dependence in non-overlapped time windows.

All along this section, we have pointed out the key state variables as determinant factors of commodity markets from related references and have also addressed a way to rely on these factors with relative market efficiency. The following section is devoted specific factors that will describe the behavior of futures prices and are known as stylized facts.

2.3 Stylized facts of agricultural commodities

This section defines the usual stylized facts of commodity prices in the literature with focus on their economic rationales. Mean-reversion, seasonality and either jumps or stochastic volatility are referred to as the main features observed in commodity markets, across a wide range of instruments, in time periods.

2.3.1 Mean-reversion

The mean-reverting behavior is often used for the modeling of the dynamic of many commodity prices as result of their economic rationales and empirical evidences. Andersson [Andersson 2007], Bessembinder et al.

[Bessembinder 1995], Geman [Geman 2009] use mean-reversion to model commodity futures prices. The concept of mean-reversion comes from prices or returns that alternate temporary at high and low levels to converge towards an equilibrium value over time. It is opposed to permanent shock effect of random walk, Andersson [Andersson 2007]. For storable commodities, mean-reversion is mainly induced by supply and demand imbalance or by the no-arbitrage relation between spot and futures prices. As illustrated by Back and Prokopczuk [Back 2013a], on one hand, prices go up when shortages occur, leading to higher investments in production facilities or more producers entering the market, which leads to higher supply; albeit with a certain lag in time. The higher supply will then push prices down again and vice versa. On the other hand, following the arguments of the theory of storage, inventory withdrawals will also lead to higher spot prices while the futures price does not change as much since the resulting higher convenience yield has an offsetting effect. Hence, convenience yield, in reflecting market's expectations of future availability of the commodity, induces mean reverting feature to positive correlation with spot price or the risk premium impact on prices.

Furthermore, a negative relationship between interest rates and prices will also induce mean-reversion phenomenon. For instance, Bessembinder et al. [Bessembinder 1995] tested investors expectation for spot price regarding mean-reversion under the risk neutral measure using term structure of futures prices. Indeed, term structure of futures prices describes several behavioral points that the investors expect spot price to exhibit it. Hence, finding an inverse relation between price levels and the term slope will then imply a low rate of appreciation of expected inter-temporal price when prices rise, and vice versa.

The mean-reversion feature is related to the notion of stationary processes that intend to capture price effects due to temporary periods of excess supply and demand. It is represented by stochastic Ornstein-Uhlenbeck (OU) process. A general Ornstein-Uhlenbeck process is the solution of the following stochastic differential equation, Barnsdorff-Nielsen and Shephard [Barndorff-Nielsen 2001]

$$\mathrm{d}X_t = \kappa(\bar{x} - X_t)\mathrm{d}t + \mathrm{d}L_t, \tag{2.11}$$

where $\kappa > 0$ is the rate of reversion to the long-run average $\bar{x}$ and $(L_t)_{t\geq 0}$ is a Lévy process with $L_0 = 0$ a.s. that is characterized by its triplet (m_t, σ_t^2, ν) on probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The parameters m_t and σ_t are respectively the drift term and diffusion coefficient of the Lévy process. The Lévy measure, ν, characterizes the number of jumps per time unit and is also referred to as intensity or activity of the process. The measure ν satisfies the conditions $\nu(\{0\}) = 0$ and $\int_{\mathbb{R}}(1 \wedge x^2)\nu(\mathrm{d}x) < \infty$.

Mean-reversion as stated in Equation (2.11) expresses the fact that changes in returns are proportional to deviation from average return with Lévy error term. Geman [Geman 2007] gives a general definition of mean-reversion of a Markov process as equivalent to a process having a finite invariant measure which characterizes the stationarity of the process also viewed as its equilibrium state. Note that if $(L_t)_{t\geq 0}$ is a Brownian motion, the process $(X_t)_{t\geq 0}$ will be the classical Ornstein-Uhlenbeck process.

To recognize the behavior of mean-reversion on data belongs to econometric issue via unit root test. Common unit root tests[9] rely on the assumption of Gaussian noise regarding the asymptotic distribution of the test.[10] That is, if Lévy noise in equation (2.11) has finite variance, both Gauss-Markov and CLT will hold. But this requires Lévy measure to be finite, $\int_{\mathbb{R}} \nu(\mathrm{d}x) < \infty$. a.s. When the variance is not finite, unit root test can still be perform based

[9] Augmented Dickey-Fuller (ADF), Kwiatkowski-Phillips-Schmidt-Shin (KPSS), Phillips-Perron, and GLS-detrended ADF test proposed by Elliot, Rothenberg, Stock (ERS) are known as unit root tests.

[10] Gaussian noise with finite variance leads to minimum variance estimator for parameters via Gauss-Markov theorem. Finite variance is required for the Central Limit Theorem (CLT) to hold in hypothesis testing to guarantee asymptotic normally distribution.

on residual augmented least squares (RALS) methodology.

Mean-reversion behavior has been discussed in futures markets literature. Andersson [Andersson 2007] has suggested a test that uses hedging errors rather than unit root test (Barkoulas et al. [Barkoulas 1997]) arguing that unit root test for mean-reverting has only very low power for application on commodity prices. Brooks and Prokopczuk [Brooks 2013] stressed difficulties in estimating several stochastic volatility models with jumps in mean-reversion framework for commodity prices. Other investigations do not reject the mean-reverting behavior for commodity prices, but they have pointed out the time varying or the stochastic equilibrium level, (Tang [Tang 2012] and Schwartz and Smith [Schwartz 2000] respectively).

2.3.2 Jump or stochastic volatility

In short intervals of time, financial markets seem to experience drastic price swings than expected. Such swings may due to either jumps or stochastic volatility. Both ump and stochastic volatility relate to extreme events and will help to represent sources of risk for risk management, portfolio allocation and derivatives pricing. Using them in the market model makes it incomplete.

The idea of stochastic volatility has emerged with shortcoming of constant volatility in the Black-Scholes model option pricing to represent persistence in the price process (Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2001]). Stochastic volatility is associated with diffusion component, thus predictable[11], while jumps correspond to sudden and unpredictable changes in prices. That is to say, a jump will occur once instantly

[11] For example, futures prices near maturity are more sensitive to new information, hence more volatile. The theory of storage predicts that shifts in supply and demand can induce stochastic convenience yield which will imply stochastic volatility because on their inverse relation. Nielsen and Schwartz [Nielsen 2004] has used this idea to model the spot volatility as a function of convenience yield level in order to avoid using information on inventory levels that may be unavailable. Geman and Nguyen [Geman 2005] derive scarcity model with inverse volatility.

and stochastic volatility characterizes the integration of process such as a piling up effects. However, jumps correspond to impact of the new information arrivals (general announcements - scheduled earnings or unscheduled news - speculative bubbles, sudden and rare events) on prices. In agricultural markets, jumps may also come from any event that will suddenly shift supply and demand[12] such as weather vagaries, decline in food production growth, excessive speculations of institutional investors etc. Mandelbrot [Mandelbrot 1963] had firstly stressed the discontinuities in commodity markets due to heavy-tailed distribution of the cotton price returns. He had found out that cotton daily prices vary more frequently than suggested by Brownian motion process. Later, Hilliard and Reis [Hilliard 1999] showed the improvement of jump-diffusion models over diffusion model in commodity markets.

The basic problem to represent extreme event in a model with of stochastic volatility or jump starts with non zero skewness and high kurtosis. Jump or stochastic volatility component can be specified in commodity price model for a (see for example Schmitz et al [Schmitz 2014]). But, as the analysis here deals with monthly volatility, we only will focus on jump feature and let the volatility to be deterministic. There are several ways to test for jumps in financial time series (Dumitru and Urga [Dumitru 2012]). Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006], Aït-Sahalia and Jacod [Aït-Sahalia 2009] and Podolskij and Ziggel [Podolskij 2010] have designed several hypothesis testing methods for this purpose among others. The Barndorff-Nielsen and Shephard (BNS) test is based on consistency of integrated volatility estimators while the Aït-Sahalia and Jacod (ASJ) test uses the power variation (that is generalized in Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006]) of processes at different time scales. The advantage of ASJ test procedure over the BNS one is that it can be also applied even if jumps have infinite activity because the estimator of realized multi-

[12] For example, Deaton and Laroque [Deaton 1992] characterized jumps as the occasional sharp price spikes due to scarcity.

power variation is robust to any jump activity. We apply, on agricultural futures returns, the jump detection tests of Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006] and Aït-Sahalia and Jacod [Aït-Sahalia 2009] that are summarized in Appendix A.5.1.

Jumps are characterized by their size and occurrence (either finite or infinite) in a time interval. The size refers to its variation and the occurrence to its activity. Jumps with finite activity are always of finite variation, while infinite activity jump processes may have either finite or infinite variation. One may have better idea on the kind of jump to specify in price model using the other test of Aït-Sahalia and Jacod [Aït-Sahalia 2011] where the test statistic is similar to the one of jump detection tests with additional parameter for arbitrary cutoff level. For, finite activity, compound Poisson process charcterizes the jump component since it is the only one process with a finite number of jumps in a finite time interval among Lévy pure jump processes. All other pure jump processes exhibit an infinite number of small jumps in any finite time interval.

In addition, we also test a la Aït-Sahalia and Jacod [Aït-Sahalia 2010] for necessity to include Brownian motion component in presence of jumps. In the absent of jumps, Brownian motion is usually included even it may be useless. The test statistic is simply the inverse of jump activity test statistic in Aït-Sahalia and Jacod [Aït-Sahalia 2011] (Le Courtois and Walter [Le Courtois 2014]).

2.3.3 Seasonality features of commodity prices

There seems not to exist a commonly accepted definition of seasonality. However, Back et al. [Back 2013b] quoted from Svend Hylleberg [13] that, seasonality is

> *"... the systematic, although not necessarily regular, intra-year move-*

[13] Svend Hylleberg, *Modelling Seasonality*, Oxford University Press, 1992.

ment caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy."

The supply process of agricultural commodities depends on the growing and harvesting periods along with the weather changes, which will drive their seasonal pattern of commodity prices. For example, the markets are inclined to push prices lower during harvest time because supply is abundant and the risk of the unknown weather interference in the yield has dissipated. Conversely, grain prices are often pressed higher during the planting and growing seasons as speculators price in weather risk premium. These events are consistent with theory of storage, Deaton and Laroque [Deaton 1992]. Meanwhile, even if commodity prices display seasonality, the cash-and-carry arbitrage relationship still holds, albeit this is not the case in financial markets (investors will easily anticipate that and take advantage).

Furthermore, the seasonality in commodity markets does not make the prices predictable. Varying factors intervene in supply and demand even if the outcome looks the same on prices. Every year, some supply and demand factors can be counted on to occur, some do not. Plus, seasonality is not guaranteed to repeat itself on average in the future, their pattern may shift[14] in time or there are underlying factors influencing prices along with the obvious fundamentals.

In the literature, Fama and French [Fama 1987] found strong seasonal variations in convenience yields for agricultural products and Sørensen [Sørensen 2002] had documented seasonal patterns in futures prices over the 1972-1997 period for soybeans, corn, and wheat of the Chicago Board of

[14]For example, sometimes the so-called corn-harvest lows occurs in mid-October, in other years it occurs in September or does not occur until December.

Trade. Karali and Thurman [Karali 2010] also found statistically significant seasonal patterns in the volatility for soybeans, corn, and wheat markets and Back et al. [Back 2013b] study how volatility seasonality affects option prices in commodity markets.

2.4 Empirical properties of agricultural futures

Commodities as primary needs contribute to the increasing interest of investors in their futures markets as save haven. Commodity instruments are strikingly different from stocks and bonds, thus on empirical properties. This motivates the demand for derivative instruments based on operational contingencies embedded in delivery of the commodity goods. Then, commodity futures prices are analyzed so as to set up appropriate portfolio strategies.

Using empirical observations guides in setting up models of prices behaviors that are required in portfolio decisions consistent with expectations. However, notice that stylized facts are so binding in that ad hoc stochastic processes with the same set of properties will not be easy to replicate. One has to rely on futures prices data to hope capturing these with a model.

Using datasets of selected commodities that we describe the construction, we investigate their empirical properties by testing the stylized facts mentioned in Section 2.3.

2.4.1 Data

Daily futures prices of selected commodities are collected from Bloomberg. These commodities are traded on American exchanges, Chicago Mercantile Exchange (CME) for grains and on Intercontinental Exchange (ICE) for soft commodities. Their inventory data come from United States Department of Agriculture (USDA) and ICE websites respectively for grains and softs.

2.4.1.1 Futures price data

We extract the settled price data from January 1990 up to August 2015 for eight agricultural futures, namely grains and softs[15] reported in Table 2.1. The table contains, for each commodity, exchange place, delivery months and tick value.

Table 2.1: Selected commodities

	Commodity	Exchance	Delivery months
Grains	Corn	CME	3, 5, 7, 9, 12
	Oat	CME	3, 5, 7, 9, 12
	Rough Rice	CME	1, 3, 5, 7, 9, 11
	Soybeans	CME	1, 3, 5, 7, 8, 9, 11
	Wheat	CME	3, 5, 7, 9, 12
Softs	Cocoa	ICE	3, 5, 7, 9, 12
	Coffee	ICE	3, 5, 7, 9, 12
	Cotton	ICE	3, 5, 7, 10, 12

The delivery month is the cash-settled month of futures contract where buyer and seller exchange cash. To save space, figures are the calendar months.

Futures contracts traded in commodity markets are of short maturity and will be active for few months only. For grain and soft commodities, futures contracts mature in calendar months with time series of their price available for shorter periods. The analysis of futures price behavior gains more insight with long historical time series. There are several methods to build up long time series with futures prices depending on the objective.

Herein, two kind of futures data will be used : expiry-month price and nearby contracts. Nearby futures prices have been used by in Bessembinder et al. [Bessembinder 1995] or as closest in Sørensen [Sørensen 2002]. The two kinds of data are build up using end-to-end concatenation that rolls from the expiring contract to next nearest contract without adjusting. The nearby futures are available in Bloomberg and we only construct the expiry-months data. The drawback of this end-to-end concatenation is that de-

[15] We use this classification following Gorton et al. [Gorton 2013].

pending on whether the market is in backwardation or in contango, the continuous price will exhibit upward or downward jumps at splice points.

The expiry-month price is a transformation of futures prices into a time series of the same delivery month over successive years to display a long period historical data. We concatenate futures contract of a specific delivery month of a year to the same futures contract of the same delivery month of the following year. For example, the corn expiry-month price, it is to concatenating the March 1990 to March 1991 and so up to March 2015. The expiry-month will serve to highlight how average price level behaves from delivery month to another (Sørensen [Sørensen 2002]).

The second category of data has the advantage to represent actual values in current market. Unlike the expiry-month time series, the distortion due to rolling over to the next nearest contract is not pronounced for nearby contract data. Particularly, as spot is elusive to define because of different locational settled prices, these datasets will provide the front nearby price as proxy for spot price like in Fama and French [Fama 1987]. However, the proportion of missing values in price time series increases with long maturities. We exclude nearby contracts with missing values over 1% proportion. Under this proportion, missing values are filled by simple linear interpolation. The fourth nearby contract of both Oat and Rough Rice encompasses more than 10% and the proportion of missing values will increase for their long maturity contracts (n° 4, 5 and 6). Hence, only the first three contracts are kept for this two commodities.

Prices of nearby contracts are represented in Appendix A.2.1 for the selected commodities over the 1990-2015 period. Simple inspection suggests two distinct periods: from 1990 to late 2006 that depicts a relatively flat trend, and from early 2007 up to 2015 with upward trend. In the early period, futures prices seems to exhibit stylized facts such as seasonality or mean-reversion as well as price spikes at some points in time. But, seasonality and mean-reversion are difficult to identify. In the second period, prices look more volatile with spikes specific to each commodity. Subsection A.5.4 below deals with statistical analysis of futures returns with aim to point out relevant prices behaviors.

A more changing price behavior occurred for the selected commodities under study with different intensities. This observation is associated with factors such as imbalance in supply and demand, political stability and speculation. Over the 1990-2006 period, most of futures prices seem to experience a unique price spike. This largely had come from a surge in price due to low acreage planted with low yield in United States during 1995-1996. In that period unfavorable factors including bad weather for corn and soybeans, low usage of stocks for wheat and oat, Brazilian frost and drought for coffee and strong export demand from China in the case of cotton. The surge in prices had resulted in low supply[16] which had implied productions of more crop the following year in order to induce price falls, Dunsby et al. [Dunsby 2008]. In 2002 and 2003, the price of soybeans and cocoa spiked due to low production of soybeans in United States and Brazil (two of the largest producers) and political unrest in Côte d'Ivoire. While in the same period wheat price decreased as a consequence of high stocks.

The 2007-2015 period encompasses the global food crisis of 2007-2008 driving by economic factors such as low global stocks/use ratios combined with uncertainty about size of crop in 2007-2008, growing food demand in developing countries (China, India, ...), lower production growth rate, biofuel production[17] based on agricultural commodities, speculation bubble, weak United States dollar, panic buying and export bans, Gutierrez [Gutierrez 2013].

[16]Production variation relies on acreage harvested and will yield per acreage via the weather, while consumption increases with large demand because of additional use in industry. Besides, the extra production of certain commodity leads to low production for other commodities in term of acreage and yields to low supply insufficient for usual demand and prices increase.

[17]Biofuel programs in the United States and European Union leads to a greater use of corn and vegetable oil which results in price increase for these commodities

2.4.1.2 Inventory Data

In general, there is, likely, no common source for inventory data, because of physical delivery at different locations. A common nomenclature for them is still at a conceptual stage. Meanwhile, using the same data source of Gorton et al. [Gorton 2013], we collect inventory levels time series on ICE website (for soft commodities) and on United States Department of Agriculture (USDA) website (for grains). The inventory data of grains are of different frequencies.[18] All data span period from March 1990 to December 2015 and observation frequencies are reported in table 2.2. Each observation period is the total of in-farm and off-farm inventories.

Table 2.2: Inventory data characteristics

	Commodity	Frequency	Period
Grains	Corn	3, 6, 9, 12	1990 - 2015
	Oat	3, 6, 9, 12	1990 - 2015
	Rough Rice	3, 8, 10, 12	1990 - 2015
	Soybeans	3, 6, 9, 12	1990 - 2015
	Wheat	3, 6, 9, 12	1990 - 2015
Softs	Cocoa*	Monthly	2002 - 2015
	Coffee	Monthly	1996 - 2015
	Cotton*	Weekly	1990 - 2002

* Data are not available over the whole the 1990 - 2015 period.

Figure 2.1 displays the inventory levels of the selected commodities. For soft commodities, the inventory levels are seasonal with slight upward trend. Meanwhile, there is a downward trend for oat inventory levels because of decrease in production as other cash crops such as corn, soybeans, and wheat have become more profitable and due to decrease in its use as a livestock feed ingredient and as a rotation crop (USDA). Also, over the same period, production activity of corn, soybeans, and wheat is more volatile with steady trend.

[18] Notice that, the links pointed in Appendix B1 of Gorton et al. [Gorton 2013] for monthly or weekly inventory data are no more available on The United States Department of Agriculture website, they have been updated to harvest seasons framework.

Most striking is world wide rice production that has experienced a new high record in 2010-2011 period due to large demand in emerging marking (China, India, Indonesia, Bangladesh, etc...) as well as in United States. Rice price had dropped down as exhibited in Figure A.3 as response to increase in supply.

2.4.1.3 Summary statistics

We provide descriptive statistics on returns of nearby futures prices in the aim to infer the behavior of agricultural prices at hands. These statistics are displayed for the whole period as well as the two distinguished periods, from 1990 to 2006 and from 2007 to 2015.

For the majority of commodities, the average returns are positive and will vary irrespective of the commodity and the nearby contract over the whole period. The same reasoning holds over the pre 2006 period, albeit the average returns of coffee and cotton are negative. After 2006, the average returns are pronounced from a short term contract to longer term what indicates large variations of futures prices in that period. Accordingly, volatility in the same period is higher than the previous period for most of commodities. Furthermore, volatility decays with maturity regardless of the commodity, what also evidences the Samuelson effect: the shorter the time-to-maturity the more volatile futures prices. However, rough rice and cocoa exhibit the converse situations. Rough rice prices have varied very little from late 2011 up to end august 2015. According to USDA report of that period, low prices of rice came from announcement of Department of Agriculture's Commodity Credit Corporation on the prevailing world market rice prices and loan deficiency payment rates applicable to 2009 crop. The cocoa case also due to little change and erosion effect of volatility.

Back to statistics, another well known property of asset returns is autocorrelated volatility as shown by the Ljung-Box test statistics of squared returns in column $Q(10)$. It expresses the fact that relatively small returns values

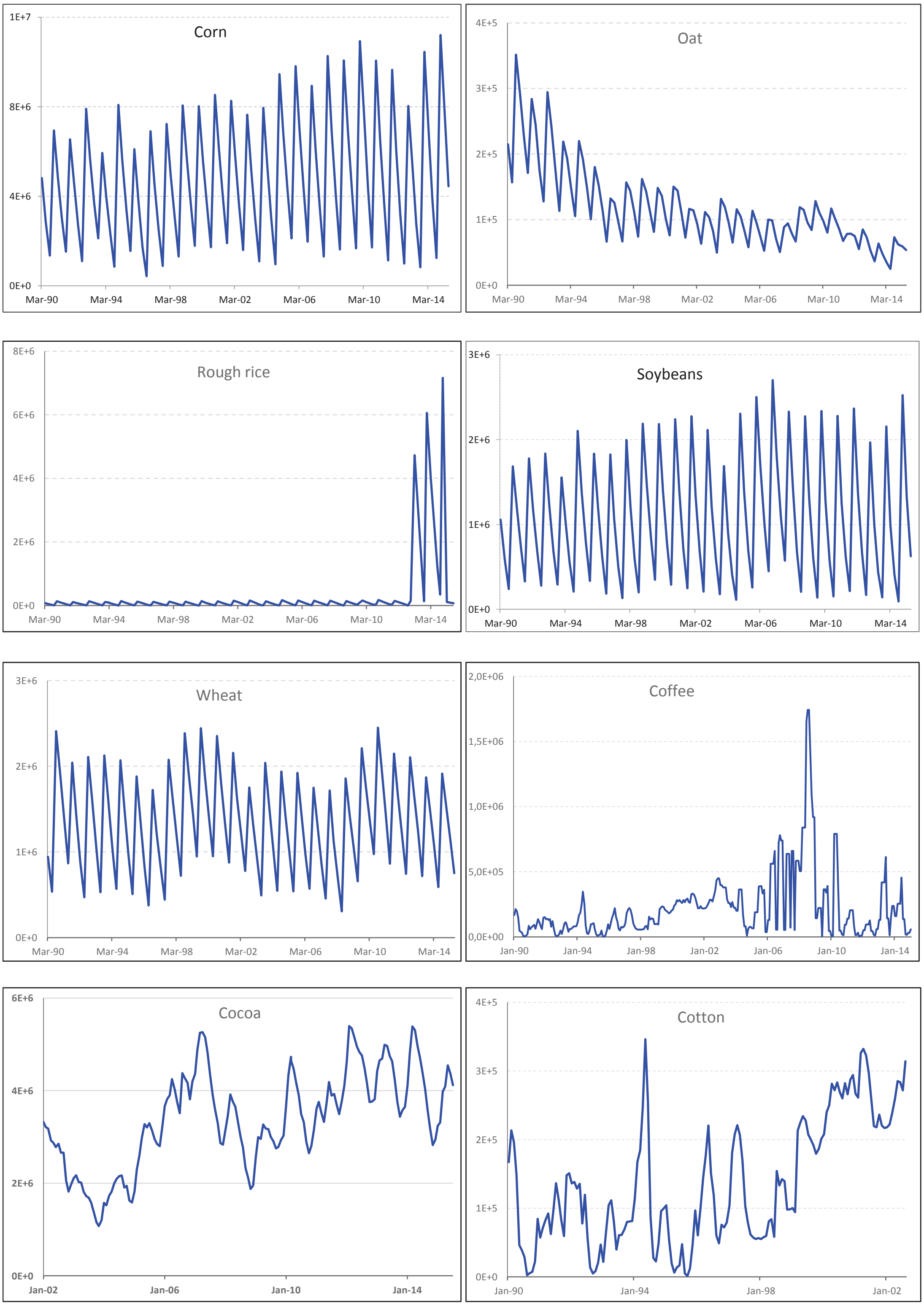


Figure 2.1: Inventory levels

of quiet periods alternate with relatively volatile ones where price variations are rather large. Mandelbrot [Mandelbrot 1963] had characterized this property as volatility clustering where high-volatility events tend to cluster in time.

The third and fourth moments together indicate shape of risk distribution for returns and are also used to reject their normality hypothesis. Indeed, commodities here are, in general, negatively skewed with high kurtosis. The negative skewness suggests that losses are more likely to occur than profits. High kurtosis that decays with the contract maturity will refer to more frequent extreme events in shorter term contracts than the longer ones, again consistent with Samuelson effect. Hence, shorter time-to-maturity are more sensitive to new information as noted by Black [Black 1976a]. But when combining negative skewness with high kurtosis, it is still not clear if extreme losses are likely to occur than extreme profits over the whole period. Leverage effect as reported in column LE, originally advocated by Black [Black 1976b] provide more information on this issue.

Leverage effect[19] reflects an asymmetric response of volatility to positive and negative past returns. Increases in volatility are larger when previous returns are negative than for the same magnitude of positive returns. However, this effect is reverse in commodity markets because the rise in volatility make prices to go up and panic is set in the markets. For instance, this seems to be the case with data at hands within the world food crisis 2007-2008 or from late 2010 up to 2013. Using t-statistic based tests, leverage effect is not significantly different from zero over the period 1990-2015 for most of the commodities. This means that drastic price moves may not come from changes in returns as assumed in 2.4.1. Large price variations may come from extra production of a specific commodity instead of another one which due to available acreage or the conjunction usage may widen the discrepancy in supply and demand. For instant, extra corn planted in United

[19] Leverage effect is correlation between the squared returns at date t and the returns at date $t-1$ that is significant, [Black 1976b], [Cont 2001].

States for ethanol production and high demand for livestock feed affects the soybean meal production (USDA). Consequently corn prices went up and farmers reacted accordingly by supplying the market with more soybean. This has increased the dependence level between different commodities.

Finally, we do not investigate autocorrelation since organized futures markets are known to be liquid and will not exhibit significant autocorrelations since Mandelbrot has argued that, "*arbitrage tends to whiten the spectrum of price changes*", Cont [Cont 2001]. Besides, for non-Gaussian time series such as the ones at hands (heavy tails due to high kurtosis), the autocorrelation function may be difficult to interpret and will not capture the dependence structure.

Table 2.3: Descriptive statistics

		N			Mean (%)			Std. dev. (%)			High moments		Extremes (%)		t-Stats	
Commodity	Nearby	Total	90-06	06-15	Total	90-06	06-15	Total	90-06	06-15	Skew	Kurt.	Min.	Max	LE(%)	$Q(10)$
Corn	1	6197	4283	1913	1.70	2.92	-0.99	27.68	23.73	34.95	-1.21	25.03	-27.62	12.76	0.44	64
	2	6197	4283	1913	1.74	2.89	-0.85	25.98	22.13	33.04	-0.38	12.44	-22.31	12.41	1.8	290
	3	6197	4283	1913	1.80	2.85	-0.53	25.04	21.10	32.16	-0.47	11.54	-17.43	10.79	1.48	372
	4	6197	4283	1913	1.93	2.67	0.29	23.61	19.52	30.89	-0.29	9.27	-15.10	10.31	0.98	631
	5	6197	4283	1913	2.00	2.45	0.91	22.12	17.93	29.42	-0.26	8.52	-14.48	9.74	-0.14	1080
	6	6197	4283	1913	1.97	2.43	0.95	20.99	16.84	28.17	-0.19	7.72	-9.25	9.65	-0.33	1930
Oat	1	6203	4281	1921	3.52	3.43	3.70	35.86	34.93	37.86	-1.14	13.75	-25.45	15.43	-0.27	108
	2	6203	4281	1921	3.01	3.27	2.43	29.78	28.71	32.06	-0.07	15.81	-16.46	12.89	-1.57	343
	3	6203	4281	1921	2.61	3.07	1.59	27.60	26.47	30.00	-0.33	6.69	-18.97	11.15	-1.47	174
Rough Rice	1	6200	4282	1917	2.18	1.86	2.86	26.84	27.55	25.20	0.11	27.29	-24.45	28.08	-1.41	57
	2	6200	4282	1917	2.10	1.87	2.60	24.13	24.32	23.72	0.08	7.86	-13.24	14.35	1.36	260
	3	6200	4282	1917	2.07	1.85	2.52	22.78	22.88	22.56	-0.29	11.18	-18.43	10.27	0.79	153
Soybeans	1	6202	4282	1919	3.12	1.16	7.66	25.42	22.97	30.20	-0.99	21.08	-23.41	20.32	-14.37*	1073
	2	6202	4282	1919	2.62	1.13	5.97	23.38	21.77	26.64	-0.64	9.21	-16.49	6.73	-4.02*	743
	3	6202	4282	1919	2.37	1.06	5.22	23.05	21.47	26.24	-0.49	7.91	-14.20	6.77	-4.30*	981
	4	6202	4282	1919	2.32	1.06	5.13	22.36	20.69	25.74	-0.32	6.53	-9.22	7.06	-2.67*	1442
	5	6202	4282	1919	2.32	1.10	5.15	21.54	19.87	24.89	-0.27	6.48	-8.11	7.04	-2.23	1697
	6	6202	4282	1919	2.34	1.10	5.11	20.75	19.08	24.07	-0.30	6.84	-9.61	7.07	-2.35	1689

Tables 2.3 and 2.4 report descriptive statistics of daily excess returns to nearby contracts for each agricultural commodity. There is in: column 1 the commodity name; column 2 the nearby contract; columns 3-5 (labeled "N") the number of daily observations in each sample; columns 6-8 the percent per annum of average excess return for each sample; columns 9-11 the annualized standard deviation (defined as the standard deviation of daily returns multiplied by the square root of 252) for each sample; columns 12-13 (labeled "High moments") skewness and kurtosis; columns 14-15 (labeled "Extremes") minimum and maximum; columns 16-17 (labeled "t-Stats") LE is leverage effect with (*) when it is significantly different from zero using ; $Q(10)$ is the Ljung-Box portmanteau test for the null hypothesis of no autocorrelation in the squared returns up to order 10. The test statistic is asymptotically χ^2 distributed with 10 degrees of freedom with critical value of 23.21 at level of 1%.

Table 2.4: Descriptive statistics (continued)

Commodity	Nearby	N			Mean (%)			Std. dev. (%)			High moments		Extremes (%)		t-Stats	
		Total	90-06	06-15	Total	90-06	06-15	Total	90-06	06-15	Skew	Kurt.	Min.	Max	LE	$Q(10)$
Wheat	1	6203	4284	1918	1.11	1.20	0.92	31.05	28.10	36.79	-0.53	18.26	-28.61	23.30	-9.34*	650
	2	6203	4284	1918	1.45	1.60	1.11	28.04	24.03	35.39	0.08	5.97	-10.91	11.08	-1.07	1102
	3	6203	4284	1918	1.89	1.91	1.86	27.02	22.68	34.81	-0.48	12.20	-23.48	9.09	-0.93	252
	4	6203	4284	1918	1.93	1.82	2.16	25.34	21.18	32.79	-0.50	11.95	-22.04	8.21	-1.33	346
	5	6203	4284	1918	1.87	1.82	1.98	23.16	19.10	30.35	-0.22	7.78	-12.16	8.15	0.81	1483
	6	6203	4284	1918	1.72	1.76	1.87	22.16	18.23	29.65	-0.22	7.78	-10.16	7.55	0.69	1483
Cocoa	1	6197	4246	1950	5.06	3.36	8.39	30.66	31.35	29.11	0.11	5.58	-10.01	12.74	1.09	339
	2	6197	4246	1950	5.03	3.41	8.21	29.36	29.68	28.65	0.05	5.62	-9.96	12.15	0.60	411
	3	6197	4246	1950	4.96	3.39	7.99	28.12	28.21	27.91	-0.01	5.52	-9.95	9.49	0.94	461
	4	6197	4246	1950	4.85	3.34	7.77	27.34	27.37	27.27	-0.02	5.57	-9.99	9.15	0.87	500
	5	6197	4246	1950	4.75	3.28	7.60	26.69	26.67	26.72	-0.04	5.61	-9.94	8.73	1.31	536
	6	6197	4246	1950	4.66	3.24	7.40	26.20	26.09	26.43	-0.05	5.68	-10.01	8.52	1.33	513
Coffee	1	6188	4252	1950	3.48	-1.16	2.01	37.69	28.33	29.70	0.88	16.39	-15.03	31.88	7.08*	886
	2	6188	4252	1950	3.47	-1.10	1.83	36.70	23.70	29.52	0.25	10.86	-20.77	23.23	-1.37	1059
	3	6188	4252	1950	3.44	-1.02	1.81	34.37	22.06	29.10	-0.06	10.94	-23.20	20.55	-4.09	853
	4	6188	4252	1950	3.39	-0.53	1.63	32.79	20.21	27.47	0.04	9.51	-18.30	19.64	-4.82	917
	5	6188	4252	1950	3.29	-0.31	1.62	31.52	18.49	25.60	0.06	7.49	-13.45	17.54	-4.89	1050
	6	6188	4252	1950	3.21	-0.18	1.26	30.66	15.94	23.88	-0.01	7.69	-15.58	17.00	-4.11	967
Cotton	1	6203	4252	1950	-0.25	-1.16	2.01	28.77	28.33	29.70	-0.93	24.04	-30.43	18.96	-6.12	98
	2	6203	4252	1950	-0.25	-1.10	1.83	25.67	23.70	29.52	-0.25	7.52	-15.86	9.89	-0.12	495
	3	6203	4252	1950	-0.19	-1.02	1.81	24.49	22.06	29.10	-0.99	16.74	-20.55	7.76	0.39	169
	4	6203	4252	1950	0.09	-0.53	1.63	22.74	20.21	27.47	-0.61	13.43	-16.80	10.76	3.2*	681
	5	6203	4252	1950	0.24	-0.31	1.62	20.98	18.49	25.60	-0.45	10.41	-13.33	9.49	-0.76	1212
	6	6203	4252	1950	0.21	-0.18	1.25	18.79	15.94	23.87	-0.02	7.42	-6.33	9.54	-1.88	3503

Table 2.5 reports the correlations matrix between the different selected commodities. The soft commodities present low correlations amongst themselves and with grains. While grains are mutually dependent up to 30%, except with rough rice. This may be due to larger production of rice in Asian countries as compared to US. Particularly, one can think interchangeability of the grains in various domain that makes the production a product in place of another one.

Table 2.5: Returns correlation matrix of nearby futures

	Corn	Oat	R. rice	Soybeans	Wheat	Cocoa	Coffee	Cotton
Corn	1							
Oat	0.46	1						
R. rice	0.19	0.15	1					
Soybeans	0.53	0.37	0.21	1				
Wheat	0.54	0.35	0.15	0.35	1			
Cocoa	0.10	0.08	0.05	0.11	0.08	1		
Coffee	0.10	0.11	0.03	0.09	0.11	0.14	1	
Cotton	0.16	0.10	0.07	0.18	0.14	0.10	0.07	1

All correlations observed are significant at significant level of 1%.

To go beyond the data description with the aim to set up commodity prices models, we apply more elaborated testing procedures in order to attain a higher level of understanding the underlying dynamics of commodities prices. This is addressed in the following subsection.

2.4.2 Testing for futures markets efficiency

The efficiency tests of futures markets starts with the checking the stationarity of the returns series. The returns are considered as realization of stationary process if unit root hypothesis is rejected. Otherwise, serial cointegration or error correction model can be run for the market efficiency test. When unit root is rejected, two efficiency tests will be performed. The first test checks for $a = 0$ and $b = 1$ in the regression model (2.10) on (stationary) returns series standing for both efficiency and unbiasedness simultaneously. However, as the regression model in (2.10) is linear the test results may not be consistent whatever their issues. The second test for market efficiency aims to include non linearity as the relative market efficiency approach. The adaptive market hypothesis concept is investigated, herein, with the Hinich portmanteau bicorrelation test decribed in Appendix A.3.

2.4.2.1 Unit root tests

Table 2.6 presents the values of test statistic for both ADF and RALS unit root tests. These values suggest the rejection of the null hypothesis that returns are integrated for the two tests. Hence, returns of commodities futures at hands can be considered as stationary. Therefore, we run the efficiency tests based on the returns series.

Table 2.6: Test statistics of ADF and RALS unit root tests

	Nearby contracts					
Commodity	1	2	3	4	5	6
Corn	-74.55	-74.34	-74.65	-11.25	-11.91	-9.94
	-79.87	-76.78	-77.12	-14.76	-13.57	-11.65
Oat	-20.32	-13.70	-12.35			
	-21.57	-15.43	-13.83			
Rough Rice	-72.91	-71.84	-72.20			
	-73.42	-75.46	-70.43			
Soybeans	-7.60	-9.22	-13.03	-10.97	-19.70	-20.60
	-9.54	-9.08	-14.41	-13.23	-20.84	-23.46
Wheat	-79.48	-76.87	-77.73	-77.62	-79.42	-79.42
	-80.32	-78.22	-79.01	-78.74	-80.46	-81.03
Cocoa	-6.27	-3.62	-5.558	-3.928	-4.09	-4.07
	-9.43	-4.39	-7.064	-5.056	-6.03	-5.98
Coffee	-10.80	-10.28	-10.30	-9.38	-10.01	-9.43
	-12.54	-14.01	-13.23	-9.98	-10.79	-13.18
Cotton	-9.09	-6.06	-5.73	-5.29	-7.53	-5.22
	-11.13	-7.94	-8.12	-8.08	-11.12	-5.02

The null hypothesis is that there is unit root against the alternative that there is no unit root. For each commodity, the test statistics are displayed on first line for ADF test and on the second line for RALS unit root test. The critical values at levels 1% and 5% are respectively -3.48 and -2.89 for ADF tes and respectively -3.571 and -3.571 for test RALS unit root test.

2.4.2.2 Efficiency tests

With stationary returns series of both spot and futures for the commodities at hands, one can estimate the regression model in equation (2.10). Meanwhile, since prices are not stationnary, the equation (2.10) has to be modified to its equivalent as follows

$$\frac{S_T - S_t}{S_t} = \widetilde{b}_0 + \widetilde{b}_1 \frac{F_{t,T} - S_t}{S_t} + e\tilde{r}r_T. \tag{2.12}$$

Then, the regression model will be run on data sample provided by equation (2.12) to be consistent with the no-arbitrage condition that futures price equals the spot price at expiry date.

The results showed in Table 2.7 do not reject that $a = 0$ is significant at 5% level, but $b = 1$ seems not to be relevant in most of cases. Therefore, spot price will be considered as unbiased estimator of futures prices. Hence, the futures markets is said to be inefficient even if they could sometimes switch to efficiency. Indeed, considering the term structure, efficiency is more expected in long term than in short term in linear framework. This assertion is not evidence with the commodities at hands. However, the rejection of linear dependence may not capture the whole structure. Non-linearity can also bias the estimates and contradicts the efficient market hypothesis. The mixed conclusions lead to use adaptive market hypothesis concept which allows to investigate relative efficiency with nonlinear test as well as degree of inefficiency.

To test for the adaptive market hypothesis with the Hinich test, one first needs to pre-whiten the data to remove the linear structure of data. Because data are stationary, we will fit the appropriate autoregressive model of order p, AR(p). For this purpose, only the front nearby prices are used to estimate the AR(p) model order[20] for each commodity. Hence, any remaining

[20]The results are the same with the other nearby prices for all the commodities under study.

Table 2.7: Linear model efficiency

Commodity		Statistics			
		a	b	R^2	RMSE
Corn	C2	7.9e-7	0.893	0.935	0.0007
	C3	8.7e-7	0.814	0.914	0.0009
	C4	-5.8e-6	0.798	0.969	0.0010
	C5	-1.3e-5	0.918	0.963	0.0010
	C6	-1.6e-5	0.908	0.968	0.0010
Oat	O2	1.8e-5	0.920	0.918	0.0012
	O3	3.7e-5	0.890	0.881	0.0014
Rough rice	RR2	9.3e-6	0.924	0.989	0.0009
	RR3	1.2e-5	0.908	0.893	0.0011
Soybeans	S2	2.4e-5	0.857	0.903	0.0007
	S3	3.8e-5	0.793	0.922	0.0008
	S4	4.1e-5	0.782	0.868	0.0009
	S5	3.9e-5	0.811	0.838	0.0009
	S6	3.5e-5	0.879	0.831	0.0010
Wheat	W2	-1.4e-5	0.901	0.951	0.0008
	W3	-2.9e-5	0.724	0.903	0.0011
	W4	-3.3e-5	0.599	0.848	0.0012
	W5	-3.6e-5	0.881	0.851	0.0012
	W5	-5.1e-5	0.859	0.861	0.0011
Cocoa	QC2	5.1e-5	0.812	0.979	0.0007
	QC3	1.1e-6	0.791	0.966	0.0007
	QC4	1.0e-6	0.738	0.956	0.0007
	QC5	1.3e-6	0.757	0.946	0.0008
	QC6	6.9e-8	0.757	0.959	0.0007
Coffee	KC2	1.5e-5	0.694	0.959	0.0012
	KC3	8.4e-6	0.709	0.948	0.0012
	KC4	4.4e-6	0.712	0.944	0.0012
	KC5	3.4e-6	0.691	0.944	0.0012
	KC6	4.2e-6	0.695	0.931	0.0012
Cotton	CT2	-1.1e-6	0.708	0.828	0.0011
	CT3	-3.5e-6	0.692	0.822	0.0012
	CT4	-1.3e-5	0.697	0.783	0.0013
	CT5	-1.9e-5	0.692	0.769	0.0013
	CT6	-1.9e-5	0.694	0.765	0.0013

R^2 is coefficient of determination of linear regression and RMSE stands for root mean squared error.

serial dependence will be to due the nonlinear structure of residuals. The whitened residuals are then analyzed following the procedure described in Appendix A.3.

To perform the H-test, we use the pre-whitened residuals from AR(p) model that are divided into monthly residuals to constitute the non-overlapped moving time windows. That is 308 non-overlapped moving time windows from January 1990 to august 2015. Table 2.8 presents the results of AR(p) model order and the H-test. The number of significant windows indicates where the presence of nonlinear serial dependence is identified along with the epochs in which it occurs. For the sample under study, such periods are indicative of evidence of adaptive market hypothesis on the commodity futures markets.

The null hypothesis of H-test is that, the time series of each window are realizations of a stationary pure white noise process that has zero bicorrelations, defined by (A.7) in Appendix A.3. The alternative hypothesis is these bicorrelations are non-zero meaning that there exists third-order nonlinear dependence in the data generation process. The rejection of the null hypothesis implies the presence of nonlinear dependence in the series and, therefore market inefficiency.

Furthermore, given the results of the number of windows in which nonlinear dependence is detected, the degree of inefficiency can thereby be estimated. This goes from 15.91% of windows for cocoa to 28.90% of the windows in the case of rough rice.

Table 2.8: Hinich non linear test results

	Commodities							
	Corn	Oat	R. rice	Soy.	Wheat	Cocoa	Coffee	Cotton
AR(p)	AR(4)	AR(5)	AR(3)	AR(5)	AR(4)	AR(3)	AR(3)	AR(5)
nb. wind.	308	308	308	308	308	308	308	308
Signif. wind.	87	60	89	62	83	49	53	54
Deg. of ineff	28.25%	19.48%	28.90%	20.13%	26.95%	15.91%	17.21%	17.53%

AR(p), nb. wind. and Signif. wind. respectively stand for AR(p) model, number of windows and number of significant windows.

2.4.3 Testing for stylized facts in commodity prices

We aim to test for continuity, mean-reversion, presence of jumps, seasonality, and inter-temporal relationship (backwardation and contango) on daily futures prices at hands. Stylized facts are driven by various factors that may be distinct. But, these features can not be tested simultaneously, hence applying the Bonferroni multiple testing procedure[21] will allow to adjust significance levels as they were performed together. So, the probability of having at least one significant result due to chance remains as an adjusted significant level. That is the significant level of each test is divided by the number of performed tests.

In order to derive the required test statistics for decisions, both nearby and expiry-month futures prices data are used. Recall that nearby futures prices contain splice points from data concatenation. In order, not to bias the testing procedures, specially in detecting jumps, the same tests are also run on a real quoted futures prices. The data of such futures are selected over the highly variation period of nearby futures prices.

2.4.3.1 Testing for mean-reversion of commodity futures prices

On evidencing the mean-reversion feature, one can[22] use the results from unit root tests. For the commodity prices at hands, the values of test statistics in table 2.6 do not reject the hypothesis that futures prices follow mean-reverting process. Both ADF and RALS tests lead to the same conclusion and RALS gives more consistency to mean-reversion behavior as ahead for whatever testing for jump will be. Indeed, the classical mean-reverting process as stated by Ornstern-Uhlenbeck has Gaussian noise and the presence of jump could affect the values of ADF test statistic while the RALS unit root test does not require to specify a density function for residuals.

[21] False discovery rate and positive false discovery rate methods give similar results.

[22] That is a non-stationary process can also contain mean-reversion (Kim and Park [Kim 2013])

Mean-reversion comes from transitory of shock effects that cause price to deviate from its underlying value and will gradually move toward the underlying fundamental value (see Yoon and Brorsen [Yoon 2005]). From Fama [Fama 1970], the efficient market hypothesis asserts that asset prices are unpredictable. Any efficiency is short-lived and could not be exploited to out-perform. So, the mean-reverting behavior violates the market efficiency hypothesis. Indeed, with mean-reversion, underlying value is predictable and this will also motivate futures hedging in commodity markets. Particularly, for rollover hedging to increase expected returns, futures price returns should follow a mean-reverting process (see Yoon and Brorsen [Yoon 2005]).

2.4.3.2 Testing for jump and their impact in agricultural market

Even the probability of having jump increase with decreasing observation time step, jumps can still be observed in daily data. For instance, Figure 5 in Aït-Sahalia [Aït-Sahalia 2004] shows that the probability that a 10% log-return involves one jump as a function of the daily sampling is about 60%. This motivates the investigation of jumps presence in daily returns of commodity futures prices at hands.

Figures in Appendix A.2.1 exhibit sudden large variations of futures prices at certain periods that suggest the presence of jumps. To check for the presence of jumps with the procedures mentioned in paragraph 2.3.2 and described in Appendix A.4, we use both nearby prices and a real quoted futures prices. Indeed, nearby futures are concatenated futures prices and the gaps at the splice points could biased the jump test if these gars are substantial on the underlying time data. Thus, jump tests are also performed on data of real quoted futures prices of a specific year which is selected on the basis of both volatility and kurtosis computed year-to-year.[23]. In so doing, only periods where the futures price varies the most are considered for each

[23]As it is well known that key indicators for the presence of jumps are volatility and kurtosis, no results are displayed

commodity.

Tables 2.9 and 2.10 display, respectively, the results of BNS and ASJ tests for the nearby futures prices. For the ASJ test, power and time scale are respectively set at $p = 4$ and $k = 3$. To save space, details of test results are moved to Appendix A.5.1.

Table 2.9: BNS jump detection test

	Commodities							
Contrat	Corn	Oat	R. Rice	Soy.	Wheat	Cocoa	Coffee	Cotton
1	9.01	11.84	11.84	3.78	6.00	7.06	4.39	7.11
	9.49	12.22	12.22	4.90	6.28	7.32	4.80	7.74
2	7.05	6.33	6.06	7.59	5.05	7.65	4.66	4.88
	7.35	6.56	6.16	7.86	5.01	7.89	4.79	4.93
3	7.78	8.16	7.17	7.87	8.04	7.34	3.91	7.11
	8.21	8.49	7.30	8.05	8.19	7.58	4.04	7.14
4	7.66			7.37	7.03	6.71	3.93	5.55
	7.98			7.55	6.99	6.89	4.12	5.61
5	5.95			6.56	5.93	6.62	2.94	4.77
	6.17			6.81	5.89	6.81	3.03	4.86
6	5.40			6.55	6.18	6.56	7.10	3.01
	5.51			6.76	6.20	5.95	7.30	3.11

The null hypothesis of no jump is rejected if the test statistic is greater than 2.807 (at 0.2%) corresponding to significant level of 1% in multiple testing. For each nearby contract, the first line corresponds to z_{TP} test statistic and the second one to z_{QP} test statistic.

For all the commodities, the BNS test rejects the null hypothesis of no jump and this regardless of the term structure. The ASJ procedure checks for either presence or absence of jumps at significant levels of 1% and 2% that corresponds to 5% and 10% respectively in multiple testing. The test statistic $\widehat{S}(4, 3, \Delta_n)_t$ converge towards 1 at low rate. This may come daily prices that are used instead of high frequency as suggested by the authors [Aït-Sahalia 2009]. The absence of jump is rejected, except for wheat on the first two contracts, oat and coffee. However, the presence of jumps is not rejected for any commodity price and, this, irrespective of their maturity. To disentangle whether presence of jumps is due to concatenation process or price shocks, we have refined the jump detection procedure by performing the same test on the real quoted futures prices. Table A.4 in Appendix A.5.1

displays the results where there is a conflicting conclusion for some commodities compared to the nearby prices case. For real quoted futures prices, corn, soybeans, wheat and cotton yield to same results, while jumps seem to be present in oat and coffee futures prices and the presence of jumps is rejected in rough rice and cocoa cases. So, concatenation method may impact the jump tests results for rough rice and cocoa, oat and coffee futures prices.

Meanwhile, putting together the jump evidence on time series of real quoted futures prices with the planting and harvesting periods in table 2.12 over a year allows to infer where jumps may originate from. Indeed, considering a the harvesting period that spans the sample period used to check for the presence of jumps on real quoted futures prices, jumps look to occur either within or between that periods depending upon the commodity. The inter-harvesting jumps are mainly due to exogenous shocks on agricultural supply or demand that inventories are stored for, while the intra-harvesting jumps may come from either external factors or the storage market value. For grain commodities, maturity effect, the seasonality in inventory levels in figure 2.1 and planting and harvesting periods in table 2.12 are part of origin of price jumps. Indeed, recorded inventory levels are the highest just after the harvesting period. As illustration, inventory level of corn is higher in December (inventory level is recorded on 1st) the harvesting period end in November.

Table 2.10: Decision of ASJ jump detection test

	Commodities							
Contract	Corn	Oat	R. rice	Soy.	Wheat	Cocoa	Coffee	Cotton
1	j	noj	j	j	noj	j	noj	j
2	j	noj	j	j	noj	j	noj	j
3	j	noj	j	j	j	j	noj	j
4	j			j	j	j	noj	j
5	j			j	j	j	noj	j
6	j			j	j	j	noj	j

The symbol "j" stands for presence of jump and "noj" means there is no jump

When the jump is considered, its activity will also matter. Hence, one has to distinguish between large jumps in finite number (finite activity) and small jumps in either finite or infinite number (infinite activity). Another test of Aït-Sahalia and Jacod [Aït-Sahalia 2011] id helpful to determine whether jumps have finite or infinite activity. The test statistic of jump activity test is the inverse of the test statistic of continuity test ([Aït-Sahalia 2010],[Le Courtois 2014]). It appears that all futures prices encompass continuous component and when jump is considered, the will be of finite activity.[24] Details for jump activity and continuous component are displayed in Appendixes A.5.2 and A.5.3.

Thus, with the above tests in 2.4.3.1, the process can be assumed to be of the type "*mean-reverting jump diffusion*" when the jump is considered. The next paragraph investigates the statistical issue of seasonal patterns for the futures prices as agricultural products are grown and harvested in a seasonal fashion.

2.4.3.3 Testing for seasonality

The seasonal fashion of growing and harvesting agricultural products affects both the price levels and volatility, Geman and Nguyen [Geman 2005]. Based on both visual inspection of graphic 2.2 and descriptive statistics on expiry-month data in Table 2.11, seasonal pattern can be considered.

We report mean and standard deviation of expiry-month prices (Sørensen [Sørensen 2002]) in table 2.11 that figure 2.2 also displays. Over a specific year, prices variations appear to be seasonal. The averages of futures prices, represented in bar chart, hit a peak in July for coffee and cotton but December for Cocoa. They reach their bottom in January except for cotton that is in December. For grains, the average futures price of rough rice and soybeans behave similar to soft commodities. These averages hits a peak in July (rough rice and soybeans) and a bottom in January. But for corn,

[24]Some of futures contract reject the null hypothesis of both finite and infinite activity

oat and wheat, the peaks are respectively in September, March and January with a bottom in December, September and November respectively. Moreover, the price variations of a year behave similarly as average futures prices in term of seasonality, even for wheat, cocoa and coffee where peaks slightly shift to May, July and September respectively. Putting altogether seasonal features with the planting and harvesting periods (reported in table 2.12) shows clear evidences that futures prices are high and more volatile just before the harvesting (in scarcity periods) and go down with high inventory levels, Black [Black 1976a].

Table 2.11: Summary statistics of Expiry-month prices

	Expiry-Month								
Commodity	Jan.	Mar.	May	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.
Corn		6410	6389	6353		6370			6353
		329.71	336.43	329.43		340.00			326.40
		142.75	147.74	137.17		149.73			137.01
Oat		6396	6337	6241		6126			6211
		205.82	206.57	205.82		200.97			201.81
		89.20	89.37	88.71		87.46			86.96
Rough rice	6226	6052	5919	5901		5769		6014	
	9.86	9.96	10.20	10.23		9.87		9.76	
	3.68	3.74	3.85	3.88		3.63		3.66	
Soybeans	6375	6374	6376	6376	6344	6272		6329	
	759.43	768.30	778.67	791.00	786.83	772.45		762.49	
	287.64	292.12	299.50	308.03	304.08	293.73		289.92	
Wheat		6455	6333	6283		6278			6517
		456.92	449.76	439.29		447.91			451.11
		180.64	182.76	177.42		180.99			180.64
Cocoa		6475	6433	6386		6339			6279
		1708.12	1713.33	1714.90		1715.78			1717.52
		671.23	672.59	673.55		673.23			672.68
Coffee		6337	6362	6473		6346			6286
		118.43	119.06	121.32		121.12			119.40
		61.41	59.02	68.23		72.93			59.20
Cotton		6371	6369	6345			6332		6306
		68.80	69.91	70.48			68.25		67.65
		18.15	20.19	20.47			17.01		16.01

For each commodity, observations number of that month within the period considered, mean futures prices and variation are displayed on first, second and third lines.

Seasonality test is also conducted on monthly volatility that is estimated from nearby futures prices. Historical volatility is computed as standard deviation of daily returns which are grouped by month for each year in order to check their seasonal pattern via Fisher test. Table 2.13 reports the values of test statistic of null hypothesis being the absence of seasonality.

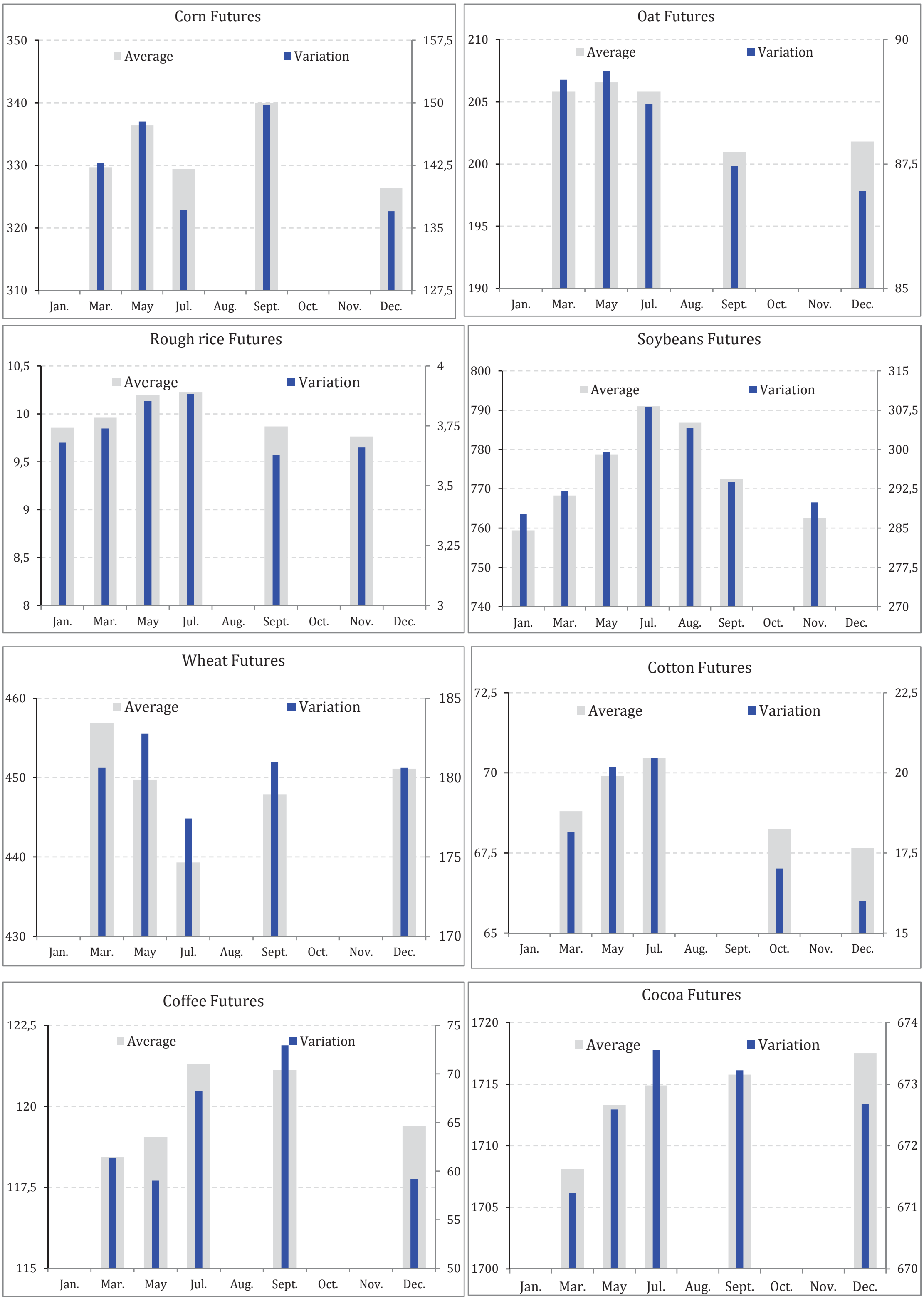


Figure 2.2: Mean and variation of futures prices: Expiry-month data

Table 2.12: Planting and harvesting seasons

Commodity	Planting	Harvesting
Corn	Apr. - Jun.	Oct. - Nov.
Oat	Apr. - May	Aug. - Sept.
Rough Rice	Mar. - May	Aug. - Oct.
Soybeans	Apr. - Jun.	Sept. - Nov.
Wheat*	Apr. - May	Aug. - Sept.
	Aug. - Oct.	May - Jul.
Cocoa		
Coffee		Nov. - Feb.
Cotton	Mar. - Jun.	Sept. - Dec.

*Winter and Spring

This is rejected except for cocoa and coffee suggesting the seasonal volatility. Graphics in Appendix A.2.2 display the monthly volatility patterns.

Table 2.13: Testing for seasonality on monthly volatility

	Commodities							
Nearbys	Corn	Oat	R. rice	Soy.	Wheat	Cocoa	Coffee	Cotton
1	9.91	6.33	8.37	6.60	4.39	0.56	1.18	3.07
2	9.51	6.77	6.98	8.92	3.95	1.38	1.67	3.28
3	8.46	7.09	6.85	7.41	4.55	1.23	1.07	3.24
4	6.71			7.73	4.85	1.10	1.05	2.42
5	8.87			7.75	3.85	1.05	1.04	3.00
6	9.79			7.39	3.71	0.99	0.83	1.93

The test compares the sum of squared errors between a model with trend and a model with trend and seasonality component. This test is called Fisher seasonality test, and it uses test statistic which is compared to values of theoretical of Fischer-Snedecor table (2.311 which is the 1% corresponding to five multiple testing of Fisher distribution with 11 and 253 degrees of freedom.). Seasonality is considered when test statistic value is greater than the theoretical value.

2.4.3.4 Inter-temporal relationship

The inter-temporal relation of futures contracts is the slope of term structure and will refer to as measure of anticipation for risk premium. Depending on the upward or downward slope, the market is said to be in situation of contango or backwardation respectively. Whatever the market situation, appropriate strategies can be based on it using additional information on inventories level[25] and futures markets.

Formally the slope is given by (Gorton et al. [Gorton 2013])

$$s(t, T_i) = \frac{F(t, T_i)}{F(t, T_1)} - 1, \qquad i > 1; \tag{2.13}$$

where $F(t, T_1)$ is nearest futures price is taken as proxy for the spot price, St_t, and $F(t, T_i)_{i>1}$ is the futures price of longer maturity contracts. Expression in equation (2.13) allows to estimate time series of slopes expressed in terms of increase proportion over the spot price for different maturities. For a given maturity, T_{i_0}, with $i_0 > 1$, if $s(t, T_{i_0})$ is positive, then the market is in contango within the period $[1, i_0]$, else the market is in backwardation. One can then check for consistency of slope sign along the maturities slope time series.

Figure 2.3 exhibits the slope time series, for different commodities futures prices with their maturities, over the 1990-2015 period. Based on graphical representations, mixed situation holds even if contango is likely to be more pronounced than backwardation except for soybeans, for which the graphical representation does not depict a clear-cut situation. One can also rely on average[26] slopes for each commodity are provided in table 2.14. On this basis, only soybeans seems to be more in backwardation than in contango; all other commodities appear to be in contango.

2.5 Conclusion

For storable commodities, we relate the market situation in term of efficiency and how fundamentals like production, inventory and spot interact to drive the futures prices. Based on analysis of commodity data prices, we draw the dynamic of the futures prices behavior. Particularly, various

[25]In the case of equity futures, the slope is solely driven by the risk-free rate through arbitrage arguments.

[26]Other statistics like median or mode can be use.

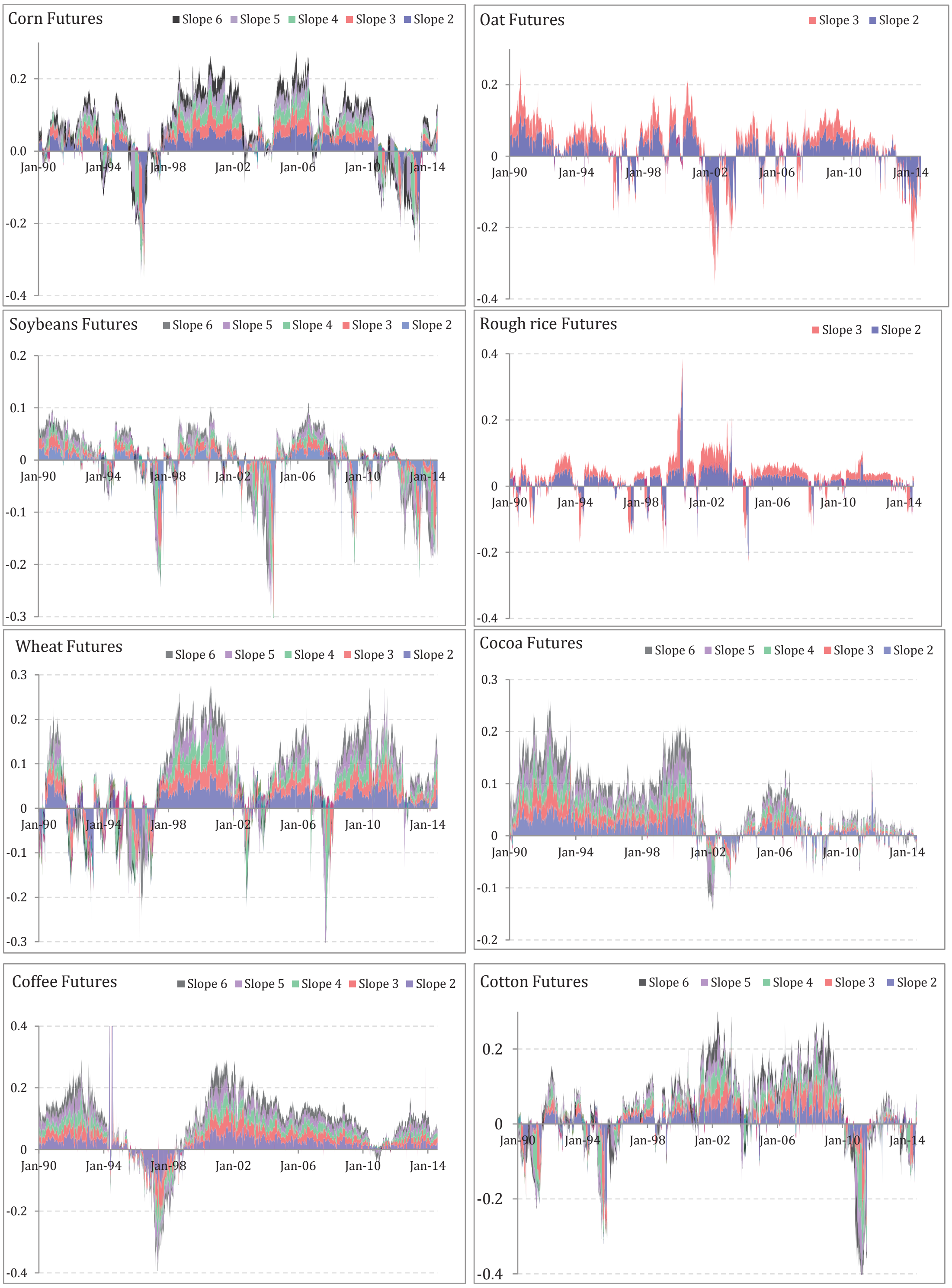


Figure 2.3: Difference between nearby futures and spot price expressed as percentage of the spot price

Table 2.14: Average difference between nearby futures and spot price expressed as percentages of the spot price

Commodity	N	Nearby contracts				
		slope 2	slope 3	slope 4	slope 5	slope 6
Corn	6198	1.992	3.653	4.724	5.554	6.512
Oat	6204	1.362	2.964			
Rough rice	6201	1.732	3.237			
Soybeans	6204	-1.102	-2.218	-3.388	-4.387	-5.188
Wheat	6203	2.009	3.310	4.279	5.559	5.523
Cocoa	6198	1.427	2.820	4.120	5.363	6.607
Coffee	6189	1.899	3.703	5.412	7.073	8.730
Cotton	6204	1.097	2.035	2.595	3.184	3.849

All average are significantly different from zero based on student test.

statistical tests are conducted. Market efficiency, mean-reverting, jump as well as seasonality in both price levels and monthly volatility can be considered for the dynamic of futures prices. Commodity markets are rejected to be efficient linearly over the whole period, instead nonlinear efficiency occur at certain periods. Besides, the unit root test used for market efficient hypothesis allows to suggest the mean-reverting behavior for all the commodity futures prices. Thus, returns on futures contract are, in some extent, predictable and futures contracts constitute thereby consistent hedging instruments against production and market risks.

Furthermore, we apply both Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006] and Aït-Sahalia and Jacod [Aït-Sahalia 2009, Aït-Sahalia 2011] tests to detect the presence of jumps. To our knowledge, jump detection procedures are not yet applied in agricultural markets even if models with jump component are suggested in many papers of agricultural commodity markets. To this end, time series of both the nearby futures prices as well as real quoted futures prices of specific year are used to distinguish whether jumps are due to concatenation process. However, the jump detection tests are originally designed for high frequency data, and we apply on daily data instead. Anyway, results allow to infer that jumps may come from both maturity and market value of storage for most of commodities and external factors like political unrest, large demand due to population growth etc... seem to affect futures prices in lesser extend.

Seasonality evidences are based on Fisher test for monthly volatility and graphic representations for long-run mean. For all the agricultural commodities, seasonality is suggested to be relevant features. Finally, we also investigate the inter-temporal relationship to address whether the term structure will depict, on average, upward or downward slope. For all the selected commodities, the converse situation is observed most of the time.

However, the identification of stylized facts does mainly contribute to appropriate investment strategy to adopt. Usually, these features are captured by parameters in price model. Hence, all these stylized facts will permit to state a model for futures prices of these agricultural commodities at hands. The following Chapter addresses the model estimation of the selected agricultural commodities.

Chapter 3

Modeling of commodity prices

Abstract: This Chapter investigates the modeling of agricultural futures prices. We agricultural prices follow mean-reverting jump-diffusion process in light of stylized facts of Chapter 2. Specifically, the drift term is periodic making the estimation to be conducted in two-stage procedure. The first step allows to estimate the equilibrium parameters related to the drift term using least square method. In the second step, we apply the particle MCMC filtering method on the residuals from the first step to obtain the remaining parameters.

Contents

3.1 Introduction

Asset prices modeling consists in estimating a set of parameters to represent key factors that drive the behavior of assets in market dynamics. Such models usually base on stylized facts and will be composed of both determinable component and error term. A process of semi-martingale class[1] is flexible enough to include all these factors together in a model. Particularly, commodity price modeling in the financial literature is addressed with Lévy processes which are Itō type semi-martingales.

The modeling of commodity prices has improved in tandem with the increased interest to the products through the trading volume and variety of contracts. Usually, the need of modeling asset prices comes with hedging and speculation strategies. On that way, the stylized facts serve as guide. The dynamic of futures prices in commodity markets can be either derived from spot price model (see for example Schwartz [Schwartz 1997] or Geman and Nguyen [Geman 2005]) or straightly posited (see Crosby [Crosby 2008]). For agricultural commodities, there are well known empirical properties related to operational contingencies embedded in delivery of the trading of their derivatives. Besides, convenience yield as utility of holding the physical good (agricultural commodities) also affects investment strategies. All these features are combined to make agricultural prices to behave with mean-reversion, seasonality and jump.

Lévy processes have statistical properties that are rich enough to include all the above mentioned features. Many studies had used Lévy processes to represent agricultural markets. For instance, Schwartz [Schwartz 1997] and Geman and Nguyen [Geman 2005] used continuous framework with Brownian representing risk factors in commodity markets. Hilliard and Reis [Hilliard 1999, Hilliard 1998], Deng [Deng 2000], Crosby [Crosby 2008] and

[1] Delbaen and Schachermayer [Delbaen 1994] had proved that the class of semi-martingale processes is optimal in self-financial strategies

recently Schmitz et al. [Schmitz 2014] included jump component to model agricultural prices.

We aim to derive a trade-off between theoretical and empirical coherences that will behave like the stylized facts of agricultural futures price, hence a price model that should include mean-reversion, seasonality and jumps. The financial literature represents mean-reversion with Ornstein-Uhlenbeck process. But this process in Gaussian and to account for fat-tailed, the error term will be replaced by Lévy process with jump. Since using the tests of Aït-Sahalia and Jacod [Aït-Sahalia 2009, Aït-Sahalia 2011] in Chapter 2 have suggested finite activity jump, the compound Poisson process is natural candidate to accommodate the jump feature in this futures price dynamics. Deterministic seasonality is incorporated by trigonometric functions on the diffusive parameter. Finally the model can be set up in form of stochastic differential equation of Lévy-driven Ornstein-Uhlenbeck process.

Data-generating system of jump-diffusion processes belongs to class Markovian processes. The optimal parameters of Markov processes are considered to be those that maximize the sample likelihood function. However, the exact maximum likelihood estimation of parameters is usually infeasible because closed-form likelihood function is available for stochastic differential equation only in very limited cases. For instance, the likelihood functions in Merton [Merton 1976] and Press [Press 1967] have infinite series representation. To overcome the estimation limits we apply a particle Monte-Carlo Markov Chains approach to draw the parameters estimate of the considered Lévy process.

The Chapter consists of three parts followed by conclusion. Section 3.2 sets up the model relative to the stylized facts of Chapter 2. Section 3.3 is devoted to the two-stage estimation procedure for the model and Section 3.4 is application on agriculture futures prices at hands.

3.2 A model for agricultural futures price

In agricultural markets, spot prices are not the same in different locations where deliveries take place at different places and will not be easy to define in agricultural markets due to location constraints. Then, we directly posit the futures price dynamic to follow mean-reverting jump-diffusion stochastic differential equation.

3.2.1 Model setting

We consider a financial market living on stochastic basis of a fixed probability space $(\Omega, \mathbf{F}, \mathbb{P})$ with information filtration $(\mathcal{F}_t)_{t\geqslant 0}$, that we assume to satisfy usual conditions.[2] Assume the market is arbitrage-free, so there will exist an equivalent martingale measure , $\widetilde{\mathbb{P}}$, of statistical measure $\mathbb{P}$ under which futures prices are martingales. However, since jump risk matters, the markets will be incomplete.

Model estimation with jump component is tedious in two ways. First, regardless the equivalent martingale measure , jumps are infrequent and will lead to estimation difficulties for adequate model precision. Second, it is not easy to choice the appropriate equivalent martingale measure under which futures prices should be martingales so as not to mislead the different risks behavior. Following empirical investigations in Chapter 2, we posit the futures price to follow mean-reverting jump-diffusion with periodic long-run mean and seasonal volatility. Seasonality in volatility is represented by trigonometric function as in Geman and Nguyen [Geman 2005], but the seasonal long-run mean is modeled as periodic long-run mean over a year.

[2]A filtered probability space that satisfies the usual conditions is complete (i.e. $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets) and right-continuous (i.e. $\mathcal{F}_t = \mathcal{F}_{t+} := \bigcap_{s>t} \mathcal{F}_s$, $\forall t$, meaning that any information known immediately after t is also known at t). The natural filtration induced by a counting process is right-continuous, and the completed natural filtration of a Lévy process is right-continuous. Together the two classes of processes cover a large chunk of the theory, and so if one of them is the driving source of randomness, it's very mild to assume right-continuity.

Denote by $F_{t,\tau}$ the price at time t of commodity futures that will mature at time τ. Let's $X_t = \ln(F_{t,\tau})$ with $F_{t,\tau} > 0, \forall t \geqslant 0$ when there is no ambiguity on the maturity τ. Under the statistical measure $\mathbb{P}$, the process X_t satisfies the following stochastic differential equation

$$\mathrm{d}X_t = \left[\mu_t - \kappa X_t - \lambda J \int z\nu(\mathrm{d}z)\right]\mathrm{d}t + \sigma_t \mathrm{d}W_t + J\mathrm{d}Y_t, \quad \mathbb{P} - a.s. \tag{3.1}$$

where κ is the rate of reversion to the long-run mean, μ_t and $\sigma_t > 0$ the volatility. Both long-run mean, μ_t, and the volatility, σ_t, are deterministic and seasonal functions of time that are stated differently. The process $W = (W_t)_{t\geqslant 0}$ is a Brownian motion under $\mathbb{P}$ and $J \in (-1, 1)$ is a jump scaling factor. The process $Y = (Y_t)_{0\leqslant t\leqslant T}$ is a compound Poisson; that is $Y_t := \sum_{k=1}^{N_t} Z_k$ where $N := (N_t)_{0\leqslant t\leqslant T}$ is a Poisson process with jump intensity $\lambda > 0$ and the process $Z := (Z_t)_{0\leqslant t\leqslant T}$ represent the jump size. The jump size process $Z := (Z_t)_{0\leqslant t\leqslant T}$ is identically and independently distributed with Lévy measure, $\nu(\mathrm{d}z)$, and also independent of counting process N_t. The Lévy measure, $\nu(\mathrm{d}z)$, satisfies the condition $\int_{\mathbb{R}} \min(|x|, 1)\nu(\mathrm{d}z) < \infty$ to represent finite variation with finite activity where $m = \mathbb{E}[Z_t] < \infty$.

The dynamic in equation (3.1) can also be written in form as the expression (2.11) of Chapter 2 where the variation of uncertainty modeled by a Lévy process, $L = (L_t)_{t\geqslant 0}$ as

$$\mathrm{d}L_t = -\lambda J \int z\nu(\mathrm{d}z)\mathrm{d}t + \sigma_t \mathrm{d}W_t + J\mathrm{d}Y_t \qquad \text{and} \qquad \mu_t = \kappa\bar{x}.$$

The periodicity of the long-run mean, μ_t, is represented by the combination of real-valued functions $e_1, \ldots, e_p$, for $p \in \mathbb{N}$,

$$\mu_t = \sum_{\ell=1}^{p} \mu_\ell e_\ell(t), \quad (\mu_1, \ldots, \mu_p) \in \mathbb{R}^p.$$

The seasonality of volatility, σ_t, is in form of exponential of trigonometric function given by

$$\sigma_t = \sigma e^{\alpha \sin[2\pi(t+\beta)]}, \quad \sigma > 0, \quad \alpha > 0, \quad \beta \in [-0.5, 0.5].$$

For modeling purposes, one needs equivalent martingale measure to the statistical measure, $\mathbb{P}$ under which the market is in equilibrium. We address the measure change from statistical measure, $\mathbb{P}$, to risk-neutral measure, $\widetilde{\mathbb{P}}$, of the process in (3.1).

3.2.2 Market price of risk

Given the model setup in (3.1), market price of commodity risk comes from both continuous and jump risks. Under $\mathbb{P}$, the market is incomplete because of two risk sources by assuming the absence of opportunity of arbitrage in the market, the equivalent martingale measure will not be unique. We assume the risk-neutral measure to be fixed by the market.

For simplicity, assume the riskless return to be zero and denote by $\widetilde{X}_t$ the log futures price under equivalent martingale measure , $\widetilde{\mathbb{P}}$

$$\mathrm{d}\widetilde{X}_t = \left[\mu_t - \kappa\widetilde{X}_t + \sigma_t\eta - \widetilde{\lambda}J\int z\nu(\mathrm{d}z)\right]\mathrm{d}t + \sigma_t\mathrm{d}\widetilde{W}_t + J\mathrm{d}\widetilde{Y}_t, \quad \widetilde{\mathbb{P}} - a.s. \tag{3.2}$$

where η is the market price for continuous risk. The stochastic process $\widetilde{W} = (\widetilde{W}_t)_{t\geqslant 0}$ is a Brownian motion under $\widetilde{\mathbb{P}}$. η only perturbs the drift term from continuous risk and there is another set of perturbations specific to jump component Y_t in equation (3.2). Namely, we assume that the jump intensity λ becomes $\widetilde{\lambda}$ under $\widetilde{\mathbb{P}}$ as follows (Aït-Sahalia and Matthys [Aït-Sahalia 2014])

$$\widetilde{\lambda} = e^a\lambda, \quad a \in \mathbb{R}.$$

The scalar a amplifies or diminishes the jump intensity. From equation (3.2) we note that perturbation of the intensity alters the drift term and will also change the frequency of jumps occurrence in the Poisson process N_t of futures price dynamic (3.1). For instance, when jumps are downward, only the compensation will lead to higher expected returns when assets prices are low and vice-versa for lower expected returns when asset prices are upward. This is consistent with the empirical risk and return trade-off observed in financial markets. In other words, compensating the jump process leads $F_{t,M}$ to carrying a risk premium for intensity misspecification. The second perturbation of jump component affects the jump size Lévy measure and we assume the Lévy measure of jump size distribution to be the same under two probability measures.

3.3 Model estimation procedure

The aim is to estimate the parameters of the process in equation (3.1) based on observation stream $\{\widetilde{X}_{t_j}\}_{j=1,\ldots,n}$. As data are generally observed discretely in time, we consider constant time step, $\Delta t > 0$; so the observation points are $\{t_j = j\Delta t,\ j = 1,\ldots,n\}$. The estimation is proceeded in two-stage framework due to seasonal long-run included in drift term in (3.1). The first step applies the approach in Franke and Kott [Franke 2013] to estimate the parameters of periodic drift term and the speed of mean-reversion of Lévy driven Ornstein-Uhlenbeck process. These parameters are also referred to equilibrium parameters since they characterize the equilibrium state. The second step uses the residuals from the first step to fit the appropriate Lévy-driven models via particle Monte Carlo Markov Chain approach.

3.3.1 Model under equivalent martingale measure $\widetilde{\mathbb{P}}$

Let's denote the vector of the parameters in the first step by

$$\boldsymbol{\vartheta} = (\mu_1, \ldots, \mu_p, \kappa) \in \mathbb{R}^p \times \mathbb{R}_*^+.$$

Then, the drift term in (3.1) could be represented in the form of inner product

$$\varphi(t,x)\cdot\boldsymbol{\vartheta} = \sum_{\ell=1}^{p} \mu_\ell e_\ell(t) - \kappa x,$$

where $\varphi(t,x)$ is $(p+1)$-dimensional function that is periodic in t to represent the seasonal pattern of long-run mean. That is, $\varphi(t,x)$ looks like

$$\begin{cases} \varphi(t,x) &= (e_1(t),\ldots,e_p(t),-x)\,, \\ \varphi(t+\delta,x) &= \varphi(t,x), \quad \forall x. \end{cases} \tag{3.3}$$

where δ is the period. Then, the model stated in equation (3.1) can also be written as follows

$$\mathrm{d}X_t = \varphi(t,X_t)\cdot\boldsymbol{\vartheta}\,\mathrm{d}t + \mathrm{d}L_t, \quad X_0 = x_0, \quad \mathbb{P}-a.s. \tag{3.4}$$

with $(L_t)_{t\geqslant 0}$ the Lévy error term. Under the equivalent martingale measure $\widetilde{\mathbb{P}}$, the equation (3.4) can be written as follows

$$\mathrm{d}\widetilde{X_t} = \varphi(t,\widetilde{X_t})\cdot\boldsymbol{\vartheta}\,\mathrm{d}t + \mathrm{d}\widetilde{L}_t, \quad X_0 = x_0, \quad \widetilde{\mathbb{P}}-a.s. \tag{3.5}$$

where

$$\mathrm{d}\widetilde{L}_t = \left[\sigma_t\eta - \lambda e^a J\int z\nu(\mathrm{d}z)\right]\mathrm{d}t + \sigma_t\mathrm{d}\widetilde{W}_t + J\mathrm{d}\widetilde{Y}_t, \quad \widetilde{\mathbb{P}}-a.s. \tag{3.6}$$

Then, the transformation of X_t into $\widetilde{X_t}$ only comes from the change of measure effect on error term $\mathrm{d}L_t$ that gives $\mathrm{d}\widetilde{L}_t$, corresponding to adjustment of market equilibrium. Actually, if $\boldsymbol{\theta}$ is the vector of parameters of $\widetilde{L}_t$, all the parameters in model (3.5) will be combined as vector $\boldsymbol{\Theta} = (\boldsymbol{\vartheta},\boldsymbol{\theta})$.

3.3.2 First step: Estimation of equilibrium parameters

The parameters $\mu_1,\ldots,\mu_p,\kappa$ characterize the equilibrium state. Franke and Kott [Franke 2013] have applied least square method to estimate periodic long-run mean of Lévy-driven Ornstein-Uhlenbeck process. The model in expression (3.2) is Lévy-driven Ornstein-Uhlenbeck process and its drift term can be expressed as linear combination of the parameters $\mu_1, \ldots, \mu_p, \kappa$.

Assume that the real-valued functions $e_1(t), \ldots, e_p(t)$ form an orthonormal system in $\mathbf{L}^2\Big([0,\delta], \frac{1}{\delta}\mathrm{d}\mu\Big)$; else, without loss of generality they can be orthogonalized via Gram-Schmidt orthogonalization algorithm,

$$\int_0^\delta e_\ell(t)e_k(t) = \begin{cases} \delta, & \ell = k \\ 0, & \ell \neq k. \end{cases} \tag{3.7}$$

Natural choices[3] for real-valued functions $e_1, \ldots, e_p$ are trigonometrical functions. For $0 \leqslant k \leqslant \lfloor p/2 \rfloor$

$$e_{2k}(t) = \sin\left(\frac{2\pi k t}{\delta}\right) \quad \text{and} \quad e_{2k+1}(t) = \cos\left(\frac{2\pi k t}{\delta}\right).$$

These functions satisfy the conditions in (3.7) and they are orthogonal in $\mathbf{L}^2\Big([0,\delta], \frac{1}{\delta}\mathrm{d}\mu\Big)$. Furthermore, assume that the total period spanned by observations T is a multiple of period δ,

$$T = n\delta, \quad \text{and} \quad n \in \mathbb{N}. \tag{3.8}$$

The least square estimator of the drift parameters of Lévy driven Ornstein-Uhlenbeck process is based on discretization of stochastic differential equation in (3.4),

$$\Delta\widetilde{X}_{(j+1)\Delta t} = \varphi(j\Delta t, \widetilde{X}_{j\Delta t})\boldsymbol{\vartheta}\,\Delta t + \Delta\widetilde{L}_{(j+1)\Delta t}, \quad j = 0, \ldots, n \tag{3.9}$$

[3]The functions $e_{2k}(t) = \sin(kt)$ and $e_{2k+1} = \cos(kt)$, $0 \leqslant k \leqslant \lfloor p/2 \rfloor$ are the basis of Fourier expansion, which is used to represent any squared integrable function. So any periodic drift can be approximated by a linear combination of these functions.

and $\boldsymbol{\vartheta}$ is found using the following objective function

$$h : \boldsymbol{\vartheta} \longmapsto \sum_{j=0}^{n} \left(\Delta \widetilde{X}_{(j+1)\Delta t} - \varphi(j\Delta t, \widetilde{X}_{j\Delta t}) \boldsymbol{\vartheta}\, \Delta t \right)^2 \tag{3.10}$$

where $n = \lfloor T/\Delta t \rfloor - 1$. The authors in [Franke 2013] have shown that, under the assumptions (3.7) and (3.8), the solution that minimizes $h(\boldsymbol{\vartheta})$ is

$$\widehat{\boldsymbol{\vartheta}} = Q_{T,\Delta t}^{-1} R_{T,\Delta t} \tag{3.11}$$

and will converge, in probability, to continuous-time least square estimator that is consistent, and asymptotically normal,

$$\boldsymbol{\vartheta} = Q_T^{-1} R_T. \tag{3.12}$$

In the expression (3.12), the matrix Q_T is given by

$$Q_T = \begin{pmatrix} T I_p & -r_T \\ -r_T' & v_T \end{pmatrix} \tag{3.13}$$

with I_p the $p-$dimensional identity matrix, r_T and v_T are defined as follows

$$r_T = \left(\int_0^T e_1(t) \widetilde{X}_t \mathrm{d}t, \dots, \int_0^T e_p(t) \widetilde{X}_t \mathrm{d}t \right)' \quad \text{and} \quad v_T = \int_0^T \widetilde{X}_t^2 \mathrm{d}t.$$

$R_T \in \mathbb{R}^{p+1}$ is the vector given by

$$R_T = \left(\int_0^T e_1(t) \mathrm{d}\widetilde{X}_t, \dots, \int_0^T e_p(t) \mathrm{d}\widetilde{X}_t\,, -\int_0^T \widetilde{X}_{t-} \mathrm{d}X_t \right)'. \tag{3.14}$$

Note that, this convergence comes from the càdlàg property of $(\widetilde{X}_t)_{0 \leqslant t \leqslant T}$

and as $\Delta t \to 0$, it holds for any $\ell, k \in \{1, \dots, p\}$

$$\sum_{j=0}^{n} e_\ell(j\Delta t) e_k(j\Delta t)\Delta t \longrightarrow \int_0^T e_\ell(t) e_k(t)\mathrm{d}t$$
$$\sum_{j=0}^{n} e_\ell(j\Delta t)\Delta \widetilde{X}_{(j+1)\Delta t} \longrightarrow \int_0^T e_\ell(t)\mathrm{d}\widetilde{X}_t.$$

3.3.3 Second step: fitting the residuals

Consider the expression in (3.6) that constitute the residuals the from first step estimation. Assume that the jump sizes follow exponential law of parameter b,

$$Z_k \sim \exp(b), \quad \text{with} \quad \nu(\mathrm{d}z) = be^{-bz}, \quad b > 0, z \geqslant 0. \tag{3.15}$$

Then, the vector of parameters for residuals in (3.6) is

$$\boldsymbol{\theta} := (\alpha, \beta, \sigma, \eta, \lambda, a, b) \tag{3.16}$$

A jump-diffusion models can be estimated using filtering methods when the closed form maximum likelihood function is intractable. Therefore, we apply a filtering approach that aims to reproduce the distributional property of parameter based on the observations. Such methods are based on Bayesian approaches and will include Kalman filter, particle filter, MCMC among others. The Kalman filter is applied in Gaussian noise context and will not be recommended in presence of jump as in this case. The particle filter is an alternative to Kalman filter where the error term is not Gaussian.

Recall that in a Bayesian approach, the parameters are considered as latent random variables, $\boldsymbol{\theta} := (\boldsymbol{\theta}_t)_{0 \leqslant t \leqslant T}$, and the posterior distribution is of interest

$$p(\boldsymbol{\theta}_{0:n}|\widetilde{L}_{0:n}) = \frac{p(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})p(\boldsymbol{\theta}_{0:n})}{p(\widetilde{L}_{0:n})}, \tag{3.17}$$

with $\widetilde{L}_{0:n}$ and $\boldsymbol{\theta}_{0:n}$ respectively stand for the vector of join random variables

$(\widetilde{L}_{t_0}, \ldots, \widetilde{L}_{t_n})$ and $(\boldsymbol{\theta}_{t_0}, \ldots, \boldsymbol{\theta}_{t_n})$.

In expression (3.17), $p(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})$ is the exact likelihood function with the resulting posterior distribution form Bayes formula $p(\boldsymbol{\theta}_{0:n}|\widetilde{L}_{0:n})$ and $p(\widetilde{L}_{0:n})$ is data marginal density function defined as

$$p(\widetilde{L}_{0:n}) = \int p(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n}) p(\boldsymbol{\theta}_{0:n}) \mathrm{d}\boldsymbol{\theta}_{0:n}.$$

Bayesian inference is addressed for model parameters' estimation with the assumption of the prior density function $p(\boldsymbol{\theta}_{0:n})$ that expresses the personal beliefs and knowledge of the phenomenon. Instead, filtering does focus on sampling from the prior density function $p(\boldsymbol{\theta}_{0:n})$. Importance sampling algorithm such as *Sequential Importance Sampling* (SIS) or *Sequential Importance Resampling* (SIR) belong to class of Monte Carlo sequential methods that are used in calculation of the posteriori distribution. In particle filter methods, a collection of weighted points, called particles, are generated recursively to approximate the distribution $p(\boldsymbol{\theta}_{0:n}|\widetilde{L}_{0:n})$. A particle is computed as soon as an observation is available and this makes the implementation of particle filter computationally intensive and may become cumbersome.

An alternative approach is the particle Markov chain Monte Carlo (PMCMC) where the $p(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})$ is replaced by its particle filter approximation $\widehat{p}(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})$. The statistical theory underlying method is detailed in Andrieu et al. [Andrieu 2010] which gives the exact approximations. The particle MCMC aims to combine particle filter and MCMC method. The main idea of PMCMC is to use Sequential Monte Carlo methods to propose some $\boldsymbol{\theta}_{0:n}$ in a Metropolis-Hastings algorithm targeting the posterior distribution of $\boldsymbol{\theta}_{0:n}$. We apply particle marginal Metropolis-Hastings (PMMH) algorithm (in Andrieu et al. [Andrieu 2010]) that leaves $p(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})$ invariant with ergodic sample under weak assumptions.

Assume that sampling from the conditional density $p(\boldsymbol{\theta}_{0:n}|\widetilde{L}_{0:n})$. It is natural to suggest a proposal density for the Metropolis-Hastings update of

$$q(\boldsymbol{\theta}'_{0:n}|\boldsymbol{\theta}_{0:n}) = q(\boldsymbol{\theta}'|\boldsymbol{\theta})p(\widetilde{L}_{0:n}|\boldsymbol{\theta}') \tag{3.18}$$

so that the only degree of freedom in the algorithm is $q(\boldsymbol{\theta}'|\boldsymbol{\theta})$. Therefore, the Metropolis-Hastings acceptance ratio is given by

$$\min\left\{1, \frac{\widehat{p}(\widetilde{L}_{0:n}|\boldsymbol{\theta}_{0:n})p(\boldsymbol{\theta}_{0:n})q(\boldsymbol{\theta}'|\boldsymbol{\theta})}{\widehat{p}(\widetilde{L}_{0:n}|\boldsymbol{\theta}'_{0:n})p(\boldsymbol{\theta}'_{0:n})q(\boldsymbol{\theta}|\boldsymbol{\theta}')}\right\} \tag{3.19}$$

3.3.4 Model validation

In model specification for asset prices, there is always a misspecification risk that could impact an investment strategy. In that case, model validation tools help to decide the consistency of the estimations on real data. We will apply residuals based test to check for parameter consistency on data at hands.

This consists in testing the residuals for stationary white noise. That is to check for zero autocorrelation for non-zero lags using Box-Pierce or Ljung-Box statistical tests. Under the null hypothesis that $\{\varepsilon_j\}$ is a sequence of independent and identically distributed random variables, the two corresponding test statistics have asymptotic chi-square distributions.

3.4 Application on agricultural futures prices

We implement the above described two-stage procedure to estimate the parameters for mean-reverting jump-diffusion in Matlab 2014. The first step is simple ordinary least square model on nearby futures prices to derive the parameters of drift term. Then, estimation in the second step on the residuals follows with application of PMMH algorithm from Andrieu et Al. [Andrieu 2010].

Recall that the stochastic process in expression (3.2) is affine jump-diffusion.

Then using the affine transformation of Duffie et al. [Duffie 2000], the model of log futures prices is given by (Aiube et al. [Aiube 2008])

$$\widetilde{X}_{t,M} = \mu_M + e^{-\kappa(M-t)}\widetilde{X}_{t,M} + A(M-t) + B(M-t) \tag{3.20}$$

where

$$\begin{aligned} A(M-t) &= -\frac{\eta}{\kappa}\left(1 - e^{-\kappa(M-t)}\right) + \frac{\sigma_t^2}{4\kappa}\left(1 - e^{-2\kappa(M-t)}\right) \\ B(M-t) &= \lambda e^a J \int_t^M \left[\exp\left(\tfrac{1}{b}e^{-\kappa(M-z)} + \tfrac{1}{2b^2}e^{-2\kappa(M-z)}\right) - 1\right] \mathrm{d}z \end{aligned}$$

3.4.1 Mean-reversion estimation: speed and periodic long-run means

The first step in estimation procedure gives the values of equilibrium parameters $\widehat{\boldsymbol{\vartheta}} = (\widehat{\mu}_1, \ldots, \widehat{\mu}_p, \widehat{\kappa})$ for each commodity. These values are reported in Table 3.1 where we only consider the significant values according to Student test, otherwise, a dash mark is reported in lieu of a value. Note that the speed of mean-reversion is the same for all maturities while the values of long-run mean clearly show consecutive up and down levels.

3.4.2 Estimates: fitting the residuals

In the second step, the values of $\widehat{\kappa}$ from the first step estimation as well as the jump scale parameter J are plugged as scalars. Specifically, we set $J = 0.8$ and will apply the PMMH algorithm as particle MCMC methods to obtain the estimates of $\boldsymbol{\theta} = (\alpha, \beta, \sigma, \eta, \lambda, a, b)$. The filtering process starts with initial vector $\boldsymbol{\theta}_0$.[4] At each time step, state variables are filtered and will constitute a collection of observations to be used as sample. Then, applying maximum likelihood estimation on filtered state variables, one can derive the target parameters. The Berndt-Hall-Hall-Hausman (BHHH) algorithm

[4]Optimizers are usually sensitive to initial conditions, so Monte Carlo can be used to choose a number of starting points in the solution space like in EM algorithm.

Table 3.1: Speed and periodic long-run mean of mean reversion

Commodity		κ	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
Corn	C1	0.01200	0.00027	-0.00006	-0.00016	-0.00029	0.00206	
	C2		0.00047	-0.00010	-0.00041	-0.00037	0.00204	
	C3		0.00035	0.00003	-0.00035	-0.00033	0.00201	
	C4		0.00034	0.00014	-0.00047	-0.00024	0.00192	
	C5		0.00025	0.00001	-0.00057	-0.00032	0.00184	
	C6		0.00024	0.00009	-0.00032	-0.00026	0.00175	
Oat	O1	0.00761	0.00016	0.00008	0.00082	-0.00067	0.00144	
	O2		0.00037	-0.00024	0.00031	-0.00011	0.00107	
	O3		0.00039	-0.00026	0.00037	0.00019	0.00094	
Rough rice	RR1	0.00206	0.00056	-0.00009	-0.00017	0.00051	0.00090	
	RR2		0.00068	-0.00002	0.00010	0.00043	0.00078	
	RR3		0.00049	0.00021	0.00031	0.00042	0.00072	
Soyebans	S1		-	-	-	-	-	-
	S2		-	-	-	-	-	-
	S3	0.00571	-0.00009	-0.00019	-0.00001	0.00012	0.00005	-0.00027
	S4		-0.00005	-0.00014	0.00003	0.00003	-0.00004	-0.00032
	S5		-0.00008	-0.00021	-0.00001	0.00007	-0.00010	-0.00039
	S6		-0.00016	-0.00012	-0.00007	0.00011	-0.00008	-0.00031
Wheat	W1	0.00839	0.00002	-0.00120	0.00059	0.00039	0.00139	
	W2		0.00021	-0.00103	0.00075	0.00035	0.00116	
	W3*	-	-	-	-	-	-	
	W4		-0.00021	-0.00088	0.00059	0.00044	0.00101	
	W5		0.00003	-0.00086	0.00045	0.00039	0.00087	
	W6*	-	-	-	-	-	-	
Cocoa	QC1	0.00627	-0.00041	0.00010	-0.00031	-0.00064	0.00087	
	QC2		-0.00036	0.00001	-0.00043	-0.00057	0.00078	
	QC3		-0.00034	-0.00004	-0.00052	-0.00053	0.00075	
	QC4		-0.00030	-0.00009	-0.00054	-0.00050	0.00073	
	QC5		-0.00032	-0.00014	-0.00054	-0.00047	0.00077	
	QC6		-0.00033	-0.00009	-0.00048	-0.00053	0.00071	
Coffee	KC1	0.00818	0.00017	-0.00012	0.00008	-0.00005	0.00173	
	KC2		0.00026	-0.00002	0.00013	0.00011	0.00167	
	KC3		0.00022	-0.00010	0.00017	0.00005	0.00155	
	KC4		0.00025	-0.00012	0.00021	0.00002	0.00151	
	KC5		0.00029	-0.00015	0.00024	-0.00001	0.00150	
	KC6		0.00024	-0.00021	0.00021	-0.00004	0.00150	
Cotton	CT1	0.00876	0.00006	-0.00027	-0.00023	0.00003	0.00209	
	CT2		-0.00022	-0.00004	-0.00010	-0.00003	0.00190	
	CT3		0.00000	-0.00020	0.00002	-0.00016	0.00208	
	CT4		0.00008	-0.00008	-0.00018	-0.00028	0.00214	
	CT5		-0.00007	-0.00007	-0.00018	-0.00021	0.00218	
	CT6		0.00008	-0.00005	-0.00006	-0.00024	0.00208	

[*] In some case, the matrix Q_T in expression (3.13) is singular.

is appropriate to give more consistent results for the likelihood function optimization.

Parameter estimates are all significant at level 5% as well as the values of Ljung-Box test statistic that suggest not to reject the null hypothesis of independence residuals at same level of significance. Furthermore the parameter estimates are closed to empirical estimates of Chapter 2.

Table 3.2: Lévy noise parameters

Commodity	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\sigma}$	$\widehat{\eta}$	$\widehat{\lambda}$	$\widehat{a}$	$\widehat{b}$	$Q(10)$
Corn	0.9112	0.0014	0.3174	0.0613	0.2106	0.0022	0.806	3.741
	[0.015]	[0.315]	[0.049]	[0.218]	[0.114]	[0.036]	[0.216]	
Oat	1.6303	0.0009	0.3208	0.0570	0.1301	0.0016	0.320	2.938
	[0.156]	[0.036]	[0.116]	[0.326]	[0.514]	[0.374]	[0.344]	
Rough rice	2.1873	0.0035	0.2435	0.0093	0.1035	-0.0013	0.402	2.249
	[0.336]	[0.245]	[0.255]	[0.349]	[0.514]	[0.054]	[0.284]	
Soybeans	0.7811	-0.0003	0.2934	0.0014	0.1407	-0.0018	0.692	1.951
	[0.279]	[0.415]	[0.219]	[0.237]	[0.614]	[0.128]	[0.009]	
Wheat	2.0230	0.0088	0.3425	0.0071	0.3007	0.0053	0.984	3.938
	[0.195]	[0.364]	[0.105]	[0.037]	[0.259]	[0.025]	[0.006]	
Cocoa	1.4914	0.0002	0.2891	0.0915	0.0952	-0.0021	0.315	1.604
	[0.278]	[0.124]	[0.096]	[0.057]	[0.150]	[0.196]	[0.514]	
Coffee	0.8457	0.0368	0.2984	0.0071	0.1203	0.0018	0.575	2.385
	[0.312]	[0.052]	[0.098]	[0.210]	[0.209]	[0.201]	[0.354]	
Cotton	2.0225	0.0016	0.2624	0.0009	0.3142	0.0119	0.475	2.080
	[0.405]	[0.421]	[0.354]	[0.319]	[0.321]	[0.322]	[0.136]	

The numbers in brackets are the standard deviation of estimate. For the Ljung-Box test, the quantile at 5% is 3.940.

3.5 Conclusion

Futures prices of agricultural commodities have been modeled as a mean-reverting jump-diffusion process with seasonality in both long-run mean and volatility. Typically, the mean-reverting process is a time varying trend with error term. Specifically for the futures prices at hands the errors term is jump-diffusion to address the non skewness and kurtosis effect as well as sudden price variation in the market. As the mean-reverting jump-diffusion process is affine jump-diffusion, we follow the transformation approach of Duffie et al. [Duffie 2000] to state the model. The estimation of parameters on futures prices data at hands is conducted in two-stage procedure due to periodic long-run mean. This permits to obtain the values for speed and periodic long-run mean in the first step using least square technique. Then, the residuals from the first step are used as input data to estimate the remaining parameters with PMCMC method. Finally, the portemanteau test of Ljung-Box is applied to show consistency of futures price model.

For all the selected commodities, the long-run varies with the delivery month with consideration of the up and down levels caused by supply and demand imbalance in mean-reversion. This allows to use mean-reversion as tool both to identify the trading range for a futures price and to compute the average price for the specific period that will take into account profits and losses. As illustration, when the current futures price is higher than the average price, the commodity is attractive for hedging. But this leads to higher supply later which will push prices down. The speed of reversion is an alert of how the market reverts to average price. Corn, wheat and cotton seem to revert faster than other commodities with significant volatility and jump intensity.

Chapter 4

Static Hedging

Abstract: We deal with the hedging issue for a producer in presence of market and production risks. We derive an optimal strategy to tackle the additional risk due to rolling over the futures contracts. In practice, these risks include market risk and mainly production risk within the inter-crop season. We address how an insurance contract could enhance the futures hedging onto further guarantee the producer revenue. In static framework, many hedging strategies have been developed in the literature but the existing measures of effectiveness are lacking. We apply ranking-based approach for various hedging strategies. This uses L-moment and will allow to rank the hedge portfolios with regard to their performance. The application on futures prices data at hands shows that taking into account market and production risks leads to better hedging strategy based on the L-performance effectiveness measure.

Contents

4.1 Introduction

The establishment of public commodity markets in the 19^{th} century has improved standardization, transparency, efficiency, as well as hedging for physical good prices. A futures contract is a standardized instrument that allows to transfer the risk of commodities from hedgers (for instance producers) to speculators which will bear these risks until delivery date. At any given time, a futures price reflects either the price expectations of buyers and sellers of a good at delivery in the future and will also contribute to establish a balance between production and consumption or the additional information about supply and demand as it becomes available. Hence, futures price is a forecast of spot price subject to changes that it is adjusted for.

Hedging in futures markets consists in a counterbalancing transaction involving a position in the futures contract that is the opposite of the position in the cash market. In agriculture, producers incur the risk of substantial lost coming from uncertainties related to adverse price variations of their crop due to bad weather, storage. These risks are also strongly correlated with the quantity, cost and quality of production. Thus, taking a counterbalanced position in futures markets provides a hedge for the good and the producer will be guaranteed to receive, at maturity of the futures contract, a predetermined price.[1] The position in futures market has to be set appropriately to reduce the risks, as much as possible, at maturity of futures contract to make the hedging strategy optimal. The choice of appropriate position in futures contract is decision making problem like portfolio management where the producer holds a non-tradable from planting time to harvest time.

A large body of literature has investigated the problem of optimal hedging strategy in futures markets. Various techniques either minimize a risk function or maximize an expected utility function of wealth in static or dynamic

[1] Alternative ways to hedge commodity production consist in stabilization funds or Government supports, see for example in Modest and Marcus [Marcus 1984].

framework. Whatever the framework, the main difference between the two approaches is that minimizing a risk function will result in pure hedge while maximizing also includes a speculative component according to risk aversion. Such risk functions include variance, semi-variance for downside risk which is generalized as lower partial moments (Chen et al. [Chen 2001] and Lien and Tse [Lien 2000]), while examples of expected utility maximization are mean-variance, mean-Gini (Shalit and Yitzhaki [Shalit 1984]). For instance, Rolfo [Rolfo 1980] had used mean-variance technique to derive optimal hedge ratio under price risk and output risk for exporter countries in agricultural markets. Besides, other methods that rely on stochastic dominance concept have been investigated.

Recall that in static framework, decision making problem relies on two dates: the initial time when the decision has to be made for the final time of delivery. The hedging strategy is dependent of the optimization technique and could be more or less effective. According to Chen et al. [Chen 2013], there is no single optimal hedge ratio that is distinctly superior to the others unless an appropriate criterion is defined. The performance measures of hedging strategies like Ederington [Ederington 1979] effectiveness, Sharpe-type ratio of Howard and D'Antonio [Howard 1987] and certainty equivalent are also misleading due to their downward bias. Then, they imply under-reported hedging strategy. To enhance the hedging performance measure, we apply L-moment to provide the ranking of different hedging strategies.

Furthermore, when the hedging horizon is longer than the futures contract maturity, the position in futures market can be maintained by holding a contract until near the maturity. This process is known as rollover strategy. It consists in closing out the active position and taking a new position in futures contract of longer maturity. The rollover incurs the risks that would not arise if the position were in a single long term futures contract. Indeed, depending on the market situation (backwardation or contango), the strategy is affected by the spread between the old contract and the new contract of longer maturity, and this gives rise to rollover risk. Particularly, the basis is further affected by the additional risk of the rollover strategy. Such risk mainly includes production risk within the inter-crop periods. Gardner [Gardner 1989] had dealt with rollover hedging when long term futures market is missing with constant output while Baesel and Grant [Baesel 1982] derived optimal sequential strategy for quantity risk. We analyze an optimal hedging strategy for both price and quantity risks that takes into account the rollover process. In so doing, the hedging approaches of Gardner [Gardner 1989] and Baesel and Grant [Baesel 1982] are combined on rollover issue. This results into optimal hedging strategy for price risk and quantity risk in term of basis hedging and production covered by insurance payoff. Finally, using the L-performance measure shows the superiority of this strategy over other strategies.

This Chapter is organized as follows. The first section states the issue of hedging strategies by and its motivations. The second one describes the existing approaches of hedge strategies in the portfolio context, the hedging effectiveness measures and therefore the shortcomings of these hedge ratios. The investigation of the hedging effectiveness measures presents a way to select the hedging strategy by ranking the hedge portfolios' performance. In the third section, optimal hedging strategy is derived, in term of basis risk for quantity risk along with the rollover process. The section four is devoted to applications on commodity data followed by a conclusion.

4.2 Related works on hedging with futures markets

Hedging belongs to risk management that fundamentally consists in transforming a state of nature to new one compatible with the expectations of economic agents. To this end, a financial hedge consists in specific position of an asset that should reduce, as much as possible, the risk incurred with another existing position. This risk comes from the mismatch between the underlying and the hedging instrument and referred to as basis risk. Specially, hedging with futures contract allows the producer to transfer his risks in market to speculators that will bear them for a premium. Commodity futures are the simplest hedging instrument and their first motive as hedging instrument lies in the need for optimal balance between tolerable risk and return that reduces risks in price, in production as well as in storage.

Many papers in the financial literature have investigated the use of futures contracts to offset uncertainties in commodity markets. The research stream on futures hedging had started with *Price Insurance Theory* and has analyzed the hedging as a way to avoid loss due to any price move related to positions in futures markets. As economic rationale for hedging, Keynes [Keynes 1930], Hicks [Hicks 1939] and Kaldor [Kaldor 1939] argued that the producers shift the risk to speculators by paying a premium. So did Working [Working 1962] for risk insurance[2], but had firstly advocated on earning returns theory where a sort of arbitrage is to enter the market only when the producer perceives a promising opportunity for profit. That is to say, a decision for hedging could also include speculation purpose and does not have to be limited to pure risk hedging only.

Later on, *Portfolio Theory* approach of Markowitz has been applied to futures hedging in order to investigate the producer's risk-return trade-off. Thus, the hedge portfolio considers in priority the asset to be hedged, the non-traded position, then the futures contract as hedging instrument. This statement can be found among other, in Rolfo [Rolfo 1980] and Ho [Ho 1984] in respectively static dynamic frameworks. The producer is then maximizing the expected utility of his wealth. For active markets, the hedging can include other traded assets to derive optimal hedging strategy like

[2] "The reason for producers to have their orders executed expeditiously is to reduce the interval in which their inventories are left uncovered, exposed to the risk of price change", Pennings and Leuthold [Pennings 2000]

in Adler and Detemple [Adler 1988a]. However, Pennings and Leuthold [Pennings 2000] noticed that William[3] had stressed the difficulty for the portfolio approach to diversify the risk in production, transport and processing (commodity availability) that inventories absorb and which will also motivates the use of futures contracts. In addition, the portfolio theory in hedging assumes the initial position of inventories to be unhedged is extremely sensitive to the predetermined position. Aside, pure insurance and portfolio approaches to hedging, there is *Loan Markets Theory* and *Liquidity Theory*.[4] Kamara [Kamara 1982] argued that the three theories contribute, in explaining why producers hedge: “the producer’s position in futures is motivated partially by the desire to stabilize income and partially by the desire to increase the expected profits”. Meanwhile, it is clear that all the approaches are all based on optimization techniques.

The literature of futures hedging also focuses on the optimization method applied to derive the static ratio. Among the various optimization techniques, the minimum-variance serves as reference for comparison with other optimization approaches. The minimum-variance approach penalizes both upside and downside deviation of returns from the mean. For instance, an agricultural producer that wants to hedge his business is much more worried by the downside shock from a target level of revenues than the upside deviation. Hence, minimum-variance hedge ratio may lead to suboptimal hedging recommendations. The mean-variance approach is consistent with expected utility theory if all uncertainty factors satisfy the location-scale[5] condition. To overcome this shortcoming, alternative approaches consistent with stochastic dominance concept have been developed. They include mean-extended-Gini (MEG), lower partial moment (LPM), Value-

[3]Williams J. C., The economic function of futures markets, Cambridge University Press, 1986.

[4]Other motivations for hedging with futures markets relate to loan markets theory and to liquidity theory. Loan markets theory refers to hedging operation by getting the accessibility for a period of time while liquidity theory is the provision that organized markets facilitate.

[5]Normal distribution is typically assumed but will not be realistic in practice.

at-Risk (VaR) and conditional Value-at-Risk (CVaR). Sequential approaches have been also investigated to take into account period and horizon effects on hedge ratios. For instance, Cecchetti et al. [Cecchetti 1988], Chen et al. [Chen 2013] and Lien and Luo [Lien 1993] derived hedge ratio in multi-period analysis, Baillie and Myers [Baillie 1991] based their analysis on conditional distribution approach (ARCH: autoregressive conditional heteroskedastic and GARCH: generalized ARCH) and Fernandez [Fernandez 2008]; Conlon and Cotter [Conlon 2012] applied wavelet decomposition to derive hedge ratio according to hedging horizon.

4.3 Some hedging approaches in static framework

We consider agricultural farmers that plant and will harvest only one commodity that requires all his financial resources so they do not diversify their wealth into other crops or financial assets. Every producer faces uncertainty with respect to both price and output and, for example, could sell futures contracts against his expected harvest, but find his actual harvest either higher or lower than expected. The farmer could hedge the risk associated to his crop through futures contracting. Then, what matters is the decision of the optimal hedge ratio that will lower as much as possible the incurred risks.

Formally, the farmer's optimal hedging problem can be stated as follows. Consider an hedging period with initial date, $t = 0$, and final date, $t = T$. At time $t = T$, the producer is expecting to sell his production of unknown quantity Q_T, at prevailing spot price S_T. Assume that there is exists an active futures market. We also consider the futures contract to live over the time horizon $[0, T]$. Denote by F_0 the futures price at time $t = 0$ and by F_T the futures price at T. If the hedging horizon is longer than T, the futures contract will be rolled over to the next period and this analysis is addressed in section 4.4. For now, the issue of the farmer, is to find the hedging strategy that will provide the optimal hedge for both price and output risks. This consists in deciding, at initial date, the position, x, in futures market. The farmer's portfolio modifies from $S_T Q_T$ to the hedge portfolio defined as follows

$$\mathcal{W}_T = S_T Q_T - x \Delta F_T, \tag{4.1}$$

where ΔF_T stands for $F_T - F_0$, the net variation of futures price with the hedging horizon. Equivalently, the hedge portfolio return is given by

$$R_h = \frac{S_T Q_T R_s - x F_T R_f}{S_T Q_T} = R_s - h R_f, \tag{4.2}$$

where R_s and R_f are respectively the spot and futures returns with

$$R_s = \frac{S_T - S_0}{S_0} \qquad \text{and} \qquad R_f = \frac{F_T - F_0}{F_0}$$

and h the hedge ratio defined by

$$h = \frac{x F_T}{S_T Q_T}. \tag{4.3}$$

There are various approaches to derive the hedge ratio, h, that rely on producer's preference according to either risk psychology or risk ordering. The risk psychology stream relates to expected utility that involves coefficients like aversion, prudence and temperance while risk ordering stream relies on stochastic dominance concept. In context of agricultural farmer, we recall the various hedge ratio according to these two streams.

4.3.1 Hedge ratios based on risk psychology

The basic hedge ratio minimizes the variance of the hedge portfolio returns R_h and had been introduced by McKinnon [McKinnon 1967] for a commodity producer. The minimum-variance hedge ratio, that we denote $h^*_{min\mathrm{V}}$, gives the position in futures contract that will make the hedge portfolio variance as small as possible to reduce the risk incurred in spot price return at maturity T. The minimum-variance hedge ratio is the solution of the following program

$$\min_h \mathbb{V}[\mathcal{W}_T] \qquad \Longrightarrow \qquad h^*_{min\mathbb{V}} = \frac{\mathbb{C}\mathrm{ov}\,[R_s, R_f]}{\mathbb{V}[R_f]} \tag{4.4}$$

This hedge ratio is pure hedge and it does not account for the portfolio expected return that allows for speculative component in the same time. The shortcoming of minimum-variance hedge ration is that, in case of multiple contracts, it is pronounced on low volatility contracts at the expense of exploiting correlation properties (Stoyanov [Stoyanov 2011]). Therefore, the mean-variance is generally seen as the extension (see for example Anderson and Danthine [Anderson 1981], Duffie [Duffie 1989]) of minimum-variance strategy that takes into account the producer's preference in terms of portfolio return and risk aversion γ.

The mean-variance is also referred to as quadratic utility when returns distribution is normally distributed. It is derived by maximizing the expected quadratic utility function,

$$\max_h \left\{\mathbb{E}[R_h] - \frac{\gamma}{2}\mathbb{V}[R_h]\right\} \quad \Rightarrow \quad h^*_{\mathrm{MV}} = \frac{\mathbb{C}\mathrm{ov}\,[R_s, R_f]}{\mathbb{V}[R_f]} - \frac{\mathbb{E}[R_f]}{\gamma\mathbb{V}[R_f]}. \tag{4.5}$$

where h^*_{MV} is composed of the pure hedge component, $h^*_{min\mathbb{V}}$, in equation (4.4) and a speculative component. Mean-variance hedge ratio allows the the risk-averse producer can hedge his income variability on the futures markets by buying or selling futures.

The amount of futures for speculation is determined by risk aversion and futures price variability. The speculative component position then converges towards zero with infinite risk aversion ($\gamma \to \infty$) or if the futures price process is martingale ($\mathbb{E}[F_T] = F_0$). That is the case where the producer is extremely reluctant to take risks or does not expect any additional return.

Pure speculative strategy holds whenever the spot and the futures are uncorrelated in case of no insurance motive. Futures trading by producers results from a mixture of hedging and speculative motives.

Similarly, using Arrow-Pratt approximation of risk premium by second-order Taylor-expansion, the producer's preference as quadratic utility function makes the mean-variance hedge ratio equivalent to the case where the portfolio returns are normally distributed. Subsequently, when the returns are not normally distributed, the mean-variance hedge ratio will be suboptimal and the quadratic utility then becomes unrealistic. In practice, normality assumption fails because fat tails distribution.

An extension of the Arrow-Pratt approximation to fourth-order Taylor expansion leads to coefficients of prudence and temperance that are respectively associated to third and fourth moments.[6] The corresponding expected utility maximization program is given by

$$\max_h \left\{ \mathbb{E}[R_h] - \frac{\gamma}{2}\,\mathbb{V}[R_h] + \frac{\chi}{6}\,\mathbb{M}_3[R_h] - \frac{\psi}{24}\,\mathbb{M}_4[R_h] \right\}, \tag{4.6}$$

where $\mathbb{M}_3$ and $\mathbb{M}_4$ are respectively third and fourth centered moments and the coefficients χ and ψ express, respectively the taste for asymmetry and aversion to fat tails. However, the optimization problem in program (4.6) is usually solved numerically. Instead, by using skewness and kurtosis in place of third and fourth moments the maximization program becomes

$$\max_h \left\{ \mathbb{E}[R_h] - \frac{\zeta}{2}\,\mathbb{V}[R_h] + \varphi\, s_3(R_h) - \frac{\xi}{2}\, s_4^2(R_h) \right\}, \tag{4.7}$$

with s_3 and s_4 being skewness and kurtosis operator respectively with modified set of coefficients for risk psychology. The hedge ratio, solution of the

[6] Alternative method based on higher moments to derive the hedge ratio without Taylor expansion has been developed in from Brooks et al. [Brooks 2012]

above program is as follows (Le Courtois and Walter [Le Courtois 2014])

$$h^*_{\text{SK}} = \frac{\mathbb{E}[R_h] + \varphi s_3(R_h)}{\zeta V[R_h] - \xi s_4^2(R_h)}. \tag{4.8}$$

This has similarity with the mean-variance hedge ratio, h^*_{MV}, in equation (4.5). The hedge ratio h^*_{SK} includes asymmetry and fat tails influences on respectively the mean and the variance of the hedge portfolio. Indeed, skewness and kurtosis together capture risk distribution of the hedge portfolio in that skewness indicates difference between profits and losses and kurtosis the occurrence of extreme events.

Other utility function can be used to derive hedge ratio in static framework. For instance, Rolfo [Rolfo 1980] had also considered logarithm preference and suggested futures contract trading as hedging instrument for variability in both the price and the production of its output. Meanwhile, alternative way to deal with hedging problem is to consider risk ordering concept.

4.3.2 Hedge ratio based on risk ordering

The risk ordering approach for hedging strategies corresponds to hedge ratios that are consistent with stochastic dominance concept[7] known to capture the properties of a distribution. Particularly, these hedge ratios rank different hedge portfolios according to preference (with only limited information about the utility function of a particular consumer) with no constraint on taste and aversion or particular distribution. They include mean-extend-Gini, lower partial moment approach (Chen et al. [Chen 2013], Lien and Tse [Lien 2000]) as well as famous risk measures in finance such as Value-at-Risk and Conditional Value-at-Risk. The purpose is then to minimize a risk specific measure.

Let's R_1 and R_2 be two random variables defined on probability space,

[7] The rationales for the stochastic dominance are well documented in Rothschild M. & Stiglitz J. E., "Increasing risk I. A definition". *Journal of Economic Theory*, 2, 225-243; 1970.

$(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geqslant 0}, \mathbb{P})$ with their respective cumulative distribution function,

$$G_1(x) = \mathbb{P}(R_1 \leqslant x) \quad \text{and} \quad G_2(x) = \mathbb{P}(R_2 \leqslant x).$$

Recall that R_1 dominates R_2 by the first-order (respectively by the second-order) stochastic dominance, SD1 (respectively, SD2) if and only if all investors that prefer more to less (respectively are risk-averse) would prefer R_1 to R_2. We denote the SD1 relation by $R_1 \succeq_{\text{SD1}} R_2$ and the SD2 relation by $R_1 \succeq_{\text{SD2}} R_2$. Formally, R_1 is said to first-order stochastically dominate R_2, if

$$G_1(x) \leqslant G_2(x), \, \forall x.$$

R_1 is said to second-order stochastically dominate R_2, if

$$\int_{\infty}^{x} G_1(x)\mathrm{d}x \leqslant \int_{\infty}^{x} G_2(x)\mathrm{d}x, \, \forall x.$$

The first-order stochastic dominance relation corresponds to all choices made by investors with monotonic expected utility function while the second-order stochastic dominance relation is all choices made by risk-averse expected-utility investors. We simply write $R_1 \succeq_{\text{SD1}} R_2$ and $R_1 \succeq_{\text{SD2}} R_2$ whenever R_1 dominates R_2 according to SD1 and SD2 respectively. Besides, the first-order stochastic dominance relation implies the second-order stochastic dominance relation,

$$R_1 \succeq_{\text{SD1}} R_2 \quad \Longrightarrow \quad R_1 \succeq_{\text{SD2}} R_2.$$

4.3.2.1 Mean-extended-Gini hedge ratio

The MEG coefficient is a non-negative, non-decreasing and bounded function of a risk parameter $1 \leqslant \delta < \infty$. Following Shalit and Yitzhaki [Shalit 1984], it can be applied to a hedge portfolio returns, R_h,

$$\Gamma_h(\delta) = \int_a^b (1 - G_h(R_h))\,\mathrm{d}R_h - \int_a^b (1 - G(R_h))^{\delta}\,\mathrm{d}R_h, \tag{4.9}$$

where a, b with $(a \leqslant b)$ are real numbers and G is the cumulative probability distribution of the portfolio return R_h. The parameter δ plays the role of risk aversion as the extend-Gini coefficient can be viewed as risk premium that should be subtracted from the expected value of portfolio. Hence, when $\delta = 1$ the investor is risk neutral and $\Gamma_h(0) = 0$; for a risk-seeker, $0 \leqslant \delta < 1$ and when $\delta > 1$, the investor is risk-averse.

Consider two portfolios, say R_1 and R_2, with their respective returns distribution G_1 and G_2. Let's $(\epsilon_n)_{n\in\mathbb{N}^*}$ be the sequence defined as follows

$$\epsilon_n = \int_a^b (1 - G_1(x))^n \, \mathrm{d}x - \int_a^b (1 - G_2(x))^n \, \mathrm{d}x. \tag{4.10}$$

Yitzhaki and Schechtman [Yitzhaki 2012] have proved that if $\epsilon_n \geqslant 0$, $R_1 \succeq_{\text{SD1}} R_2$ and $R_1 \succeq_{\text{SD2}} R_2$. Consequently, mean-extended-Gini coefficient $\Gamma_\delta(R_h)$ is the risk measure and can be minimized to achieve an optimal hedging strategy[8], h^*_{MEG}. However, as the mean-extended-Gini coefficient in equation (4.9) is difficult to evaluate in practice since there is no explicit analytic formula, Shalit and Yitzhaki [Shalit 1984] have suggested the following expression

$$\Gamma_h(\delta) = -\delta \mathbb{C}\text{ov}\left(R_h \,,\, (1 - G(R_h))^{\delta-1}\right) \tag{4.11}$$

that leads to the optimal hedge ratio (Shalit [Shalit 1995]) given by

$$h^*_{\text{MEG}} = \frac{\mathbb{C}\text{ov}\left(R_s \,,\, (1 - G(R_h))^{\delta-1}\right)}{\mathbb{C}\text{ov}\left(R_f \,,\, (1 - G(R_h))^{\delta-1}\right)}. \tag{4.12}$$

Therefore, the mean-extended-Gini hedge ratio can be estimated under assumption of probability distribution.

[8]The producer can also obtain an efficient set based on each value of δ. The efficient set is progressively reduced when the producer performs the mean-extended-Gini analysis for different values of δ and retains only the intersection of the efficient sets.

4.3.2.2 Hedging with lower partial moment

Lower partial moment belongs to class of downside risk measures. Downside risks only focus on the losses and then considers the worse case scenarios from a target level of revenues. The lower partial moment is characterized by two parameters, the target level return, c, that determines the shortfalls and the power, n, of the shortfalls. The lower partial moment of the hedge portfolio returns, R_h, is defined by

$$\ell_n(c, R_h) = \int_{-\infty}^{c} (c - R_h)^n \, \mathrm{d}G(R_h), \quad n \in \mathbb{N}, \tag{4.13}$$

where the cases $n < 1$ and $n > 1$ characterize, respectively a risk seeking investor and implies risk averse investor (Fishburn [Fishburn 1977][9]). Note that semi-variance is a special case of lower partial moment approach, with $c = 0$ and $n = 2$, $\ell_2(2, \cdot)$.

Furthermore, the lower partial moment satisfies the first and second order stochastic dominance relations and can be used as risk measure. Bawa [Bawa 1978] showed that n^{th} order lower partial moment is consistent with stochastic dominance of the $(n+1)^{th}$ order. Lien and Tse [Lien 2000] had observed that, when $n > 1$, the n^{th} order lower partial moment is given by

$$\ell_n(c, R_h) = \mathbb{E}\left\{[\max(0, c - R_h)]^n\right\}. \tag{4.14}$$

The first order condition with right to the hedge ratio is

$$-n\mathbb{E}\left\{[\max(0, c - R_h)]^{n-1} R_f\right\} = 0,$$

with the second order condition always satisfied (positive).

[9] The Fishburn risk measure has the same form but allows for a non integer, positive power function.

4.3.2.3 Hedge ratio based on VaR and CVaR

From Ogryzak and Ruszczyński [Ogryczak 2002] Value-at-Risk and conditional Value-at-Risk (hence, from now on VaR and CVaR) satisfy SD1 and SD2 properties respectively. VaR and CVaR belong to the class of downside risk measures when dealing with hedging. They measure the "potential losses" associated with a risky position on a predefined horizon, at a given risk level $\alpha \in (0,1)$. Specially, VaR indicates the potential loss of amount at probability $1-\alpha$ for a strategy over a specified time horizon, while the CVaR, as an extension of VaR, gives the total amount of a given loss event. Formally, the VaR at probability level α for the hedge portfolio returns R_h is defined as

$$\mathrm{VaR}_{\alpha}(R_h) = \inf\left\{x \in \mathbb{R}, \mathbb{P}(R_h > x) \leqslant 1-\alpha\right\}, \tag{4.15}$$

and the corresponding CVaR is as follows

$$\mathrm{CVaR}_{\alpha}(R_h) = \mathbb{E}\left[-R_h \middle| -R_h > \mathrm{VaR}_{\alpha}(R_h)\right]. \tag{4.16}$$

Thus, given an amount, VaR stresses how often a portfolio could loose and CVaR will indicate the potential loss beyond a given amount. Meanwhile, VaR lacks the sub-additivity property, which is fundamental for portfolio diversification and will provide no information about the portfolio losses corresponding to period of predefined risk. The CVaR is sub-additive and accounts for tail risk. Hence, it allows portfolios optimization as shown in Rockafellar and Uryasev [Rockafellar 2002]. Besides, CVaR overcomes lack of sub-additivity and indifference to tail losses, but will require a large size data for consistent estimation even more sensitive to estimation errors than VaR.

The optimization problems[10] for VaR and CVaR are given by

$$\begin{aligned} h^*_{\mathrm{VaR}} &= \arg\min h \in \mathbb{R}\, \mathrm{VaR}_\alpha(R_h) \\ h^*_{\mathrm{CVaR}} &= \arg\min h \in \mathbb{R}\, \mathrm{CVaR}_\alpha(R_h) \end{aligned} \tag{4.17}$$

However, one knows that VaR and CVaR measures depend on the distribution of the hedge portfolio returns which would lead hedging strategy extremely dependent on predetermined risk level α and distribution.

Overall, each hedge ratio leads to a specific hedging strategies that depends upon the approach used. That is to say with the same data, the hedging strategy to apply, among the above described, remains elusive. A criteria to distinguish them is hedging effectiveness that evaluates the hedging performance.

4.3.3 Hedging effectiveness

Hedging performance is a measure of hedging effectiveness that serves as criterion to compare the consistency in both estimation and post samples of different hedge ratios. There are three main measures of effectiveness in financial literature that relate to futures hedging: Ederington [Ederington 1979] measure, Howard and D'Antonio [Howard 1987] Sharpe-type measure and the certain equivalent measure. Ederington [Ederington 1979] has first defined effectiveness measure to indicate the reduction effect provided by the futures contract in term of the percentage reduction of the hedge portfolio variance over the spot asset variance,

$$\mathbf{HE}_{\mathrm{ED}} = 1 - \frac{\mathbb{V}[R_h]}{\mathbb{V}[R_s]}. \tag{4.18}$$

[10]These problems require to assume that VaR_α and CVaR_α are continuously differentiable in h and that the distributions of the spot return R_s and futures return R_f have positive density.

According to this measure, in equation (4.18), a hedge ratio is deemed better than another if it leads to a smaller variance of the hedge portfolio.

The Howard and D'Antonio [Howard 1987] measure takes into account both expected return and volatility of hedge portfolio,

$$\mathbf{HE}_{\text{SH}} = \frac{\mathbb{E}[R_h] - r}{\sigma_h} - \frac{\mathbb{E}[R_s] - r}{\sigma_s}, \tag{4.19}$$

where r is the risk-free interest rate and σ_h and σ_s are respectively the return volatilities of hedge portfolio and the spot.

The third criteria of hedging effectiveness is based on the certainty equivalent measure and is defined such that position in futures contract equates its same expected utility.

$$\mathbb{E}[u(R_s + e)] = \mathbb{E}[u(R_s - hR_f)],$$

where u is an increasing and concave utility function and e is the certainty equivalent.

Recall that, in all the above three cases, hedging effectiveness is based on the estimated hedge ratio. Lien [Lien 2006, Lien 2012] has shown that all these measures are unreliable because they are downward biased leading to under-reported hedging strategy. Specifically, the Ederington measure is likely to perform only with minimum-variance hedge ratio. In the case of portfolio non normality (as results of spot and futures returns' distribution asymmetry and fat tails), the Sharpe-type hedging effectiveness will fail to onsider relevant properties of portfolio.

To overcome this limits, we propose a ranking based measure of hedging effectiveness by applying the L-performance defined with regard to L-moment approach. The advantage of using L-moment relies on their consistency in estimation. The L-performance measure ranks different hedge portfolios regardless the methodology of the hedge ratio. Darolles et al.

[Darolles 2009] have used L-performance criterion to rank hedge funds on different portfolio strategies just as Sharpe-ratio ranking.

Let's denote by $\mathbf{HE}_{\text{LP}}$ the effective L-performance and by $L^h_{q,p}$ and $L^s_{q,p}$ be L-performance of respectively hedge portfolio and spot asset for given q and p. The L-performance effectiveness is defined by

$$\mathbf{HE}_{\text{LP}} = L^h_{q,p} - L^s_{q,p} \tag{4.20}$$

where the L-performances $L^h_{q,p}$ and $L^s_{q,p}$ are presented in Appendix 4.21.

To estimate a L-performance, consider a sample of independent and identically distributed returns r_i, $i = 1, \ldots, N$ with their order statistics: $r_{1:N} \leq \ldots \leq r_{N:N}$. The estimator of L-performance is a ratio of the two linear combinations of order statistics given by

$$\widehat{L}_{q,p,N} = \frac{\sum_{i=1}^{N} r_{i:N} P_{1,p}\left(\frac{i}{N}\right)}{\sum_{i=1}^{N} r_{i:N} P_{2,q,p}\left(\frac{i}{N}\right)}, \tag{4.21}$$

for L-performance defined[11] on $u \in (0,1)$, $p \in \mathbb{N}$ and $0 \leq q \leq p-1$ and polynomials $P_{1,p}$ and $P_{2,q,p}$ described as follows

$$P_{1,p}(u) = \frac{(2p+1)!}{p!} u^p (1-u)^p.$$

$$P_{2,q,p}(u) = \frac{(2p+1)!}{q!(2p-q)!} \left[u^{2p-q}(1-u)^q - u^q(1-u)^{2p-q}\right]$$

The L-performance estimator is consistent and asymptotically normal, under standard regularity conditions, Darolles et al. [Darolles 2009].

In the aims of comparing the hedge ratios as well as the hedging effectiveness, we compute the different hedge ratios using commodity futures describe in Chapter 1. The estimation procedures are addressed in Appendix

[11] Notice that, in practice, it is insightful to consider several different pairs of parameters q, p to obtain alternative rankings of portfolios with respect to $\widehat{L}_{q,p,N}$.

B.1. These applications is addressed in section 4.5.

4.3.4 Limits of existing hedge ratios

So far, the issue of hedging commodity revenue with futures market has considered both maturity date and delivery date as similar. In reality, these dates may differ mainly when there is no futures contract of long maturity. One could think about a producer that has to set a hedging strategy for his activity against either adverse price and yield variation over the planting season with available futures contracts which mature earlier than the delivery date, say T. In this situation, the above described hedge ratios in section 4.3 will not be effective due to dates mismatch between the position in futures market and cash position. Typically, when the production period exceeds maturity date of the active futures contract, the producer will usually initiate a rollover strategy. It consists in closing out the position in the nearby futures few days prior to its maturity date and taking another position in a contract with longer maturity. The rollover strategy is subject to additional basis risk. Gardner [Gardner 1989] had suggested a rollover marketing strategy as a way to efficiently hedge against the additional basis risk in this situation. However, his strategy includes constant outputs neglecting the production risk that matters for storable commodities.

In agricultural markets, price and output uncertainties are interrelated in that prices react inversely to large variations of output (Conroy and Rendleman [Conroy 1983]). The combination of these two risks rises the problem of the appropriate position in hedging instruments, specially for the futures contract with longer maturity that should also account for the additional basis risk in the rollover process. But, for the rollover strategy, production risk is also relevant in inter-crop period for stock and the coming crop year on uncertainties related to weather conditions that could lead to imperfect hedge. Hence, hedging in rollover strategy need to be extended to tackle the inter-crop season to further guarantee revenue over long time exposure.

Additional hedging instruments are insurance products designed[12] for agricultural producers that have the opportunities to purchase an insurance contract. In the next section we investigate the optimal hedging strategy that include an insurance policy.

4.4 Optimal sequential hedging

The optimal sequential hedging combines the approach of Gardner [Gardner 1989] on rolling over futures contracts and the sequential hedging of Baesel and Grant [Baesel 1982] to derive a hedging strategy that accounts for both price and production risk using futures and insurance contracts.

4.4.1 The Strategy

Consider an agricultural producer that plans to sell[13] his crop for the $T > 2$ coming years. Since futures contracts are available with short maturities, using a rollover strategy on their positions, they can lock in price, in the first year, for the T coming years. Multiyear futures contracts, or sequential rollovers as a substitute, make more sense, along Working's [Working 1953] lines, as a device for locking in receipts within an T-years period, argued [Gardner 1989].

In rollover strategy, a producer faces additional risks including stocks deterioration, low revenue for the coming crop years due to production risk like weather conditions, etc. The producer can purchase an insurance contract to further hedge his revenue. On using insurance to reduce these risks, the producer need an optimal policy together with the futures hedging strategy.

[12]For instance, in United States, the Risk Management Agency has introduced together with private insurance companies, a variety of agricultural insurance contracts. The producer can buy insurance contract based on individual yields (Income Protection, IP, or Revenue Assurance, RA) or on aggregate yields (Group Risk Income Protection, GRIP), see Mahul and Wright [Mahul 2003].

[13]We consider the harvest time and sale time to be the same. Otherwise, sale takes place later and there will be only price risk to manage with storage cost.

An insurance policy is described by the couple $(I(\cdot), prem)$ where $I(\cdot)$ and $prem$ are respectively the risk-neutral indemnity payed to risk-averse producer and insurance premium. We assume; at any time t, the premium to depend on actuarial value of the policy and the indemnity function to be non-negative and less than the insured value (see Mahul and Wright [Mahul 2003]),

$$0 \leqslant I_t(x) \leqslant x, \quad \forall x \geqslant 0 \qquad \text{and} \qquad prem = \zeta \mathbb{E}[I_t(x)] \tag{4.22}$$

with $\zeta(0) = 0$, $\zeta'(x) > 1$ for all $x > 0$ is deterministic loading factor. An optimal insurance contract for a crop year is the insurance premium and the indemnity function that maximize the producer's expected utility of gross revenue under the above mentioned constraints:

$$\max_{I_t(\cdot), prem} \mathbb{E}\Big[u\Big(R_t + I_t(\cdot) - prem\Big)\Big] \tag{4.23}$$

with R_t the producer gross revenue at time t. The indemnity function depends on insurance contract and the wealth process is function of both the indemnity and the marketing strategy.

Consider a revenue insurance (like Income Protection, IP, or Revenue Assurance, RA) where the producer chooses a proportion of the expected revenue to insure. In such insurance contract, the price at which the crop is valued moves with price changes in the market. Therefore, the producer will receive indemnity equal to the difference between the percentage of the value he has insured and the revenue at end of period, if only if the former is greater than the latter. For simplicity, assume the expected revenue at time t to be the average revenue at time $t-1$,

$$\mathbb{E}_{t-1}[S_t Q_t] := F_{t-1,t}\bar{Q}_{t-1},$$

with $\mathbb{E}_{t-1}$ being the conditional expectation on information available at $t-1$

and $\bar{Q}_{t-1}$ is the average output[14] over the period $[0, t-1]$. The indemnity schedule $I_t(\cdot)$ is as follows

$$I_t(S_t, Q_t; \upsilon_t) = \left[\upsilon_t F_{t-1,t}\bar{Q}_{t-1} - S_t Q_t\right]^+, \quad \upsilon_t \in (0, 1] \tag{4.24}$$

where the notation $[\cdot]^+$ stands for $\max(0, \cdot)$ function. The producer has to decide the proportion υ_t of his expected revenue to choose according to his hedging strategy with future contract.

Consider a producer following the marketing strategy as in Gadner [Gardner 1989], his wealth will rely on the sequential rollover strategy for achieving the whole T-years period hedge. That is, in planting season, T crops, each of quantity Q_t, $t = 1, \ldots, T$ are sold for delivery at the beginning of the next crop year. If $F_{t,t+1}$, $t \in \{0, 1, 2, \ldots, T-1\}$, is the price of futures contract traded at t for delivery at $t+1$. The producer's wealth at initial time, $t = 0$, is given by

$$\mathcal{W}_0 = F_{0,1} Q_{1,T}, \tag{4.25}$$

with $F_{0,1}$ being the price of futures contract traded at initial time for delivery at end of the first crop year and $Q_{1,T}$ is the total quantity for first year to T. More generally, we denote the total quantity for period from $t = j$ to $t = T$ by

$$Q_{j,T} = \sum_{t=j}^{T} Q_t, \quad j \in \{1, \ldots, T\}.$$

At inception of the hedging strategy, no indemnity could be received but the decision about υ_1 is made for $t = 1$ by paying a premium $prem$. Determining the premium at a time step is a pricing issue and relates to insurance company. So, we neglect the term $prem$. Indeed, we assume that the producer has already select his insurance contract and he does know the correspond-

[14] In more realistic case, the expression $\mathbb{E}_t[S_t Q_t]$ is estimated in a surrounding geographic area and in futures market. This estimators of individual yield and price are imperfect and will affect the optimal insurance policy. We consider the insurance contract, already chosen by the producer and we assume that $\mathbb{E}_{t-1}[S_t] = F_{t-1,t}$ (no arbitrage condition) and no correlation between S_t and Q_t.

ing premium. We then focus on deciding the optimal futures hedge and proportion of the expected revenue following market outcome. This aims at comparing how insurance contract will impact the the hedging strategy in futures market.

At time $t = 1$, a number of T contracts are bought at price $F_{1,1}$. At the same time, the first crop Q_1 is sold at spot price S_1 and the remaining contracts $T - 1$ of total quantity $Q_{2,T}$ are rolled over by selling futures for delivery at T. The wealth, $\mathcal{W}_1$, at end of the first crop year, is

$$\mathcal{W}_1 = \mathcal{W}_0 - F_{1,1}Q_{1,T} + S_1Q_1 + F_{1,2}Q_{2,T} - cQ_{1,T} + I_1(S_1, Q_1; \upsilon_1) \tag{4.26}$$

where $F_{1,1}Q_{1,T}$ is the cost of buying back the initial sales, S_1Q_1 is the spot market revenue, $F_{1,2}Q_{2,T}$ the sale of period ahead futures, and $cQ_{1,T}$ is transaction costs (brokerage fees and opportunity cost of margin funds), with c the amount of these costs per bushel traded. The expression in (4.26) can also be written in the following form,

$$\mathcal{W}_1 = \mathcal{W}_0 + sp_1Q_{2,T} - cQ_{1,T} + \max\Big(b_1Q_1\,,\ \upsilon_1F_{0,1}Q_0 - F_{1,1}Q_1\Big), \tag{4.27}$$

where b_1 and sp_1 are respectively the basis and the spread[15] and Q_0 has to be set.

Analogously, at any time $t < T$, there are $T - t - 1$ contracts bought back at price $F_{t,t}$ and Q_t will be sold at spot price S_t with the remaining contracts $T - t$ of total quantity $Q_{t,T}$ rolled over by selling futures for delivery at T. The wealth, $\mathcal{W}_t$, at end of the t^{th} crop year follows as

$$\mathcal{W}_t = \mathcal{W}_{t-1} + sp_tQ_{t+1,T} - cQ_{t,T} + \max\Big(b_tQ_t\,,\ \upsilon_tF_{t-1,t}\bar{Q}_{t-1} - F_{t,t}Q_t\Big). \tag{4.28}$$

[15]The basis at time t on futures contract for delivery at $t+1$ is $b_t = S_t - F_{t,t+1}$ and the spread between the period ahead futures and nearby futures prices is $sp_t = F_{t,t+1} - F_{t,t}$

At the final time T, the wealth, $\mathcal{W}_T$, over the T-years period is given by

$$\begin{aligned} \mathcal{W}_T &= \mathcal{W}_{T-1} - F_{T,T}Q_T + S_TQ_T - cQ_T + I_T(S_T, Q_T; \upsilon_T) \\ &= \mathcal{W}_{T-1} - cQ_T + \max\Big(b_TQ_T\,,\, \upsilon_T F_{T-1,T}\bar{Q}_{T-1} - F_{T,T}Q_T\Big) \end{aligned} \tag{4.29}$$

The sequence of quantities $(Q_t)_{t=1\ldots,T}$ can be determined by using the backward recursive technique. At each step, as soon as the quantity, Q_t, is obtained, the insurance policy, υ_t, can be settled afterward.

4.4.2 The solution

Consider a producer with quadratic utility where the objective to maximize the expected wealth subject to wealth variance constraint,

$$\mathbb{E}\Big[u(\mathcal{W}_t)\Big] = \mathbb{E}[\mathcal{W}_t] - \frac{\gamma}{2}\mathbb{V}[\mathcal{W}_t], \quad t \in \{1, \ldots, T\} \tag{4.30}$$

where γ is the risk aversion parameter of the producer. Note that the producer aversion may change from period to period in the rollover process, but since it is a given parameter, herein we will let it constant over the whole period.

Since the $\max$ function is not differential along the line $x = y; \forall x, y \in \mathbb{R}$ and the optimization problem boils down to two cases. That is, the producer's wealth, over each period $[t, t+1]$, $t \in \{1, \ldots, T-1\}$, depends upon

(i) either the revenue at end of each period, if it is greater than the proportion of expected revenue,

(ii) or a part of expected revenue, if it is greater than the revenue at end of each period.

So, the wealth at any time does not include the revenue at end of each period and a proportion of expected revenue all together. However insurance contract is purchased by the production whatever his expectation in the market. Particularly, the case (ii) addresses a guarantee against production risk when the crop yield is lower than expected.

Let's start the optimization problem at final time of the hedging horizon T and apply backward recursion to determine the quantities for earlier dates. Using expression (4.29), the quantity Q_T and the wealth $\mathcal{W}_T$ are such that one has:

- in case (i), $\upsilon_T F_{T-1,T}\bar{Q}_{T-1} < S_T Q_T$,

$$\mathbb{E}\Big[u(\mathcal{W}_T)\Big] = \mathcal{W}_{T-1} + Q_T(\mathbb{E}[b_T] - c) - \frac{\gamma Q_T^2}{2}\mathbb{V}[b_T]; \tag{4.31}$$

  with the optimal quantity for futures contract given by

$$Q_T^{*,\phi} = \frac{\mathbb{E}[b_T] - c}{\gamma \mathbb{V}[b_T]} \tag{4.32}$$

- in case (ii), $\upsilon_T F_{T-1,T}\bar{Q}_{T-1} > S_T Q_T$,

$$\mathbb{E}\Big[u(\mathcal{W}_T)\Big] = \mathcal{W}_{T-1} + \upsilon_T F_{T-1,T}\bar{Q}_{T-1} - Q_T(\mathbb{E}[F_{T,T}] + c) - \frac{\gamma Q_T^2}{2}\mathbb{V}[F_{T,T}] \tag{4.33}$$

  where the optimal quantity in futures contracts and insurance policy are respectively given by

$$\begin{aligned} Q_T^{*,\upsilon} &= -\frac{\mathbb{E}[F_{T,T}] + c}{\gamma \mathbb{V}[F_{T,T}]} \quad \text{and} \\ \upsilon_T^* &= 1 - \frac{\mathbb{E}[F_{T,T}]Q_T^{*,\upsilon}}{F_{T-1,T}\bar{Q}_{T-1}} \end{aligned} \tag{4.34}$$

The proportion, υ_T^*, of the expected revenue is given at optimal crop yield according to the expected futures price. Observe that, in the case (ii) where revenue insurance payoff is paid to the producer, crop yield will be so low to make the insurance payoff as maximum as possible, (see (4.24)). The insurance payoff is a decreasing function of the crop yield until a trigger function (see Mahul and Wright [Mahul 2003]). Since the worse case scenario would be no crop yield and the producer will have bought the insurance contract at the total expected revenue, the proportion, υ_T^*, is given by expression (4.34).

Similarly, to find the optimal hedging strategy at any time t prior to final time T, consider the two cases (i) and (ii) with their corresponding expected utility expressions at time t. For $t < T$, we follow the backward recursion and replace Q_{t+1} by Q_{t+1}^*, determined earlier, in expression (4.28). It gives raise to

- in case (i), $\upsilon_t F_{t-1,t}\bar{Q}_{t-1} < S_t Q_t$,

$$\mathbb{E}\Big[u(\mathcal{W}_t)\Big] \equiv Q_t(\mathbb{E}[b_t] - c) - \frac{\gamma}{2}\Big\{Q_t^2\mathbb{V}[b_t] + 2Q_tQ_{t+1,T}^*\mathbb{C}\mathrm{ov}(b_t, sp_t)\Big\}; \quad (4.35)$$

with the optimal quantity of futures contract being

$$Q_t^{*,\phi} = \frac{\mathbb{E}[b_t] - c}{\gamma\mathbb{V}[b_t]} - \frac{Q_{t+1,T}^*\mathbb{C}\mathrm{ov}(b_t, sp_t)}{\gamma\mathbb{V}[b_t]} \quad (4.36)$$

- in case (ii), $\upsilon_t F_{t-1,t}\bar{Q}_{t-1} > S_t Q_t$,

$$\mathbb{E}\Big[u(\mathcal{W}_t)\Big] \equiv -Q_t(\mathbb{E}[F_{t,t}]+c) - \frac{\gamma}{2}\Big\{Q_t^2\mathbb{V}[F_{t,t}] - 2Q_tQ_{t+1,T}^*\mathbb{C}\mathrm{ov}(F_{t,t}, sp_t)\Big\} \quad (4.37)$$

where the optimal quantity in futures contracts and insurance policy at time t are respectively given by

$$\begin{aligned} Q_t^{*,\upsilon} &= -\frac{\mathbb{E}[F_{t,t}] + c}{\gamma\mathbb{V}[F_{t,t}]} + \frac{Q_{t+1,T}^*\mathbb{C}\mathrm{ov}(F_{t,t}, sp_t)}{\gamma\mathbb{V}[F_{t,t}]}, \quad \text{and} \\ \upsilon_t^* &= 1 - \frac{\mathbb{E}[F_{t,t}]Q_t^{*,\upsilon}}{F_{t-1,t}\bar{Q}_{t-1}}. \end{aligned} \quad (4.38)$$

Let Q_t^*, $t \in \{1, \ldots, T\}$, be the optimal quantity to rollover from nearby futures contract to new one with,

$$Q_t^* = \begin{cases} Q_t^{*,\phi} & \text{if crop yield is lower than expected} \\ Q_t^{*,\upsilon} & \text{otherwise.} \end{cases}$$

Over the hedging horizon, $[0, T]$, the optimal quantity depends upon risk aversion, transaction costs, the spread and either the basis or the futures price at end of period. Specially, when the crop yields are lower than expected, indemnity is paid to the producer based on the proportion, υ_T^* as compensation. This guarantees the producer in the situations when drastic weather conditions hold leading to low revenue.

The component with the spread, sp_t reflects profit and loss relation in futures market at the same time, scaled by the expected optimal hedge at future dates. Particularly, at any time t prior to final time T, the optimal quantity, Q_t, differs from the optimal quantity at T, Q_T with adjustment term of the hedge at future dates. Hence, at the end of hedging horizon, the optimal quantity does not include the spread since the producer will close the hedge and will not consider another futures contract in this strategy.

4.5 Empirical applications

Recall that two categories of price data where used in Chapter 2, the nearby contract prices with the front contract as proxy for spot price and expiry month prices. We will apply the hedge strategies by considering the last nearby as futures contract. In order to compares strategies of hedge ratios with the sequential hedging strategy, we restrict the period of analysis to three years, that is from August 1^{st}, 2012 to July 31^{th}, 2015. We exclude soybeans meal commodity as the results look similar to soybean case.

Table 4.1: Estimation of Hedge ratios

	minV		MV		SK		MEG		LPM		VaR		CVaR	
Commodity	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.	est.	eff.
Corn	1.081	0.797	1.915	0.322	1.916	0.322	1.162	0.737	1.062	0.797	1.091	0.797	1.194	0.798
	0.000	1.471	0.001	3.903	0.001	3.903	0.000	1.907	0.000	1.307	0.000	1.557	0.000	1.544
	0.007	1e-04	0.012	1e-04	0.012	1e-04	0.008	1e-04	0.008	1e-04	0.008	1e-04	0.021	1e-04
Oat	1.127	0.494	1.604	0.406	1.605	0.406	1.253	0.417	1.336	0.477	1.139	0.494	1.148	0.434
	1e-04	0.730	2e-04	1.146	2e-04	1.146	1e-04	0.865	1e-04	0.945	1e-04	0.742	1e-04	0.782
	0.018	1e-04	0.020	1e-04	0.020	2e-04	0.018	1e-04	0.019	1e-04	0.018	1e-04	0.018	1e-04
R. rice	0.8002	0.472	2.395	-1.40	2.399	-1.42	0.481	0.326	0.354	0.326	0.818	0.472	0.838	0.397
	-1e-04	2.077	7e-04	5.657	7e-04	5.655	-3e-04	0.187	-4e-04	0.086	-1e-04	2.198	-1e-04	2.219
	0.009	0.001	0.019	3e-04	0.019	3e-04	0.011	0.000	0.010	0.000	0.009	1e-04	0.009	1e-04
Soybeans	-0.11	0.006	1.067	-0.66	1.067	-0.66	-0.13	0.009	-0.10	0.006	-0.09	0.005	-0.07	0.003
	-5e-04	-0.20	0.000	1.886	0.000	1.886	0.000	1.286	-5e-04	-0.18	-5e-04	-0.16	-5e-04	-0.16
	0.016	0.000	0.020	1e-04	0.020	1e-04	0.016	0.000	0.016	0.000	0.015	0.000	0.017	1e-04
Wheat	1.101	0.825	1.841	0.452	1.840	0.453	0.815	0.698	0.844	0.780	1.108	0.825	1.121	0.7253
	1e-04	2.899	4e-04	3.922	4e-04	3.924	0.000	0.932	0.000	0.827	1e-04	2.959	1e-04	2.5589
	0.007	1e-04	0.013	2e-04	0.013	2e-04	0.009	1e-04	0.008	1e-04	0.007	1e-04	0.008	1e-04
Cocoa	1.227	0.838	-2.32	-6.19	-2.33	-6.19	1.283	0.816	1.235	0.838	1.209	0.838	1.259	0.738
	-1e-04	-13.1	0.002	-3.38	0.002	-3.38	-1e-04	-12.8	-1e-04	-13.4	0.000	-12.4	0.000	-11.8
	0.004	-3e-04	0.027	0.000	0.027	0.000	0.004	0.000	0.004	0.000	0.004	0.000	0.004	0.000
Coffee	1.125	0.979	0.840	0.916	0.839	0.916	1.124	0.857	1.082	0.977	1.123	0.979	1.133	0.868
	0.000	-0.40	1e-04	1.576	0.000	1.575	0.000	0.506	0.000	0.806	0.000	-0.37	0.000	-0.37
	0.004	-1e-04	0.007	0.000	0.007	0.000	0.003	0.000	0.004	0.000	0.003	0.000	0.004	0.000
Cotton	1.146	0.536	1.420	0.505	1.420	0.505	0.714	0.431	0.814	0.491	1.149	0.536	1.367	0.503
	0.000	2.030	0.000	2.026	0.000	2.026	0.000	1.341	0.000	1.541	0.000	2.033	0.000	2.033
	0.009	1e-04	0.009	1e-04	0.009	1e-04	0.009	1e-04	0.010	1e-04	0.009	1e-04	0.009	1e-04

For each approach, the first column is labeled "est." as estimates of hedge ratio, average return and standard deviation for the corresponding hedge portfolio are given below on second and third lines respectively.
The second column displays the estimates of effectiveness measures of Ederington [Ederington 1979], $\mathbf{HE}_{\mathrm{ED}}$, Howard and D'Antonio [Howard 1987], $\mathbf{HE}_{\mathrm{SH}}$, and L-performance, $\mathbf{HE}_{\mathrm{LP}}$ respectively on first, second and third lines.
We consider the riskless interest rate to be $r = 0$.

Table 4.1 displays estimates of hedge ratios as presented in Section 4.3. It is clearly apparent that mean-variance and skweness-kurtosis based methods seem to be more consistent that other in that Sharpe ratio measure is higher for the hedge ratios in the first two method than the others. Besides, these two hedge ratios are relatively closed. The other methods also look similar with more differences. This classification is more pronounced in case of rough rice and cocoa when opposite position is suggested by the two classes of hedge ratios. In the optimization program, the hedge ratios from mean-variance and skweness-kurtosis methods maximize the objective function while the other methods minimize their objective function. This shows how hedging strategy depends on the optimization program with regard to the producer's preference. Besides, as expected, even hedge ratios may vary substantially from one approach to another, there is no clear cut to stand for method of optimization based on effectiveness measures. Indeed, the L-performance effectiveness measure is closed to zero for all these strategies.

The sequential hedging strategy requires assumption of distribution for spot and futures prices. Instead, we assume historical returns over the three-years period for both spot and futures prices. We set the Q_0 as in (4.27) to the average output until july 2012. Herein, the rollover dates is chosen arbitrary[16] at end of July for all commodities. Table 4.2 exhibits results of optimal sequential hedging strategy which are more consistent, based on L-performance effectiveness, than those of Table 4.1. Measures of L-performance effectiveness are all significantly greater than zero for all the commodities. Besides, the $\mathbf{HE}_{\text{ED}}$ measure is sensibly lower for the sequential strategy, what suggests the superiority of L-performance over the other effectiveness measures.

Besides, note that adding an insurance contract to futures contract in rollover hedging seems to decrease the number of futures contracts when low crop is expected. This effect of insurance contract is illustrated in the

[16] In more reality case, rollover date are published

Table 4.2: Optimal sequential hedging strategy

Commodity	Q_1^*	Q_2^*	Q_3^*	$\mathbf{HE}_{\mathrm{ED}}$	$\mathbf{HE}_{\mathrm{SH}}$	$\mathbf{HE}_{\mathrm{LP}}$
Corn	1.402	1.402	0.486	0.313	1.735	2.013
	0.614	0.615	0.715			
Oat	0.691	0.872	-1.501	0.402	1.808	2.371
	0.765	0.815	0.565			
R. rice	0.277	0.273	0.392	0.676	2.049	5.452
	0.603	0.591	0.652			
Soybeans	0.101	0.091	0.687	0.595	1.176	3.715
	0.893	0.912	0.852			
Wheat	0.302	0.309	0.767	0.292	3.810	4.174
	0.593	0.612	0.552			
Cocoa	0.422	0.417	0.668	0.682	1.210	3.152
	0.851	0.863	0.801			
Coffee	1.810	1.793	1.610	0.362	2.144	3.143
	0.551	0.563	0.611			
Cotton	0.154	1.154	1.319	0.427	1.385	2.763
	0.901	0.552	0.549			

We set transaction costs to zero ($c = 0$);
Q_1^*, Q_2^*, Q_3^* are displayed on first line for each commodities. There are respectively the number of futures contracts for july 2013, 2014 and 2015.
$\mathbf{HE}_{\mathrm{LP}}$ is L-performance effectiveness measure.

hedging strategies for the first two years and the first year respectively for oat and cotton. Therefore, combining futures and insurance contracts will further reduce market and production risk. Specially, insurance contract is addressed in low crop yield situation.

4.6 Conclusion

The purpose of hedging has received many contributions in literature of futures market. For storable commodities, the hedging issue is of specific in that the asset is often non-traded. Futures contracts are the usual instruments to cover the farmer from the losses, but their use requires appropriate hedging strategies because of market moves. Hence, there is no guarantee of achieving the goal of reducing the price risk with only futures hedge. Indeed, when harvest fails, the the losses increase at final time and farmer may go bankruptcy. Other hedging strategies have been studying; the rollover strategy that consists in lock in price for longer time period by sequentially closing position in nearby contract and taking other in new futures contract along long time period.

We have investigated different approaches of hedging with futures contract in agricultural markets. We have first described the existing approaches that do not consider output risk due to production contingencies like bad weather condition, pests infestation for stored goods. In these strategies output is considered as deterministic and strategies strongly depend on the approaches. Besides, there is no clear cut for a best approach over the other based the existing effectiveness measures.

Since most of producers in agriculture strongly rely on revenue from their activity, production risk is relevant. Hence, management of risk should include production risk contribute to avoid substantial losses on final income. In addition, the rollover strategy is subject to larger production risk within the inter-crop periods and then and additional basis risk.

We derive sequential optimal hedging with rollover process that takes into account production risks. The strategy requires both futures market and insurance contract that are combined to further guarantee the producer a level of gross revenue. The resulting hedge depends on spread between nearby futures and new futures contracts as well as either the basis or the expected futures price at end of each period such that when the crop yields are lower than expected indemnity is paid to producer as compensation by insurance company.

In order to distinguish the hedging approaches, we estimate hedging ratio from described static hedging strategies for commodity data at hands and described in Chapter 2. The results show how difficult is to select the best strategies based on the existing effectiveness measure. However, the application of L-performance measure is significant on the sequential strategy with insurance contract.

Hedging in static framework only requires the price distribution at initial and final dates of each period to compute various moment for various hedge ratios. Futures prices ae settled daily and one may gain additional information about price behavior by using price pattern over the hedging horizon. Then, the hedging problem in continuous time framework by modeling the prices is addressed in following chapter 5.

Chapter 5

Dynamic hedging

"You can think of hedging as a bit like sailing. You can move where you want almost regardless of the direction of the wind, by adjusting the sail so that you make use of the component that you need".

Alexander Lipton, Merrill Lynch

Abstract: We investigate the optimal hedging strategy for an producer in continuous time framework with a position in futures market over a crop year. We consider the agricultural prices to follow mean-reverting jump-diffusion process in light of empirical analysis of Chapter 2. The optimal hedging strategy is conducted with expected utility maximization. The hedging strategy with only futures contract remains deficient. Then, a put option is be written to further reduce risks faced the producer. The optimal hedging strategy is achieved by using the approach of expected utility maximization. It turns out that only the future contract is insufficient to cover the risks incurred by a producer. Thus, it is written put option to further reduce the risk of loss of a producer.

Contents

5.1 Introduction

The modeling in continuous time has been pioneered by Merton [Merton 1969] financial economics with the intertemporal consumption and portfolio choice problem of an investor in a stochastic dynamic programming setting. Later, Ho [Ho 1984] and Adler and Detemple [Adler 1988a, Adler 1988b] used this setting for hedging purpose in commodity markets. They had dealt with finding the optimal position in futures contract that reduces the risks incurred by non-traded in portfolio context. Furthermore, the empirical analysis in Chapter 2 has stressed about including jumps as sudden and substantial of price variation to improve hedging strategy in agricultural markets. This chapter investigates the extend to which hedging strategy in continuous time setting will take into account behavior of agricultural prices at hands.

The problem hedging in agricultural market consists in reducing the risk incurred by non-traded asset with appropriate positions in other assets or derivatives such as futures contract and option. Such issue relates to commodity producers who need to secure their future incomes. Generally, perfect hedge is not feasible and optimal hedging strategy is derived instead. In continuous time framework, the optimal hedging problem in commodity market has been studied via expected utility maximization and Ho [Ho 1984] and Adler and Detemple [Adler 1988a, Adler 1988b] had considered asset price driven by only Brownian risk. We extend the problem by adding jump risk and consider asset price dynamics to follow mean-reverting jump-diffusion processes. In doing so, the market is incomplete due to jump risk as well as imperfect correlation between the two asset. Hence, the equivalent martingale measure is no more unique in no-arbitrage condition. The imperfect correlation implies basis risk that Monoyios [Monoyios 2004] has investigated with put option under Brownian motion risk. The expected utility maximization is conducted under minimal martingale measure to result in optimal strategy. Such portfolio strategies with in jump-diffusion process have been also investigated by Aït-Sahalia and Matthys [Aït-Sahalia 2014] with robust martingale measure under growth entropy criteria.

The structure of this is as follows. The first section derives hedging strategy with futures contract presence of consumption rate. Since, this trading strategy strongly relies on market dynamics as well as quantity risk, and in the second section, we include put option to further reduced cash position risk.

5.2 Hedging jump risk in commodity market

Consider an economy that lives on stochastic basis of a fixed probability space $(\Omega, \mathbf{F}, \{\mathcal{F}_t\}, \mathbb{P})$ with finite horizon $[0, T]$. Information is gathered as filtration $\{\mathcal{F}_t; t \in [0, T]\}$ generated by Brownian motions and Poisson processes and satisfies usual conditions.

5.2.1 Models setting

Consider two asset prices $(S, F) := (S_t, F_t)_{0 \leqslant t \leqslant T}$ that follow mean-reverting jump-diffusion process. The asset with price S_t is a non-traded asset and F_t represents futures price process that is traded in the market. The two price processes are mean-reverting jump difuusion and satisfy the stochastic differential equations, for $0 \leqslant t \leqslant T$,

$$\frac{\mathrm{d}S_t}{S_{t-}} = \left[\kappa_1(\mu_{1,t} - \ln S_t) - \lambda_1 J_1 \int z\nu_1(\mathrm{d}z)\right]\mathrm{d}t + \sigma_{1,t}\mathrm{d}W_{1,t} + J_1\mathrm{d}Y_{1,t}, \tag{5.1}$$

$$\frac{\mathrm{d}F_t}{F_{t-}} = \left[\kappa_2(\mu_{2,t} - \ln F_t) - \lambda_2 J_2 \int z\nu_2(\mathrm{d}z)\right]\mathrm{d}t + \sigma_{2,t}\mathrm{d}W_{2,t} + J_2\mathrm{d}Y_{2,t} \tag{5.2}$$

where $(W_1, W_2) = (W_{1,t}, W_{2,t})_{0 \leqslant t \leqslant T}$ is 2-dimensional Brownian motion with constant correlation $-1 \leqslant \rho \leqslant 1$, such that $\mathrm{d}W_{1,t}\mathrm{d}W_{2,t} = \rho\mathrm{d}t$. For $i = 1, 2$, the parameters κ_i and μ_i are constants and represent, respectively, the speed of mean-reversion and the long-run mean return. The coefficient $\sigma_{i,t}$ is the deterministic volatility function of asset i. The process $Y_i := (Y_{i,t})_{0\leqslant t\leqslant T}$ is a compound Poisson; so, $Y_{i,t} := \sum_{n=1}^{N_{i,t}} Z_{i,n}$ with independent jump counting process $N_i := (N_{i,t})_{0\leqslant t\leqslant T}$ and jump size process $Z_i := (Z_{i,t})_{0\leqslant t\leqslant T}$. N_i is a Poisson process with intensity $\lambda_i > 0$ and $Z_i := (Z_{i,t})_{0\leqslant t\leqslant T}$ is independent process distributed with Lévy measure $\nu_i(\mathrm{d}z)$ that satisfies the condition $\int_{\mathbb{R}} \min(|x|, 1)\nu_i(\mathrm{d}z) < \infty$. Also, we assume the independent jump components for the two assets with their price dynamics (5.1) and (5.2) written under the statistical measure $\mathbb{P}$.

Besides, consider that there is costless information process, $Q := (Q_t)_{0\leqslant t\leqslant T}$, about the delivery is continuously gathered and allows the hedger to revise his beliefs. The process Q is diffusion satisfies the stochastic differential equation

$$\frac{\mathrm{d}Q_t}{Q_t} = \alpha \mathrm{d}t + \varsigma \mathrm{d}W_{3,t} \tag{5.3}$$

where α and ς are respectively the average and the diffusion coefficients of Q; the process $(W_{3,t})_{0\leqslant t\leqslant T}$ is a standard Brownian motion process correlated to Brownian motion $(W_{2,t})_{0\leqslant t\leqslant T}$ of futures price dynamic by constant ϱ, $-1 \leqslant \varrho \leqslant 1$. The process $(Q_t)_{0\leqslant t\leqslant T}$ allows to update anticipations about yield at harvest-time.

Consider an hedger with deterministic consumption rate $c := (c_t)_{0\leqslant t\leqslant T}$ at time t that are committed to the terminal value, at time T, of a position in cash position $S_T Q_T$. Besides, trading takes place continuously in two securities, a risk-less bond at constant interest rate, $r \geqslant 0$, and in the futures contract F_t. A futures position is taken by marking to market a margin account according to a process $\omega := (\omega_t)_{0\leqslant t\leqslant T}$ that constitutes credits and debits at time t. The credits (or debits) are added at the constant continuously compounding rate r to the hedger's margin account V_t. We assume that losses bringing the account to a negative level are covered by borrowing at the same interest rate, and ignore transactions costs and other institutional features. Then, the margin account has the following form

$$V_t = \int_0^t e^{r(t-s)}\omega_s \mathrm{d}F_s - \int_0^t c_t \mathrm{d}t \tag{5.4}$$

where the re-investment at time s of $\omega_s \mathrm{d}F_s$ at rate r leads to $e^{r(t-s)}\omega_s \mathrm{d}F_s$ at time t. Then, applying Itō's Lemma gives its stochastic differential equation representation in the following form

$$\begin{aligned} \mathrm{d}V_t = \Big[rV_t - c_t + \omega_t F_t \kappa_2(\mu_{2,t} - \ln F_t) - \omega_t F_t \lambda_2 J_2 \int z\nu_2(\mathrm{d}z)\Big]\,\mathrm{d}t \\ + \omega_t F_t \sigma_{2,t} \mathrm{d}W_{2,t} + \omega_t F_t J_2 \mathrm{d}Y_{2,t} \end{aligned} \tag{5.5}$$

Given a strategy ω_t in futures contract, in the absence of any income derived outside the investments in these two assets, the hedger's wealth, $\mathcal{W}_t$, satisfies the stochastic differential equation

$$\mathrm{d}\mathcal{W}_t = \mathrm{d}(S_t Q_t) + \mathrm{d}V_t \tag{5.6}$$

In equation (5.6), the expression $\mathrm{d}(S_t Q_t)$ operates as a pure information process in that the two dynamics (5.4) and (5.6) live in the same filtration as proven in Adler and Detemple [Adler 1988a].

Like in Chapter 3, consider there is an equivalent martingale measure under which the price dynamics are martingale. Since the market is incomplete, the equivalent martingale measure is not unique and one need *minimal martingale measure* under which hedging strategy will be optimal with these dynamics. This minimal martingale measure comes from entropy criteria over the set of equivalent martingale measures (see Aït-Sahalia and Matthys [Aït-Sahalia 2014] and Zariphopoulou [Zariphopoulou 2001]). Herein, we assume the risk-neutral measure to be minimal martingale measure for simplicity. Then, the dynamic (5.5) under the minimum martingale measure is posited to be

$$\begin{aligned}\mathrm{d}V_t = \Big[rV_t - c_t + \omega_t F_t \Big(\kappa_2(\mu_{2,t} - \ln F_t) - \sigma_{2,t}\eta - \lambda_2 e^a J_2 \int z\nu_2(\mathrm{d}z)\Big)\Big]\,\mathrm{d}t \\ + \omega_t F_t \sigma_{2,t} \mathrm{d}\widetilde{W}_{2,t} + \omega_t F_t J_2 \mathrm{d}\widetilde{Y}_{2,t}.\end{aligned} \tag{5.7}$$

Under the minimal martingale measure, the hedger problem is to find a trading strategy $\{c_t, \omega_t\}_{0\leqslant t\leqslant T}$, among the admissible strategies, that maximizes his expected utility

$$\mathcal{J}(V,F,S,Q,t) = \max_{\{c_u,\omega_u\}_{t\leqslant u\leqslant T}} \mathbb{E}_t^0 \left[\int_t^T U(c_u,u)\mathrm{d}u + B(\mathcal{W}_T)\right] \tag{5.8}$$

subject to the boundary condition

$$\mathcal{J}(V,F,S,Q,T) = B(\mathcal{W}_T,T), \quad \text{with} \quad \mathcal{W}_T = V_T + S_T Q_T \tag{5.9}$$

where $\mathbb{E}_t^0$ denotes the expectation conditional on the information available at time t under the minimal martingale measure. The total wealth at end of period, T, is given by the sum $\mathcal{W}_T = V_T + S_T Q_t$ and $U(\cdot,t)$ is an instantaneous utility function for consumption such that

$$\frac{\partial U}{\partial c} > 0 \quad \text{and} \quad \frac{\partial^2 U}{\partial c^2} < 0.$$

The constraint in (5.9) is the boundary condition with the bequest utility function, $B(\cdot,T)$, of wealth assumed to be concave. This condition takes into account the hedger modification with regard to intermediate cash positions in non-traded asset.

5.2.2 Optimal hedging strategy

The optimal hedging strategy is the solution of (5.8) and will give the optimal consumption rate c_t^* and position in futures contract ω_t^* that, together, maximizes expected utility of the hedger's wealth. We derive Hamilton-Jacobi-Bellman (HJB) equation characterizing the optimal solution by using stochastic dynamic programming and Itō lemma for semi-martingale processes

$$
\begin{aligned}
0 = \max_{c_t, \omega_t} \Bigg\{ & U(c_t,t) + \mathcal{J}_V \left[rV_t - c_t + \omega_t F_t \left(\kappa_2(\mu_{2,t} - \ln F_t) - \sigma_{2,t}\eta - \lambda_2 e^a J_2 \int z\nu_2(\mathrm{d}z) \right) \right] \\
& + \frac{1}{2}\mathcal{J}_{VV}\omega_t^2 F_t^2 \sigma_{2,t}^2 + \mathcal{J}_{VF}\omega_t F_t^2 \sigma_{2,t}^2 + \mathcal{J}_{VS}\omega_t F_t S_t \rho \sigma_{1,t}\sigma_{2,t} + \mathcal{J}_{VQ}\omega_t F_t Q_t \varrho \varsigma \sigma_{2,t} \\
& + F_t \lambda_2 e^a \int \Big[\mathcal{J}(V_{t-} + V_{t-}\omega_t J_2 z, t) - \mathcal{J}(V_{t-}, t) \Big] \nu_2(\mathrm{d}z) \Bigg\}
\end{aligned} \tag{5.10}
$$

subject to terminal condition in (5.9). We subscript value function $\mathcal{J}$ with the variable to represent its partial derivative with regard to this variable. In (5.10), we neglect the terms that are not affected by the control variables.

The maximization program can be expressed in two optimal decision rules (consumption rate and futures positions) that can be determined separately with respect to derived utility. The first optimization program is

$$
0 = \max_{c_t} \Big\{ U(c_t,t) - \mathcal{J}_V c_t \Big\}. \tag{5.11}
$$

which leads to

$$
U_c(c_t^*, t) = \mathcal{J}_V \qquad \Longrightarrow \qquad c_t^* = U_c^{-1}\Big(\mathcal{J}_V\Big) \tag{5.12}
$$

because in expression (5.11), an increase in consumption would lead to an increase in utility while a decrease in wealth would lead to decrease in derived utility.
The second optimization program is given by

$$
\begin{aligned}
\max_{\omega_t} \Bigg\{ & \mathcal{J}_V \omega_t F_t \Big[\kappa_2(\mu_{2,t} - \ln F_t) - \sigma_{2,t}\eta\lambda_2 e^a J_2 \int z\nu_2(\mathrm{d}z) \Big] + \frac{1}{2}\mathcal{J}_{VV}\omega_t^2 F_t^2 \sigma_{2,t}^2 \\
& + \mathcal{J}_{VF}\omega_t F_t^2 \sigma_{2,t}^2 + \mathcal{J}_{VS}\omega_t F_t S_t \rho \sigma_{1,t}\sigma_{2,t} + \mathcal{J}_{VQ}\omega_t F_t Q_t \varrho \varsigma \sigma_{2,t} \\
& + F_t \lambda_2 e^a \int \Big[\mathcal{J}(F_{t-} + F_{t-}\omega_{t-} J_2 z, t) - \mathcal{J}(F_{t-}, t) \Big] \nu_2(\mathrm{d}z) \Bigg\}.
\end{aligned} \tag{5.13}
$$

To solve (5.12) and (5.13), additional assumptions on hedger's preferences and jump-size distribution are required. We consider a hedger with preferences represented by exponential utility functions for both instantaneous and terminal utility functions

$$\begin{aligned} U(x,t) &= -e^{-rt-\gamma x} && (5.14) \\ B(x,T) &= -e^{-rT-\gamma x} && (5.15) \end{aligned}$$

with constant risk aversion parameter $\gamma \in (0,1)$. The function $B(\cdot,T)$ is assumed to be similar to the instantaneous utility function $U(\cdot,t)$ for simplification to avoid analysis of trade-off between the utility of inter-temporal consumption and the utility of the terminal wealth.

Under exponential utility, we conjecture that one can find a solution to (5.10) of the form (see Monoyios [Monoyios 2004] and Zariphopoulou [Zariphopoulou 2001])

$$\mathcal{J}(V,F,S,Q,t) = -e^{-\gamma\varphi(t,T)v}\left(\mathbb{E}^0_{t,y}\left[e^{-\chi(T-t)-\gamma\varpi^2 S_T Q_T}\right]\right)^{1/\varpi^2} \tag{5.16}$$

where $\varphi(t,T) = e^{r(T-t)}$, $_{0\leqslant t\leqslant T}$ and $V_t = v$ is the time t endowment of wealth. $\mathbb{E}^0_{t,y}$ denotes expectation under the minimal martingale measure conditional on $S_T g(Q_T) = y$ with χ defined market price of risk come from both continuous and jump components according to the minimal martingale

$$\chi = \frac{1}{2}\eta^2\varpi^2 + \lambda_2(e^a - 1), \quad \varpi^2 = 1 - \rho^2.$$

Then, the differentiations of (5.16) lead to

$$\begin{aligned} \mathcal{J}_V(V,F,S,Q,t) &= -\gamma\varphi(t,T)\mathcal{J}(V,F,S,Q,t) \\ \mathcal{J}_{VV}(V,F,S,Q,t) &= \gamma^2\varphi^2(t,T)\mathcal{J}(V,F,S,Q,t) \\ \mathcal{J}_{VF}(V,F,S,Q,t) &= -\gamma\varphi(t,T)\mathcal{J}_F(V,F,S,Q,t) \\ \mathcal{J}_{VS}(V,F,S,Q,t) &= -\gamma\varphi(t,T)\mathcal{J}_S(V,F,S,Q,t) \\ \mathcal{J}_{VQ}(V,F,S,Q,t) &= -\gamma\varphi(t,T)\mathcal{J}_S(V,F,S,Q,t) \end{aligned} \tag{5.17}$$

Plus, we assume that the jump-sizes follow exponential distribution with parameter $b > 0$

$$\nu_2(\mathrm{d}z) = be^{bz}, \quad z < 0. \tag{5.18}$$

Using (5.16) and the differentiation of value function allow to write

$$\begin{aligned} 0 = -\gamma\varphi(t,T)\omega_t\Big[\kappa_2(\mu_{2,t} - \ln F_t) - \sigma_{2,t}\eta - \frac{\lambda_2 e^a J_2}{b}\Big] + \frac{1}{2}\gamma^2\varphi^2(t,T)\omega_t^2 F_t\sigma_{2,t}^2 \\ - \gamma\varphi(t,T)\frac{\mathcal{J}_F}{\mathcal{J}}\omega_t F_t\sigma_{2,t}^2 - \gamma\varphi(t,T)\frac{\mathcal{J}_S}{\mathcal{J}}\omega_t S_t\rho\sigma_{1,t}\sigma_{2,t} \\ - \gamma\varphi(t,T)\frac{\mathcal{J}_Q}{\mathcal{J}}\omega_t Q_t\varrho\varsigma\sigma_{2,t} + \lambda_2 e^a\int\Big[e^{-\gamma\varphi(t,T)F_t - \omega_t - J_2 z} - 1\Big]\nu_2(\mathrm{d}z). \end{aligned} \tag{5.19}$$

We fix $J_2 = -1$ to consider only negative jumps and take the differential with respect to ω_t yields to

$$\begin{aligned} \gamma\varphi(t,T)\omega_t F_t\sigma_{2,t}^2 + \frac{\lambda_2 e^a F_t b}{\gamma\varphi(t,T)\left(b - \gamma\varphi(t,T)F_t\omega_t\right)^2} = \Big[\kappa_2(\mu_{2,t} - \ln F_t) - \sigma_{2,t}\eta - \frac{\lambda_2 e^a}{b}\Big] \\ + \frac{\mathcal{J}_F}{\mathcal{J}}F_t\sigma_{2,t}^2 + \frac{\mathcal{J}_S}{\mathcal{J}}S_t\rho\sigma_{1,t}\sigma_{2,t} \\ + \frac{\mathcal{J}_Q}{\mathcal{J}}Q_t\varrho\varsigma\sigma_{2,t}. \end{aligned} \tag{5.20}$$

Let

$$\begin{aligned} &A(b) = \gamma^2\varphi^2(t,T)b^2, \quad B(b) = -2\gamma^3\varphi^3(t,T)b, \quad C(b) = \gamma^3\varphi^3(t,T), \\ &D(b) = -\frac{\kappa_2}{\sigma_{2,t}^2}\Big(\mu_{2,t} - \ln F_t\Big) + \frac{\eta}{\sigma_{2,t}} + \frac{\lambda_2 e^a}{\sigma_{2,t}^2}\left(\frac{1}{b} + F_t\right) \\ &\qquad - \frac{\mathcal{J}_F}{\mathcal{J}}F_t - \frac{\mathcal{J}_S}{\mathcal{J}}\frac{S_t\rho\sigma_{1,t}}{\sigma_{2,t}} - \frac{\mathcal{J}_Q}{\mathcal{J}}\frac{Q_t\varrho\varsigma}{\sigma_{2,t}} \end{aligned} \tag{5.21}$$

then equation (5.20) is cubic polynomial in $\omega_t F_t$ and we set $x_t := \omega_t F_t$,

$$C(b)x_t^3 + B(b)x_t^2 + A(b)x_t + D(b) = 0. \tag{5.22}$$

To solve equation we apply Cardano's method of trigonometric approach. Let

$$p(b) = \frac{B(b)}{C(b)}, \quad q(b) = \frac{A(b)}{C(b)}, \quad m(b) = \frac{D(b)}{C(b)}$$

Besides, let

$$G(b) = \frac{3q(b) - p^2(b)}{9}, \quad R(b) = \frac{9p(b)q(b) - 27m(b) - p^2(b)}{54} \quad \Delta = G^3(b) + R^2(b) \tag{5.23}$$

such that the solution of equation (5.22) subject to the integral convergence condition $b - \gamma\varphi(t,T)x_t > 0$ is given by

$$x_t^* = 2\sqrt{-G^3(b)}\cos\left[\frac{1}{3}\arccos\left(\frac{R(b)}{\sqrt{-G^3(b)}}\right)\right] - \frac{p(b)}{3} \tag{5.24}$$

The optimal investment strategy x_t^* refers to drift, jump intensity compensation and cash position portfolio weights. To see that, only the coefficient $D(b)$ in (5.21) that intervenes in x_t^* via the coefficient $R(b)$ is dependent from the market moves. Indeed, as soon as the risk aversion is set up and the average jump-size is estimated, x_t^* will still vary as function of cash position. The trading strategy is also affected by mean-reversion speed, periodic long-run, seasonality in volatility and jump-sizes. In fact, assuming that proportions of indirect utilities (of futures and spot prices and quantity) over the function value may be interpreted as parameters that evolve inversely with risk aversion, the optimal investment strategy in futures contract strongly relies on expected cash positions as well as wealth.

Then, using futures contract allows to lower part of the risks because when the futures price and production risks are highly, correlated either positively or negatively (adverse position in futures contract), it leads to a important reduction of the total incurred risk. However jump risk in futures price may deter their position adversely yielding to loss depending on the jump-sizes.

Furthermore, the optimal consumption rate c_t^* is also function of both expected cash position and endowment of $V_t = v$ at time t,

$$c_t^* = \frac{1}{\gamma}\ln(\gamma) - \varphi v + \frac{1}{\gamma\varpi^2}\ln\left(\mathbb{E}_{t,y}^0\left[e^{-\chi(T-t)-\gamma\varpi^2 S_T Q_T}\right]\right) \tag{5.25}$$

The producter's optimal consumption rate and strategies depend upon his wealth position. When his wealth is uncertain, he would consume less and would take more important position in futures. That is, when the producter incurs risk, he will hedge more for future consumption in order to lower risk unanticipated fall in consumption.

At the end, the using position in futures contracts as hedging strategy need to be performed since both wealth and cash position uncertainties will substantially affect the strategies. An alternative is to add another derivative on the non-traded asset. We address the case of European put option in following section.

5.3 Hedging basis risk with put option

Previously, cash position relates to non-traded and futures contract is used as hedging instrument against adverse moves of its position. In this section, we add to the initial investment strategy an European put option written on the non-traded asset with payoff $h(S_T)$ at maturity T, where h is a continuous function.

In this framework the investor's account is credited at time T with π units of the option payoff $h(S_T)$. Then, the hedger's optimization problem is now the trading strategy $\widetilde{x} := (\widetilde{x}_t)_{0 \leqslant t \leqslant T}$ in the class of admissible strategies to achieve the supremum

$$\mathcal{J}^{\pi}(t, v, y) := \max_{\widetilde{x}_t} \mathbb{E}^{0}_{t,v,y}\Big[u\left(V_T + \pi h(S_T)\right)\Big] \tag{5.26}$$

where V_t follows the same stochastic differential equation in (5.7) and $\mathbb{E}^0_{t,v,y}$ denotes expectation under the minimal martingale measure conditional on information available at t, $V_t = v$ and $S_t = y$. In this strategy, the consumption rate is neglected. Besides, one requires the initial endowment of the option $\pi h(S_T)$ to be bounded below, which covers long positions in calls and puts, short positions in puts but excludes short call positions (see Monoyios [Monoyios 2004]).

With the same assumptions of section 5.2, the application of Itō formula on (5.26) leads to the same differential equation in (5.19) subject to $V_t = v$ and $S_t = y$. Therefore, the conjecture of the value function with option credit at maturity T is still given by

$$\mathcal{J}^{\pi}(t,v,y) = -e^{-\gamma\varphi(t,T)v}\left(\mathbb{E}^{\mathbb{Q}}_{t,y}\left[e^{-\chi(T-t)-\gamma\varpi^2\pi h(S_T)}\right]\right)^{1/\varpi^2}. \tag{5.27}$$

Then, to derive optimal hedging strategy $\tilde{x}_t$, the option price is necessary.

5.3.1 The asking price of the put option

Recall that the option price with jump-diffusion process relies on the assumption of jump-size distribution. We consider the jump-size to follow exponential distribution of parameter b and the option price can be determined with Kou [Kou 2002] model. The Kou option price model is double exponential distribution for the jump-size. Herein, we only focus on negative jumps.

Consider a put option where $h(S_T) = (K-y)^+$ for $K > 0$. The price of a European put of maturity T and strike K is given by (see Kou [Kou 2002] and Kou and Wang [Kou 2004])

$$P_T(k) = e^{-rT}\mathbb{E}^0\left[\left(e^k - S_0 e^{S_T/S_0}\right)^+\right] \tag{5.28}$$

where $k = \ln(K)$. Hence, the utility indifference asking price at time $t \leqslant T$ of a European put with payoff $h(S_T) = P_T(k)$ is given by (see Monoyios [Monoyios 2004])

$$p^a(t,y) = \frac{e^{-r(T-t)}}{\gamma\varpi^2}\ln\left(\mathbb{E}^0_{t,y}\left[e^{\gamma\varpi^2 P_T(k)}\right]\right). \tag{5.29}$$

The greek, Δ, the sensitivity of the option with respect to underlying price, can be estimated by inverting the derivative of the option's Laplace transform (see Kou [Kou 2007])

$$\frac{\partial P_T(k)}{\partial S_0} = \mathcal{L}_\xi^{-1}\left(\hat{f}_P(\xi)\right), \quad \xi > 1. \tag{5.30}$$

with $\mathcal{L}_\xi^{-1}$ the Laplace inversion with respect to ξ. The Laplace transform with respect to k for the put option $P_T(k)$ is

$$\hat{f}_P(\xi) = \int_{-\infty}^{\infty} e^{-\xi k} P_T(k) = e^{-rT}\frac{S_0^\xi}{\xi}\exp\left[\psi(-(\xi-1)T)\right], \quad \xi > 1 \tag{5.31}$$

Function $\psi(z)$ is the Lévy exponent of characteristic function of the spot price process obtained via affine transformation as in Chapter 3.

5.3.2 Optimal hedging strategy

In the framework of (5.26), consider the situation where there is no position in option in the investment strategy; that is $\pi = 0$

$$\mathcal{J}(t,v) := \mathcal{J}^0(t,v,y). \tag{5.32}$$

This situation exactly corresponds to the investment strategy in (5.24) where hedging is carried out with only futures contract. Furthermore, following the analysis in Monoyios [Monoyios 2004], we consider the situation of a debit of one unit of the option payoff $h(S_T)$ which corresponds to $\pi = -1$. The classical definition of the utility indifference selling price or simply the *ask price* of the claim, $p^a(t,y)$, is the solution of

$$\mathcal{J}(t,v) = \mathcal{J}^{-1}\left(t, v + p^a(t,y), y\right) \tag{5.33}$$

Let $\tilde{x}^{-1} := (\tilde{x}_t^{-1})$, $0 \leqslant t \leqslant T$ be the optimal trading strategy for the problem with value function $\mathcal{J}^{-1}\left(t, v + p^a(t,y), y\right)$. The difference between the optimal strategy for $\pi = -1$ and $\pi = 0$, represents the additional position taken in the hedging instrument as a result of the sale of the claim. That is,

$$\widetilde{x}_t^h = \widetilde{x}_t^{-1} - \widetilde{x}_t, \tag{5.34}$$

where $\widetilde{x}_t^h$ is hedging strategy associated to the sale of the claim at the ask price $p^a(t, y)$. The strategy, $\widetilde{x}_t^h$, corresponds the classical option hedging strategy when futures price is perfectly correlated to spot price ($\rho = \pm 1$) and perfectly correlated to quantity process ($\varrho = \pm 1$). Since the jump components of the two assets are independent, they will not affect this relationship. Therefore, the differential of (5.33) with respect to y yields

$$\mathcal{J}_y^{-1}\left(t, v + p^a(t, y), y\right) = -\mathcal{J}_v^{-1}\left(t, v + p^a(t, y), y\right) p_y^a(t, y) \tag{5.35}$$

where $p_y^a(t, y)$ is derivative with respect to $S_T = y$.

Then, a fundamental result from Monoyios [Monoyios 2004] follows. The hedging strategy for the sale of the put option at the asking price $p^a(t, y)$ at time $t \in [0, T]$ is to hold ω_u^a shares of the futures contract at time $u \leqslant t$, given by

$$\omega_u^a = \frac{\rho \sigma_{1,u} S_u}{\sigma_{2,u} F_u} \frac{\partial p^a(u, y)}{\partial y} \tag{5.36}$$

where $p_y^a(t, y)$ is function of delta hedging,

$$\begin{aligned} \partial p_y^a(t, y) &= e^{-r(T-t)} \frac{\partial P_T(k)}{\partial S_0} \\ &= \mathcal{L}_\xi^{-1}\left(\hat{f}_P(\xi)\right), \quad \xi > 1 \end{aligned} \tag{5.37}$$

The strategy in (5.36) is discounted delta hedge. Delta hedging strategy expresses a corresponding change in the underlying price to how much of the change will be reflected in the option price. The discount factor relates to maturity effect in that hedging position is further tempered with longer time-to-maturity. Hence, the option really affects the hedging closer to maturity.

5.4 Conclusion

We have derived optimal hedging strategy with asset price following mean-reverting jump-diffusion process. This market is incomplete and the hedging problem is conducted in expected utility maximization framework under minimal martingale measure. We consider exponential utility function under which a distortion method can be used to conjecture the value function. The solution to optimization program follows with further assumptions on jump-size distribution.

In the first instance, we consider hedging with futures contract and strategy strongly depends on market moves and production risk. To further reduce the basis risk, European put option has been considered later. Optimal hedging strategy with put option results in hedging with futures plus a discounted delta hedging strategy. The delta hedging strategy with option is well known to reduce the price risk of underlying asset.

Chapter 6

General Conclusion

All around the world, production of commodities relies on many uncertain factors. These factors may become unfavorable for a producer, specially in agriculture that mainly serves for human diet. The main risks in agricultural production coming from changes in prices and outputs than expected. Indeed, at harvest time, prices could be unfavorable with regard to costs and the expected yield crop. Then, the producer should look for a way to guarantee as much as possible his revenue against these risks. The financial market is an alternative to transfer these risks to investors that are able bear them for a premium paid by producers. Commodity markets allow producers, in need, to hedge their crop revenue, but they also serve as save haven for investors.

In financial markets, various instruments can be used to develop hedging strategies. Such strategies mainly include derivatives on spot price like futures, forwards or options contracts. The derivatives allow to postpone delivery at future date and this at predetermined price. Producers could lock in price for a certain maturity date. Besides, because derivatives are speculative but also subject to various risk which may result in important loss, a producer with positions in the above mentioned derivatives could also, nevertheless, look for additional revenue in the market. Another hedging instrument is insurance contract that is like option, but it will be specially used to address production risk in case of low crop yield. The insurance contract guarantees the underwritten proportion of expected yield crop.

In this thesis, the study have been set out to investigate the modeling agricultural futures prices and the issue of hedging strategies in portfolio context. Hedger is a producer whose portfolio strategy is consisted of non-traded asset as spot price and futures, options or insurance contracts to cover his revenue against adverse price moves or production deficiency. Hedging strategy requires prior market investigation in order to address appropriate expectations. In commodity markets, stylized facts are informational to draw dominant economic rationales. Empirical analysis on past data is usual way to draw economic factors as model and then, expectations will be made following predictions, preferences and contexts. Accordingly, appropriate hedging strategy iss developed with regard to the producer expectations. To this end, we have investigate hedging issue in both static and dynamic frameworks.

The thesis has explored the following aspects:

- ◇ *Behavior of commodity prices*: Using econometric tools on commodity prices data at hands, we have checked for well known stylized facts in literature: mean-reversion, seasonality and presence of jumps. The tests on daily prices are conducted in such that all the features can be considered all together in price model as economic factors. These tests have shown that, commodity market is likely to be inefficient with mean-reversion and finite jump activity. Mean-reversion feature reflects the equilibrium state of price that temporary deviates from its fundamental value. This relies on how supply and demand imbalance will fluctuate to converge towards the equilibrium price. Plus, there is seasonal behavior in both the long-run of mean-reversion and monthly volatility to relate the calendar shape on prices. That is to say that seasonal patterns come from respectively delivery of futures contracts and crop year. Jumps come from relevant changes due to the sudden news and especially in inter-crop season periods. They represent additional market risk to manage in dynamic hedging.

◇ *Modeling the agricultural futures prices*: In light of the highlighted stylized facts in Chapter 2, mean-reverting jump-diffusion process with periodic long-run mean and seasonal volatility is posited as model for the agricultural prices at hands. Such dynamic without jump is similar to those that have been investigated earlier in the literature where a deterministic function is considered for the seasonal component in the trend (see Geman and Nguyen [Geman 2005]). Jumps occur randomly and infrequently and their consideration will make difficult the estimation of the process under study with maximum likelihood. Hence, we carry out the estimation in two-stage procedure. In the first step, the speed of mean-reversion as well as periodic long-run mean parameters are estimated with least square technique. In the second step, we apply particle MCMC method to estimate the remaining parameters with the residuals of the first step. Particle MCMC are applied, instead of Kalman filter, because non Gaussian noise. Particle MCMC is proven to be robust Bayesian method in that the exact likelihood function of the measure process conditional on parameters of interest is approximated by particle filtering which is known to be more consistent. The parameters are then considered as latent random variables. The implementation of this estimation procedure gives parameter estimates that are validated with Ljung-Box test on residuals.

◇ *Static optimal hedging strategy*: In static framework, many approaches according to the hedger's preference have been investigated. They lead to optimal hedge ratios that strongly depend on the approach. Hence, it is difficult to distinguish which strategy is the best among all the hedge ratios based on the Ederington performance measure.

To overcome this limit of effectiveness measure, we have suggested L-performance measure to rank strategies according to their performance. Then, we have derived the optimal hedging strategy with futures contract in presence of market risk and output risk when long term futures market is missing. Such strategy is implemented with rollover process. The rollover hedging consists in switching from nearby futures contract to more distant futures contract. This is to maintain the position in futures contract along with hedge portfolio horizon. However, this strategy incurs additional risk, the rollover risk, because of price spreads between the nearby futures contract and the futures with longer maturity. The optimal hedging strategy in rollover performs on other strategies on both market and output risks in in intra- and inter- crop year. Using L-performance to rank strategies according to their performance, it comes out that the combination of futures and insurance contracts is the best hedging strategy over the others. Specifically, insurance contract guarantee a proportion of crop revenue when the crop yield is lower than expected.

◇ *Dynamic optimal hedging strategy* : To account for daily settlement and the stylized facts tested in chapter 2, we investigate the hedging strategy in continuous framework. The hedging horizon considered is similar to a crop year. The optimal strategy in continuous time framework is conducted in utility maximization setting where the producer is looking for the dynamic position in futures contract and consumption rate. In this hedging situation, since the spot and futures are not perfectly correlated and there is presence of jumps, the market is incomplete. And this is so when the cash market is non-traded asset. The optimization is conducted under the minimal martingale measure. We show that hedging with futures contract can be improved with additional put option in the hedge portfolio.

Future direction of research

We have investigated hedging strategy to reduce risks related to adverse move that could lower the income of a producer in agricultural markets. This issue is extensible to consider multi-commodity hedging with regard to situation of many goods as a way diversify production risk. In this case, correlation risk also matters and it will be interesting to analyze, the particular case of incomplete market in term of number of futures contracts to consider for the optimal hedge.

On another side, depending on the framework, the hedging problem can also be extended. In static framework, an alternative to improve hedging strategy could be to include, a non-path dependent claim on spot price to manage basis and rollover risks. Finally, in continuous time framework, the study lacks of application on market data. Such empirical investigation would definitely be a complement to address commodity management policy. Specifically, it will allow the analysis how dynamic strategies behave in both intra-crop and inter-crop year for long hedging horizon with missing futures markets. To this end, the redundancy of insurance contract is stressed in term of linearity and non-linearity in the hedging strategy.

Appendix A

Appendix of Chapter 1

A.1 Basic on commodity markets

A commodity is a physical good that is often (but not always) a primary input to production processes that generate a refined good. There are different kinds of commodities traded and three classes[1] may be distinguished: energy (crude oil, natural gas, ...), metal (silver, gold, copper, ...), and agriculture (corn, wheat, soybean, cocoa, ...).

There is an increasing appetite of investors for commodity markets as save haven and this contributes to uncertainty associated to their economy as well as their price fluctuations. Relevant commodity markets are in the USA: CME Group (Chicago Mercantile Exchange), [Gorton 2013]. For example, in the USA, commodity markets are regulated by the *Commodity Futures Trading Commission* - (CFTC). There are other organized markets, in Europe (the NYSE Liffe and the London Commodity Exchange - LCE), in Osaka (Kansai Commodities Exchange, KANEX). Market actors consist of hedgers and speculators. Hedgers are producers, traders and agribusinesses while speculator are market liquidity providers such as participants

[1] An other classification of commodities exists as soft and hard that distinguishes perishable commodity from the others. Soft commodity include consumption commodities that depend on weather like agriculture and livestock and hard commodities are energy and metals (precious and industrial).

in Hedge Funds.

Commodities are effectively traded in physical markets for immediate delivery, but also they are also exchanged in financial markets as financial instruments. The financial instruments of commodity markets are spot and its derivatives that are used for both hedging and speculation. Forwards, futures, and options are the most common and traditional hedging instrument although more complex derivatives such as swaps, exotic options, real options, and credit derivatives have emerged in recent years. Futures contracts are the most important instruments traded for agricultural commodities as they are highly regulated. In following paragraphs, spot and traditional instruments assets are described.

A.1.1 Spot price

A spot price reflects all the characteristics that satisfy required quality of a commodity good. It is the current market price at which a commodity is traded at a given location. Due to wide swings[2] in either demand or supply, spot prices are not easy to define. This makes the level of volatility also difficult to forecast accurately because of swings in production and consumption. Finally, spot price appears to be strongly correlated with unexpected inflation.

For this reason, most of financial exchanges relate to futures (or forward) contracts instead of cash market. Indeed, the main feature of financial derivatives is the exchange of the underlying asset that takes place at future date upon an agreed price. Forward and futures are the instruments to hedge and to trade risk for a maturity.

[2]Extreme weather (drought, frost, or thunderstorms) can reduce the harvest or even destroy it entirely, new technologies that help for production, political risks may disrupt production and distribution, changes in taste and consumption patterns.

A.1.2 Forward contract

A forward contract is an agreement between two parties to buy or sell an asset (of any kind) at an agreed future date and at a specified price called exercise price. In commodity markets, forward contracts involve physical settlement at maturity and most of them are cash-settled. At inception of forward contract, no initial payment and no transfer of ownership of the underlying are required because exercise price is equal to initial forward price in order to fix the value of forward contract at initial date.

Formally, consider a forward contract written at initial time $t = 0$ and that will mature at T with $t \in [0, T]$ being the current time. Let's K be the exercise price, $G_{t,T}$ the forward price at time t that will mature at T and V_t^G the market value of forward contract. At inception of forward contract, we have

$$V_0^G = 0 \qquad \text{and} \qquad K = G_{0,T}. \tag{A.1}$$

To find the forward price, assume the financial market is arbitrage free, complete and perfect with $b_{t,T}$ the value at time t default-free discount bond that pays one numÃ©raire at T. Let S_t be the spot price at t which equates the forward price at maturity T.

$$S_T = G_{T,T} \qquad \text{and} \qquad V_T^G = G_{T,T} - K. \tag{A.2}$$

At time $t = 0$, conditions (A.1) and (A.2) combined with no-arbitrage assumption leads to,

$$V_0^G = S_0 - Kb_{0,T} = 0 \quad \Longrightarrow \quad G_{0,T} = \frac{S_0}{b_{0,T}}. \tag{A.3}$$

$G_{0,T}$ being the equilibrium forward price at inception and more generally,

$$G_{t,T} = \frac{S_t}{b_{t,T}} \quad \text{for} \quad t \in [0, T]. \tag{A.4}$$

This pricing of forward contract also works even if there are storage costs associated with holding the underlying asset, Jarrow and Oldfield [Jarrow 1981].

A forward contract is traded between two parties. At maturity, there are always a looser and a winner. The former gives to the latter $G_{T,T} - G_{0,T}$ in one of the two following forms:

- ⋄ effective delivery of the underlying asset at $S_T = G_{T,T}$ against payment of $G_{0,T}$,
- ⋄ or by payment of:
    - ⋆ $G_{T,T} - G_{0,T}$ by the seller to the buyer if $G_{T,T} > G_{0,T}$,
    - ⋆ or $G_{0,T} - G_{T,T}$ by the buyer to the seller if $G_{0,T} < G_{T,T}$.

In practice, it is not easy to unwind a forward contract before it matures. For example an opposite position at time t in forward contract will just fixe its values at $G_{t,T} - G_{0,T}$ at that time.

Forward contracts incur default risk because they are traded over-the-counter and are not easy to offset. Futures contracts, instead may be an alternative to forward contracts.

A.1.3 Futures contract

Futures contracts have originated with agricultural commodities (corn, oats, and wheat) in U.S. Chicago Board of Trade (CBOT). Seemingly, futures contracts are like forward that are traded in organized markets[3] and are standardized in their expiration date, the size of contract, quality of physical commodity. Futures contract are traded with initial margin[4] from two counterparts and are daily settled. Initial margins are held by a clearinghouse

[3] Such organized markets are Chicago Board of Trade (CBOT), Euronext Paris SA, etc...

[4] Margins in futures trading significantly differ from the margin in stock or bond trading and it is low (usually 7% to 10% of the value of the contract being traded).

being the intermediate and regulator. This makes the value of a futures contract to be marked to market with daily settlement of profits and losses. If $F_{t,T}$ is the price of a futures contract at time $t \in [0,T]$, the daily margin paid or received by the buyer is $F_{t,T} - F_{t-1,T}$ while the seller disburses or collects $F_{t-1,T} - F_{t,T}$. The daily settlement implies the value of futures contract to be zero everyday.

Futures contracts more liquid and less risky than forward in that they can easily be cancelled at any time before maturity. Futures contract may be seen as many forward contracts, each forward maturing everyday until the expiry of futures contract. Otherwise, forward and futures look similar because they indicate price expectations and the economy's direction since they are viewed as forecast of future spot price in the short-run (Houthakker [Houthakker 1968], Chow et al. [Chow 2000]). However, they are not similar as they may appear. The main difference between them is that futures contracts are daily settled whereas forward contracts are settled at maturity. This implies futures prices to be greater (less) than forward prices if the risk-free interest rate is stochastic and positively (negatively) correlated to spot price.[5] They differ in taxation, transactions costs and margin rules and market standards. Futures contracts present low counterpart risk because of the clearinghouse.

Practically, a buyer pays $G_{0,T}$ for forward contract and $F_{T,T}$ for futures contract. Chow et al. [Chow 2000] and Pindyck [Pindyck 2001] argued that there no significant difference exists between forward and futures prices for most of commodities. Another famous derivatives traded in commodity markets are options.

[5] *"To see why this is so, note that if the interest rate is non-stochastic, the present value of the expected daily cash flows over the life of the futures contract will equal the present value of the expected payment at termination of the forward contract, so the futures and forward prices must be equal. If the interest rate is stochastic and positively correlated with the price of the commodity (which we would expect to be the case for most industrial commodities), daily payments from price increases will on average be more heavily discounted than payments from price decreases, so the initial futures price must exceed the forward price"*, Pindyck [Pindyck 2001].

A.1.4 Options

Option are more flexible than both forwards and futures as they give buyers the right, not the obligation, to buy or sell underlying asset at strike price in future date. The price to pay is that options come to be riskier but they offer diversification in portfolio with limited upside or downside risk.

Commodity options constitute an exception[6] on how to value derivatives with a different, deeper approach because they are not easily transferable into future like other financial assets. For instance, agricultural products are perishable and may need additional costs for storage. Moreover, the no-arbitrage relationship between futures and spot may not hold because of location specificity of the commodity and the intricacy of storage. Consequently, hedging dynamically an underlying that may not be owned (illiquid) requires some precautions. Particularly, pricing and hedging commodity options should deal with every expiry like a separate underlying security and with a specific arbitrage involving the physical delivery. This is challenging issue, but pricing options on futures contracts represents an alternative.

As futures are more liquid than spot as financial instruments, commodity options are written on futures with the option maturity usually shorter than futures contract maturity. Option prices can be derived easily with no-arbitrage condition because a futures contract is a martingale under the risk neutral measure. Besides, modeling spot prices is difficult as the factors that contribute to equilibrium are not easy to determine.

Agricultural options are largely exchanged by the CME group (created by the merging of Chicago Mercantile Exchange (CME) and the Chicago Board of Trade (CBOT)) in the U.S. and Euronext Liffe outside the U.S. The underlying products include dairy products, cocoa, coffee, sugar, soybean products, corn, wheat, live cattle, and lean hogs. There are many others options

[6] "*You can own all the oil you need in Rotterdam; but, if your delivery is in New York tomorrow, you will have a problem*", Nassim Taleb in Foreword of Geman [Geman 2009].

traded in commodity markets, beside classical call and put options. An interesting one is spread option such as a plain vanilla option written on the difference between two futures prices (or, on the difference between two spot prices). It is traded for a diverse range of products used as hedging instruments for variety of risks, correlation and lock in revenues. Some examples include options on inter-commodity spreads (cracks and sparks), intra-commodity spreads (quality), calendar spreads, and locational spreads. Another type of derivatives are swaps. Commodity swaps are similar to an interest rate swaps, but the parties will exchange a fixed price with a floating or variable price for the commodity.

In commodity markets, trading options allows for hedging, speculation and diversification. They offer the possibility to hedge market risk but they may depict high leverage bet on the price direction for speculation. For instance, producers can use calendar spreads on two futures contracts to hedge in a market that tends to swing between backwardation and contango.

A.2 Pattern exhibits for commodity futures

A.2.1 Commodity futures prices

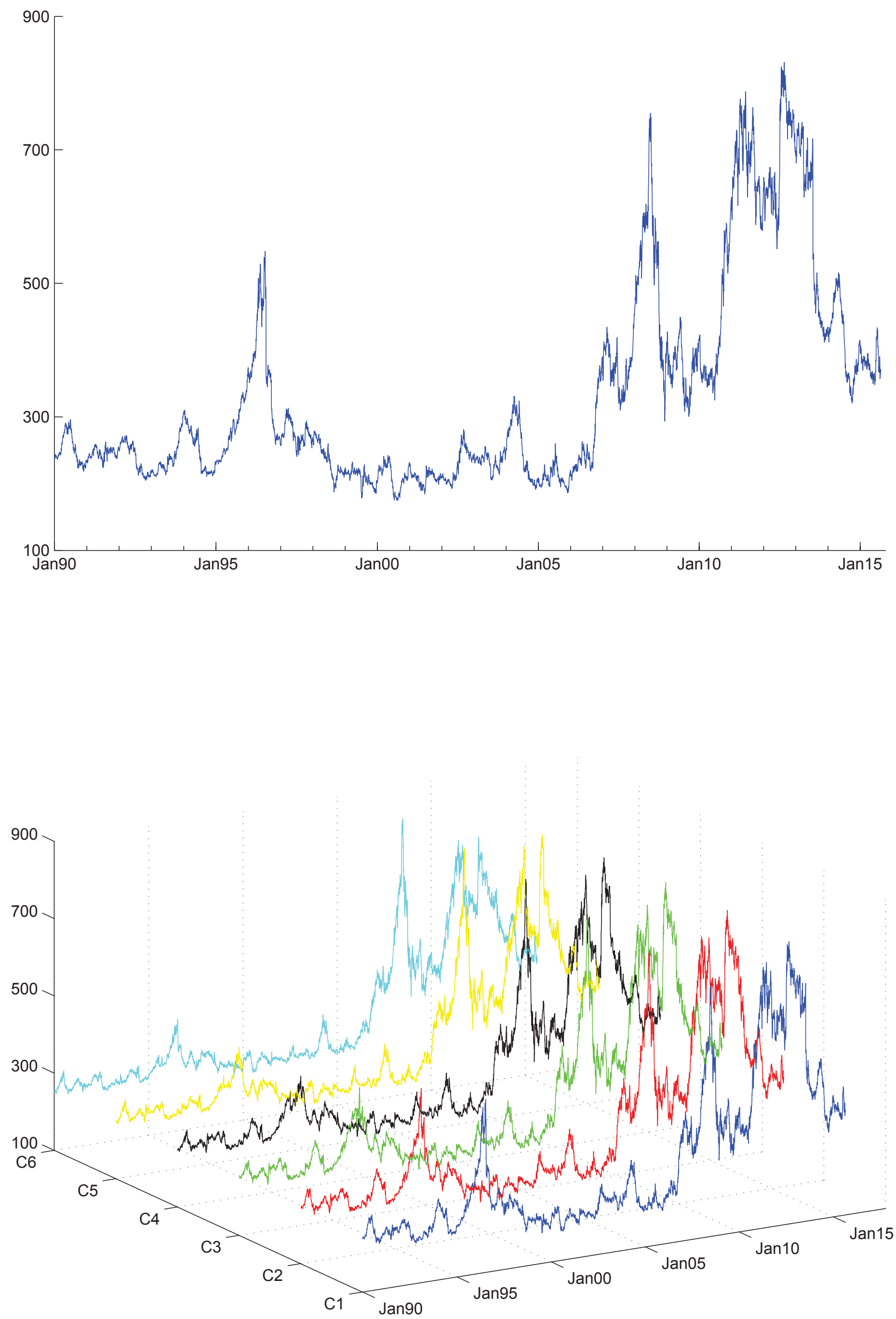
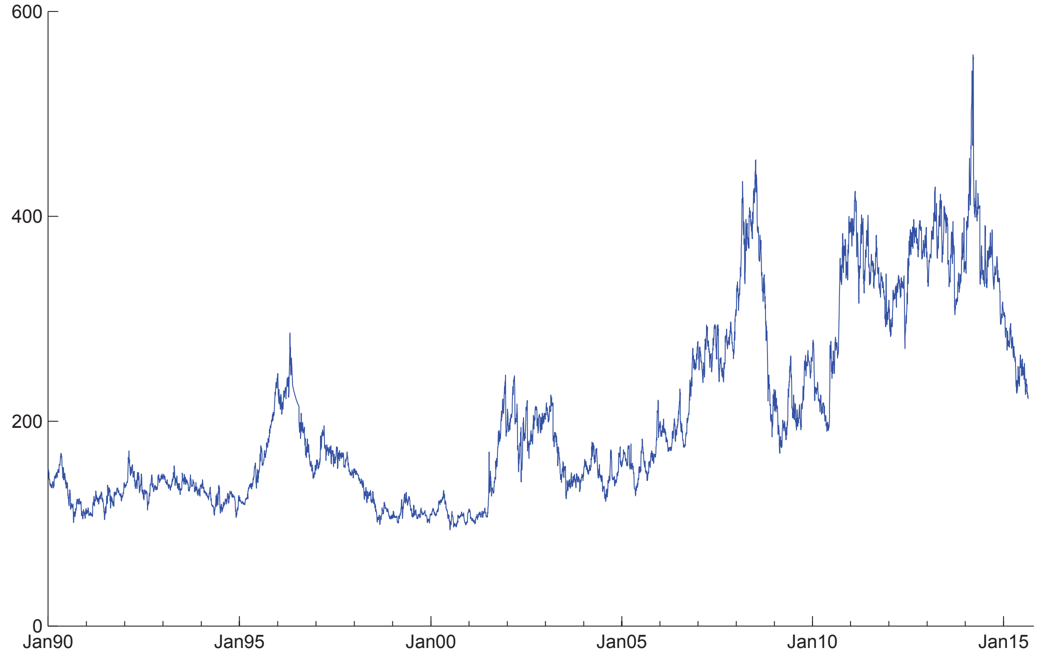


Figure A.1: Corn futures prices



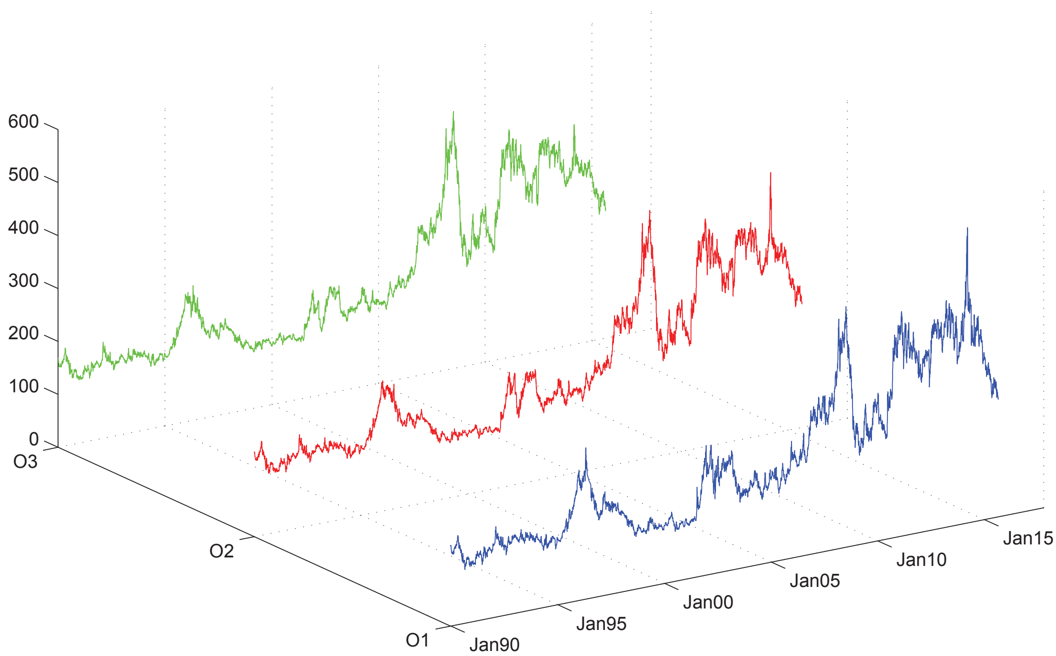
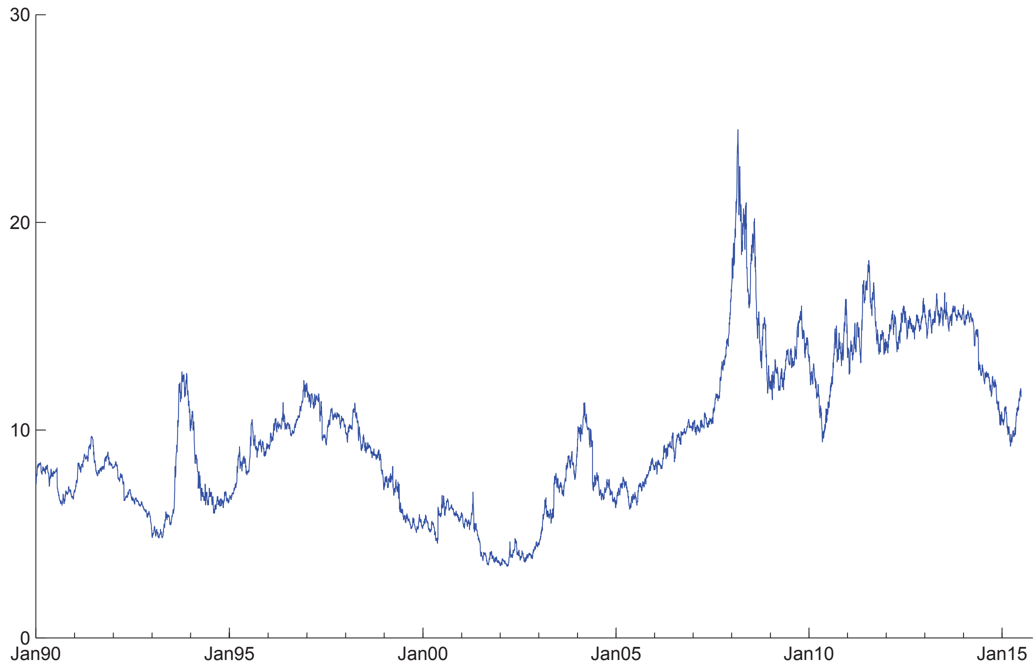


Figure A.2: Oat futures prices



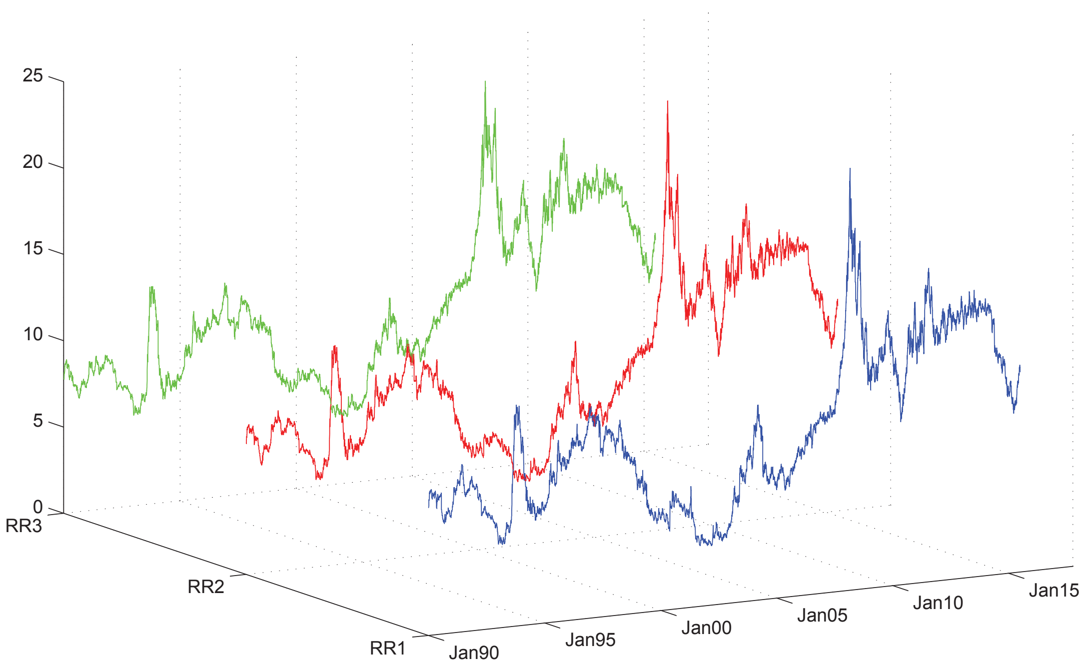


Figure A.3: Rough rice futures prices

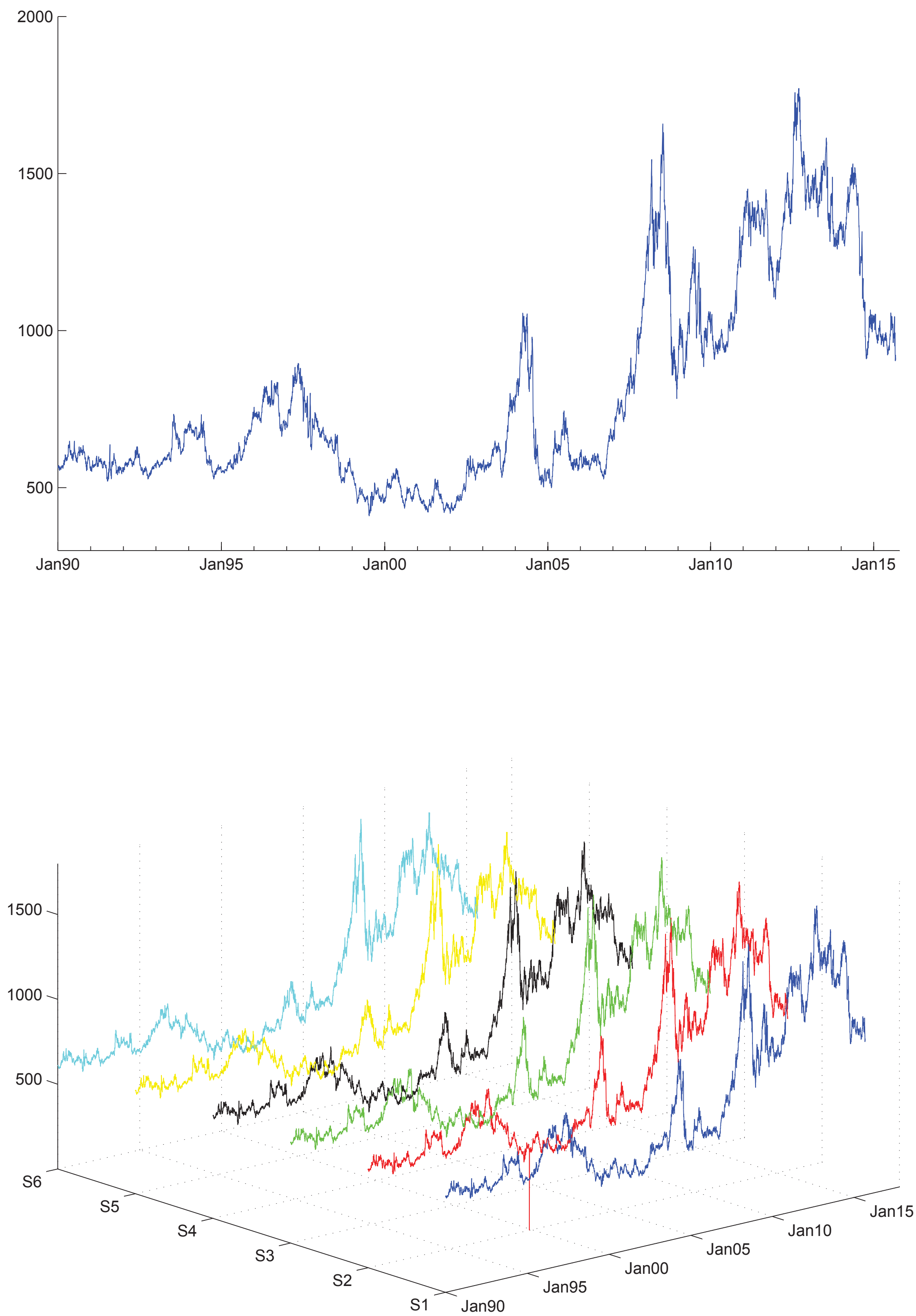


Figure A.4: Soybeans futures prices

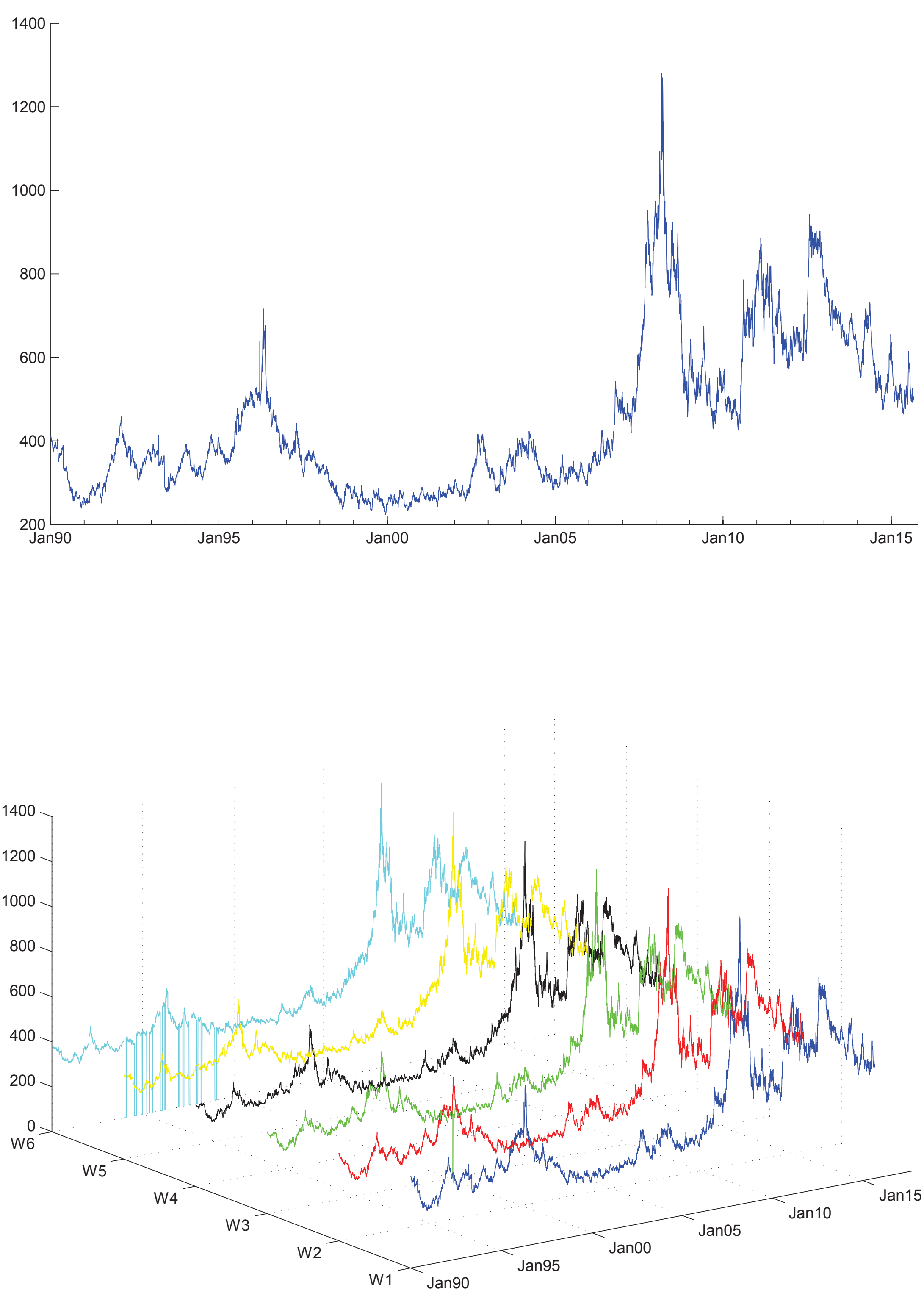
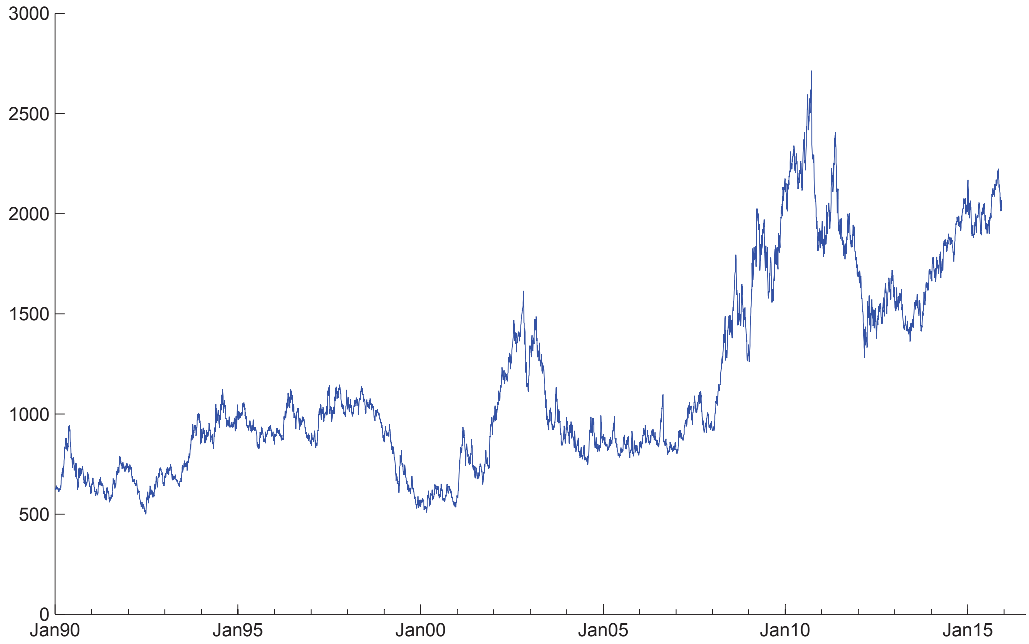


Figure A.5: Wheat futures prices



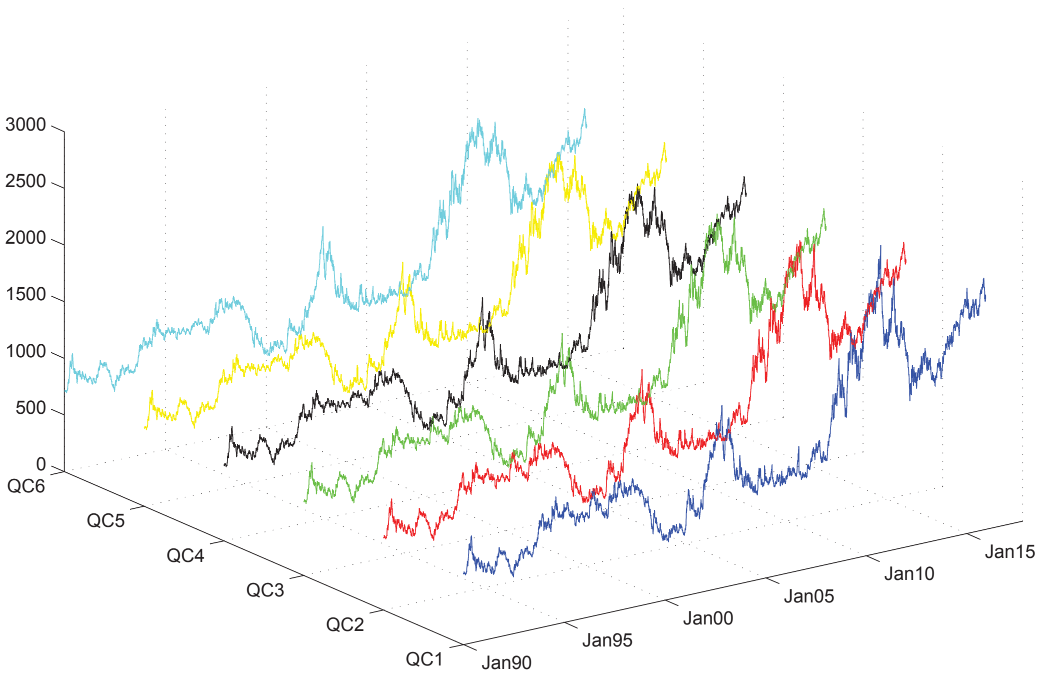
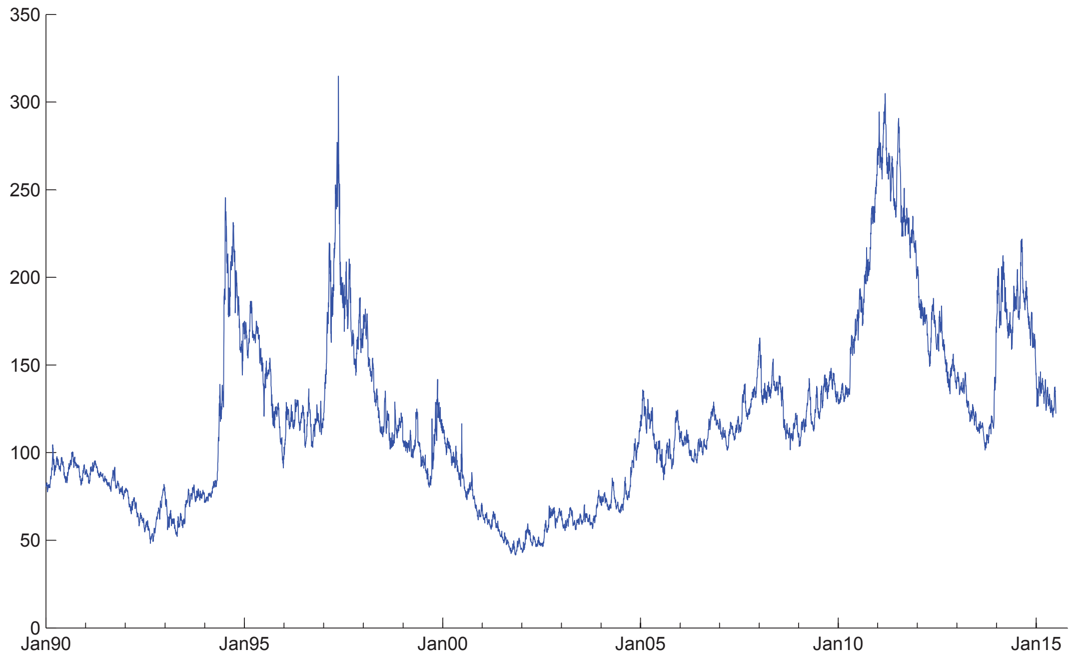


Figure A.6: Cocoa futures prices



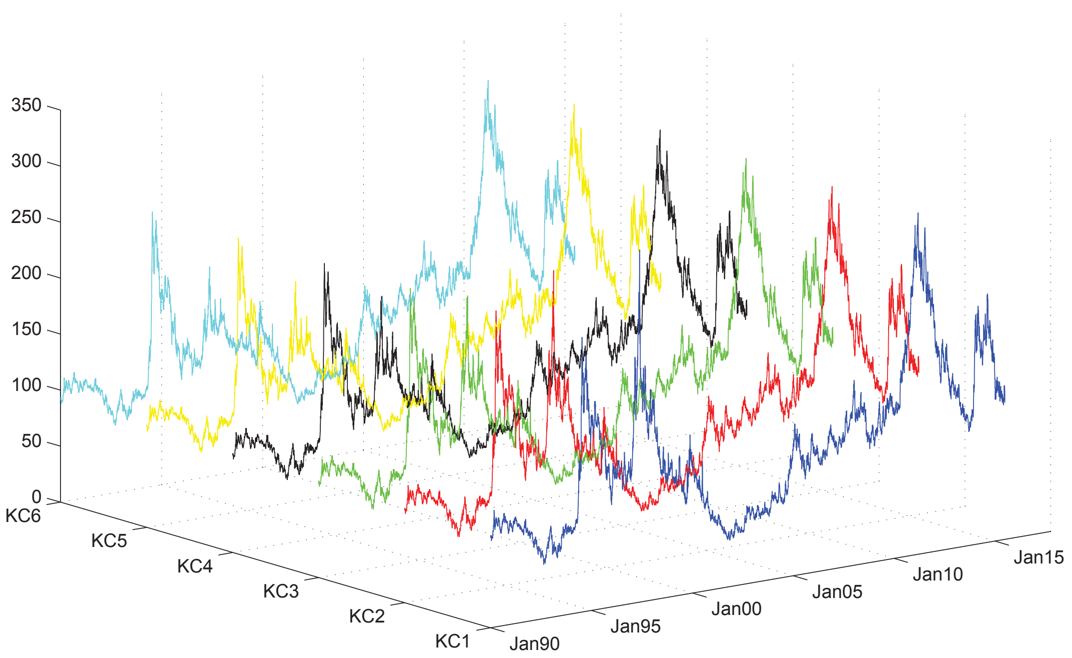
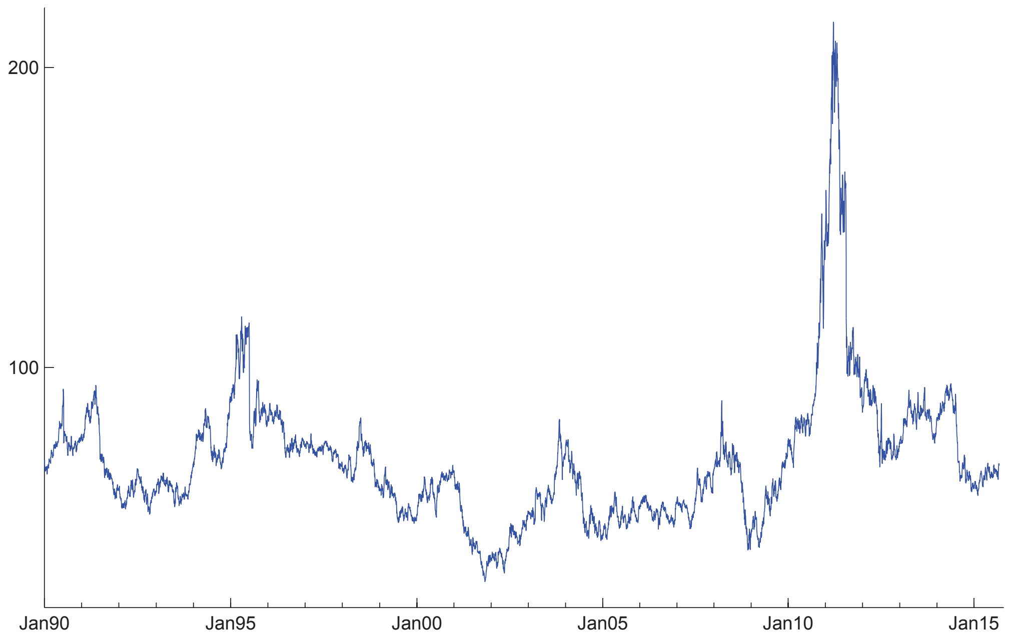


Figure A.7: Coffee futures prices



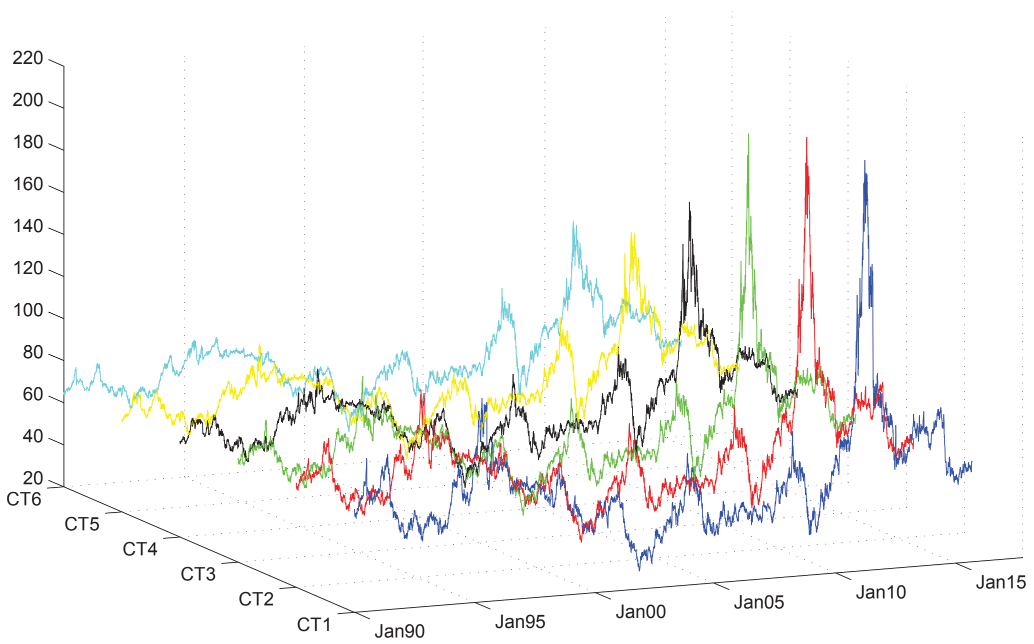


Figure A.8: Cotton futures prices

A.2.2 Monthly volatilities of commodity futures

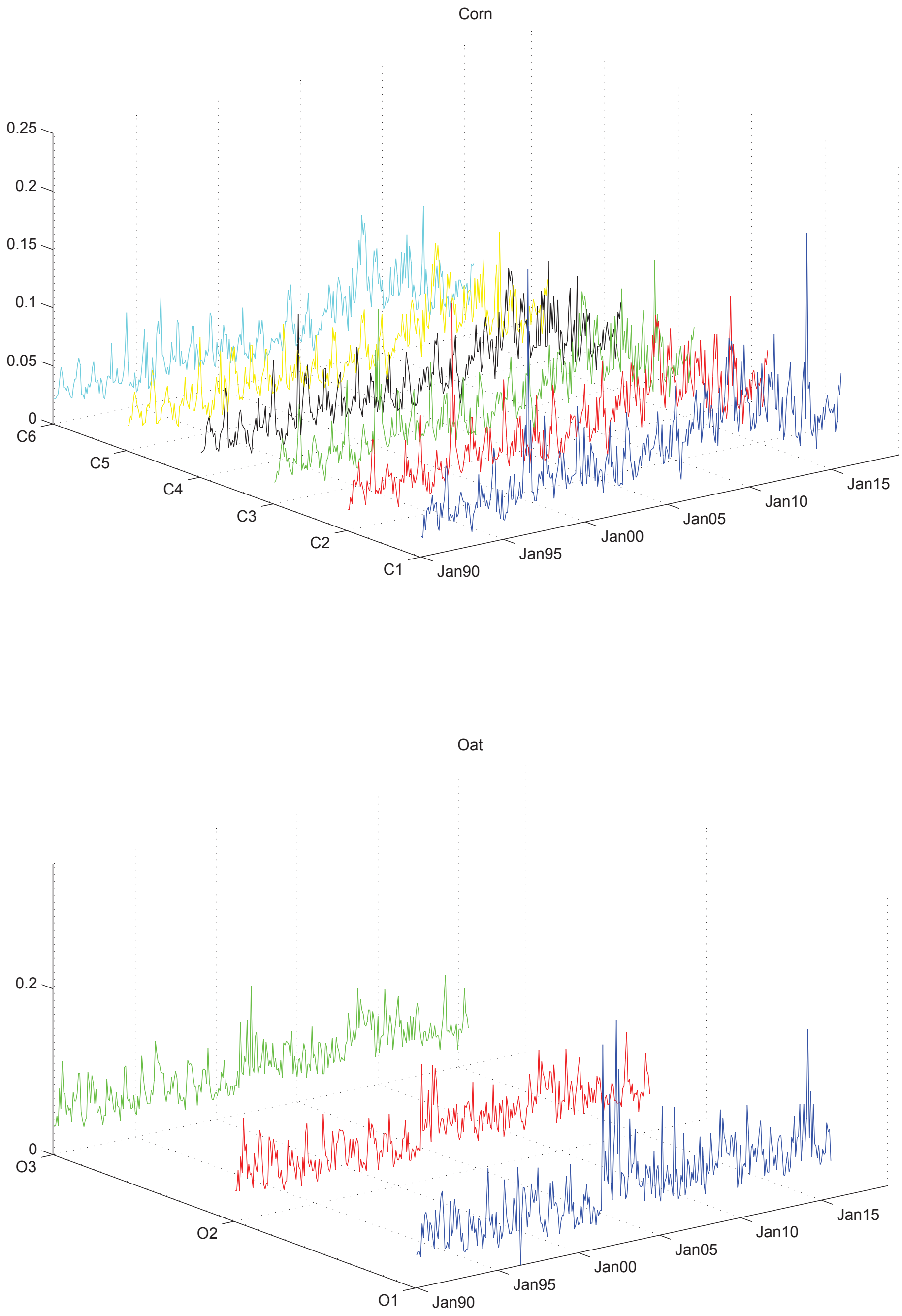


Figure A.9: Corn and Oat monthly volatilities

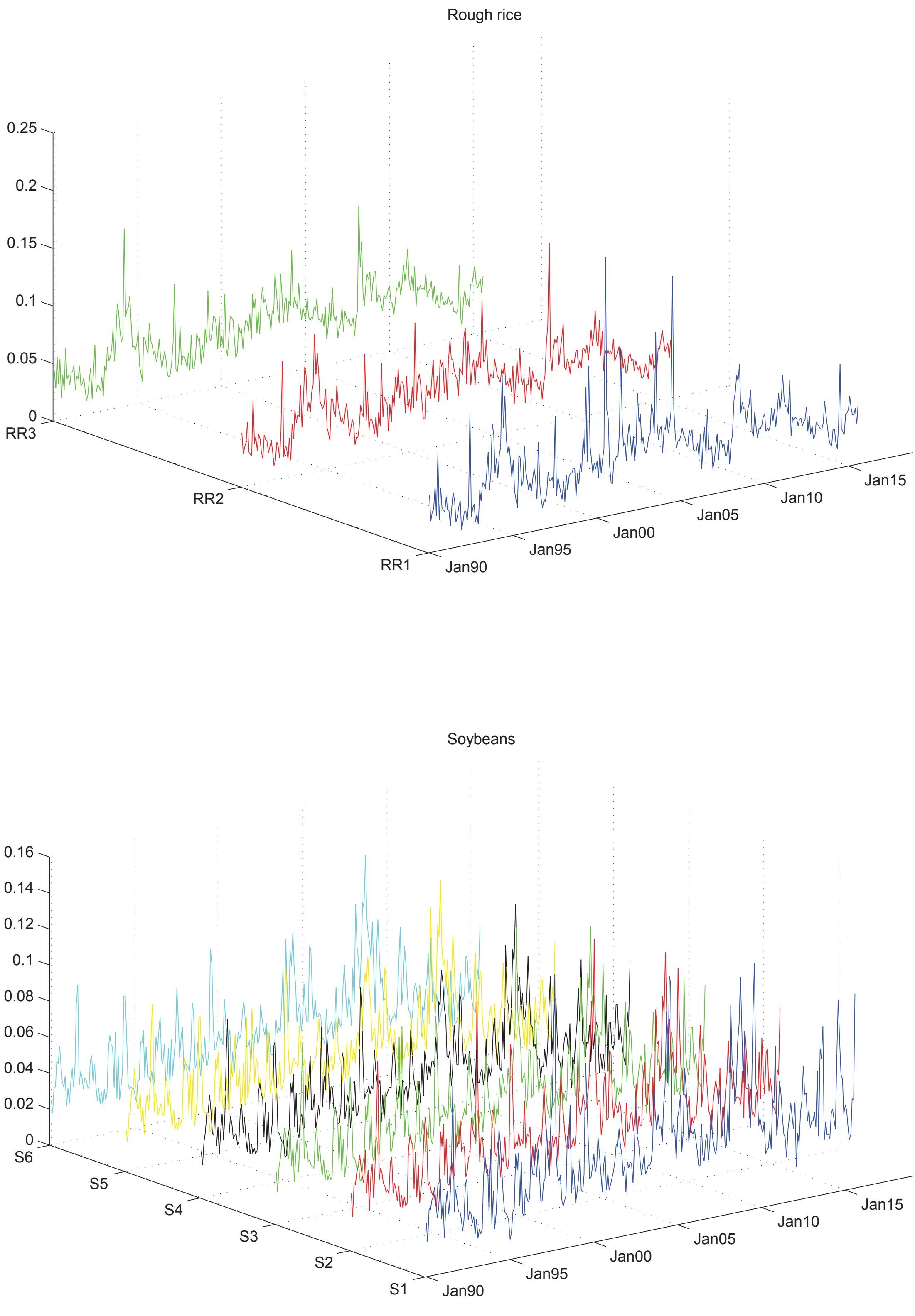
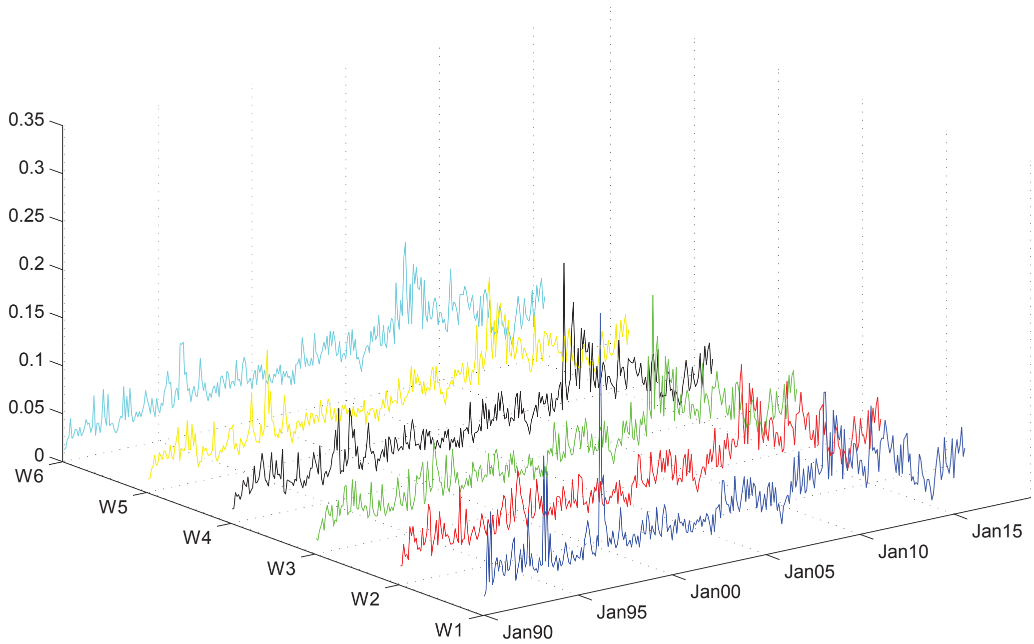


Figure A.10: Rough rice and Soybeans monthly volatilities

Wheat



Cocoa

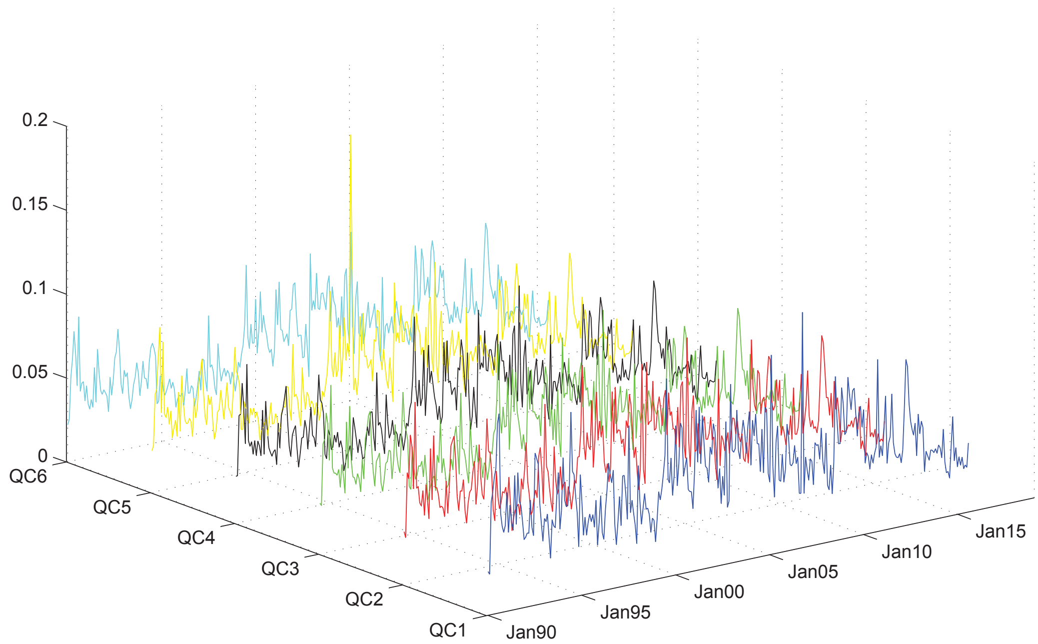


Figure A.11: Wheat rice Cocoa Soybeans monthly volatilities

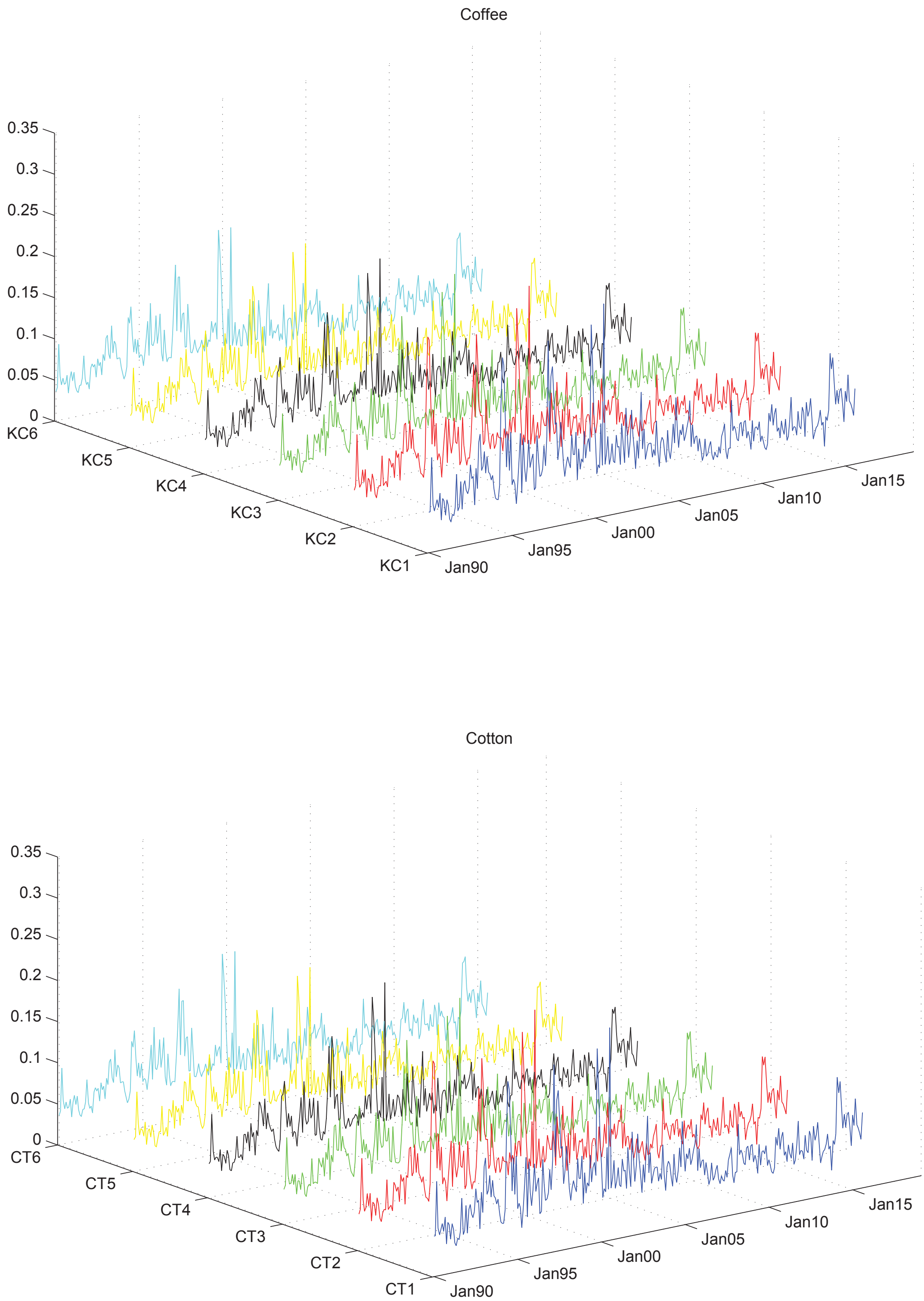


Figure A.12: Coffee and Cotton monthly volatilities

A.3 The Hinich portmanteau bicorrelation test

In practice, a certain number of price values, say $n \in \mathbb{N}$, are discretely observed for any asset over a given period $[0, t]$. Let $\Delta_n = t/n$ be the mesh; if $n \to \infty$[7], then $\Delta_n \to 0$ and one usually uses data with equidistant observation times $i\Delta_n$, $i = 0, 1, \ldots, n$.

Let $(X_t)_{t \geq 0}$ denote the log-price process of which increment between two instants is referred to as returns as follows

$$\Delta_i^n X := X_{i\Delta_n} - X_{(i-1)\Delta_n}, \qquad \text{for} \qquad i = 1, 2, \ldots, n \tag{A.5}$$

The Hinich portmanteau bicorrelation test (H-test) is used to detect epochs of transient dependence in a discrete-time pure white noise process and involves a procedure of dividing the full sample period into equal length non-overlapping moving time windows on each of which the portmanteau bicorrelation statistic is computed, to detect nonlinear serial dependence.

Let the sequence $(\epsilon_{i\Delta_n})_{i=1\ldots,n}$, be the errors terms of the regression (2.10) on spot and futures returns. If we denote by ℓ the window length, then the k^{th} window is $\left\{\epsilon_{(k\ell+1)\Delta_n}, \epsilon_{(k\ell+2)\Delta_n}, \ldots, \epsilon_{(k+1)\ell\Delta_n}\right\}$. The sequence of standardized errors $(\nu_{(k\ell+i)\Delta_n})_{i=1\ldots,\ell}$ is given by

$$\nu_{(k\ell+i)\Delta_n} = \frac{\epsilon_{(k\ell+i)\Delta_n} - m_{\epsilon,\ell}}{s_{\epsilon,\ell}} \tag{A.6}$$

where $m_{\epsilon,\ell}$ and $s_{\epsilon,\ell}$ are the expected value and the standard deviation of each window process, respectively. The null hypothesis of H-test for each window is that $(\nu_{(k\ell+i)\Delta_n})_{i=1\ldots,\ell}$ are realizations of a stationary pure white noise process with zero bicorrelation. The bicorrelation is defined by

$$\wp_\nu(j,q)\hat{\text{A}} \ = \hat{\text{A}}\ \mathbb{E}\left[\nu_{(k\ell+i)\Delta_n}\nu_{(k\ell+j)\Delta_n}\nu_{(k\ell+q)\Delta_n}\right] \quad \text{for} \quad i < j < q < l, \tag{A.7}$$

[7]In reality $n < \infty$ and assumption of $n \to \infty$ is made for convergence convenience. Hence, only the observation times that are smaller than or equal to t are considered.

where l is the number of lags in each window. The alternative hypothesis is that the process generated in the window is random with some non-zero bicorrelations, that is, there exists third-order nonlinear dependence in the data generation process.

The H-statistic, used to detect nonlinear dependence within a window,

$$H\hat{A} = \hat{A} \sum_{q=2}^{L}\sum_{j=1}^{q} \frac{(\ell - q)\wp_{\nu}^{2}(j,l)}{n-q}, \quad \text{with} \quad l\hat{A} := \hat{A}\, \ell^{0.4} \tag{A.8}$$

that follows a Chi-2 distribution $\chi^2_{l(l-1)/2}$. A window will be statistically significant if the null hypothesis is rejected at the given threshold level.

A.4 Testing for jumps on discretely observed data

A jump in a process X is defined by

$$\Delta X_t = X_t - X_{t-}$$

where $X_{t-} = \lim_{u \uparrow t} X_u$ and $t-$ refers to as immediate instant before time t. Note that jump differs from an increment of $(X_t)_{t\geq 0}$ between two instants.

Testing for discontinuities is to decide whether the price dynamic should include jump component or not. If the log-price process $(X_t)_{t\geq 0}$ is assumed jump-diffusion, then the jump component will be included depending on its importance basing on statistical tests.

$$\mathrm{d}X_t = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t + \mathrm{d}J_t \quad \text{with} \quad J_t = \sum_{j=1}^{N_t} y_{t_j}, \tag{A.9}$$

where μ_t is the drift term and in equation (2.11), $\mu_t = \kappa(\bar{x} - X_t)$ with $\mathrm{d}L_t = \sigma_t \mathrm{d}W_t + \mathrm{d}J_t$ and σ_t being the diffusion parameter and $(Wt)_{t\geq 0}$ a Brownian motion. $(J_t)_{t\geq 0}$ is the jump process with y_{t_j} representing the jump size at time t_j and N_t the random number of jumps up to time t.

The integrated volatility relates to quadratic variation of process X which is given as follows

$$[X]_t = \int_0^t \sigma_s^2 \mathrm{d}s + \sum_{j=1}^{N_t} y_{t_j}^2 \tag{A.10}$$

where $\sum_{j=1}^{N_t} y_{t_j}^2$ is the jump component. An estimator of quadratic variation is the realized variance (RV)

$$RV_t = \sum_{i=1}^{n} (\Delta_i^n X)^2 \xrightarrow[n\to\infty]{\mathbb{P}} [X]_t \tag{A.11}$$

and a consistent estimator of integrated volatility is realized bipower variation (BV) which is robust to jumps in limit.

$$BV_t = \frac{\pi n}{2(n-2)} \sum_{i=2}^{n} |\Delta_i^n X||\Delta_{(i-1)}^n X| \xrightarrow[n\to\infty]{\mathbb{P}} \int_0^t \sigma_s^2 \mathrm{d}s. \tag{A.12}$$

The BNS test consists of comparing realized volatility and realized bipower variation by using the relative jump

$$RJ_t := \frac{RV_t - BV_t}{RV_t} \tag{A.13}$$

that asymptotically converges towards normal distribution with appropriate variances, [Barndorff-Nielsen 2006]. This leads to two test statistics[8] z_{TP}

[8] The two test statistics depend respectively realized Tri-Power Quarticity and realized Quad-Power Quarticity

$$TP_t := n\mu_{4/3}^{-3} \frac{n}{n-2} \sum_{i=3}^{n} |\Delta_{(i-2)}^n X|^{4/3} |\Delta_{(i-1)}^n X|^{4/3} |\Delta_i^n X|^{4/3} \xrightarrow[n\to\infty]{\mathbb{P}} \int_0^t \sigma_s^4 \mathrm{d}s,$$

$$QP_t := n\mu_1^{-4} \frac{n}{n-3} \sum_{i=4}^{n} |\Delta_{(i-3)}^n X||\Delta_{(i-2)}^n X||\Delta_{(i-1)}^n X||\Delta_i^n X| \xrightarrow[n\to\infty]{\mathbb{P}} \int_0^t \sigma_s^4 \mathrm{d}s.$$

where $\mu_k = 2^{k/2}\Gamma\left(\frac{k+1}{2}\right)/\Gamma\left(\frac{1}{2}\right)$. Test statistics depend on $\upsilon = TP_t, QP_t$ and are then given by

$$z_\upsilon := \frac{RJ_t}{\sqrt{\left(\frac{\pi}{2} + \pi - 5\right)\frac{1}{n}\max\left(1, \frac{\upsilon}{BV_t^2}\right)}} \xrightarrow[n\to\infty]{\mathcal{L}} \mathcal{N}(0,1).$$

and z_{QP} that depend on estimator of integrated quarticity $\int_0^t \sigma_s^4 \mathrm{d}s$. The test statistics are compared with the normal distribution under the null hypothesis there are no jumps.

Furthermore, Barndorff-Nielsen and Shephard [Barndorff-Nielsen 2006] have generalized the bipower variation concept as realized power variations which can be generally used to estimate $\int_0^t \sigma_s^p \mathrm{d}s$ in the presence of jumps. It is computed as sums of products of adjacent absolute returns to a certain power to be fixed. Further, authors have argued that multipower of absolute variations can separate the continuous part of the quadratic variation, [Aït-Sahalia 2009].

The realized power variation of order $p > 0$ of process $(X_t)_{t \geq 0}$ is given by

$$\widehat{B}(p, \Delta_n)_t = \sum_{i=1}^{\lfloor t/\Delta_n \rfloor} |\Delta_i^n X|^p \,, \tag{A.14}$$

where $\lfloor x \rfloor$ stands for the integer part of $x \in \mathbb{R}$.

AÃ¯t-Sahalia and Jacod [Aït-Sahalia 2009] used result of multipower absolute variations to show that when $p > 2$ with the process $(X_t)_{t \geq 0}$ jumping, $\widehat{B}(p, \Delta_n)_t$ is invariant to sampling scale modifications. However, this does hold for continuous processes. Thus, the ASJ test compares $\widehat{B}(p, \Delta_n)_t$ at two different time scales 1 and $k > 1$ by using the test statistic

$$\widehat{S}(p, k, \Delta_n)_t = \frac{\widehat{B}(p, k\Delta_n)_t}{\widehat{B}(p, \Delta_n)_t}$$

which converges for $p > 3$ towards normal distribution . In the presence of jumps, both $\widehat{B}(p, k\Delta_n)_t$ and $\widehat{B}(p, \Delta_n)_t$ should converge towards the same value, giving $\widehat{S}(p, k, \Delta_n)_t \to 1$ with $\Delta_n \to 0$. However, when there are no jumps $\widehat{S}(p, k, \Delta_n)_t$ will also converge, but it does depend on the Δ_n-scaling parameter k. This gives the possibility to specify either absence or presence for null hypothesis both using the same test statistic $\widehat{S}(p, k, \Delta_n)_t$ with different critical values (respectively $c^c_{n,t}$ and $c^j_{n,t}$).

Let u_n be a cutoff sequence that converges towards zero. Define,

$$\widehat{b}(p, \Delta_n, u_n)_t = \sum_{i=1}^{\lfloor t/\Delta_n \rfloor} |\Delta_i^n X|^p \, \mathbb{1}_{\{|\Delta_i^n X| \leq u_n\}},$$

and the following test statistics,

$$\widehat{S}(p, k, \Delta_n, u_n)_t = \frac{\widehat{b}(p, \Delta_n, u_n)_t}{\widehat{b}(p, k\Delta_n, u_n)_t},$$

$$\tilde{s}(p, k, \Delta_n, u_n)_t = \frac{\widehat{b}(p, k\Delta_n, u_n)_t}{\widehat{b}(p, \Delta_n, u_n)_t}$$

and

$$\bar{s}_t = \frac{\widehat{b}(p', k\Delta_n, u_n)_t \, \widehat{b}(p, \Delta_n, u_n)_t}{\widehat{b}(p', \Delta_n, u_n)_t \, \widehat{b}(p, k\Delta_n, u_n)_t}.$$

A.5 Empirical results

A.5.1 Details of ASJ [Aït-Sahalia 2009] jump test

The decision rule of no jump is

$$\begin{cases} \widehat{S}(p, k, \Delta_n)_t < c^c_{n,t} & \text{for absence of jumps} \\ \widehat{S}(p, k, \Delta_n)_t > c^j_{n,t} & \text{for presence of jumps} \end{cases} \tag{A.15}$$

where $c^c_{n,t}$ and $c^j_{n,t}$ are critical values. We fix $p = 4$ and $k = 3$

Table A.1: Grain commodities

			Absence of jumps			Presence of jumps		
Commodity	Nearby	$\widehat{S}(p,k,\Delta_n)_t$	$c^c_{n,t}$ (1%)	$c^c_{n,t}$ (2%)	p_c	$c^j_{n,t}$ (1%)	$c^j_{n,t}$ (2%)	p_j
Corn	C1	0.737	2.44	2.56	2e-11	3541592	2759354	0.50
	C2	0.767	2.43	2.55	7e-11	1548164	1206218	0.50
	C3	0.817	2.41	2.54	4e-10	848166	660830	0.52
	C4	0.695	2.42	2.55	2e-11	399371	311161	0.57
	C5	1.164	2.42	2.55	1e-7	139192	178651	0.51
	C6	0.990	2.37	2.51	7e-8	87725	112593	0.59
Oat	O1	3.390	2.53	2.63	0.91	227844	177520	0.49
	O2	4.475	2.51	2.62	0.95	62618	80369	0.48
	O3	3.915	2.48	2.59	0.99	120275	93710	0.49
Rough rice	RR1	1.130	2.34	2.49	7e-4	35399	27581	0.49
	RR2	1.17	2.40	2.53	2e-4	10085	7858	0.49
	RR3	1.120	2.37	2.51	0.001	19773	15406	0.47
Soybeans	S1	0.879	2.40	2.53	2e-6	349187	272062	0.51
	S2	1.265	2.39	2.52	6e-6	73284	57098	0.53
	S3	1.196	2.38	2.52	2e-6	88055	68607	0.50
	S4	0.886	2.38	2.52	0.003	89226	69518	0.49
	S5	1.232	2.38	2.51	0.005	91314	71145	0.49
	S6	1.606	2.36	2.50	1e-4	54716	42631	0.49
Wheat	W1	3.781	1.71	1.78	0.81	1344911	1726175	0.53
	W2	4.399	1.70	1.77	0.92	332776	259275	0.49
	W3	0.836	1.69	1.76	1e-13	3382231	2635191	0.59
	W4	0.764	1.67	1.74	2e-10	4649968	3622921	0.52
	W5	1.13	2.34	2.48	3e-4	1507133	1174250	0.53
	W6	1.02	2.24	2.38	0.002	2382949	3748292	0.39

The probability that the null hypothesis is true is p-value; p_c is the p-value of null hypothesis of absence of jumps and p_j is the p-value of null hypothesis of presence of jumps. Significant levels 1% and 2% correspond to 5% and 10% respectively in multiple testing.

Table A.2: Soft commodities

			Absence of jumps			Presence of jumps		
Commodity	Nearby	$\widehat{S}(p,k,\Delta_n)_t$	$c^c_{n,t}$ (1%)	$c^c_{n,t}$ (2%)	p_c	$c^j_{n,t}$ (1%)	$c^j_{n,t}$ (2%)	p_j
Cocoa	QC1	0.784	2.51	2.62	5e-22	7747	6036	0.57
	QC2	0.956	2.52	2.61	2e-12	9421	12091	0.53
	QC3	0.892	2.50	2.61	2e-10	11833	9219	0.53
	QC4	0.931	2.51	2.61	4e-18	11179	8710	0.53
	QC5	0.921	2.50	2.61	1e-17	16664	12984	0.52
	QC6	0.997	2.49	2.60	2e-14	23222	18093	0.51
Coffee	KC1	3.188	2.54	2.64	0.75	293781	228893	0.50
	KC2	3.587	2.56	2.66	0.98	205690	160259	0.49
	KC3	4.068	2.56	2.66	0.99	178867	139360	0.49
	KC4	4.521	2.57	2.67	0.99	111064	86533	0.49
	KC5	6.595	2.56	2.67	1.00	112704	87811	0.49
	KC6	4.358	2.55	2.65	1.00	147559	114967	0.49
Cotton	CT1	0.916	2.58	2.67	1e-26	299703	233507	0.50
	CT2	0.852	2.59	2.68	1e-30	58292	45417	0.51
	CT3	0.727	2.59	2.68	8e-34	109524	85333	0.51
	CT4	0.739	2.59	2.68	9e-34	151973	118407	0.50
	CT5	0.823	2.59	2.68	9e-34	228067	177694	0.50
	CT6	0.748	2.58	2.67	7e-31	17899	13946	0.53

Table A.3: Single futures contract jump test

	Descriptive statistics					BNS		ASJ				
	N	Mean(%)	Std.(%)	Skew.	Kurt.	z_{TP}	z_{QP}	$\widehat{S}(p,k,\Delta_n)_t$	$c^c_{n,t}$	p_c	$c^j_{n,t}$	p_j
CH2014	565	-7.16	20.84	0.328	6.187	2.880	2.885	0.983	1.934	1.8e-3	66.0	0.150
CK2014	607	-5.18	20.38	0.266	5.956	3.191	2.337	0.979	2.017	2.5e-4	35.5	0.148
CN2014	648	-14.42	19.93	0.207	5.830	2.855	2.785	1.012	2.003	3.7e-4	32.2	0.195
CU2014	671	-16.08	19.23	0.142	5.714	2.904	2.214	0.993	2.081	3.8e-2	46.0	0.181
CZ2014	755	-10.91	18.79	0.191	4.801	2.968	2.622	0.989	1.819	1.4e-2	76.0	0.159
OH2015	500	-17.23	21.72	-0.356	4.621	2.807	2.914	0.998	2.062	7.1e-4	17.06	0.146
OK2015	500	-20.19	22.78	-0.182	4.802	2.451	2.525	0.972	2.091	2.7e-4	42.43	0.165
ON2015	707	-20.32	22.61	-0.185	4.785	2.653	2.710	1.009	2.158	2.2e-5	25.35	0.139
OU2015	750	-20.65	23.29	0.014	5.428	2.731	2.817	1.012	2.133	6.2e-4	92.40	0.123
OZ2015	457	-10.93	23.19	0.276	4.446	2.236	2.290	0.983	1.985	2.2e-3	34.01	0.127
RRF2009	292	6.61	28.34	-0.501	3.387	1.782	1.884	NaN	-	-	-	-
RRH2009	290	-21.51	29.66	-0.308	2.617	1.161	1.230	NaN	-	-	-	-
RRK2009	293	-29.40	29.96	-0.098	2.581	1.387	1.387	NaN	-	-	-	-
RRN2009	292	-36.20	28.32	-0.047	2.646	1.553	1.487	NaN	-	-	-	-
RRU2009	293	-28.42	22.97	0.093	3.390	1.122	1.172	NaN	-	-	-	-
RRX2009	462	-0.22	23.18	-0.131	5.021	1.347	1.762	NaN	-	-	-	-
SF2014	545	5.22	16.70	-0.007	4.070	2.383	3.140	0.979	2.165	1.7e-3	88.49	0.218
SH2014	586	6.88	16.05	-0.184	3.774	2.376	2.803	0.983	2.232	6.4e-4	63.33	0.131
SK2014	628	9.52	15.69	-0.287	3.770	2.060	2.081	1.043	2.241	3.8e-5	75.84	0.192
SN2014	669	2.95	15.55	-0.292	3.661	2.574	2.480	0.991	2.270	6.5e-4	90.84	0.157
SQ2014	692	2.36	15.29	-0.321	3.848	2.316	2.321	1.065	2.277	4.1e-3	85.44	0.189
SU2014	712	-3.90	15.62	-0.621	5.582	2.533	1.977	1.007	2.278	4.1e-3	50.05	0.223
SX2014	757	-4.03	15.41	-0.201	5.355	2.046	1.988	0.991	2.288	1.9e-3	82.42	0.125

For the BNS test, the null hypothesis of no jump is rejected if the test statistic is greater than 1.96 at 5%. For ASJ test, p_c is the p-value of null hypothesis of absence of jumps and p_j is the p-value of null hypothesis of presence of jumps. Significant levels 1% and 2% correspond to 5% and 10% respectively in multiple testing.

Table A.4: Single futures contract jump test

	Descriptive statistics					BNS		ASJ				
	N	Mean(%)	Std.(%)	Skew.	Kurt.	z_{TP}	z_{QP}	$\widehat{S}(p,k,\Delta_n)_t$	$c^c_{n,t}$	p_c	$c^j_{n,t}$	p_j
WH2011	419	5.95	31.07	0.061	4.836	1.630	1.854	NaN	-	-	-	-
WK2011	462	4.87	31.64	-0.086	4.877	1.177	1.263	NaN	-	-	-	-
WN2011	504	3.97	32.30	-0.238	5.229	4.710	4.706	1.003	2.125	8.3e-3	85.81	0.153
WU2011	547	4.21	30.80	-0.188	5.239	5.137	5.171	0.983	2.133	6.1e-4	66	0.213
WZ2011	611	-3.66	30.18	-0.221	5.196	4.764	4.890	0.979	2.185	5.1e-3	31.35	0.179
QCH2012	492	-17.43	27.00	0.318	4.620	1.957	1.950	NaN	-	-	-	-
QCK2012	493	-16.81	27.69	0.309	4.198	0.956	0.944	NaN	-	-	-	-
QCN2012	493	-20.19	28.06	0.305	3.949	0.658	0.648	NaN	-	-	-	-
QCU2012	492	-6.18	28.17	0.267	3.786	0.907	0.907	NaN	-	-	-	-
QCZ2012	490	-10.36	27.55	0.215	3.745	0.436	0.436	NaN	-	-	-	-
KCH2014	747	-11.40	28.24	0.808	7.324	2.317	2.493	0.967	1.957	1.1e-3	61.82	0.123
KCK2014	748	-12.41	30.25	0.715	6.828	2.768	2.859	0.981	2.023	2.2e-4	99.14	0.151
KCN2014	749	-12.70	30.32	0.653	6.549	2.266	2.387	1.003	2.087	3.2e-4	27.14	0.128
KCU2014	748	-7.31	30.56	0.690	6.456	3.071	2.279	0.996	2.113	1.2e-3	391.38	0.161
KCZ2014	747	-11.20	30.99	0.601	6.036	2.935	3.080	1.012	2.148	3.4e-4	27.87	0.129
CTH2012	740	8.61	27.62	-0.049	4.294	2.174	2.433	1.003	2.109	1.5e-3	602.45	0.150
CTK2012	742	3.47	25.71	-0.028	4.387	3.382	2.634	0.994	2.144	3.9e-4	220.22	0.229
CTN2012	739	-3.38	25.95	-0.161	4.950	2.279	2.716	1.014	2.143	4.1e-4	350.16	0.134
CTV2012	741	-4.30	24.60	-0.146	4.995	3.303	2.907	0.987	2.158	2.2e-3	66.10	0.142
CTZ2012	740	-4.84	23.06	-0.139	5.305	4.806	4.086	0.985	2.072	5.2e-4	640.50	0.119

The null hypothesis of no jump is rejected if the test statistic is greater than 1.96 at 5%.
Month codes: January(F); February(G); March(H); April(J); May(K); June(M); July(N); August(Q); September(U); October(V); November(X); December(Z).

A.5.2 Details of jump activity test, [Aït-Sahalia 2011]

The decision rule is

$$\begin{cases} \widetilde{s}(p,k,\Delta_n)_t < c_{n,t}^{fA} & \text{for finite activity} \\ \bar{s}(p',k,\Delta_n)_t < c_{n,t}^{infA} & \text{for infinite activity} \end{cases}$$

where $c_{n,t}^{a}$ is the critical value and $p' > p > 3$. The values of test statistic for $p = 4, p' = 5$, $k = 3$ are computed for cutoff u_n proportional to estimated integrated volatility, see [Le Courtois 2014] and [Aït-Sahalia 2010]

Table A.5: Testing for jump activity

		Finite activity				Infinite activity			
Commodity	Nearby	$\tilde{s}(p,k,\Delta_n)_t$	$c_{n,t}^{fA}$ (1%)	$c_{n,t}^{fA}$ (2%)	p_{fA}	$\bar{s}(p,k,\Delta_n,u_n)_t$	$c_{n,t}^{infA}$ (1%)	$c_{n,t}^{infA}$ (2%)	p_{infA}
Corn	C1	2.315	1.86	2.11	0.161	0.114	1.69	1.76	7e-24
	C2	2.797	1.92	2.16	0.378	0.122	1.65	1.73	4e-19
	C3	2.616	1.80	2.07	0.299	0.118	1.60	1.69	3e-15
	C4	2.708	2.13	2.32	0.290	0.131	1.77	1.82	4e-40
	C5	2.408	1.91	2.15	0.185	0.120	1.68	1.75	1e-24
	C6	2.049	1.52	1.69	0.067	0.116	1.78	1.82	1e-43
Oat	O1	1.775	1.54	1.71	0.062	0.341	1.28	1.30	6e-34
	O2	2.427	1.58	1.75	0.173	0.405	1.22	1.25	2e-20
	O3	2.483	1.63	1.77	0.189	0.421	1.25	1.28	2e-24
Rough rice	RR1	1.967	1.04	1.27	0.111	0.358	1.19	1.22	3e-18
	RR2	2.535	0.82	1.07	0.310	0.396	0.98	1.04	5e-07
	RR3	2.673	0.87	1.12	0.361	0.401	1.05	1.10	8e-09
Soybeans	S1	1.800	1.28	1.48	0.052	0.372	1.17	1.21	5e-16
	S2	2.245	1.17	1.38	0.169	0.396	1.07	1.12	2e-09
	S3	1.973	1.44	1.63	0.062	0.419	1.14	1.18	1e-12
	S4	2.014	1.87	2.00	0.021	0.448	1.34	1.36	3e-52
	S5	2.176	1.94	2.07	0.035	0.462	1.37	1.39	4e-79
	S6	2.017	1.71	1.87	0.037	0.444	1.26	1.29	2e-24

The probability that the null hypothesis is true is p-value; p_{fA} is the p-value of null hypothesis of finite activity and p_{infA} is the p-value of null hypothesis of infinite activity. Significant levels 1% and 2% correspond to 5% and 10% respectively in multiple testing

Table A.6: Testing for jump activity

Commodity	Nearby	Finite activity				Infinite activity			
		$\tilde{s}(p,k,\Delta_n)_t$	$c_{n,t}^{fA}$ (1%)	$c_{n,t}^{fA}$ (2%)	p_{fA}	$\bar{s}(p,k,\Delta_n,u_n)_t$	$c_{n,t}^{infA}$ (1%)	$c_{n,t}^{infA}$ (2%)	p_{infA}
Wheat	W1	1.670	1.80	1.94	0.005	0.379	1.31	1.34	1e-45
	W2	1.850	1.85	1.99	0.010	0.399	1.32	1.34	9e-48
	W3	1.855	1.85	1.88	0.009	0.404	1.36	1.38	2e-78
	W4	1.906	1.73	1.88	0.022	0.403	1.29	1.31	2e-35
	W5	1.559	1.16	1.38	0.034	0.360	1.15	1.19	2e-14
	W5	1.659	1.26	1.34	0.034	0.841	0.85	0.88	8e-04
Cocoa	QC1	1.968	1.73	1.73	0.023	0.402	1.25	1.28	1e-25
	QC2	1.819	1.73	1.77	0.022	0.152	1.59	1.66	1e-13
	QC3	1.929	1.76	1.87	0.021	0.152	1.81	1.86	3e-28
	QC4	1.904	1.81	1.85	0.025	0.171	1.76	1.82	1e-13
	QC5	2.035	1.80	1.90	0.024	0.167	1.81	1.82	8e-23
	QC6	2.001	1.87	1.88	0.031	0.162	1.72	1.78	2e-20

Table A.7: Testing for jump activity

		Finite activity				Infinite activity			
Commodity	Nearby	$\tilde{s}(p,k,\Delta_n)_t$	$c_{n,t}^{fA}$ (1%)	$c_{n,t}^{fA}$ (2%)	p_{fA}	$\bar{s}(p,k,\Delta_n,u_n)_t$	$c_{n,t}^{infA}$ (1%)	$c_{n,t}^{infA}$ (2%)	p_{infA}
Coffee	KC1	1.999	1.86	1.53	0.073	0.001	11.2	11.8	6e-17
	KC2	1.951	1.31	1.81	0.043	0.032	2.42	2.54	1e-16
	KC3	2.001	1.66	1.87	0.083	0.042	2.32	2.44	8e-14
	KC4	1.945	1.53	1.81	0.063	0.038	2.25	2.39	2e-12
	KC5	1.913	1.55	1.76	0.123	0.035	2.32	2.44	8e-14
	KC6	1.791	1.33	1.63	0.134	0.028	2.18	2.32	8e-11
Cotton	CT1	2.151	1.81	2.03	0.093	0.041	2.50	2.61	3e-19
	CT2	2.234	2.03	2.13	0.123	0.052	2.40	2.51	1e-15
	CT3	2.166	1.57	2.05	0.073	0.039	2.46	2.57	1e-17
	CT4	2.236	1.46	2.13	0.193	0.040	2.63	2.72	6e-26
	CT5	2.226	1.24	2.12	0.154	0.035	2.71	2.79	1e-31
	CT6	2.139	1.01	2.02	0.212	0.026	2.74	2.89	1e-35

A.5.3 Details of continuity test, [Aït-Sahalia 2010]

Decision rule for null hypothesis of Brownian presence is

$$\widehat{S}(p, k, \Delta_n, u_n)_t < c_{n,t}$$

where $c_{n,t}$ is critical value. All values of test statistic for $p = 1.5$, $k = 3$ are computed for cutoff u_n proportional to estimated integrated volatility, see [Le Courtois 2014] and [Aït-Sahalia 2010]

Table A.8: Grains: Test of continuity

Commodity	Nearby	$\widehat{S}(p, k, \Delta_n, u_n)_t$	$c_{n,t}$ (1%)	$c_{n,t}$(2%)	p-value
Corn	C1	1.497	1.483	1.491	0.031
	C2	1.490	1.482	1.502	0.021
	C3	1.506	1.482	1.502	0.063
	C4	1.502	1.482	1.503	0.050
	C5	1.514	1.483	1.503	0.104
	C6	1.541	1.483	1.503	0.352
Oat	O1	1.414	1.398	1.504	0.037
	O2	1.326	1.336	1.504	0.026
	O3	1.330	1.312	1.504	0.035
Rough rice	RR1	1.088	1.074	1.105	0.021
	RR2	1.102	1.073	1.104	0.045
	RR3	1.091	1.069	1.081	0.032
Soybeans	S1	1.467	1.399	1.407	0.479
	S2	1.453	1.398	1.406	0.301
	S3	1.462	1.400	1.408	0.413
	S4	1.471	1.400	1.401	0.525
	S5	1.472	1.399	1.407	0.535
	S6	1.478	1.399	1.407	0.613
Wheat	W1	1.415	1.400	1.408	0.035
	W2	1.511	1.488	1.496	0.066
	W3	1.492	1.487	1.081	0.032
	W4	1.424	1.400	1.408	0.063
	W5	1.515	1.487	1.494	0.097
	W6	1.441	1.400	1.408	0.175

A.5.4 Descriptive statistics of inventory data

Table A.9: Softs: Test of continuity

Commodity	Nearby	$\widehat{S}(p,k,\Delta_n,u_n)_t$	Presence of Brownian $c^c_{n,t}$ (1%)	$c^c_{n,t}$(2%)	$p-value$
Cocoa	QC1	1.655	1.578	1.587	0.729
	QC2	1.659	1.580	1.586	0.773
	QC3	1.662	1.579	1.587	0.807
	QC4	1.673	1.578	1.586	0.906
	QC5	1.680	1.579	1.586	0.943
	QC6	1.671	1.580	1.587	0.886
Coffee	KC1	1.658	1.579	1.586	0.267
	KC2	1.702	1.581	1.588	0.293
	KC3	1.684	1.580	1.587	0.259
	KC4	1.697	1.581	1.587	0.289
	KC5	1.697	1.580	1.588	0.387
	KC6	1.723	1.582	1.588	0.499
Cotton	CT1	1.652	1.578	1.586	0.291
	CT2	1.609	1.409	1.408	0.580
	CT3	1.655	1.491	1.498	0.599
	CT4	1.671	1.492	1.499	0.609
	CT5	1.697	1.493	1.500	0.653
	CT6	1.689	1.492	1.499	0.709

Table A.10: Descriptive statistics of inventory level

Commodity	N	Mean	Std dev	Min	Q1	Median	Q3	Max
Corn	100	4743725	2846263	425942	2113724	4315442	6912303	11202714
Oat	100	117944	60818	24744	75742	104349	144094	351709
Soybeans	100	1071825	738412	91960	428649	992264	1672896	2701366
Rough Rice	78	64402	53923	1412	17348	56581	110673	166660
Wheat	100	1337111	570871	305818	873616	1347378	1819065	2449617
Cocoa	308	3173384	1145267	122000	2660242	3325039	3897938	5393598
Coffee	225	2685033	1462312	321	1622432	2712325	3829498	5092735
Cotton	649	133839	90486	1205	60370	106545	213132	350604

Appendix B

Appendix of Chapter 3

B.1 Estimation of hedge ratios

In practice, the estimation of hedge ratio depends on the methods that is adopted to compute the hedge ratio. Herein, we describe some estimations methods for existing approach for hedge ratio with no quantity risk.

The minimum-variance hedge ratio is simply estimated by linear regression of spot returns on futures returns

$$r_{s,t} = a + \beta r_{f,t} + \varepsilon_t, \tag{B.1}$$

where a the intercept, β an estimate of h_{MV}, ε the error term and t is the observation time. While the linear regression is easy to implement by ordinary least square technique, it relies on no exhaustive assumptions which makes the estimated hedge ratio critical on statistical basis. Error term in equation (B.1) is often heteroskedastic and ordinary least square approach is based on unconditional mean and variance instead.

In the expected utility approach, appropriate utility function and distribution are usually guested to achieve closed form solution. Otherwise, numerical approximation usually allows to derive the hedge ratio.

The estimation of the mean-extended-Gini hedge ratio, is usually based on empirical distribution function of R_h

$$\widehat{\Gamma}_h(\delta) = \frac{\delta}{N}\left\{\sum_{i=1}^{N} r_{h,i}[1-\widehat{G}(r_{h,i})]^{\delta-1} - \frac{1}{N}\left(\sum_{i=1}^{N} r_{h,i}\right)\left(\sum_{i=1}^{N}[1-\widehat{G}(r_{h,i})]^{\delta-1}\right)\right\} \tag{B.2}$$

where N is the sample size and $r_{h,1},\ldots,r_{h,N}$, the observations of hedge portfolio returns. Then mean-extended-Gini coefficient, $\widehat{\Gamma}_h(\delta)$ is minimized as risk measure function. Alternatively, Shalit [Shalit 1984] had used another formula whose estimation is as follows

$$\widehat{h}_{\text{MEG}} = \frac{\sum_{i=1}^{N}(r_{s,i}-\bar{r_s})(d_i-\bar{d})}{\sum_{i=1}^{N}(r_{f,i}-\bar{r}_f)(d_i-\bar{d})} \tag{B.3}$$

with $d_i = [1-\widehat{G}(r_{h,i})]^{\delta-1}$ and $\bar{d} = \sum_{i=1}^{N} d_i/N$.

The lower partial moment hedge is approximated either on basis of the empirical distribution or the kernel estimation, Lien and Tse [Lien 2000]. The empirical distribution approach leads to

$$\widetilde{\ell}_n(c,r) = \frac{1}{N}\sum_{r_{h,i}<c}(c-r_{h,i})^n, \tag{B.4}$$

and the kernel estimation consists in substituting the probability density function of the portfolio returns by a kernel density function[1],

$$\widehat{\ell}(n,\bar{r},G) = \frac{1}{N\varpi}\sum_{i=1}^{N}\int_{-\infty}^{\bar{r}}(\bar{r}-r)^n\,k\left(\frac{r-r_i}{\varpi}\right)\mathrm{d}r, \tag{B.5}$$

with k is the kernel function and ϖ is the bandwidth. By plugging $z =$

[1]The density function of R_h can be estimated by the kernel method

$$\widehat{g}(R_h) = \frac{1}{N\varpi}\sum_{i=1}^{N} k\left(\frac{R_h-r_{h,i}}{\varpi}\right).$$

$(r - r_i)/\varpi$ into the integral, we have

$$\widehat{\ell}_n(\bar{r}, G) = \frac{1}{N} \sum_{i=1}^{N} l_n(c, r_{h,i}), \tag{B.6}$$

with

$$l_n(c, r_{h,i}) = \int_{-\infty}^{(c - r_{h,i})/\varpi} (c - z\varpi - r_{h,i})^n \, k(z) \mathrm{d}z. \tag{B.7}$$

Setting $n = 2$ and assuming that the portfolio returns and the futures returns are independent, then hedge ratio is the same as the of semi-variance will be the same as the minimum variance hedge ratio, Lien and Tse [Lien 2002].

Traditional way to estimate the hedge ratios from VaR and CVaR is numerical optimization, unless convenient distributions is use to get closed form solution.

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Modélisation et stratégies de couverture des matières premières agricoles

Introduction générale

Les prix des marchandises sont soumis à des variations de niveaux de production de leurs actifs sous-jacents, ainsi que des facteurs liés à leur économie tels que les saisons calendaires ou les campagnes agricoles, la consommation et les politiques de l'offre et l'équilibre de la demande, les stocks ... Ainsi, les matières premières encourent-elles les risques à la fois de marché et de production. Surtout que la majeure partie des revenus des producteurs est représentée par le rendement de leurs cultures. Toute évolution défavorable des prix aura une incidence sur leurs revenus. D'une part, la mondialisation des marchés des matières premières pourvoit des produits dérivés comme les futures, forwards ou des options pour se protéger contre ces risques en les cédant aux investisseurs qui recherchent des opportunités de spéculation. D'autre part, les matières premières peuvent également être stockées afin d'éviter les perturbations dues aux pénuries qui génèreraient des coûts de portage résultant de la détérioration de la qualité des produits.

Dans la production agricole, un moyen d'éviter de tels coûts de portage est d'entrer sur les marchés financiers avec des dérivés ayant leurs valeurs déterminées, en quelque sorte, par les prix de ces biens physiques pour une date future. C'est à dire qu'avoir des stocks de produits, afin de faire face à des épisodes de pénuries éventuelles à l'avenir, contribue ainsi à la logique de la relation entre le prix au comptant et le prix à terme. Pour les matières premières, l'impact de la variabilité des prix sur l'économie réelle est plus grand. Les variations des prix de matières premières se rapportent à chaque entité économique ; des individus aux organisations et pays. Par conséquent, la gestion des risques dans l'économie des matières premières s'avère très importante. Les individus ont besoin de gérer ces risques pour couvrir leurs revenus, les entreprises pour protéger leur base de fond et leur compétitivité, et les pays pour protéger leur stabilité macroéconomique. Particulièrement, les produits agricoles sont concernés, car ce sont des ressources naturelles consommés dans l'alimentation humaine de base. Ils sont également utilisés à d'autres fins. Par exemple, le maïs est utilisé dans tout ; les édulcorants artificiels, les sources de carburant ainsi que dans le papier et les conteneurs. Les marchés à terme sont, à la fois, des organisations de gestion des risques et de découverte des prix[2]. Sur ces marchés, les anticipations stratégiques des acteurs du marché interagissent pour former le "mécanisme des prix" pour refléter un large éventail d'informations à venir sur les conditions du marché.

Les contrats à terme sont principalement utilisés comme instruments de couverture de l'exposition en trésorerie, mais s'ils ne correspondent pas à l'exposition directe des prix des produits de base; Ils représentent des paris sur les prix au comptant espérés. Par exemple, un producteur de blé qui plante une culture fait un pari que le prix du blé ne baissera pas si bas qu'il aurait été préférable de ne pas avoir planté son blé. Ce pari est inhérent à l'activité agricole, mais l'agriculteur peut préférer ne pas le faire. Par conséquent, il peut couvrir ce pari en vendant un contrat à terme le blé. À part

[2]Blau [Blau 1944] (p. 1) avait déclaré que *"commodity futures exchanges are market organizations specially developed for facilitating the shifting of risks due to unknown future changes in commodity prices; i.e., risks which are of such a nature that they cannot be covered by means of ordinary insurance."*

la gestion du risque de prix, il existe de nombreuses externalités positives associées à la couverture avec les contrats de futures. Rappelons que les contrats à terme dans les échanges des matières premières ont lieu avec la standardisation en volume et en qualités afin d'améliorer l'efficacité de leurs extractions, leurs distributions et les processus de consommation. Par conséquent, les fonctions de couverture et de formation des prix des marchés à terme améliorent l'efficacité des opérations de production, de stockage et de commercialisation. La couverture assure également la continuité des flux de trésorerie car elle permet de mettre à l'abri le producteur des mouvements volatiles des prix et lui garantira des flux de revenus ininterrompus et stables apportant une certaine certitude dans le processus de production. Cela se revient à la certitude dans la planification de la production à un prix minimum garanti en utilisant des contrats à terme.

En outre, les marchés à terme agricoles servent également de couverture efficace contre l'inflation, puisque les prix des aliments font partie de ceux qui grimpent souvent en premier. Gorton et Rouwenhorst [Gorton 2006] ont souligné que les rendements à terme des matières premières et l'inflation sont positivement corrélés en grande échelle sur le long terme. En effet, les rendements des matières premières à terme sont volatiles par rapport à l'inflation, leurs corrélations à long terme permettant de mieux capter les propriétés inflationnistes d'un investissement en matières premières. Les futures agricoles peuvent aussi réaliser de bonnes performances lorsque la population mondiale croît, ou lorsque la croissance de la classe moyenne entraîne une demande accrue. Enfin, les contrats de futures agricoles fonctionnent parfois comme une couverture contre la volatilité sur les marchés boursiers: les tensions géopolitiques sur les marchés émergents et frontaliers découlent souvent de pénuries alimentaires. Sur les marchés agricoles, les contrats à terme permettent de se couvrir contre les risques de marché et de production et leur fonction de formation des prix est intégrée dans les processus de changement de prix et des imprévus de production. Le risque de production entraîne des risques supplémentaires qui peuvent réduire davantage le rendement des cultures et affecter les prix espérés. En effet, les gains et pertes des futures sont réglés quotidiennement et comme tout actif financier, les prix des matières premières peuvent changer soudainement et avec des variations importantes ; généralement, en raison de nouvelles annonces ou de réformes, de troubles politiques ou des aléas climatiques.

Un exemple frappant est l'instabilité politique en Côte d'Ivoire, où les prix du cacao avaient atteint un sommet entre 2002 et 2003. Une autre illustration de ces mouvements de prix provient des aléas climatiques (production de café pendant les gelées brésiliennes) ou d'une demande d'exportation supplémentaire rapide (dans le cas de la Chine) qui peut avoir un impact sur la production finale. Par conséquent, l'utilisation de contrats à terme contre tous ces risques en même temps peut ne pas être aussi efficace et peut même s'avérer néfaste. La variation soudaine et significative des prix sur une très courte période de temps est représentée un saut de prix. Par conséquent, la situation dans laquelle une position unique en contrat à terme est prise pour couvrir tous ces risques en même temps, conduit à un marché incomplet en raison de multiples sources de risques. Même s'il existe une stratégie de couverture optimale pour ces risques, il sera difficile à implémenter.

La littérature financière décrit cette situation de mesure martingale non unique. En effet, tous les processus de risque devraient être martingales sous une mesure martingale équivalente. Dans une telle stratégie, la mesure n'est pas unique puisqu'il existe plus d'une source de risques à couvrir avec une seule variable d'état. La question primordiale sur le marché des matières premières agricoles est la couverture contre les variations défavorables de prix avec comme principal avantage une réduction significative du risque de perte via une stratégie de portefeuille. Dans les théories classiques de stratégies de couverture, celle de l'assurance de prix justifie leur mérite par leur garantie minimale contre les risques inhérents aux fluctuations des prix. A juste titre, Keynes [Keynes 1930], Hicks [Hicks 1939] et Kaldor [Kaldor 1940] ont soutenu les stratégies de couverture comme des outils d'atténuation de risques. Ensuite, la théorie du portefeuille, initiée par Markowitz, envisageait les stratégies de couverture comme une garantie contre les risques tout en intégrant la réduction des risques à la maximisation de l'utilité espérée. Ce cadre théorique a été utilisé en application par Johnson [Johnson 1960] et Stein[Stein 1961] pour expliquer les straté- gies de couverture à des fins de performance de rendements et d'atténuation de risques. C'est dire donc qu'il n'existe pas de stratégie de couverture efficace qui éliminerait complètement tout risque. Une stratégie de couverture permettrait, plutôt, de transformer un risque inacceptable en une forme acceptable : comme la cession de risque investisseurs sur les marchés de matières premières via les contrats à terme.

Toutefois, si une couverture inefficace entraînerait des pertes pouvant entraîner la faillite. Ce fut le cas de Metallgesellschaft AG, le plus gros conglomérat allemand qui avait frôlé la faillite après avoir subi des pertes avoisinant 1,5 milliard de dollars US sur ses transactions de dérivés énergétiques en décembre 1993. À l'époque, le prix du pétrole avait baissé tandis que le marché à terme est passé de la situation de déport en situation de report et la combinaison de ces mouvements de marché a conduit à de sérieuses pertes sur les positions à terme pour Metallgesellschaft AG. Aussi, une couverture dépend-t-elle fortement de la situation d'application, ainsi que des coûts associés aux stratégies mises en œuvre.

Sur les marchés à terme, l'incompatibilité des positions entre l'actif sous-jacent et le contrat à terme rend la stratégie de couverture moins efficace, puis les risques ne seront pas suffisamment compensés. Toute différence entre les positions du sous-jacent et de l'instrument de couverture entraîne un risque de base qui peut entraîner des gains ou des pertes générés par la position couverte. Particulièrement sur les marchés de matières premières, un contrat à terme échu n'existe plus. Pour conserver un engagement à long terme, il faut définir un processus de stratégie de rollover sur les positions des contrats à terme. Cela consiste à renverser la position dans le contrat à terme proche et reprendre une autre position sur un contrat à plus long terme. L'écart entre ces deux positions constitue un risque supplémentaire appelé risque de *rollover*. Il peut provenir de situations de marché ou de contingences de production.

Par exemple, sur les positions de vente à terme, la situation la plus idéale serait un marché en report avec une tendance baissière des prix puisqu'il offre deux occasions de générer des gains. Une tendance baissière de prix qui générera évidemment des gains pour la position de vente à terme et un marché en report qui permet à la couverture de vente des contrats à terme à un prix au comptant plus bas, permettant ainsi plus de gains lorsque le prix au comptant diminue. La situation inverse générera un risque de base. En outre, si la production est inférieure aux prévisions, elle sera d'autant plus importante. Le risque de base et le risque de roulement sont vraiment inquiétants et sont la conséquence d'une stratégie de couverture inefficace. Le risque de base a été introduit dans la littérature des marchés à terme par Paroush et Wolf [Paroush 1989, Paroush 1992] pour montrer leur influence sur la couverture et la production optimale ainsi que d'autres paramètres d'allocation. Mais, plus tôt, Holthausen [Holthausen 1979] avait étudié des modèles de couverture et de production en l'absence de risque de base pour montrer dans quelles circonstances l'agent en sur ou sous couverture garantit entièrement sa production et l'effet de l'augmentation de l'aversion au risque sur la couverture. De façon plus générale, la question de la couverture des contrats à terme a fait l'objet de diverses approches. Les stratégies de couverture dépendent de leur approche d'optimisation utilisée et ceci est plus marquant dans un cadre statique. Plusieurs papiers dans la littérature sur les questions de couverture avec contrats à terme se sont concentrés sur l'aspect de modélisation et cela en tandem avec des mesures d'efficacité. Les mesures actuelles d'efficacité de couverture ne sont pas cohérentes car aucune des approches d'optimisation ne permet d'obtenir la stratégie de couverture la plus performante.

Dans le cadre statique, les techniques d'optimisation sont basées sur la variance minimale, la moyenne-variance, la semi-variance, la moyenne-Gini et la semi-variance généralisée. La variance minimale est l'approche la plus standard et n'inclut pas les aspects du portefeuille comme les rendements espérés ou la psychologie du risque. Les autres méthodes sont ensuite développées pour l'améliorer en tenant compte de ses caractéristiques pertinentes. La variance moyenne intègre le rendement espéré et l'aversion au risque. Par exemple, Rolfo [Rolfo 1980] a utilisé l'approche moyenne-variance pour obtenir une stratégie optimale de couverture pour le risque de prix et le risque de production des pays exportateurs de produits agricoles. Le concept de semi-variance ne tient compte que du risque à la baisse, plutôt que des bénéfices et des pertes considérées dans la stratégie de couverture pour réduire les pertes moyennes. De plus, la semi-variance généralisée, également appelée approche par moments partiels inférieurs (LPM), a été appliquée dans la littérature de couverture sur des contrats à terme (Chen et al. [Chen 2001], par exemple dans Lien et Tse [Lien 2000]) afin d'intégrer la psychologie du risque au concept de dominance stochastique. L'approche de Gini moyenne est également compatible avec la dominance stochastique ainsi que l'utilité espérée ; en particulier cela est utilisé lorsque la moyenne-variance ne fonctionne pas à cause de non-normalité des rendements ou d'estimateurs biaisés dans le cadre de moindres carrés ordinaires, voir Shalit et Yitzhaki [Shalit 1984]. Les autres contributions reposent sur l'aspect inconditionnel de la volatilité en tant mesure de risque à partir des approches de distribution conditionnelle telles que ARCH (Autoregressive Conditional Heteroskedasticity), le modèle GARCH (Baillie et Myers [Baillie 1991]) ou modèle multi-période (Cecchetti et al. [Cecchetti 1988], Chen et al. [Chen 2013] et Lien et Luo [Lien 1993]). Par ailleurs, Fernandez [Fernandez 2008] et Conlon et Cotter [Conlon 2012] ont montré les effets de l'horizon sur le ratio de couverture en utilisant les ondelettes[3]. En temps continu, Ho [Ho 1984] et Adler Detemple [Adler 1988a] sont des pionniers à étudier sur la couverture avec les contrats à terme sur les marchés de matières premières. Leurs papiers ont établi des stratégies optimales de couverture via la méthode de programmation dynamique telle qu'appliquée par Merton [Merton 1971] dans le contexte de gestion de portefeuille. Plus précisément, le portefeuille de couverture comprend principalement en trésorerie l'actif non cessible, le contrat futures et les autres actifs tels les options, les actions ou les obligations. Les auteurs ont considéré une dynamique brownienne pour représenter les sources de risque dans leurs approches et ont relevé l'inefficacité de la stratégie optimale de couverture.

Le but de cette thèse est de développer des stratégies de couverture dans un contexte de portefeuille, pour les matières premières stockables. En particulier, la question de la couverture sur les marchés financiers est prise en compte pour les produits agricoles. Les stratégies de couverture visent à réduire, autant que possible, les pertes ainsi que les dépenses défavorables en raison des fluctuations des prix du marché et des décisions de production. L'objectif fondamental des instruments de couverture est de fournir une contre-position qui réduirait les pertes en partie ou en totalité selon la nature de la couverture. Avant la plantation et la récolte, un producteur doit décider comment garantir ses revenus. Le portefeuille de couverture est analysé dans des cadres statiques et dynamiques.

En statique, le problème de couverture est exposé et certaines approches ainsi que leurs applications empiriques sont présentées. Principalement, nous avons établi des stratégies optimales combinant contrats à terme et d'assurance pour le risque de marché ainsi que le risque de production dans le processus de rollover. Puisque, les décisions prises dans un cadre statique ne tiennent pas compte des modèles de rétroaction le long de la période de

[3] La méthode des ondelettes est un raffinement de l'analyse de Fourier qui décompose les séries chronologiques en composantes haute et basse fréquences qui correspondent respectivement aux variation de court et long termes.

couverture, la même question est étudiée en dynamique.

En effet, la dynamique des prix influe considérablement sur les stratégies de portefeuille au cours de la période de couverture. En temps continu, nous analysons les insuffisances de la stratégie de couverture qui proviendrait soit de l'aspect de modélisation (représentation du risque par le mouvement brownien seulement), soit des instruments de couverture. D'une part, les stratégies de couverture dynamique tiennent compte des mouvements de marché et en particulier ceux liés aux sauts des prix. Cela donne une analyse plus fine du comportement des prix à terme de matières premières, telle que soulignée au chapitre 2. Les tests de détection de sauts de Barndorff-Nielsen et Shephard [Barndorff-Nielsen 2006], AïtSahalia et Jacod [Aït-Sahalia 2009] sont appliqués sur des données réelles et la composante de saut s'avère pertinente pour le prix des contrats à terme utilisés.

Cependant, la couverture avec le contrat à terme seul doit être améliorée. D'autre part, une stratégie de couverture alternative consiste à inclure une option écrite sur l'actif non cessible. Le portefeuille de couverture avec option renforce la stratégie de couverture en réduisant davantage les incertitudes. La thèse est organisée en quatre chapitres. Le premier chapitre applique les tests statistiques récents sur les prix à terme de produits agricoles. L'étude empirique suggère que les prix à terme suivent un processus de « retour à l'équilibre avec saut-diffusion » avec la volatilité et la moyenne à long terme saisonnières.

Ensuite, le deuxième chapitre traite de l'estimation du modèle avec une procédure en deux étapes. Premièrement, l'estimation de la vitesse de retour à l'équilibre et les moyennes périodiques est abordée en utilisant les moindres carrés ordinaires. Deuxièmement, les résidus de la première étape permettent d'estimer les paramètres restants avec la méthode de particule MCMC. Les troisième et quatrième chapitres étudient 204 les stratégies de couverture, respectivement, statique et dynamique. Principalement, la couverture statique comprend les risques de marché et de production avec les applications, tandis que dans la stratégie de couverture dynamique, on ajoutera au contrat à terme, une option écrite sur l'actif non cessible.

Résumé Chapitre 1

Les facteurs économiques des matières premières agricoles sont déterminés par la production, les stocks et le prix au comptant, qui concourent tous ensemble aux caractéristiques des prix futures. En utilisant la littérature associée, nous décrivons le comportement des prix de matières premières agricoles et les implications économiques correspondantes, en mettant l'accent sur leurs faits stylisés. Il s'agit en particulier du retour à l'équilibre, de la saisonnalité et des sauts observés sur les prix de futures de céréales et de matières premières dites « douces » à travers divers tests économétriques. Ces tests sont menés de telle sorte que les caractéristiques soulignées peuvent pourront être étudiées ensembles sur les prix de futures.

Enfin, nous avons également montré que la structure par terme des matières premières étudiées tend plutôt à être en report.

Résumé Chapitre 2

Ce chapitre étudie la modélisation des prix agricoles de futures supposés suivre le processus de retour à l'équilibre avec saut comme mis en évidence au chapitre 2. Précisément, comme le terme de la dérive est aussi saisonnière, l'estimation est faite en utilisant une procédure en deux étapes. La première étape permet d'estimer certains paramètres du terme de dérive par la méthode des moindres carrés. Dans la deuxième étape, on applique une méthode de filtrage récente sur les résidus de la première étape afin d'estimer les paramètres restants.

Résumé Chapitre 3

Plusieurs stratégies de couverture peuvent être initiées en statique selon les différentes approches de ratios de couvertures. Cependant, il est difficile de choisir parmi ces ratios de couverture, la meilleures selon la préférence de l'agent et le contexte, et ce, même en se basant sur les mesures d'efficacité existantes. Nous proposons une alternative en ordonnant les portefeuilles de couverture selon leur performance avec les L-moments. Dans une seconde partie nous étudions la couverture d'un producteur en présence des risques de marché et de production dans une stratégie de "*rollover*". Nous traitons la stratégie de couverture pour le risque additionnel dû au retournement de positions en contrat de futures. En pratique ces risques sont, pour un producteur, celui de marché et surtout le risque de production en inter-campagne agricole.

Nous montrons comment un contrat d'assurance approprié peut davantage améliorer la couverture avec le contrat futures en garantissant le revenu du producteur. L'application sur les données collectées montrent ainsi que la prise en compte du risque de marché et du risque de production s'avèrent la meilleure stratégie de couverture à partir de l'approche par L-moments.

Résumé Chapitre 4

L'étude faite ici porte sur la stratégie de couverture optimale en temps continu avec une position en contrat de futures le long d'une campagne agricole. À la lumière de l'analyse empirique du 2, nous avons considéré une dynamique de retour à l'équilibre pour les prix de titres agricoles. La stratégie de couverture optimale est obtenue en utilisant l'approche de

l'utilité espérée à maximiser. Il s'avère seul, le contrat future est insuffisant pour couvrir les risques encourus par un producteur. Ainsi, une option de vente est-elle écrite pour davantage réduire les risques de perte d'un producteur.

Conclusion Générale

Partout dans le monde, la production de produits de base repose sur de nombreux facteurs incertains. Ces facteurs peuvent devenir défavorables pour un producteur, spécialement en agriculture qui sert principalement à l'alimentation humaine. Les principaux risques en production agricole proviennent des variations non anticipées des prix et des récoltes. En effet, au moment de la récolte, les prix pourraient être défavorables par rapport aux coûts et le rendement de culture attendu. Le producteur, devrait alors trouver un moyen de garantir, autant que possible, son revenu contre ces risques. Le marché financier est une alternative pour transférer ces risques aux les investisseurs qui sont en mesure de les supporter moyennant une prime payée par les producteurs. Les marchés des matières premières permettent ainsi aux producteurs, dans le besoin de couvrir leurs revenus de culture, mais servent aussi de valeurs refuges aux investisseurs.

Sur les marchés financiers, divers instruments peuvent être utilisés pour élaborer des stratégies de couverture. Ces stratégies comprennent principalement les dérivés sur le prix au comptant comme les contrats à terme (futures et forwards) ou les options. Les dérivés permettent de reporter la livraison à une date future et ce, à un prix prédéterminé. Les producteurs pourraient bloquer le prix pour une certaine date d'échéance. En outre, parce que les produits dérivés sont spéculatifs, mais aussi soumis à divers risques qui peuvent entraîner une perte importante, un producteur, peut néanmoins, prendre des positions dans les dérivés mentionnés ci-dessus dans l'optique de revenus supplémentaires sur le marché.

Un autre instrument de couverture est un contrat d'assurance qui est comme option, mais il est spécialement utilisé pour traiter les risques de production en cas de rendement de cultures faible. Le contrat d'assurance garantit la proportion souscrite de rendement espéré de culture. Dans cette thèse, nous avons fait l'étude de la modélisation des prix agricoles à terme et abordé la question des stratégies de couverture dans un contexte de gestion de portefeuille. L'agent en couverture est un producteur dont la stratégie portefeuille est composée de l'actif non cessible représenté par le prix au comptant et de contrat de futures, d'option ou de contrat d'assurance pour couvrir ses revenus contre les mouvements de prix dé- favorables ou un déficit de production.

La stratégie de couverture exige une analyse de marché préalable afin de répondre aux attentes appropriées. Sur les marchés des matières premières, les faits stylisés permettent de d'expliquer les comportements prix. L'analyse empirique sur les données passées est usitée pour ressortir les facteurs économiques sous forme de modèle, puis les anticipations se forment en suivant les prévisions, les préférences et le contexte. Ensuite, la stratégie de couverture appropriée est établie selon les anticipations des producteurs. À cette fin, nous avons analysé la stratégie couverture pour un producteur agricole dans les cadres statique et dynamique.
La thèse a exploré les aspects suivants : Comportement des prix des matières premières : en appliquant des méthodes économétriques sur les données de prix de matières premières choisies, nous avons testé les faits stylisés bien connus dans la littérature : le retour à l'équilibre, la saisonnalité et la présence de sauts.

La thèse a exploré les aspects suivants

◇ *Comportement des prix des matières premières* : En appliquant des méthodes économétriques sur les données de prix de matières premières choisies, nous avons testé les faits stylisés bien connus dans la littérature : le retour à l'équilibre, la saisonnalité et la présence de sauts. Les tests sur les prix quotidiens sont effectués de telle sorte que toutes les caractéristiques seront considérées ensemble dans le modèle de prix comme les facteurs économiques. Ces tests ont montré que, le marché de matières premières agricole est plutôt inefficient avec un phénomène de retour à l'équilibre et de sauts par moment. Le retour à l'équilibre reflète l'état du prix d'équilibre qui dévie temporaire de sa valeur fondamentale. Ceci repose sur la façon dont le déséquilibre entre l'offre et la demande fluctue pour converger vers le prix d'équilibre. De plus, il y a le comportement saisonnier à la fois, sur la moyenne de long terme du retour à l'équilibre et sur les volatilités mensuelles pour représenter les variations calendaires des prix. Ce qui veut dire que la saisonnalité provient respectivement des échéances de contrats à terme et des campagnes agricoles. Quant aux sauts, ils sont dus aux changements pertinents en raison des nouvelles soudaines et ce spécialement en inter campagnes. Ils représentent le risque de marché supplémentaire à gérer en couverture dynamique.

◇ *Modélisation des prix à terme agricoles* : À la lumière des faits stylisés mis en évidence dans le chapitre 2, un processus de retour à l'équilibre avec saut-diffusion est retenu pour les prix agricoles où la moyenne à long terme et la volatilité sont saisonnières. Cette même dynamique sans saut est similaire à celle étudiée plus tôt dans la littérature où une fonction déterministe est considérée pour la composante saisonnière dans la tendance (voir Geman et Nguyen geman2005soybean). Les sauts se produisent de façon aléatoire et irrégulière et leur considération rend difficile l'estimation du processus avec le maximum de vraisemblance. Par conséquent, nous avons procédé à une estimation en deux étapes. Dans la première étape, la vitesse de retour à l'équilibre, ainsi que les paramètres périodiques de moyenne à long terme sont estimés par moindres carrés. Dans la deuxième étape, nous appliquons la méthode de particules MCMC pour estimer les paramètres restants en utilisant les résidus de la première étape. La méthode de particules MCMC est appliquée au lieu du filtre de Kalman, parce que le bruit est non gaussien. Le particule MCMC est avéré être une méthode bayésienne robuste car la fonction de vraisemblance exacte du processus de mesure conditionnellement aux paramètres d'intérêt est approchée avec le filtre particulaire qui est connu pour être plus cohérent. Les paramètres sont alors considérés comme des variables aléatoires latentes. La mise en œuvre de cette procédure d'estimation donne des estimations de paramètres qui sont validés avec le test de Ljung-Box sur les résidus.

◇ *Stratégie statique de couverture optimale* : Dans le cadre statique, de nombreuses approches selon la préférence de l'agent en couverture ont été étudiés. Ils conduisent à des ratios de couverture optimales qui dépendent fortement de leur approche. Ainsi, il est difficile de distinguer quelle stratégie est la meilleure parmi tous les ratios de couverture en utilisant la mesure de la performance d'Ederington. Pour surmonter cette limite de la mesure d'efficacité, nous avons proposons la mesure L-performance pour ordonner les stratégies en fonction de leur performance. Ensuite, nous avons obtenu la stratégie de couverture optimale en contrat à terme en présence du risque de marché et du risque de production lorsque pour les maturité courtes. Cette stratégie est mise en œuvre avec le processus de retournement de position rollover. Cette couverture consiste à passer du contrat de futures dont l'échéance est proche pour un contrat de futures d'échéance plus longue. Ceci permet de maintenir la position en couverture sur tout l'horizon du portefeuille. Cependant, cette stratégie encourt le risque supplémentaire, dit de risque de retournement, en raison des écarts de prix entre le contrat d'échéance proche et le contrat de plus long terme. La stratégie de couverture optimale en rollover s'avère meilleure que les autres stratégies en terme de risque de marché et de risque de production en campagne agricole ou inter-saison. En utilisant, la mesure de L-performances pour ordonner les performances de stratégies, il ressort que la stratégie de rollover combinant le contrat de futures et d'assurance est la meilleure stratégie de couverture. Particulièrement, le contrat d'assurance garantit une partie des recettes des cultures lorsque le rendement des cultures est plus faible que prévu.

◇ *Stratégie dynamique de couverture optimale* : Stratégie dynamique de couverture optimale : Pour tenir compte des appels de marges quotidiennes et les faits stylisés testés au chapitre 2, nous avons étudié la stratégie de couverture en temps continu. L'horizon de couverture considéré est similaire à une période de campagne agricole. La stratégie optimale en temps continu est faite via maximisation de l'utilité espérée où le producteur recherche la position dynamique en contrat de futures et le taux de consommation. Dans cette situation de couverture, puisque le spot et le contrat de futures ne sont pas parfaitement corrélés et puis la présence de sauts, le marché est incomplet. Et il en est ainsi lorsque le marché au comptant est un actif non cessible. L'optimisation est réalisée sous la mesure martingale minimale. Nous montrons que la couverture avec contrat de futures peut être améliorée une option de vente supplémentaire dans le portefeuille de couverture.

Extension et orientation de recherche

Nous avons étudié la stratégie de couverture afin de réduire les risques liés aux évolutions défavorables qui pourraient réduire le revenu d'un producteur sur les marchés agricoles. Cette question est extensible pour inclure la couverture dans le cadre de multi-production à l'égard de la situation de diversification du risque de production. Dans ce cas, le risque de corrélation est également important et il sera intéressant d'analyser le cas particulier du marché incomplet en terme de nombre de contrats de futures à envisager pour la couverture optimale.

D'un autre côté, selon le cadre statique ou dynamique, le problème de couverture peut également être étendu. Dans le cadre statique, une alternative pour améliorer la stratégie de couverture pourrait être d'inclure, un produit dérivé dépendant que de la valeur finale du prix au comptant pour la gestion des risques de base de de retournement de positions. Enfin, en temps continu, l'étude n'a pas d'application sur les données du marché. Une telle analyse empirique serait certainement un complément pour aborder la politique de gestion des matières premières. Plus précisément, il aurait permis de voir comment l'évolution des stratégies dynamiques à la fois en inter et intra campagne agricole pour les horizons de couverture long. À cette fin, la notion de redondance du contrat d'assurance et du contrat de l'option serait soulignée en terme de linéarité et de non-linéarité dans la stratégie de couverture

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Abstract: In agricultural markets, producers incur price and production risks as well as other risks related to production contingencies. These risks impact the producer activity and could decrease his income. The globalization of markets, particularly those of agricultural commodities, provides hedging instruments including futures contracts which will serve to develop a hedging strategy. However, the situation whereby single futures contract-based positions could offset many risks leads to incomplete market. Especially, an producer looking for better hedging strategy could also include insurance, option contract or mutual funds to further guarantee his income, especially when crop yields are lower than expected.

We investigate the hedging strategies in static framework as well as in continuous time framework. Prior, we analyze the behavior of agricultural prices using various statistical approaches and suggest appropriate price modeling for data at hands. The static hedging strategy also accounts for rollover process which gives raise to additional risks due to spread between new futures and nearby futures and inter-crop hedging. We particularly address hedging strategy that combines futures and insurance contracts. Since decisions making in static framework does not include price changes along the hedging horizon, optimal hedging strategy in continuous time framework will take into account jumps and seasonality by combining futures and option contracts.

Résumé : Sur les marchés agricoles, les producteurs encourent les risques de prix et de production ainsi que d'autres types de risques liés aux aléas de production. Ces risques impactent l'activité du producteur et pourraient diminuer ses revenus. La mondialisation des marchés, en particulier ceux des matières premières agricoles, permet de développer une stratégie de couverture en utilisant des instruments comme les contrats à terme. Cependant, la situation selon laquelle une position basée seulement sur un contrat futures devrait couvrir tous les risques entraîne un marché incomplet. Le producteur en recherche de meilleure stratégie de couverture pour ajouter un contrat d'assurance ou d'option pour garantir davantage ses revenus, surtout lorsque les rendements des cultures prévus diminuent.

Nous étudions, ici les stratégies de couverture dans le cadre statique, ainsi que dans le cadre de temps continu. Avant, nous analysons le comportement des prix des matières premières agricoles en utilisant diverses approches statistiques afin de suggérer la modélisation des prix adéquate aux données. La stratégie de couverture statique comprend également le processus de retournement de positions qui pourrait entraîner d'autres risques supplémentaires en raison de l'écart entre les nouveaux contrats à terme et des contrats à terme à proximité ainsi que la couverture inter-culture. Nous proposons une stratégie de couverture qui combine des contrats futures et d'assurance. Comme la prise de décisions dans le cadre statique ne tient compte des mouvements quotidiens de prix le long de l'horizon de couverture, la stratégie de couverture optimale en temps continu combine des positions en contrat à terme et options tout en prenant en compte les sauts et la saisonnalité dans la dynamique des prix.