# Game-theory approaches to study and influence interactions among self-interested agents in wireless networks

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Mention : Informatique

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Ecole Doctorale – MATISSE

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# *Abstract*

Almost all modern mobile devices are equipped with a number of various wireless interfaces simultaneously, so that each user is free to select between several types of wireless networks. This opportunity raises a number of challenges, since in general selfish choices do not lead to a globally efficient repartition of users over networks. In order to study this problem, we split the general users allocation subject into three subtopics. At first, we study how the users are making network selection decision, which information is available for them and by which means. We develop a model, where users decision are lead by ratings of available networks and prices they have to pay. At the second step we already study the outcome of selfish users behavior, which we found to be inefficient. We decide to introduce a specific taxation policy, which takes into account the users diversity in price (or QoS) perception, and lead to an optimal situation, when the total QoS experienced by users is maximized. The last subtopic covers the problem of providers interaction, which has a crucial impact on users welfare. We study both models with static and mobile users, and for the former case we propose a novel model of Internet access providers competition in the vehicular networks.

# *Résumé de la thèse en Français*

Depuis quelques années, les appareils de communication mobiles parviennent à gérer simultanément plusieurs interfaces réseau, donnant ainsi aux utilisateurs mobiles la possibilité de choisir facilement entre plusieurs types de points d'accès et de technologies disponibles. On peut naturellement s'attendre à ce que les différentes applications d'un appareil mobile utilisent différentes interfaces réseau simultanément. Un utilisateur mobile peut donc tirer profit de la diversité des technologies disponibles, exploitant leurs avantages pour chaque application ou besoin de service spécifique. Notons que cette augmentation des possibilités de choix stimule également la concurrence entre les fournisseurs de réseaux sans fil, qui conduit généralement à une baisse des prix d'accès et une amélioration de la qualité de service (QoS).

Le concept de "Always best connected" [1] a été récemment introduit. Par cette expression, on comprend un système qui sélectionne automatiquement à chaque instant le réseau le plus approprié pour un utilisateur, en tenant compte de ses préférences (tels que la disposition à payer, le niveau de consommation d'énergie, etc), ainsi que les exigences de son applications (par exemple de seuil tolérable de délai, le débit disponible, la gigue). Toutefois, un mécanisme basé sur ce concept, bien que défini pour satisfaire les utilisateurs, pourrait conduire à des situations où certaines technologies seraient surexploitées et d'autres sous-utilisées par rapport à une allocation optimale. En effet, la conjonction de décisions individuellement optimales (ici, le choix d'un point d'accès pour chaque mobile/application) ne conduit généralement pas à un optimum global. Cette utilisation inefficace correspondant à une qualité de service moindre, il peut alors être dans l'intérêt des fournisseurs d'accès d'introduire des incitations afin d'influencer les décisions prises par les utilisateurs.

Un grand nombre d'études montrent en effet l'inefficacité potentielle dans les scénarios où les utilisateurs font égoïstement leurs décisions d'association aux points d'accès. Cela a incité la communauté scientifique à regarder de plus près le processus de décision des utilisateurs, et en particulier les méthodes qui pourraient aider les fournisseurs à influencer le comportement des utilisateurs d'une manière souhaitable. La littérature scientifique fournit plusieurs enquêtes consacrées au problème de la répartition des utilisateurs dans les réseaux d'accès sans fil (comme [2–5]).

Le problème de répartition des utilisateurs dans un réseau y est généralement présenté comme suit. Un certain nombre de points d'accès sans fil coexistent dans une zone géographique. Ces points d'accès peuvent mettre en œuvre la même –ce type de réseau est appelé **homogène**– ou différentes technologies d'accès sans fil (par exemple WLAN, WiMAX, UMTS, et plus récemment, LTE) –constituant ainsi un réseau **hétérogène**– afin de fournir un accès à Internet. Les utilisateurs situés à l'intérieur des zones de couverture de ces points d'accès cherchent à établir une connexion, et pour ce faire doivent choisir un ou point(s) d'accès (ou plusieurs, en cas de multihoming).

Les points d'accès ont généralement des zones de couverture qui se chevauchent ou même coïncidant, et donc les utilisateurs situés dans des zones d'intersection sont en mesure de choisir entre plusieurs réseaux. Cependant, dans certains scénarios les utilisateurs ont la possibilité de choisir même si les zones de couverture ne se chevauchent pas : par exemple, dans le cas de réseaux véhiculaires, les utilisateurs mobiles peuvent rencontrer successivement plusieurs points d'accès et pour chacun décider de s'y connecter ou non.

Dans ce travail de thèse, nous étudions le problème de la répartition des utilisateurs dans les réseaux sans fil hétérogènes. Nous avons séparé ce problème en trois thèmes interconnectés que nous avons abordés séparément. Cette fragmentation est fondée sur l'échelle de temps à laquelle est considéré le problème.

L'échelle de temps la plus petite que considérons correspond aux décisions prises par les utilisateurs eux-mêmes : nous nous concentrons sur la façon dont les utilisateurs sélectionnent parmi plusieurs alternatives, et sur quel type d'information est donné sur ces alternatives avant ce choix. Dans le cas d'utilisateurs statiques, cette décision se limite à un choix entre un certain nombre de réseaux disponibles. Dans le chapitre 2, nous présentons et comparons les différentes approches considérées dans la littérature pour ce cas.

Dans le Chapitre 3 nous étudions un système où une entité tierce est chargée de recueillir les informations fournies par les utilisateurs sur la qualité de service dont ils ont bénéficié lors de leur connexion, et de propager cette information sous forme d'un score aux autres utilisateurs qui auront à effectuer un choix. Ainsi, les utilisateurs suivants pourront baser leur choix de point d'accès sur un compromis entre la qualité de service qu'ils peuvent attendre (estimée par les scores reçus) et le prix qu'ils auront à payer. En modélisant l'arrivée d'utilisateurs au cours du temps par un processus de Poisson et en supposant un temps de connexion distribué selon une loi exponentielle, nous pouvons estimer la répartition de la demande entre les points d'accès à laquelle les fournisseurs peuvent s'attendre pour un profil de prix fixé. Ce type de système est intéressant en raison du fait que le choix est simplifié pour les utilisateurs, qui disposent d'une estimation du service auquel s'attendre. En outre, le système est auto-régulé (trop de demande sur un point d'accès conduit à une dégradation de son score et par conséquent à une réduction de sa demande), et conduit à une distribution de la demande qui n'est pas trop éloignée de la situation optimale (en terme de somme des scores). Nous montrons également que même lorsque les utilisateurs entrent dans le système et le quittent au cours du temps (et la répartition des utilisateurs n'est jamais parfaitement stable), il est possible d'avoir de bonnes estimations du nombre d'utilisateurs sur chaque point d'accès.

Dans le Chapitre 4 nous considérons un modèle simplifié de choix des utilisateurs, qui nous permet une étude analytique complète. Dans un premier temps, les utilisateurs sont supposés être non-atomiques, c'est-à-dire que leurs décisions individuelles ont une influence négligeable sur la qualité perçue par les autres. Nous avons supposé que tous les utilisateurs connaissent le niveau de qualité de service dans les réseaux disponibles, leurs choix étant des compromis entre la qualité de service et le prix à payer. Cependant, les utilisateurs diffèrent dans leur sensibilité relative à ces deux quantités ; nous modélisons cette diversité en considérant différentes catégories d'utilisateurs, chacune avec une valeur de sensibilité au prix. La concurrence entre les utilisateurs se modélise alors comme un jeu de routage, qui est connu pour avoir un équilibre.

Dans ce cadre, nous considérons alors l'échelle de temps supérieure : puisqu'on peut prédire la répartition des utilisateurs dans le système à partir du comportement égoïste des individus, comment pouvons-nous les inciter à « coopérer » en vue d'atteindre une situation globalement optimale ? Nous avons utilisé le prix imposé sur chaque point d'accès comme outil d'incitation. En définissant le coût social comme la somme des temps de latence subis par tous les utilisateurs du système, nous obtenons des expressions analytiques pour les taxes optimales sous certaines hypothèses (leur existence étant établie par des travaux précédents), et décrivons l'algorithme pour les calculer.

Nous avons montré par simulation que ce système basé sur les prix fonctionne même dans le scénario réaliste où les arrivées et les départs d'utilisateurs au cours du temps sont aléatoires (le modèle analytique considère la demande des utilisateurs comme étant statique). Nous proposons également une nouvelle interprétation de la perte d'efficacité due à la non-coordination entre utilisateurs, en la reformulant en termes de surdimensionnement (capacité de transmission qui pourrait être économisée en coordonnant les utilisateurs) ou bien de demande supplémentaire qui pourrait être servie sans surcoût grâce à la coordination. Ces interprétations peuvent permettre aux fournisseurs d'estimer l'intérêt économique d'introduire une forme de coordination entre utilisateurs (par exemple par les prix).

L'échelle de temps la plus grande considérée dans cette thèse correspond à la concurrence entre les fournisseurs d'accès. Nous nous concentrons sur ce sujet dans le Chapitre 5, où deux cas sont traités séparément. Dans une première partie, nous étendons le modèle proposé dans le Chapitre 3, en permettant aux fournisseurs de choisir les tarifs d'accès, ce qui influence la demande disponible (la demande totale étant alors supposée élastique, c'est-à-dire dépendante du prix). Les fournisseurs cherchent ici à maximiser leurs revenus ; nous étudions le jeu simultané dans le cas où l'infrastructure des fournisseurs diffère (un fournisseur ayant une capacité de traitement supérieure à l'autre), et comparons l'issue de la compétition à une situation de monopole (où un seul fournisseur possède toutes les infrastructures).

La deuxième partie du chapitre 5, étudie la concurrence entre fournisseurs d'accès dans les réseaux véhiculaires. Nous considérons un cas simple avec seulement deux points d'accès, appartenant à deux fournisseurs, disposés le long d'une route. Les utilisateurs se déplacent dans les deux sens, et voient un fournisseur avant l'autre ; ils doivent alors décider d'accepter ou non de payer le prix observé. Les fournisseurs, sachant cela, doivent prendre en compte les deux sens de flux d'utilisateurs : ceux qui n'ont pas vu de concurrent, et ceux ayant vu le concurrent auparavant et qui ont soit refusé de payer (ce qui signifie que leurs contraintes de prix sont bas) soit été rejetés en raison des contraintes de capacité du concurrent. Nous étudions la concurrence comme un jeu simultané sur les prix, et montrons qu'il existe deux équilibres, dans lesquels le fournisseur qui fixe sont prix à un niveau bas gagne des revenus plus élevés que son concurrent.

En outre, nous étudions le cas où les utilisateurs peuvent modifier leurs préférences de prix après le premier fournisseur rencontré : cela peut représenter le fait que les utilisateurs deviennent prêts à payer davantage car les chances de trouver une autre opportunité d'accès diminuent. Nous constatons alors un phénomène intéressant : si les utilisateurs acceptent de payer davantage, dans le jeu de compétition sur les prix les fournisseurs peuvent se retrouver avec un équilibre où tous deux fixent un prix inférieur (au cas sans variation de préférence), un des fournisseurs subissant même une perte dans son revenu.

Enfin nous considérons le cas où des points d'accès des deux fournisseurs fonctionnent sur la même fréquence et ainsi interfèrent, nuisant aux communications des clients du concurrent (si l'on ne considère que la liaison descendante). Nous étudions dans ce contexte le problème de la localisation optimale des points d'accès, et mettons en évidence plusieurs stratégies possibles dans le jeu de compétition entre fournisseurs. D'autre part, si les points d'accès sont contrôlés par un seul fournisseur (monopole), il peut être rentable de fixer un prix élevé sur l'un de ses points d'accès afin de saturer l'autre point d'accès : de la sorte le fournisseur pourrait gagner plus de revenus, en exploitant au maximum l'hétérogénéité de la volonté à payer des utilisateurs. De nouveau dans le cas de deux fournisseurs en compétition, dans le jeu de compétition sur les prix nous observons que lorsque la demande des utilisateurs est faible, le fournisseur qui fixe le prix en premier va facturer un prix élevé que son concurrent ne cherche à interférer avec lui pour diminuer sa capacité et détériorer ses revenus. Par rapport au cas sans interférences, cela conduit donc à une situation paradoxale, où la concurrence amène l'un des fournisseurs à augmenter son prix.

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# Abbreviations

| | |
|---|---|
| **AHP** | **A**nalytic **H**ierarchy **P**rocess |
| **AP** | **A**ccess **P**oint |
| **ELECTRE** | **El**imination and **C**hoice **Tr**anslating Priority |
| **GRA** | **G**rey **R**elational **A**nalysis |
| **HSDPA** | **H**igh **S**peed **D**ownlink **P**acket **A**ccess |
| **ITS** | **I**ntelligent **T**ransportation **S**ystems |
| **LTE** | **L**ong **T**erm **E**volution |
| **MAC** | **M**ultiple **A**ccess **C**ontrol |
| **MADM** | **M**ultiple **A**ttribute **D**ecision **M**odel |
| **MD** | **M**atching **D**egree |
| **MDP** | **M**arkov **D**ecision **P**rocess |
| **MEW** | **M**ultiplicative **E**xponent **W**eighting |
| **NE** | **N**ash **E**quilibrium |
| **OBY** | **O**n **B**oard **U**nit |
| **PoA** | **P**rice **Of** **A**narchy |
| **PoS** | **P**rice **Of** **S**tability |
| **QoE** | **Q**uality **Of** **E**xperience |
| **QoS** | **Q**uality **Of** **S**ervice |
| **RSU** | **R**oad **S**ide **U**nit |
| **SAW** | **S**imple **A**dditive **W**eighting |
| **SINR** | **S**ignal to **I**nterference plus **N**oise **R**atio |
| **SIR** | **S**ignal to **I**nterference **R**atio |
| **SMDP** | **S**emi **M**arkov **D**ecision **P**rocess |
| **SNR** | **S**ignal to **N**oise **R**atio |
| **TOPSIS** | **T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution |

| | |
|---|---|
| **TCP** | **T**ransmission **C**ontrol **P**rotocol |
| **UDP** | **U**ser **D**atagram **P**rotocol |
| **UGT** | **U**ser **D**atagram **P**rotocol |
| **UMTS** | **U**niversal **M**obile **T**elecommunications **S**ystem |
| **VANET** | **V**ehicle **A**d-hoc **NET**work |
| **VoIP** | **V**oice **o**ver **I**nternet **P**rotocol |
| **V2I** | **V**ehicle **T**o **I**nfrastructure |
| **V2V** | **V**ehicle **T**o **V**ehicle |
| **WE** | **W**ardrop **E**quilibrium |
| **WCDMA** | **W**ideband **C**ode **D**ivision **M**ultiple **A**ccess |
| **WiMAX** | **W**orldwide **I**nteroperability for **M**icrowave **A**ccess |
| **WLAN** | **W**ireless **L**ocal **A**rea **N**etwork |
| **WMAN** | **W**ireless **M**etropolitan **A**rea **N**etwork |
| **WMC** | **W**eighted **M**arcov **C**hain |

# Chapter 1

# Introduction

## 1.1 General background

The last years witnessed a tendency in the world of mobile devices, towards an increase in the number of different wireless network interfaces handled simultaneously. This variety of technologies gives mobile users a possibility to easily choose between several types of access points available. We could expect in the nearest future that it will be also possible that different applications on a mobile device use different network interfaces simultaneously. A mobile user could profit from the diversity of available technologies, exploiting their advantages and drawbacks and taking into account concrete application or service needs. Moreover, this increase of choice opportunities gives a stimulus to a competition between wireless network providers, which usually leads to a decrease in access prices and to Quality of Service (QoS) enhancement.

Quite recently the “Always best connected” [1] concept was introduced. By this term we understand a system, which in every moment of time automatically selects the most suitable network for a user, taking into account his preferences (such as desirable cost, level of power consumption, etc.) as well as the requirements of his applications (e.g. tolerable delay threshold, available rate, jitter). However, a mechanism based on this concept, even if aiming to satisfy the users, could drive the system to a situation where some technologies would be overused and some other under-utilized. This inefficient use could further lead to a QoS degradation and it is in the interest of providers to improve their resource management by giving users some incentives influencing their decisions.

A large number of studies indeed show potential inefficiency in scenarios where users selfishly make their access point association decisions. This fact strongly incentivizes the scientific community to look closer on the users decision process, and specifically, on the methods which could help providers to influence the users behavior in a desirable way. Scientific literature provides several surveys devoted to the users allocation problem in the wireless access networks (like [2], [3], [4], [5] ).

## 1.2 Problem description

The general users allocation problem scenario looks as follows. A number of wireless access points coexist in some area. These access points can implement the same - this type of network is called **homogeneous** - or different wireless access technologies (e.g. WLAN, WiMAX, UMTS, and most recently, LTE), thus constituting a **heterogeneous** network, to provide an Internet access. The users, which are located inside the coverage areas of these access points are willing to establish an Internet connection, and to do so they have to choose one or several (in case of multihoming) access point(s) to connect to among the available ones.

The access points usually have some overlapping or even coinciding coverage areas, and thus users situated in intersection areas are able to choose among several networks. However in other scenarios the users also have an opportunity to choose, even when coverage areas do not overlap: e.g. in the case of vehicular networks, mobile users sequentially meet several access points and for each access point they have to decide whether it is worth connecting to it or not. Figure 1.1 shows an example of heterogeneous network scenario, when different types of wireless networks coexist in the same area. Note, that in this manuscript we use terms "mobile use", "user", "customer", "mobile station" interchangeably, and by them we understand entities searching for an Internet connection. By "network access point", "network owner", "provider" if not specified, we understand an entity which provides users with an Internet connection on some predefined conditions. By "agent" or "player" we understand an entity, that makes an action or strategic decision in game theoretical environment, which can be both users or providers, depending on the model considered. In game theory models these entities are assumed to be selfish or self-interested, which means that they aim to rationally maximize their individual welfare without regard on impact they may cause on others.

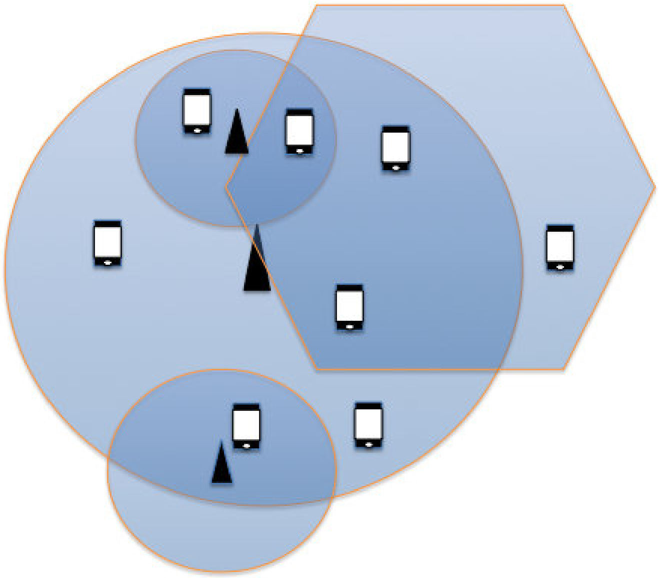

FIGURE 1.1: Users allocation problem general topology

A number of crucial questions arise regarding the problem of users allocation in the wireless network, which we could divide into two sets: the first one concerns users (or consumers) interests and the other one is focused on network facilities owners (i.e., network and service providers, government regulators, etc.) interests.

At first, to analyze user behavior we have to answer the following questions:

1. *What do the user want?*

   We have to understand what each individual user needs. We assume that every user wants to connect to the Internet, but particular needs could differ. We have to classify possible user goals, trying to reveal whether he wants to download a file, check his email, make a video call, etc., because these may influence further the network selection process.

2. *What are the users sensitive to?*

   Depending on their goals, the users could have different Internet connection requirements, e.g., video calling users wants to have low jitter, though they could tolerate a moderate connection rate, while users aiming to download heavy files prefer a high speed Internet connection. We need to define the parameters users are sensitive to.

3. *How do the users make their decisions?*

   Obviously, knowing all parameters the users are sensitive to is not enough for appropriate network selection. We need to model the way users are choosing one alternative above others, having information about network parameters, or some "beliefs" (like in Bayesian games described later on) about their values, as well as to determine their relative importance. Moreover we have to strictly define which kind of information is available for users and by which means. Some information could be advertised by network providers (e.g., cost or average connection speed), other types of information could be predicted based on statistics from previous sessions, or through active probing like in [6]. Every time we assume that users have some additional information, we have to understand that in real scenarios most probably this information advertising could lead to a serious communication overhead (i.e., there is a trade-off between information accuracy and overhead).

It could seem that users allocation concerns the users only: they aim to establish an Internet connection and they make the final decision about access points association. This is not true due to several reasons. First of all, the users behavior is provider-driven, because providers organize infrastructure and facilities, and users behave as a response to the situation resulting from providers actions. In addition, network providers have their own incentives and goals, and they could change the network conditions after observing user behavior in order to reach their objectives. Second, providers may make the association decisions: in models like [7] and [8], a user submits a bid for the services he would like to get, and then providers decide how to treat the user's request.

Analogically, we define a number of questions we want to answer when dealing with Internet access providers with regard to the user allocation problem:

1. *What do the providers want?*

   Somehow the users and the providers aims coincide: both types of entities want users to have an Internet connection. Nevertheless, this is not the providers primary goal - more often they aim to maximize their revenues. Still there can be variants: providers could propose fixed duration contracts and the main issue for him in this case is to ensure the users satisfaction or to decrease current maintenance costs (e.g., the power control). Therefore, we have to precisely define the final aim provider wants to achieve.

2. *What is the outcome of users behavior?*

   In real life, individuals' rational actions could lead to an unsatisfying situation. When a number of entities share the same source and every one of them tries to

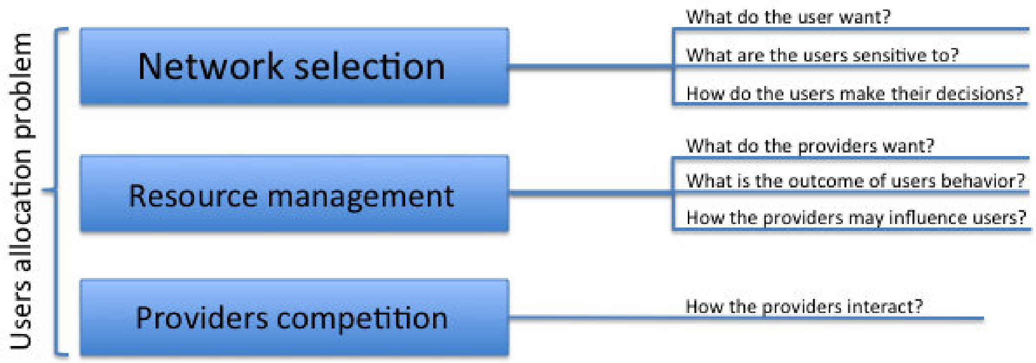


FIGURE 1.2: Users allocation problem subdivision

satisfy its own requirements or needs, it could lead to global dissatisfaction, due to the fact that each individual entity does not think about the consequences of his actions. This fact is also known as "the tragedy of commons" [9]. We need to investigate how far the outcome of such selfish behavior is from an optimum situation regarding some global objective function, quantifying satisfaction of provider's aims.

3. *How could the providers influence users?*

   Whatever providers goals are, it is important to know how they could elicit the users to act in a desirable way. We want to investigate by which means providers could incentivize users to change their original selfish decisions. Most frequently, incentives are introduced through encouragements or penalties, however we will also discuss systems where different types of incentives are considered.

4. *How do the providers interact?*

   When several providers coexist, their behavior may change dramatically. Every provider has to take into account the behavior of competitors, in order to predict his own revenue and to achieve his goals. Moreover, in the situation of competition, providers have to struggle with each other to attract users. It is matter of fact that competition in market usually enhances the quality of proposed goods, but whether it is the case for the wireless internet access market has to be investigated. We have to study this interaction of neighboring providers, as well as its impact on user behavior.

We could group the aforementioned questions into broader research topics, as illustrated on Figure 1.2:

1. **Network selection problem:**

   In this area we include all questions regarding the decision a user has to make. We have to know which network parameters are more important to users, what are the constraints of his mobile device, budget, time, etc. We have to define which information is available to users and by which means: whether it is advertised by the network provider, given by some web service, observed through probing or from previous experience. Finally, the main problem is to understand, having all this, how the user chooses an access point to connect to.

2. **Resource management problem:**

   – Efficiency analysis: Given the fact that each user tries to selfishly choose the best solution, we have to investigate to which outcome this kind of behavior could lead. This is up to provider(s) to define a criterion of optimal resource usage. In majority of research, unregulated selfish behavior of users is found to imply strong QoS degradation.

   – Fixing inefficiency: If the outcome of users selfish behavior is inefficient regarding some provider's target function (e.g., revenue, power consumption), the provider may influence users in order to make them change their decisions (it is not possible to change the selfish nature of users behavior, but one could give some incentives to influence it).

3. **Providers interaction problem:**

   Effective resource management is a good solution for a provider when he is alone, i.e. when he is a monopolist in some area. The situation gradually changes when a competitor appears: providers now have to find a way to achieve their goals, taking into account that they have to compete for users. Obviously, the providers competition can has a considerable impact on users behavior and thus we have to include it into consideration.

## 1.3 Thesis plan and contributions

In this thesis we tackle the users allocation problems in different scenarios. We consider cellular, urban and vehicular wireless networks, trying to predict and analyze an outcome of selfish behavior of involved entities.

Chapter 2 discusses the main relevant publications on the topic. We first survey the works, focussed on the users side of the problem: mainly it concerns the network selection decision they have to make. Then we described articles on competition between users, both trying to measure inefficiency of users behavior and ways to fix it. Finally, we look on the models where the competition between providers is studied.

In Chapter 3 we describe a system, where users produce network selection knowing both prices and ratings of networks. These ratings are averaged QoS experienced by users in previous time slots and are gathered by a special middle controller. We study the dynamics of the system and propose a way to predict the outcome of users decisions.

Chapter 4 is devoted to the resource management problem: we study there how Internet access provider can minimize the total latency experienced by users through appropriate taxes introduction. Users perceive the pricing in a different manner, thus we divide them in a number of classes with the same price sensitivity. We apply results from the routing games theory, and derive analytical expression for the optimal taxes.

In the next Chapter, we focus on the competition between Internet access providers. This competition could arise in various settings, and we consider two cases: when users are static and when users are moving. For the static users case we apply the results for the rating-based model from Chapter 3, and for the mobile users we consider a new model of providers competition in vehicular networks.

In the Chapter 6 we make a conclusion, briefly resume the work done and propose future research directions.

The results described in this thesis work were published in

– V. Fux and P. Maillé. A rating-based network selection game in heterogeneous systems. In *Proc. of NGI*, 2012

– V. Fux, P. Maillé, J.-M. Bonnin, and N. Kaci. Efficiency or fairness: managing applications with different delay sensitivities in heterogeneous wireless networks. In *Proc. of WoWMoM*, 2013

– V. Fux and P. Maillé. Incentivizing efficient load repartition in heterogeneous wireless networks with selfish delay-sensitive users. In *Proc. of ICQT*, 2013

– V. Fux, P. Maillé, and M. Cesana. Price competition between road side units operators in vehicular networks. In *Proc. of IFIP Networking*, Trondheim, Norway, June 2014 and these articles are currently in process:

– V. Fux, P. Maillé, and M. Cesana. Road side units operators in competition: a game-theoretical approach. Journal article, 2014

– V. Fux. RSU deployment problem: unfair and aggressive competition with help of interference. submitted to NetGCoop, 2014

# Chapter 2

# State of the Art

In this chapter we discuss already existing solutions for all three dimensions of the users allocation problem. Starting from the lowest time scale we at first consider various approaches in the area of network selection: the main accent is made on Multiple Attribute Decision Models (MADMs), which prescribe how a user will choose one network among the available ones, knowing different characteristics of them. Further we survey the articles studying the possible outcomes of selfish users behavior in heterogeneous networks as well as the ways access providers can influence their customers. Some basic game theory definitions are given prior to it. Then, we overview models which study competition between providers: they struggle for users, aiming to maximize their individual profits. Finally, we provide a short overview of research challenges with which the research community currently deals.

## 2.1 Network selection problem

The network selection problem deals with how a user should select the most suitable network among a set of alternatives. A large number of articles tackle the network selection problem with a help of already existing tools from the decision theory such as multiple attribute decision models. They propose a way to evaluate each available network based on a number of observed parameters and their relative importance. In what follows we describe the major MADMs and discuss which parameters may be important for users. Finally, we survey several other approaches to the network selection problem, which are presented in the scientific literature.

Following [3] which provides a good overview of network selection decisions models, we divide the decision criteria that mobile users have in the network selection problem into four groups:

1. **User preferences.** These criteria are the constraints, imposed by user. The most important criterion here is the budget the user wants to spend for the proposed service. Other criteria here are the Quality of Experience (QoE) expectation (subjective measure of the quality of a service), the time of connection (for the cases when day/night pricing differs) and etc.
2. **Application requirements.** Depending on the application running, the requirements for a network connection could differ. The most popular example is the difference between video call service and simple file download requirements. In the first case, an application needs small delay and jitter values with an admissible rate, while in the latter case it is more sensitive to the available throughput.
3. **Device constraints.** Given an extensively increasing market of mobile devices, the users could have devices implementing totally different technologies. Each device could have its own constraints: the most obvious constraints are the supported wireless connection interfaces, but here we also include battery level, screen size, etc.
4. **Network parameters and conditions.** By network parameters we understand some static information, e.g., network type and energy consumption. By network conditions we mean information which changes over time and could vary from user to user, like signal level, delay, jitter, rate, cost, etc.

The parameters could also be classified by the way their values are perceived by users:

– **Beneficial parameters** The bigger the value of this parameter, the better for users.

– **Cost parameters** The lower the value of this parameter the better for users.

– **Nominal-the-best parameters** The closer the value of this parameter to a some predefined value, the better for users.

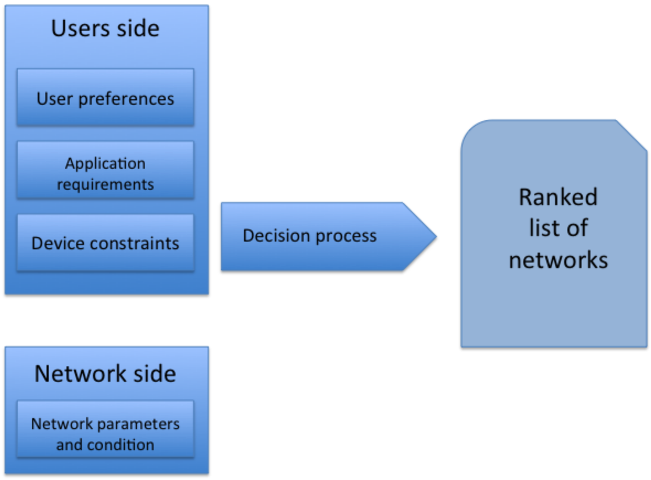


FIGURE 2.1: Decision process in MADM

### 2.1.1 Multiple Attribute Decision Models (MADM) application to the Network Selection problem

Some of the aforementioned parameters could be known by users: they could be directly advertised by the network provider, gathered by probing or estimated from previous experience. Given a number of parameters, a user has to decide which network to connect to. The multiple attribute decision models describe different ways of how he could do it. These algorithms provide a way to evaluate and to rank available networks by their suitability for concrete user needs.

The main drawback of MADMs is that as a rule they do not take in consideration the outcome of users decisions, i.e, they focus only on the welfare of an individual user, and do not consider how his decision could impact the welfare of the whole population. This is important because if all users apply the same mechanism to select the most suitable network, most probably some changes in it could lead to increase of the satisfaction level of every user in the system, especially if this mechanism will take into account negative externalities caused by the selfishness of users.

The other problem with MADMs is that they are hard to evaluate. One of the attempts to compare different decision models was made in [2]. However, in that work the decision model were compared with the same weight values, which could be seen as not the adequate simulation setting - clearly, in different models weights have different meanings and influence. More accurate simulations should consider the models trained on some set of examples of different applications, and only when the appropriate weights for each application are determined, they could be compared. Moreover, it is not clear how one could evaluate different selection decisions - most likely one has to make tests regarding the Quality of Experience, in which users after sending to an algorithm their preferences and choosing an application they want to use, report their quality evaluation. But this evaluation is clearly subjective, and needs quite an accurate study we do not consider here.

The rest of the subsection will be devoted to the MADMs description and their application examples to the network selection problem. Due to the reason stated above we do not aim to compare them; our main objective is to show different ways of producing the network selection decision.

#### 2.1.1.1 The Simple Additive Weighting (SAW) and Multiplicative Exponent Weighting (MEW) methods

The Simple Additive Weighting is the most simple and widely used decision model. When only two decision parameters are considered, authors use various names for the method, thus it is quite difficult to give concrete references, especially given the simplicity of the approach.

In all methods described below, a user tries to choose a network from a set $M$. There is a number of criteria $N$ (like delay, jitter, cost), which a user can observe, $x_{ij}, j \in N, i \in M$. For each criterion, the user has some preferences, expressed as weights $w_j, j \in N$. Then the score a network receives regarding that user is equal to the weighted sum of the criteria values:

$$\text{Score}^i_{SAW} = \sum_{j \in N} w_j \bar{x}_{ij}, \tag{2.1}$$

where $\bar{x}_{ij}$ is the appropriately normalized parameter depending on its type - whether it is beneficial (the bigger the better) or the cost (the lower the better) parameter.

Multiplicative Exponent Weighting is the other one simple and commonly used decision model. In contrast to SAW, MEW as the score of a network takes a product of network parameters, taken in the power of corresponding weights. Using the same notation, the score of a network is:

$$\text{Score}^{i}_{MEW} = \prod_{j \in N} \bar{x}_{ij}^{w_j}. \tag{2.2}$$

Further, the network with higher score is selected; if the connection failed, the next one from the score list is taken and so on.

#### 2.1.1.2 Technique for Order Preference by Similarity to Ideal Solution Algorithm (TOPSIS)

Each network in TOPSIS [16] is viewed as a parameter vector of size $N$. The main idea of this approach is to find a network which is the closest to the best solution, and the farthest from the worst one. As in the previous two models, the normalized parameters are computed. Further, a matrix consisting of elements $v_{ij} = \bar{x}_{ij} \cdot w_{ij}$ is considered, where $\bar{x}_{ij}$ and $w_{ij}$ are the normalized parameter $j$ and the weight of parameter $j$ for network $i$, respectively. Further, the best and the worst solutions are constructed, taking the best (worst) value among all networks for each parameter $j \in N$. Finally the network with the biggest score defined as:

$$\text{Score}_{TOPSIS} = \frac{d^-}{d^+ + d^-} \tag{2.3}$$

is chosen, where $d^-$ and $d^+$ is the Euclidean distance to the worst and the best solution respectively.

#### 2.1.1.3 Analytic Hierarchy Process (AHP) and Grey Relational Analysis (GRA)

If in the previous methods the criteria weights are given as inputs, in this approach the authors in [17] propose a method to compute them. In the AHP method, a user should produce pairwise comparisons and evaluate a relative importance of parameters by assigning values from 1 to 9, and then the algorithm calculates the final weights. Later, the Grey Relational Analysis is used, which aims to find a similarity between available alternatives and the best solution. As in the previous methods the normalization of parameters is needed. Unlike other approaches, the authors consider one more type of parameters - nominal-the best, which means the closer the current value to the nominal one, the better. Further, they define the best solution $x_0$, consisting of the best values for each parameter among all alternatives' parameters. Then the score for network $i \in M$ is calculated as follows:

$$\text{Score}^i_{GRA} = \frac{1}{N} \sum_{j=1}^{N} \frac{D_{min} + D_{max}}{D_{ij} + D_{max}}, \tag{2.4}$$

where $D_{ij} = w_{ij}|x_{0j} - \bar{x}_{ij}|$, $D_{max} = \max_{i \in M, j \in N} D_{ij}$ and $D_{min} = \min_{i \in M, j \in N} D_{ij}$.

Clearly, the Grey Relational Analysis could be used with predefined weights, as well as the Analytic Hierarchy Process could be used in conjunction with other decision models.

#### 2.1.1.4 Elimination and Choice Translating Priority (ELECTRE)

This method originally proposed in [18] was first applied with slight modifications to the problem of network selection in [19]. At the first step of the mechanism a reference network is chosen. In [19] as the reference network an unreal network with a desired set of attributes is considered (similar to what we called best solution in the previous method). Further, for each network and each attribute an absolute difference is computed, followed by normalization. Then, the normalized values are multiplied by the criteria weights.

On the next step, concordance and discordance sets are constructed:

$$\text{Cset}_{kl} = \{j : (w_j \bar{x}_{kj}) \geq (w_j \bar{x}_{lj})\}$$
$$\text{Dset}_{kl} = \{j : (w_j \bar{x}_{kj}) < (w_j \bar{x}_{lj})\},$$

where $k, l \in M$ are the compared networks, $\bar{x}_{lj}$ is the normalized value of the difference between attribute $j$ of network $l$ and the reference network.

Based on these sets, the concordance and discordance matrices $C$ and $D$ are constructed. The elements $k, l$ in the matrices look as follows:

$$C_{kl} = \sum_{j \in Cset_{kl}} w_j$$
$$D_{kl} = \frac{\sum_{j \in Dset_{kl}} |w_j \bar{x}_{kj} - w_j \bar{x}_{lj}|}{\sum_{j \in N} |w_j \bar{x}_{kj} - w_j \bar{x}_{lj}|}$$

On the last step the net concordance and discordance indexes are calculated. The net concordance (discordance) index corresponds to a measure of dominance (weakness) of network $k$ over other networks compared with the measure of dominance of other networks over network $k$:

$$C_k = \sum_{l=1, l \neq k}^{M} C_{kl} - \sum_{l=1, l \neq k}^{M} C_{lk}$$
$$D_k = \sum_{l=1, l \neq k}^{M} D_{kl} - \sum_{l=1, l \neq k}^{M} D_{lk}$$

A network with the highest net concordance index and the lowest discordance index should be chosen. If there is no such network, the networks could be ranked by both parameters, and then the network with the highest average ranking is the best one.

#### 2.1.1.5 VIKOR

One application of this mechanism to the problem of network selection is described in [20]. Similarly to previous methods, at first it is necessary to determine the best and the worst values for each criterion:

$$x_j^+ = \{(\max_{i \in M} x_{ij} | j \in N_b), (\min_{i \in M} x_{ij} | j \in N_c)\},$$
$$x_j^- = \{(\min_{i \in M} x_{ij} | j \in N_b), (\max_{i \in M} x_{ij} | j \in N_c)\},$$

where $N_b$ and $N_c$ are the sets of benefit and cost criteria, respectively (the authors do not consider nominal-the best parameters).

Further, for each network the authors compute several values, based on which the ranking would be made:

$$S_i = \sum_{j \in N} w_j \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-},$$
$$R_i = \max_{j \in N} [w_j \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}],$$
$$Q_i = \gamma(\frac{S_i - S^+}{S^- - S^+}) + (1 - \gamma)(\frac{R_i - R^+}{R^- - R^+}),$$

where

$$S^{+} = \min_{i \in M} S_i, \quad S^{+} = \max_{i \in M} S_i,$$
$$R^{+} = \min_{i \in M} R_i, \quad R^{+} = \max_{i \in M} R_i.$$

Here $\gamma$ is a parameter such that $0 \leq \gamma \leq 1$. On the final step, the authors propose two conditions, such that if both of them are satisfied, a network with the smallest value of $Q_i$ is chosen and if not, several networks are proposed as output (for detailed description see [20]).

#### 2.1.1.6 The weighted Markov chain (WMC) method

The idea of the WMC approach presented in [21] is to rank networks according to stationary probabilities of the Markov chain. The transition matrix is constructed, in which each element $t_{ij}$ represents the probability of transition from network $i$ to network $j$. At the initial step all values in the matrix are equal to 0. Then, for each criterion $q$ a ranking list should be obtained:

$$\tau_q = [i_1 \geq i_2 \geq \ldots \geq i_M], \tag{2.5}$$

where " $\geq$ " denotes an appropriate ordering relation, depending on the type of parameter (note that this is general enough to include all three types of parameters) and $\tau_q(i)$ denotes the ranking of network $i$ from the point of view of factor $q$.

Then, for each $t_{ij}$ it is necessary to make an update following the rule:

$$t_{ij} = t_{ij} + \frac{w_q}{\tau_q(i)}, \quad \text{if } \tau_q(i) \geq \tau_q(j). \tag{2.6}$$

Finally, the authors compute the stationary probabilities:

$$\pi_j = \sum_{i=0}^{M} \pi_i t_{ij}, \quad \sum_{j=0}^{M} \pi_j = 1, \tag{2.7}$$

and the ranking list is a list of networks with decreasing probabilities.

#### 2.1.1.7 Matching degrees and Weighted Bipartite Graph Algorithm

In [22] the authors assume that there is a special entity gathering network parameters at some predefined periods of time. The system obtains the users requirements in the beginning of each connection. At the first step, the authors use two exponential smoothing, which allows them to predict network parameters between gathering periods (the authors consider that network parameters could significantly change from the moment the network status is collected to the moment when a handover is performed). Then, the authors calculate the Matching Degree (MD) between each user requirements and each network predicted parameters, taking into account weights obtained from Analytic Hierarchy Process. If we denote the user requirements by the vector $X = \{x_1, \dots, x_N\}$ and the network parameters by the vector $Y = \{y_1, \dots, y_N\}$, then the similarity $s_j$ for each parameter $j \in N$ is defined as follows:

$$s_j = \begin{cases} \frac{x_j - |x_j - y_j|}{x_j}, & 0 \le y_j \le 2x_j, \\ 0, & y_j > 2x_j. \end{cases}$$

And finally, the MD between the current user and the network is simply a weighted sum:

$$MD = \sum_{j=1}^{N} w_j s_j. \tag{2.8}$$

Somehow, the MD calculation is close to the idea of the TOPSIS algorithm, where the distance from the best solution is computed. But in [22] the authors complement the MADM. They also propose an algorithm (Weighted Bipartite Graph Algorithm), which maximizes the sum of users MDs and acceptance rate, with respect of a constraint condition.

#### 2.1.1.8 Spearman footrule based algorithm

The last approach we consider in this section was proposed in [23]. The main idea is to construct a ranking that would be the closest possible to all ranking lists based on each decision criterion separately. The authors assume that for each criterion $j \in N$ it is possible to make an ordering by its appropriately normalized value. The Spearman footrule defines a distance between two ranks in the following way:

$$D(\tau_x, \tau_y) = \frac{2}{M(M-1)} \sum_{i=1}^{M} (\tau_x(i) - \tau_y(i)), \tag{2.9}$$

where $\tau_x(i)$ denotes the position of network $i$ in the ranking made by parameter $x$.

Further, the optimal ranking is computed:

$$\tau^* = \arg\min_{\tau_x} \sum_{j=1}^{N} D(\tau_x, \tau_j). \tag{2.10}$$

Analogically, the same approach could be used when decision criteria have different weights, which will contribute to the computation of the optimal ranking. The authors also propose an algorithm for optimal ranking computation.

### 2.1.2 Other network selection and handoff algorithms

In this section we consider several network selection algorithms which take into account the dynamics of the system, contrary to MADMs, which simply rank all the available networks by their suitability and do not consider the previous history of network parameters and do not predict their future values.

One of the most popular ways to model the dynamics of users decision involves Markov Process applications. The common idea is to consider states of the system, each of them representing a decision epoch. In each decision epoch a system/provider decides how to treat a newly arriving user: whether he has to be rejected or connected to a concrete network based on the load distribution. The optimal policy then is the set of actions in each state.

This approach can be found in [24]: in the model time is splitted into decision epochs, and the connection time of each user assumed to be geometrically distributed. Figure 2.2 illustrates this process, where by $S_t, 0 \leq t \leq T$ we denote the random variable which contains the state of the process at time epoch $t$ (it includes the current network used by a user and all networks parameters taken in consideration) and $A_t$ is the action chosen by the user at time $t$.

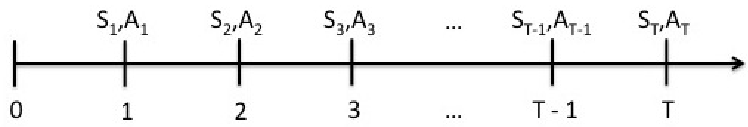


FIGURE 2.2: MDP time scale

The user reward function is the difference between the link reward function $f(S_t, A_t)$, which reflects the QoS provided by the chosen network during time interval $(t, t+1)$ and the signaling cost function $g(S_t, A_t)$, which takes non-zero values only in the case when the network is different from the one chosen in the previous time epoch:

$$r(S_t, A_t) = f(S_t, A_t) - g(S_t, A_t).$$

As the link reward function $f(\cdot)$ the authors consider a weighted sum of the utility functions of different network parameters (their number can vary). The idea is to find optimal decision rules for each state. By decision rules the authors understand a procedure for action selection in each state at a specified decision epoch $\delta_t : S \to A$, where $S$ is the state space and $A$ is the action space. A set of decision rules for each time epoch gives a policy $\pi = (\delta_1, \ldots, \delta_T)$.

The authors try to find a policy that maximizes the expected total reward for the time a user spends in the system, and propose an algorithm to compute such an optimal stationary policy.

Quite similar ideas appear in [25]. The authors consider two access points, implementing different access technologies (namely WLAN and HSDPA) and having the same coverage area. This area is divided into several rings, and for each ring the users inside it have the same achievable throughput and moreover, the users arrival process is the same in all rings: they are Poisson arrivals.

Service rates are assumed to differ among access points. For WLAN the authors assume users to receive an equal service rate, while for HSDPA an equal transmission time interval is supposed. The user satisfaction is a function of the throughput and the global reward. The authors aim to maximize the difference between the total users satisfaction and the penalty for user rejection and propose an iteration algorithm in order to determine the optimal policies.

In [26] the authors consider a system in which WLAN, WiMAX and cellular networks form a heterogeneous environment, and providers jointly decide who will serve the next arriving users. The original idea is that the user allocation has two steps: the offline stage, where the system computes the optimal policies for each possible state (similar to [24]) and writes them to a public table, and the online stage, when providers for each newly arriving user check this table to decide whether they should serve this user or not.

The restless bandit approach [26] provides an indexable rule - for each network in a particular state it attaches an appropriate index. The network with the lowest index will serve an arrived user. The authors show how to compute these indices and check efficiency of the restless bandit system approach through simulations. In comparison with "existing scheme", which is not described well in the article, this approach showed significant gain in terms of expected reward.

Unfortunately, the authors were not clear enough regarding the session types they consider - it looks that depending on the type of wireless network, the users of the same session type have different QoS requirements. This differentiation is strange and needs some clarification.

### 2.1.3 Summary

In this section we described the most popular approaches from the decision theory and a few method involving Markov Process modeling. MADMs have an obvious disadvantage that they make a decision, taking into account only the current system parameters and do not include into consideration previous history of each network. Therefore MADMs also do not consider an outcome of all population behavior, they just prescribe how to produce a network selection decision. The approaches based on Markov Processes also do not consider this issue, they rather prescribe what a user should do in the case when he knows his current state and probability distribution of the next states to occur. However in real-world systems the current state information is quite hard to get. This is why in Chapter 3 we study a rating-based system, where users leave their feedbacks about the QoS they experience, and a third party entity forms a special rating of networks, which also includes previous history of each particular network QoS.

## 2.2 Game Theory basics

The next sections describe works devoted to the investigation of competition between mobile users looking for Internet connection, and Internet access providers aiming to directly or indirectly maximize their profit. The natural way of studying competition between selfish entities is to apply game theoretical models, in which players are competing users or providers, and the satisfaction level from their decisions is described by a payoff function.

Game theory allows us to predict the consequences of users behavior, assuming that all players are rational. That means that a player knows all alternatives available to him and has clear preferences. In what follows we consider different game models, which have various competition rules. But the basic game structure is almost the same for all undermentioned models:

**Definition 2.1.** A strategic game (see [27]) is a tuple $< N, M, P >$, where $N$ is a finite set of players, $M = M_1 \times M_2 \times \ldots M_n$ is the strategy space, with $M_i$ denoting the set of actions (strategies) available to user $i$, and $P = (P_1, P_2, \ldots, P_n)$ is the vector of players payoff functions, which represent their preference relations.

Besides strategic games we will also consider some implementations of extensive games, where players do not choose actions simultaneously, but do it in a sequence. This group of games contains quite complex models and we will focus only on leader-follower games, where one player chooses his action before his opponent.

### 2.2.1 Steady state

The main concept we will focus in the following models is the steady state of a game, i.e., a situation when no player could increase his payoff by unilaterally changing his strategy. In the case of a simple simultaneous game this steady state is called Nash equilibrium:

**Definition 2.2.** A Nash equilibrium of a strategic game is a strategy profile $m^* \in M$ such that for every player $i \in N$:

$$P_i(m^*_{-i}, m^*_i) \geq P_i(m^*_{-i}, m_i) \text{ for all } m_i \in M_i,$$

where $m_i^*$ stands for a vector of strategies of all players except player $i$.

Sometimes players can observe some random variable and taking into account that the other player also observes its values, play different strategies. In this case the steady state situation is called Correlated Equilibrium (CE):

**Definition 2.3.** Correlated equilibrium [27] of a strategic game $< N, M, P >$ consists of:

– a finite probability space $(\Omega, \pi)$, where $\Omega$ is a set of states and $\pi$ is a probability measure on $\Omega$

– a information partition $\mathcal{H}_i$ of $\Omega$ for each player $i \in N$

– a function $\sigma_i : \Omega \to M_i$, with $\sigma_i(\omega) = \sigma_i(\omega')$ whenever $\omega \in H_i$ and $\omega' \in H_i$ for $H_i \in \mathcal{H}_i$. This function $\sigma_i$ is the strategy of user $i$

such that for every $i \in N$ and any other strategy $\sigma_i'$ we have:

$$\sum_{\omega \in \Omega} \pi(\omega) P_i(\sigma_{-i}, \sigma_i) \geq \sum_{\omega \in \Omega} \pi(\omega) P_i(\sigma_{-i}, \sigma_i')$$

When the number of users is relatively large and the individual impact of users is negligible, we call them *non-atomic* players. In the case of routing games (and congestion games), where users want to send their flows to some destination, by choosing paths with minimum total cost, we have a special equilibrium, also known as Wardrop equilibrium:

**Definition 2.4.** At a Wardrop equilibrium, the cost of every used route is less or equal to the cost of any unused route.

### 2.2.2 Equilibria evaluation

Often, an equilibrium situation of a game is inefficient: probably the players will gain more in total if they decided to cooperate, or their selfish behavior leads to negative externalities, which may be fixed by giving appropriate incentives. Moreover, it may happen that a game possesses several equilibria. Thus, sometimes we need some way to evaluate/characterize an equilibrium of a game.

For comparison purposes we define Pareto efficiency:

**Definition 2.5.** The strategy profile $m^*$ is Pareto efficient if there is no strategy profile such that at least one player has strictly higher payoff, while no other player has strictly lower payoff:

$$\nexists\, m \in M, j \in N : \forall\, i \neq j\ P_i(m) \geq P_i(m^*) \text{ and } P_j(m) > P_j(m^*)$$

Note that not every Nash equilibrium is Pareto efficient.

If we have some efficiency measuring function of strategy profiles, which we want to maximize, then we may evaluate possible selfish an equilibria inefficiency using the so-called Price of Anarchy.

**Definition 2.6.** If we denote by $M^{Nash}$ the set of Nash equilibria, then Price of Anarchy regarding some efficiency measuring function $F$ (the bigger value the better) is

$$PoA = \frac{\max_{m \in M} F(m)}{\min_{m \in M^{Nash}} F(m)}.$$

A high Price of Anarchy means that selfish equilibria are inefficient, while close to 1 value implies that selfish equilibria are close to optimal situation. Sometimes the authors use the so-called Price of Stability:

**Definition 2.7.** If we denote by $M^{Nash}$ the set of Nash equilibria, then the Price of Stability regarding some efficiency measuring function $F$ is

$$PoS = \frac{\max_{m \in M} F(m)}{\max_{m \in M^{Nash}} F(m)}.$$

## 2.3 Users competition analysis and resource management

In this section we discuss different works focused on the possible outcomes of mobile users making selfish decisions. Some authors claim that the selfish equilibrium of a game is a situation when everyone is satisfied and the only question is how to reach this situation avoiding the long period of convergence. That means that the final steady state is not so easy to reach, and some distributed network selection algorithms may be needed in order to decrease the number of handovers of users between several networks.

Other authors try to analyze an equilibrium of a game, using a global metric (e.g., total users utility). In most cases selfish behavior leads to a situation different from the optimal one (with high PoA) and in this situation an intervention from the provider is needed. Knowing the current inefficient situation and the desirable outcome, providers may introduce some penalties or incentives in order to fix the inefficiency.

### 2.3.1 Congestion games

In a subset of strategic games called congestion games [28] there is a set of alternatives, which are congestable. It means that the utility of a user when he chooses an alternative depends only on this alternative characteristics, and on the number of other users who made the same selection.

Formally, a congestion game is a tuple $< N, M, C >$, where $N$ is the set of users, $M$ is the set of alternatives (networks in our case) and $C$ is the vector of cost functions, such that $c_j = c_j(n_j)$, where $n_j$ is the number of users who selected alternative $j$. If players are able to select some subset of alternatives, then the cost function is just a sum of costs from all selected alternatives. This class of game is especially interesting because in [28] the author showed that all finite congestion games have a pure strategy Nash equilibrium.

It is quite natural to apply the congestion games models to users competition analysis: usually users compete for scarce radio resources, and the higher load on the access point implies the lower QoS (and thus the lower game payoff). E.g, reference [29] considers an interference-based network selection game - which is an instance of the congestion game - where a user selects an access point, preferring the one which operates on a frequency with the smallest number of interferers. The authors propose a way of calculating the Nash equilibrium as a solution of the mathematical programming problem; as a quality measure of users allocation, the average number of interferers is considered.

In [30] the same authors try to find PoA and PoS bounds for three different user cost functions, depending on the throughput (which depends on the distance between a user and an AP) and the congestion on an AP. Despite that theoretical inefficiency bounds appeared to be quite high, simulations show that in realistic scenarios the inefficiency due to selfish users behavior is negligible and thus no intervention from the provider is needed. This work was extended in [31], where a more detailed comparison through simulations is provided for the same cost function types.

Throughput-based utility function is also considered in [32]. The authors study the user allocation problem focusing on the comparison between two multiple access protocols: namely TDMA and HSDPA. In the game considered, the users are competing for accessing different base stations, trying to maximize a difference between the obtained throughput and the power cost. With a slight modification of the model, the authors manage to transform it into a congestion game, for which they use a simple algorithm for finding the Nash equilibrium. Analogically in [33] the users are assumed to be sensitive to the throughput of a connection and the price they have to pay. There the authors emphasize that NE is quite difficult to reach in a distributed manner, thus they focus on a more general case of equilibrium - Correlated Equilibrium.

### 2.3.2 Routing games

A wide range of works is devoted to a special case of congestion games - called routing games - in which a network (graph) is considered. In general, there is a graph (directed or undirected) $G =< V, E >$, where $V$ is the set of vertices and $E$ denotes the set of links. Users are willing to route their flows from the source to the destination, with the aim to minimize the experienced delay or latency. There can be several commodities, meaning several source-destination pairs $\{s_w, t_w\} \in W$. Sometimes commodities differ not only by their source and destination, but also by some other specific parameters, e.g. the sensitivity to a possible monetary cost imposed by a provider. An important theoretical study for multicommodity setting is shown in [34], [35]: they prove the existence of taxes (monetary costs the users have to pay for routing their flows on each link), such that a selfish users equilibrium will be optimal from the point of view of the total latency minimization. Other important results about PoA bounds and optimal equilibria in multi commodity routing games can be found in [36] and [37].

A routing problem with multiple commodities $k \in W$ (origin-destination pairs) is investigated in [38], where the users are sensitive to the latency they experience, which is additive. Two types of traffic are considered which have different utility functions: inelastic traffic has a step function utility, while for elastic traffic the utility function is nondecreasing and concave. Only one provider is considered, who aims to maximize his profit. As the social optimum the authors consider the total users utility without taking into account prices payed by them. For inelastic traffic the authors proved that for the monopoly price, which the provider chooses in order to maximize his revenue, there exists a Wardrop equilibrium (which is not true in general) and moreover, this equilibrium is socially optimal. Thus, there is no need for any optimization of traffic - due to the inelastic nature of traffic, meaning that a user will refuse to maintain connection when the rate is below some threshold, the provider sets a price such that all users distribute in an optimal way. For elastic traffic the authors prove the existence of Wardrop equilibrium and find that it can be inefficient.

In this thesis we also apply the routing games model to the problem of network selection. In the model proposed in Chapter 4 we study the selfish users allocation between two access points. We assume that these access points belong to the same entity, which is interested in social welfare optimization (minimization of total experienced latency). Due to theoretical results from [34], [35] we know that the optimal taxes exist, and additionally to results of [38] we aim to determine their closed-from expression.

### 2.3.3 Bayesian games and auctions

In real word systems the preferences of mobile users may vary. For example, if the current active application of a user is VoIP, then the user may prefer a network with lower delay/jitter, while when he lunches video streaming content he may prefer the one with better throughput. When we model strategic behavior of users in such scenarios, we have to take into account that every individual is aware only of his own preferences, but can hardly guess those of others. For this situation it is convenient to apply Bayesian games.

In the simplest variant of Bayesian game each player has a *type* - a variable determining his preferences, which influences his utility function. The set of types of all players is called a state of the game, and each player has a priory beliefs about the real state as a probability measure on the set of all possible states. All players then try to maximize their expected payoffs.

One example of Bayesian game application can be found in [39]. The article studies a model where users are sensitive to the bandwidth they are allocated (bandwidth is shared equally between users), and every user has his own bandwidth requirements, and these requirements are private information they do not share with each other. The authors consider a Bayesian network selection game, where each user $i$ has bandwidth requirement $b_i$ (which is the user's type) and the utility of connection to network $j$ is:

$$u_i^j = \begin{cases} U(\tau_i^j) - P_j, & \text{if } \tau_i^j \geq b_i, \\ -P_j, & \text{otherwise,} \end{cases}$$

where $\tau_i^j$ is the allocated bandwidth from network $j$, $P_j$ is the price charged for network $j$, and $U(\tau_i^j)$ is the utility from the allocated bandwidth. The strategy of a user here is a mapping from the type (requirement) space to the action space, and the action here is the probability distribution over available networks.

One quite obvious application of Bayesian games is auctions. Indeed, for auctions usually users are aware of their own preferences for some good (e.g., the maximum cost they want to pay for it), but can only guess about how the same good is valued by other competitors. This type of model is investigated in [40]: there is only one wireless network and a number of users are competing for the bandwidth. The wireless network provider organizes an auction, where each user $i$ bids a time interval $t_i$ (his type) he would like to stay connected and the price $p_i$ per unit of bandwidth per unit of time, which he would like to pay (which should be bigger than the minimum price threshold fixed by the provider). Further, each user obtains a bandwidth value proportional to his bid.

For our study we preferred to apply routing games model rather than Bayesian games, since the former have all necessary theoretical bases about the optimal taxes existence. The fact that all users types are deterministic doesn't harm our model: in Chapter 4 we assume that there are several classes of users (having some individual parameters) and we consider that they just simply connect to the network with the lowerst cost. So each user can try to connect to all available networks in order to check the QoS (or it can be a passive probing) and make a choice based on this information, without the need to know the parameters distribution among all users.

### 2.3.4 Population games

In population (evolutionary) games there is a set of $Q$ classes of non-atomic players. Each class has its own strategy set $M_q$ and mass or size $d_q$. The way each class is distributed among its available strategies is called strategy distribution vector $y_q = \{y_q^1, y_q^2, \ldots, y_q^{M_q}\}$, with the condition $\sum_{i=1}^{M_q} y_q^i = d_q$.

Reference [41] proposes an example of population games application to the case of multi-homing (each user being able to split his demand between several APs) network selection. The authors consider two transport layer models: UDP and TCP; they influence the throughput of an individual user. The population is divided in classes, each class is a group of people sharing the same parameters (like physical layer rate, frame size and available APs). In the game considered, each user is willing to maximize his utility function, which is the difference between the achieved throughput and the cost imposed by the network provider. This cost consists of two parts: the first one is called "cost-price" - it is the cost of externalities caused by all users belonging to the same class, and the second part is the usual price, charged by the provider.

Other population games applications can be found in [42] and [43]. In [42], populations are users situated within the same area, or differently speaking, having access to the same set of access points. The authors apply replicator dynamics in order to achieve an evolutionary equilibrium (which is the fixed point for the replicator dynamics, for details see [42]). Further they also study the Nash equilibrium of the game, considering whole classes of users as players. In [43] the population is the ratio of users, choosing concrete network and the authors propose an algorithm to reach an equilibrium situation.

Actually, the model we consider in Chapter 4 is an instance of a population game as well: we have several non-atomic classes, and the users in the same class can choose different alternatives.

### 2.3.5 Other strategic games

In this subsection we describe some other game theoretical approaches, which we find to be useful to mention since they present different points of view on the user allocation problem.

A basic game-theoretical network selection scheme appears in [44]. The idea is that users report their request type to the system and further, the coexisting networks compete for users requests. The authors assume that each user gives different preference numbers to all networks, depending on they parameters. The provider payoff is preference number of users he serves. The game is played in rounds: at each round, every provider selects a request to serve; if the request is chosen to be served by one network, the others can not choose it. This model has a number of drawbacks and is quite naive, but is interesting as one of the first steps of game theory applications to the network selection problem. This model has an extension, proposed in [45]. There the authors use Analytical Hierarchy Process and Grey Relational Analysis to quantify the suitability of a network for a particular request.

In [46], users association is made taking into account both the utility of a user and the congestion of a network. The decision of user association is not distributed since users communicate with all available networks in order to know their current loads. The authors consider three different types of access points: WCDMA, IEEE 802.16 WMAN and IEEE 802.11 WLAN.

An example of the distributed network selection algorithms is shown in [47]. Several Internet access points coexist in the same area, and each access point's coverage is separated into zones of identical throughput. The users are sensitive to the throughput they receive, which depends on the number of users connected to the access point. The authors propose a Nash Learning algorithm, which converges to a Nash equilibrium. However they state that the selfish behavior of users may lead to a inefficient situation, and thus some special rewards should be introduced. They consider an algorithm, in which users are not competing for throughput, but for a reward. For this purpose they apply marginal cost pricing [48], which assigns a fee for a user to balance the loss of throughput caused by his choice.

In [49] the authors consider a game between users with intra-cell optimization and find that the equilibrium reached is the optimal one from the point of view of total users utility. A multi-cell network with several base stations is considered, where the users are non-atomic. The users are divided in a number of classes, having the same rate vector for users inside a class. Two scheduling policies are considered as in [25]: equal time and equal throughput allocations. All users are aiming to maximize their throughputs, which depend on the policy and the class they belong to. In the case of equal time allocation, the authors find that the Nash equilibrium is unique and that it also maximizes the total utility of users.

A distinguishing work is presented in [6], where the authors compared different probing schemes. They modeled the access point selection as a dynamic load-balancing game with slotted time, where $n$ players (each one having work of size $w$ units in each period of time) are selecting among $m$ access points. The users are sensitive to the delay they experience being connected to network $a$ at time period $t$:

$$d_{a,t} = (s_t + u_t)w + pq_t,$$

where $s_t$ and $u_t$ are the jobs assigned at period $t$ and assigned on the previous periods but not yet processed (due to maximum jobs per period constraints) respectively. If a user makes $q_t$ probes of different networks, then he has an additional cost $pq_t$, where $p$ is the size of a probe.

A case of multiple access points selection is also considered, when each user has $j$ jobs to process. Denoting by $\pi_{i,t}$ user $i$ jobs assignment at time $t$ (and $\pi_{-i,t}$ is all other users assignments) and by $A_{i,t}$ the set of networks that receive jobs from user $i$ at time $t$, we could express the total delay over all periods $t \in T$, which user $i$ wants to minimize as follows:

$$D_i = \sum_{t \in T} \left[ (|A_{i,t}| - 1)\delta + \arg \max_{a \in A_{i.t}} d_a(\pi_{i,t}, \pi_{-i,t}) \right],$$

where $\delta$ is a cost of managing connection with several networks. The user experiences the maximum delay over all selected access points. Two information models are compared: the bulletin board model, in which users are informed of the delay of each network at the end of each time slot; and the probing model, where users have an information only about probed networks or those to which they were connected at the previous time slot. Several probing policies are considered :

1. **Naive probing policies.** These are two policies: one prescribes not to probe any network and the other one says to probe all of them
2. **Freshness-based policy.** A network whose information is less up to date is probed

3. **Variance-based policy.** The users are probing at each time slot a network with the biggest variance of probing information.

as well as six association policies (two of each type, corresponding to the cases of multiple and single network selection scenarios):

1. **Random policy.** A network to associate with is choosen randomly
2. **Hedge Algorithm.** A no-regret learning algorithm is applied for network selection
3. **Expected delay minimization.** The users are selecting a network with minimum expected delay.

Strictly speaking, the authors do not investigate competition among users, but compare different "association policy - probing policy" pairs with the help of game theory. The authors produce simulations and compute users payoffs for each policy pair and then find mixed strategy equilibria as well as point out dominated strategies. Based on these results the authors find that the users should either not probe at all, or probe the least up to date network, and that the preferable association policy is to minimize the expected delay. This work presents a new interesting research direction of the network selection problem, studying the way users may obtain the information about networks states.

In [50] only one access point is considered; users are sensitive to their signal-to-interference-ratio (SIR), and their strategies are the power levels on which their mobile devices operate. The authors proved that the simultaneous game between users has a Nash equilibrium, which is unique. Further they show that this Nash equilibrium is not Pareto efficient. Then they consider a modified game, where users are sensitive to price, which is proportional to their power levels and some common pricing coefficient $c$. Finally, the authors proposed an algorithm, reaching Nash equilibrium, and numerically found a value of $c$ such that no other $c$ can increase the revenue for *all* users. In [51] the authors consider a multi access points version of the same model.

The other type of users competition is considered in [52], where users are free to choose a connection rate for their VoIP applications. In the proposed model only one access point is considered, but still the work is interesting, due to experiment results, which show that even if users are able to freely choose their connection rates, this does not lead to a congestion. Moreover, the authors find that the Nash equilibrium between users is close to the optimum situation. However, the article shows results only with a small number of users, and it is difficult to predict how the situation will change when the population is relatively large.

In some papers, the users are assumed to perform additional actions in order to connect to a desirable network. In [53] this kind of scenario is considered: users receive information about the current load and the geographical location of all available access networks, and then decide which one to connect to, taking into account the distance they need to travel for it. More formally, the cost user $i$ will experience to connect to network $j$ is defined as follows:

$$c_{i,j} = \alpha \cdot n_j + D_{i,j},$$

where $n_j$ is the load on network $j$, $D_{i,j}$ is the cumulated distance user has to travel in order to have an access to network $j$ (it takes into account the distance already passed) and $\alpha$ is a weighting parameter, which is assumed to be non negative.

The authors assume that the users make their decision in a sequence and have perfect information about the previous actions of all others (which is a quite limiting assumption, taking into account that users are unable to make any action during other users movement). Under these strong assumptions, the game considered appeared to have the Nash equilibrium. Moreover, the authors proposed an intuitive myopic algorithm, which is proved to lead to the Nash equilibrium.

### 2.3.6 Summary

In this section we described various game theoretical approaches and their application in user allocation studies. Each model application implies its own assumption, but in most cases the authors found selfish user behavior lead to suboptimal outcomes. Some authors claim that this inefficiency is small, and thus it is not worth trying to incentivize users to change their decisions. However, in Chapter 4 we describe how the inefficiency of users behavior can be interpreted into potential revenue losses. Thus even in the case when the inefficiency seems to be small, provider still can be interested in fixing it, since the revenue losses are incrementing. One of the most natural ways to influence customer's behavior is through monetary penalties or rewards. This is why Chapter 4 focuses on taxation policies.

## 2.4 Competition among providers

Provider competition may crucially impact the welfare of their customers. Often, the providers competition arises in situations when the APs they own have overlapping coverage areas. However it is not always the case. For example, in vehicular networks, where users are highly mobile, competition arises even without overlapping coverage areas: mobile users move by a road, and meet APs in sequence. Thus, users unserved by the first provider met may be served by the next one.

We consider two main types of approaches focusing on provider competition. The first group studies provider interactions as a game with some specially introduced payoff function, which is quite similar to what we surveyed in the previous section. We denote this type of competition as one-level game. The second group of works takes the result of underlying users competition and further considers it as a prediction of providers revenues. Then, these revenues are used as the payoff function in the providers game. This second type of games is called hierarchical games and these games are especially interesting because they allow to implicitly observe how providers competition influences user behavior and vice versa.

### 2.4.1 One level games

In [7] the so-called bankruptcy game is considered in order to model the bandwidth allocation and the admission control problems. A user coming into the system with several heterogeneous access points requests some amount of bandwidth and the access points operators want to provide as much bandwidth units as possible. In the decorations of bankruptcy game, the user is considered as a bankrupt entity, requested bandwidth as the money he has to return to the creditors (access points owners). The operators may form coalitions, and when they do so, their payoffs increase. The optimal bandwidth share is found due to Shapley value ([54]) and simulations show that it can decrease the connection blocking probability. This work shows an interesting interpretation of the user allocation problem, which, however, seems to us unrealistic.

The other one-level providers game is proposed in [55]. The authors introduce the bandwidth demand function which says how many bandwidth units the users want to buy, given the providers prices. The users are naturally divided into two classes:

the premium users which have access only to one big provider, and best-effort users which may choose between two providers. Then, the authors consider two types of competition: simultaneous and leader-follower games, and in both cases the providers want to maximize their revenues by playing with prices for bandwidth unit.

### 2.4.2 Hierarchical games

In [56] a hierarchical game is considered, where on the first level mobile users are selecting among two available base stations, preferring the one which provides the highest SINR. Contrary to most works, the authors focus on the uplink transmission, and thus the entities which are producing interference are the mobile users by themselves (not the base stations). The users are distributed with uniform density on a segment of specified length $[-L, L]$ and are able to connect to both base stations, without any distance restrictions.

Two scenarios are considered: in the first one, the base stations are assumed to operate on the same frequency and thus all users are interfering with each other, and in the second scenario, the base stations operate on different frequencies, and in this case a user experiences interference only from users belonging to the same network as he does.

In the one-frequency case, the authors find an interesting feature: it appears that the sets of users choosing the same base station (cells in the terminology of the authors) could be non-convex. This happens when one base station (assume it situated on the right side of the segment, close to $L$) is located at a large distance, and thus interference on it is not as big as on the base station which is in the middle of users segment. Thus, users which are on the left edge of the segment, though being far away from the distant base station would prefer it due to its low interference level. In this case, the set of users choosing this distanced base station would be a union of two subsegments of original user segment.

On the second level of the hierarchical game the authors investigate how base station owners should locate their equipment in order to maximize the throughput of users, associated with their base station. Both the cooperative and non-cooperative types of games are considered. The authors managed to find equilibrium points due to analytical and numerical studies, considering the one-frequency case as well as the two-frequency case. It appears that in the non-cooperative game the players tend to place the base station closer than in the cooperative case, thus leading to a less efficient situation from the point of view of users throughput (which depends on SINR).

The other hierarchical game can be found in [43]. On the first level an evolutionary game is considered, for which the authors propose new dynamics and find stationary points. These stationary points are viewed as the outcomes of users competition and are helpful for the providers revenue prediction in the pricing competition game. In this hierarchical game one provider is considered to be a neutral provider, meaning that his aim is not to maximize his revenue, but to regulate the market. The authors show how this regulator can influence the market, increasing the average utility of users and the total revenue gained by providers.

In [29] the authors consider competition between providers on top of an interference-based network selection game. This underlying game is the congestion game, where users try to minimize the interference they experience, which depends on how many users transmit on the same frequency. Thus, for providers it is crucial to choose an appropriate operating frequency. The authors prove that the Nash equilibrium in the providers frequency game is Pareto-Optimal, and deduce that the proposed system has a nice feature to be self-regulated, meaning that even if all participants behave selfishly, it does not imply that they harm each other.

### 2.4.3 Summary

In the thesis we decided to study two different models: where the users are static and where there are highly mobile. For the first case, the model we consider is quite close to the approaches described in this section: two access points have an overlapping coverage area, and the users located in the intersection are able to choose which access point they want to connect to. Later, the outcome of their decisions is used in order to predict the providers' revenues in the situation of competition. Thus, the first model we study in Chapter 5 is the hierarchical game. For the second case we consider a novel model of Internet access providers competition in vehicular networks. All users are highly mobile there, thus we can not use the approaches presented before: the users make a binary decision for each access point they see. They decide whether they want to pay the price charged by the provider, and if not, they continue moving by the road to the next Internet access provider.

## 2.5 Challenges and open questions

Given a large number of opportunities and the variety of available network access technologies, a user searching for a network connection has to deal with a quite challenging problem. In order to select the most suitable network, he has to understand which networks parameters are important for him, and moreover, what priority or weight he should assign to each of these parameters. There are a number of sophisticated algorithms, which define how a user should prioritize the available networks, taking into account the current application needs, the location, the level of the mobile device battery charge, etc. In general, these algorithms provide a list of available networks, sorted by their suitability; the user's mobile device automatically selects the best one. These systems are clearly user-centric, and here comes their disadvantage: trying to satisfy every individual user in a distributed maker, they do not take into account how the proposed selections could influence the system as a whole. It could appear that some network access point would be overused, yet being the most suitable due to some reasons, and that a negligible offloading of this network would drastically enhance the performance for a large number of users.

For this reason, when we consider the users allocation problem, in order to make a proper investigation we have to consider a heterogeneous network as a whole system. It is necessary to avoid unbalanced resource utilization, that may result from the user-centric approach. Obviously, users are not able to coordinate by themselves, that is why the majority of works consider network providers as the entities responsible for the regulation. The objective of the regulator could vary: a provider could aim to maximize users' QoS (or some function of it), to optimize the energy consumption, or simply to increase its own profit, etc. As users are behaving selfishly, trying to ensure good performance for themselves and thus competing with each other for scarce resources, it is natural to apply Game Theory tools to study such situations. In general, users are sensitive to a number of parameters, some of which are congestion-dependent and some of which (such as prices) can be regulated or influenced by providers.

However providers by themselves are not so free in setting prices. When a provider constitutes a monopoly in some area, he can manage his resources more easier than in a scenario when he has several competitors. In the latter case providers may compete for users, and this could lead to prices decrease as well as to QoS degradation (due to a higher number of users attracted), which constitutes an important part of the users allocation problem.

In what follows we try to enumerate the most important research challenges in the users allocation problem:

1. **Modeling complex networking phenomena**

   In order to investigate all the consequences of selfish users behavior, we have to know how individual decisions influence different network parameters. It is hard to include all physical phenomena in an analytical model; it makes the model more realistic and easier to apply, though increases computational complexity and in most of the cases makes the analytical study impossible.

2. **Solving optimization problems in real time**

   Given that in practical scenarios the time for making decisions is very small, it is necessary to make the trade-off between computation speed and distance to the optimal solution. It may appear that in some situations, due to computational limitations of devices it is not worth performing elaborate optimization algorithms.

3. **Dealing with the lack of information available to users**

   In current systems, users have quite poor information about access points, like signal level, name, technology and price. In the nearest future, social services may offer users the possibility to leave their feedback in manually through scores or automatically with connection statistics. In the first case, with all networks there would be associated a rank, based on which users will make network selection decisions. In the second case, an algorithm can use previous data for the prediction of the possible QoS level at AP. When one of these systems is deployed, providers would be interested not only in short term revenue (e.g. in the situation when a user connects due to low price and the QoS is low as well), but also in maintaining the acceptable QoS level, which strongly affects their future gain. Finally, if a user plans to stay connected for a long period of time, he may produce a probing of available networks (this approach is studied in [6]).

4. **Dealing with the lack of information available to network providers**

   Typically, a provider is not aware which application his customer is using (he could try to deduce it by incurred load). Some authors propose systems where users implicitly claim the type of application they want to use, in order to produce appropriate resource management. But some users may want to launch several application simultaneously, or in sequence, which means that they have to send session information several times. Moreover, it means additional load on the uplink channel. Internet access providers may use some users statistics, but this in turn implies some errors and may be not optimal.

5. **Influencing users behavior without harming to the quality of connection, provider's revenue, and users welfare**

   Most commonly, as an influencing means authors consider price or tax charged (per packet, per time unit, etc.) on different networks. Obviously, playing with prices will give us the necessary effect - users would prefer the cheapest network, but one has to remember that playing with price could have negative externalities, such as reluctance of users to stay in a system with dynamically changing prices and revenue degradation. Moreover, providers calculate their prices based on a large number of parameters, and prices change involves a complex risk-aware computations. In this case more accurate economical models are needed.

6. **Considering other scenarios of providers competition**

   Most works dealing with providers competition consider scenarios when coverage areas of access points have an intersection, and thus users have a choice between several networks. However, in some systems this situation is very rare but yet competition between providers exists. This is the case for vehicular networks, where APs may not intersect, but due to highly mobile users, there is always competition for them between providers.

In this thesis we partially tackle these challenges. E.g, for our providers interaction study we take into consideration the interference the users may experience from a closely located access point, which allows us to study one more interesting leverage in pricing competition. We cope with the lack of information available to users in Chapter 3, where we propose a rating based system, where each user leaves a feedback about the QoS he experienced, which is further processed by a third part entity, resulting in a network rating. We also try to model the insufficient or not-up-to-date information available to providers in Chapter 4, when the computation of taxes (imposed in order to lead the system to the optimal situation) is based on some approximated data. In the same Chapter we show that the optimal taxes can be adjusted, in order to ensure a minimum level of revenue, thus allowing the provider to keep his revenue level unchanged after optimization. Finally, Chapter 5 contains a different from the "classical" scenario of providers competition: we study a vehicular network, where users are highly mobile and thus competition arises even if providers' access points have no intersection in coverage areas.

# Chapter 3

# Dynamic adaptation of user decisions through a noncooperative game

In this section we describe a network selection mechanism, where users share their experience about connection quality at different Internet access points. The special entity which we call central controller gathers this users feedback and transforms it into a rating of an access point, which impacts network selection decision of further arriving users. We do not focus on the technical aspects of the proposed mechanism, rather on the dynamics of users behavior in this kind of system. We aim to study a steady state in the proposed system from the point of view of the total users welfare: whether selfish decisions of users are optimal in this setting? The results of this work will be further used in Chapter 5, where we will consider a game between providers taking into account the steady state of users competition.

## 3.1 Model

### 3.1.1 Network topology

In this model we consider a system consisting of two networks. For the sake of simplicity, we assume that both networks have the same coverage area, as illustrated in Figure 3.1.

Note that the model provided above could be easily extended to case, when access points have only partial overlapping. We also consider that all mobile users own technologies allowing them to connect to both networks.

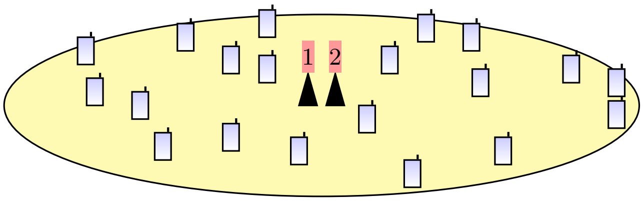


FIGURE 3.1: Network topology considered.

### 3.1.2 User behavior

Users are sensitive to the QoS they experience, and to the price they are charged for the service. While the latter is clearly advertised by the networks, the former is less obvious to determine, since QoS estimations based on probing often involve some nonnegligible amount of uncertainty, due to the rapid changes in radio conditions. To cope with that problem, we consider a controller that computes in real time an *averaged* (over all users) value of the QoS level of each network, and propagates those levels to all users in the system. That average value will be called the *rating* of the considered network: it can be computed based on some feedback of the experienced QoS from all users (hence the averaging), or directly calculated by central controller based on the number of connected users. The details of that aspect are beyond the scope of this paper: we focus here on the dynamics implied by the rating scheme, and will consider that this rating depends on the level of congestion of each network (i.e., the number of connected users).

We consider that time is slotted; at each time slot users that are present in the system make a choice. Recall that the final decision is left to the user herself (instead of an algorithm implemented within the mobile terminal). To describe user behavior, we use the well-known logit model [57], where each user chooses a network based on its quality and price, but also on other individual criteria that we model as random variables (see [57] for details). In the case of two networks, the probability that a user $j$ chooses network $i \in \{1, 2\}$ then equals, at each time slot:

$$p_i^j = \frac{e^{(V_i^j - s^j P_i)}}{e^{(V_1^j - s^j P_1)} + e^{(V_2^j - s^j P_2)}}, \tag{3.1}$$

where $V_i^j$ is the current quality of network $i$ for user $j$, $P_i$ is the price per time slot of network $j$, and $s^j$ is the price sensitivity of user $j$ (that will be assumed to follow a given distribution over the user population).

### 3.1.3 Perceived quality and loyalty effect

At each time slot, the central controller gathers information about the QoS experienced by users, and updates the network ratings. We chose the following update mechanism for the rating $Q_i^t$ at time $t$:

$$Q_i^t = \beta \cdot Q_i^{t-1} + (1-\beta)\bar{Q_i}^{t-1}, \tag{3.2}$$

where $Q_i^{t-1}$ is the rating of network $i$ on period $t-1$, $\bar{Q_i}^{t-1}$ is the (estimated) QoS computed by the central controller at period $t-1$, and $\beta \in (0,1)$ is a memory coefficient, that prevents ratings from changing too fast after a temporary QoS variation. It is easy to see that a bigger $\beta$ reduces the oscillations in $Q_i$, but in the other hand the information about the network congestion state then becomes less representative of the current situation.

The quality value $V_i^j$ in (3.1) can be considered as a simple rating (this quality value being then the same for all users), or alternatively we could consider this value to vary from user to user. More precisely, we will consider in this paper that $V_i^j$ contains a QoS-related term $Q_i$, that is modulated by the network (if any) that the user was attached to in the previous time slot. This way, we are modeling some *loyalty effect*, meaning that a user is reluctant to switch networks once he is connected to one. More precisely, we consider that the quality of network $i$ considered in (3.1) by user $j$ is $V_i^j = Q_i(1+\alpha)$ if user $j$ was with network $i$ during the previous time slot, and $V_i^j = Q_i$ otherwise. The parameter $\alpha > 0$ can be interpreted as the loyalty value (or some cost corresponding to switching networks) of users. It introduces a bias in (3.1), that favors the decision to stay with the same network. Note that in this paper, we assume that all users have the same loyalty value $\alpha$.

The intuition about this system is that it should be self-regulating, i.e., independently of the QoS function used (delay, interference level, available bandwidth, ...), users should end up being distributed over the networks, in accordance with the quality and price levels. In other words, a situation where most users constantly choose the cheapest network is not possible in our system, because the rating of the congested network will degrade significantly, and consequently less users will choose that network in the next round.

### 3.1.4 User arrival and departure processes

We consider that at each time slot, the number of new users entering the game (i.e., willing to benefit from the service) is randomly distributed, following a Poisson distribution with mean value $\lambda$.

Users leave the system after some (randomly distributed) time. We assume that this service duration follows a memoryless distribution, i.e. at each time slot there is a probability $1-q$ that the user ends its service (call) at the end of the slot, independently for each user participating in the system. It is easy to see that the expected number of users in the system then converges to $\frac{\lambda}{1-q}$.

## 3.2 Analytical results for fixed network prices

In this section, we analyze the lower level of the game, that is the one played among users, selecting their network based on prices and quality. We therefore assume in this section that the prices $P_1$ and $P_2$ are fixed and constant. In that context, we derive some analytical results regarding the steady-state situation of the stochastic process defined in Section 5.3.2.

### 3.2.1 Existence of a stationary distribution

We first remark that the discrete-time process $(n_1^t, n_2^t)$, giving the evolution of the number of users connected to each network, is a Markov chain. Indeed, at each time slot the quality of service $Q_i^j$ considered by users for their next decision, and the number of new arrivals, only depend on the current state (and not on the previous ones). Since those values are the only ones determining the distribution of $(n_1^{t+1}, n_2^{t+1})$, the process satisfies the Markov property. It is easy to check that this Markov chain is irreducible and aperiodic: just consider that any transition $(n_1, n_2) \to (0, 0)$ has a non-zero probability, as well as any transition $(0, 0) \to (n_1, n_2)$.

To establish that the Markov chain is ergodic (and thus, admits a stationary distribution), it remains to show that at least one state is positive recurrent. This can be done easily by considering the state $(0, 0)$, which allows us to reason only on the total number of users regardless of their network choice. The total number of users in the system is itself a (discrete-time) Markov process, that is irreducible and aperiodic, and obviously positive recurrent since the number of users converges to the finite value $\lambda/(1 - q)$, as pointed out in Subsection 3.1.4. Therefore all its states are recurrent, including the state with no users that coincides with the state $(0, 0)$ of the process $(n_1^t, n_2^t)$.

Consequently, the process $(n_1^t, n_2^t)$ is an ergodic Markov chain, that therefore admits a stationary distribution: after some time, the probabilities of visiting each state $(n_1, n_2)$ do not change. In particular, we can then claim that the number of users in each network has a mathematical expectation, around which it will oscillate during a process trajectory.

### 3.2.2 Expected number of users in each network

For simplicity reasons, we first consider the case without loyalty effect (i.e., $\alpha = 0$), so that the perceived network ratings are the same for all users, i.e., $V_i^j = Q_i$. We assume the price sensitivities of users to be uniformly distributed on the interval $[a, b]$, for $0 \leq a < b$. Thus, when the quality scores $(Q_1, Q_2)$ of the previous time slot and the number of users $n^t$ in the network are given, the mathematical expectation of the number of users choosing network $i$ at time slot $t$ is, with $\bar{i} := \{1, 2\} \setminus \{i\}$:

$$\begin{aligned}
\mathbb{E}[n_i^t] &= \sum_{j=1}^{n^t} \mathbb{E}_s[\mathbb{E}[\mathbb{1}_{\{\text{user } j \text{ selects network } i\}} | s_j = s]] \\
&= n^t \int_a^b \frac{e^{(Q_i - xP_i)}}{e^{Q_1 - xP_1} + e^{Q_2 - xP_2}} \frac{1}{(b-a)} dx \\
&= n^t \left[ 1 + \frac{1}{(P_{\bar{i}} - P_i)(b-a)} \ln \frac{1 + e^{Q_{\bar{i}} - Q_i} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}} - Q_i} e^{-a(P_{\bar{i}} - P_i)}} \right]
\end{aligned} \tag{3.3}$$

when $P_1 \neq P_2$. If $P_1 = P_2$, then

$$E[n_i^t] = \frac{n^t}{1 + e^{Q_{\bar{i}} - Q_i}} \; . \tag{3.4}$$

When there is some loyalty effect (i.e., $\alpha > 0$), the computation is a bit more complicated since the perceived rating is user-specific: we have $V_i^j = Q_i(1 + \alpha \mathbb{1}_{\{j \in N_i^{t-1}\}})$, where $N_i^t$ represents the set of users connected to network $i$ during time slot $t$, and $N^t := N_1^t \cup N_2^t$. We then have for a user present at time slot $t$,

$$\begin{aligned}
j \in N_i^{t-1} \quad &\Rightarrow \quad p_i^j(t) = \frac{e^{(Q_i(1+\alpha) - s^j P_i)}}{e^{(Q_i(1+\alpha) - s^j P_i)} + e^{(Q_{\bar{i}} - s^j P_{\bar{i}})}} \\
j \in N_{\bar{i}}^{t-1} \quad &\Rightarrow \quad p_i^j(t) = \frac{e^{(Q_i - s^j P_i)}}{e^{(Q_i - s^j P_i)} + e^{(Q_{\bar{i}}(1+\alpha) - s^j P_{\bar{i}})}} \\
j \notin N^{t-1} \quad &\Rightarrow \quad p_i^j(t) = \frac{e^{(Q_i - s^j P_i)}}{e^{(Q_i - s^j P_i)} + e^{(Q_{\bar{i}} - s^j P_{\bar{i}})}}.
\end{aligned}$$

Consequently, we have, if we define $m_i := |N_i^{t-1} \cap N_i^t|$,

$$\begin{aligned}
\mathbb{E}[n_i^t] \quad = \quad & m_i \mathbb{E}\left[\frac{e^{(Q_i(1+\alpha) - s^j P_i)}}{e^{(Q_i(1+\alpha) - s^j P_i)} + e^{(Q_{\bar{i}} - s^j P_{\bar{i}})}}\right] + \\
& m_{\bar{i}} \mathbb{E}\left[\frac{e^{(Q_i - s^j P_i)}}{e^{(Q_i - s^j P_i)} + e^{(Q_{\bar{i}}(1+\alpha) - s^j P_{\bar{i}})}}\right] + \\
& (n^t - m_1 - m_2) \mathbb{E}\left[\frac{e^{(Q_i - s^j P_i)}}{e^{(Q_i - s^j P_i)} + e^{(Q_{\bar{i}} - s^j P_{\bar{i}})}}\right],
\end{aligned}$$

where the three summands respectively represent the expected number of users which were in network $i$ and did not change their choice, the expected number of users which migrated from network $\bar{i}$ to network $i$, and the expected number of newly arrived users that chose network $i$.

After some algebra, we obtain, conditionally on $n^{t-1}$, $m_1$ and $m_2$, and on the values of $Q_1$ and $Q_2$ at the previous time slot,

$$\begin{aligned}
\mathbb{E}[n_i^t] = & m_i \left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}} - Q_i(1+\alpha)} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}} - Q_i(1+\alpha)} e^{-a(P_{\bar{i}} - P_i)}}\right] \\
& + m_{\bar{i}} \left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}} - Q_i} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}}(1+\alpha) - Q_i} e^{-a(P_{\bar{i}} - P_i)}}\right] \\
& + (n^t - m_1 - m_2) \left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}} - Q_i} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}} - Q_i} e^{-a(P_{\bar{i}} - P_i)}}\right],
\end{aligned}$$

for the case when $P_1 \neq P_2$, with $K_i := \frac{1}{(P_{\bar{i}}-P_i)(b-a)}$. When prices are equal, we have

$$\mathbb{E}[n_i^t] = \frac{m_i}{1+e^{Q_{\bar{i}}-Q_i(1+\alpha)}} + \frac{m_{\bar{i}}}{1+e^{Q_{\bar{i}}(1+\alpha)-Q_i}} + \frac{n^t - m_i - m_{\bar{i}}}{1+e^{Q_{\bar{i}}-Q_i}}.$$

Finally, to have results conditionally on the user repartition at time slot $t-1$ only, we can plug in the previous expression the relations $\mathbb{E}[m_i] = n_i^{t-1}q$ and $\mathbb{E}[n^t] = qn^{t-1} + \lambda$, where we recall that $q$ is the probability that a user continues his service at the next time slot and $\lambda$ is the expected number of new entrants at each time slot.

### 3.2.3 Average churn rate

In this subsection, we focus on the phenomenon of *churn*, that is, the fact that users switch networks during their communication. This can be due to the mobility of users (that is not considered here), to some temporary changes in the network conditions (reflected by a change in the quality values $(Q_i)$), or to some user-specific criteria.

Quantifying the occurrence of that phenomenon is of crucial importance to the network management, since switching networks incurs energy-costly procedures to perform the handover. The frequency of churns is therefore directly linked to the overall energy consumption of the global network.

Using the same method as before, the expectation of the number of network changes $h^t$ at time slot $t$ (conditionally on the situation at time slot $t$) can be computed:

$$\begin{aligned} \mathbb{E}[h^t] &= n_1^{t-1}q\left[1 - K_1 \ln \frac{1+e^{Q_1(1+\alpha)-Q_2}e^{-b(P_1-P_2)}}{1+e^{Q_1(1+\alpha)-Q_2}e^{-a(P_1-P_2)}}\right] \\ &\quad + n_2^{t-1}q\left[1 + K_1 \ln \frac{1+e^{Q_2(1+\alpha)-Q_1}e^{-b(P_2-P_1)}}{1+e^{Q_2(1+\alpha)-Q_1}e^{-a(P_2-P_1)}}\right] \end{aligned}$$

still with $K_1 = \frac{1}{(P_2-P_1)(b-a)}$.

### 3.2.4 Illustrations

In this subsection, we present some simulations that illustrate the selection game we have defined, and the analytical results of this section. Two cases are considered: one without loyalty effect ($\alpha = 0$), and one with a loyalty value $\alpha = 3$. Unless specified otherwise, the parameters used in the simulations are the following:

– range of the price sensitivity values $s^j$: $[a, b] = [0, 0.4]$,

– average number of new entrants per time unit: $\lambda = 200$,

– probability of leaving the system at the end of the current time slot: $1 - q = 0.2$,

– quality score of network $i$ of the form [1] $\bar{Q}_i = 1 - (n_i^t/C_i)^2$, with $C_i$ the capacity of network $i$,

– networks of respective capacities $C_1 = 1000$, $C_2 = 600$,

– respective prices of each network $P_1 = 9$, $P_2 = 8$,

– memory effect in the computation of $Q_i$ in (3.2): $\beta = 0.9$.

Figure 3.2 shows the evolution of the number of users in each network, without any loyalty effect. We remark that due to the inner probabilistic nature of user choices, those numbers do not converge to a given value. However, after a few iterations the system is close to its steady state, and the number of users in each network oscillates around their expected value. Note here that the expectation on each iteration is computed from (3.3)-(3.4), but using previous iteration's expectations $\mathbb{E}[n_i^{t-1}]$ instead of the real values $n_i^{t-1}$. Therefore, the curves for $\mathbb{E}[n_1^t]$ and $\mathbb{E}[n_2^t]$ are completely deterministic. We observe that those expected values are very good estimators of the average values of $n_1^t$ and $n_2^t$, respectively.

We plot in Figure 3.3 the corresponding values of the ratings $(Q_1, Q_2)$, computed over time following (3.2). Similarly to the number of users in each network, after the starting phase where ratings are high due to the small number of users, ratings stabilize around a constant value, still with oscillations. Note however that the amplitude of the oscillations are smaller than for the number of users, due to the memory effect introduced in (3.2) that smoothes the variations.

Figures 3.4 and 3.5 are the counterparts of Figures 3.2 and 3.3, but with a loyalty value $\alpha = 3$. We remark as expected that oscillations still take place, but to a smaller extent with respect to the no-loyalty case. Notice also that the loyalty phenomenon affects not only the number of handovers (the churn effect), but also the average balance between networks: users tend to go more to network 1 when the loyalty effect is introduced. The explanation of this is as follows: without the loyalty effect the majority of users already

1. Note that we could also consider totally different forms for $Q_1(n_1)$ and $Q_2(n_2)$, that could reflect the different technologies used in the heterogeneous network. With the form taken here, the only heterogeneity lies in the capacity differences among networks.

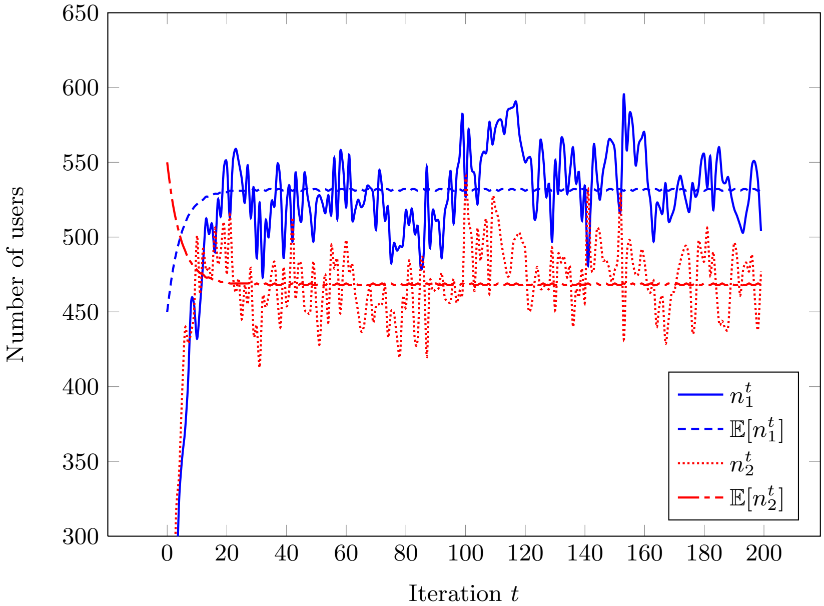


Figure 3.2: Number of users in each network, without loyalty effect.

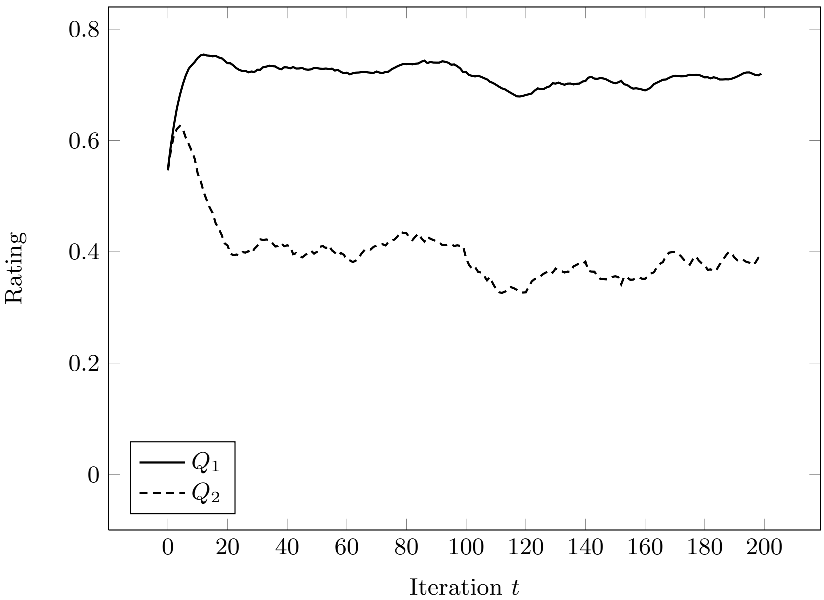


Figure 3.3: Rating dynamics, without loyalty effect.

used to prefer the first network, and the loyalty effect then retains them from changing networks. Users spend less time "exploring" the other network, and prefer to stick to their current one (in most cases, their preferred one). Another direct consequence is that the loyalty effect tends to reduce the difference in the steady-state ratings $Q_1$ and $Q_2$: users mostly preferring network 1 and churning less, that network becomes more congested, hence a reduction in its rating.

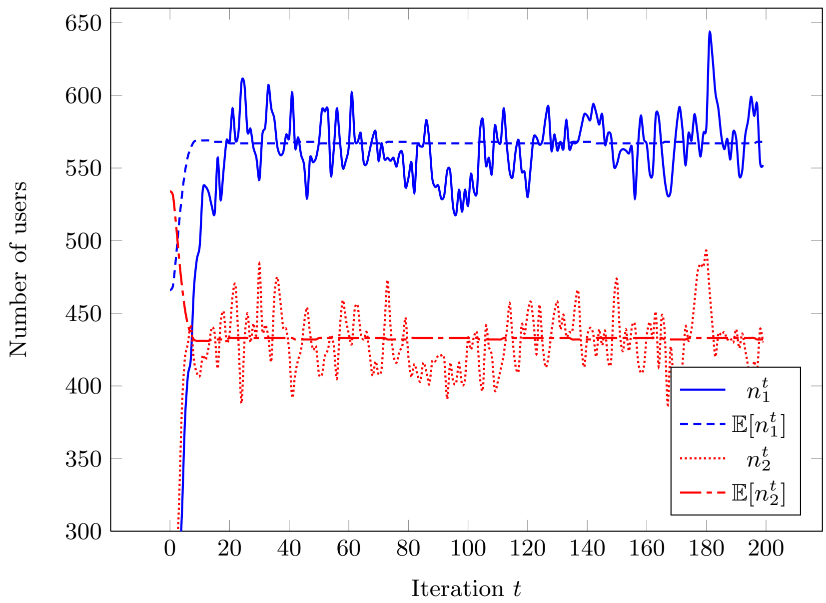


FIGURE 3.4: Number of users in each network, with loyalty effect ($\alpha = 3$).

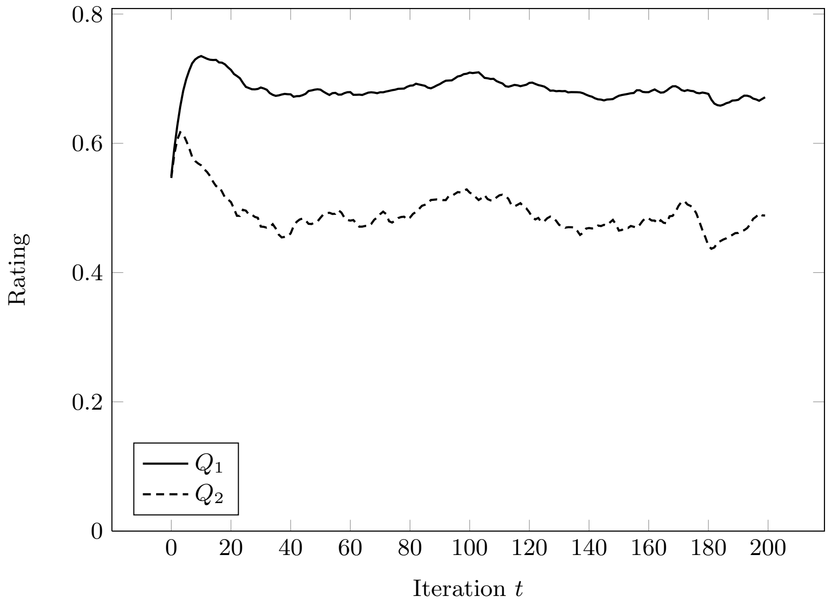


FIGURE 3.5: Rating dynamics, with loyalty effect ($\alpha = 3$).

Finally, Figure 3.6 illustrates the dependence of the loyalty coefficient on the churn phenomenon: as expected, a larger reluctance to switch networks reduces churn significantly, even if the other network is temporarily more attractive.

### 3.2.5 Computing the steady-state user distribution

The simulation results of Subsection 3.2.4 suggest that the mathematical expectations of the number of users (computed by recursively estimating the number of users at each

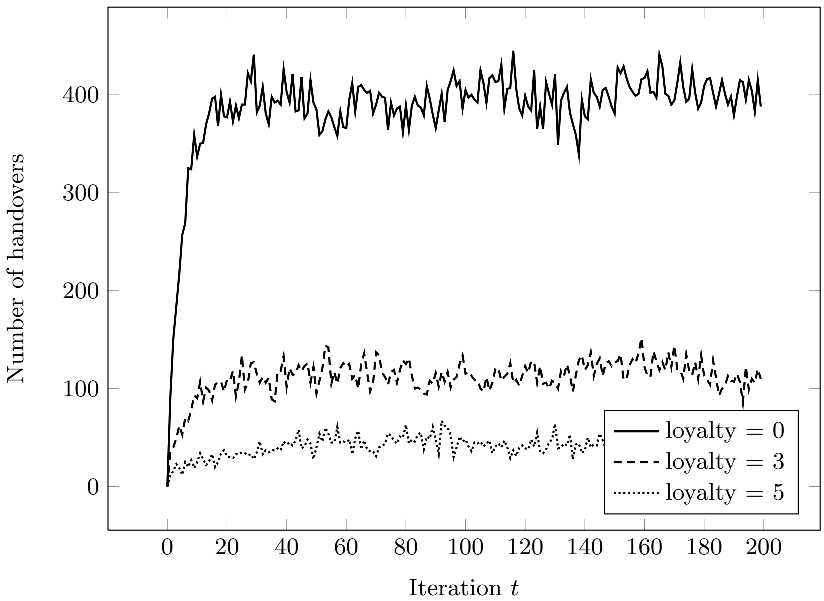


FIGURE 3.6: Number of handovers per time slot.

time slot) are very close to the steady-state average values. This is partially due to the memory effect $\beta$: when $\beta$ tends to 1 then the quality values $Q_i$ converge to a fixed value. Considering that limit case when $\beta \to 1$, we expect that without loyalty effect, the average number $n_i^*$ of users in network $i$ is close to the solution of the following fixed-point equation when $P_1 \neq P_2$:

$$n_i^* = \frac{\lambda}{1-q}\left[1 + K_i \ln \frac{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}e^{-b(P_{\bar{i}}-P_i)}}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}e^{-a(P_{\bar{i}}-P_i)}}\right], \tag{3.5}$$

and when $P_1 = P_2$ :

$$n_i^* = \frac{\lambda}{1-q}\frac{1}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}}, \tag{3.6}$$

with $K_i = \frac{1}{(P_{\bar{i}}-P_i)(b-a)}$

With some loyalty effect, that fixed-point equation becomes:

$$\begin{aligned} n_i^* = & n_i^* q \left[1 + K_i \ln \frac{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)(1+\alpha)}e^{-b(P_{\bar{i}}-P_i)}}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)(1+\alpha)}e^{-a(P_{\bar{i}}-P_i)}}\right] \\ & + \left(\frac{\lambda}{1-q} - n_i^*\right) q \left[1 + K_i \ln \frac{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)(1+\alpha)-Q_i(n_i^*)}e^{-b(P_{\bar{i}}-P_i)}}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)(1+\alpha)-Q_i(n_i^*)}e^{-a(P_{\bar{i}}-P_i)}}\right] \\ & + \lambda \left[1 + K_i \ln \frac{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}e^{-b(P_{\bar{i}}-P_i)}}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}e^{-a(P_{\bar{i}}-P_i)}}\right] \end{aligned} \tag{3.7}$$

when $P_1 \neq P_2$ and

$$n_i^* = \frac{n_i^* q}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)(1+\alpha)}} + \frac{n_{\bar{i}}^* q}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)(1+\alpha)-Q_i(n_i^*)}} + \frac{\frac{\lambda}{1-q} - n_i^* q - n_{\bar{i}}^* q}{1+e^{Q_{\bar{i}}(n_{\bar{i}}^*)-Q_i(n_i^*)}}. \tag{3.8}$$

Remark that Equations (3.5) - (3.8) can be solved numerically.

### 3.2.6 Price of Anarchy

It is interesting to evaluate, how far the steady state in users rating based dynamics from optimal distribution. We will focus on optimization of the total users welfare, which is the sum of quality of connection scores for all users:

$$W = n_1 Q(n_1) + n_2 Q(n_2),$$

and as a quality function we use the same as in simulations latency function : $Q(n_i) = 1 - (\frac{n_i}{C_i})^2$. For this quality function it is easy to find optimal distribution:

$$W' = 3(\frac{n-n_1}{C_2})^2 - 3(\frac{n_1}{C_1})^2 = 0 \Longleftrightarrow$$
$$n_1^2(C_1^2 - C_2^2) - 2nC_1^2 n_1 + C_1^2 n^2 = 0,$$

where $n$ is the total number of users in the system. From last equation we deduce that

$$n_i^{opt} = \frac{nC_i}{C_1 + C_2}.$$

We calculated the PoA of the users dynamics for both when loyalty effect takes place and nor, assuming that providers charge equal prices $P_1 = P_2$. Thus, we applied the equations (3.6) and (3.8). The results are shown on Figures 3.7 - 3.8. We observed that for simulation settings we have the PoA is lower when users are loyal, however the dependance of PoA on $\alpha$ is not obvious.

## 3.3 Summary

We have introduced a model of network selection by wireless users in an heterogeneous network. In that system, users make their choice based on networks' ratings, that

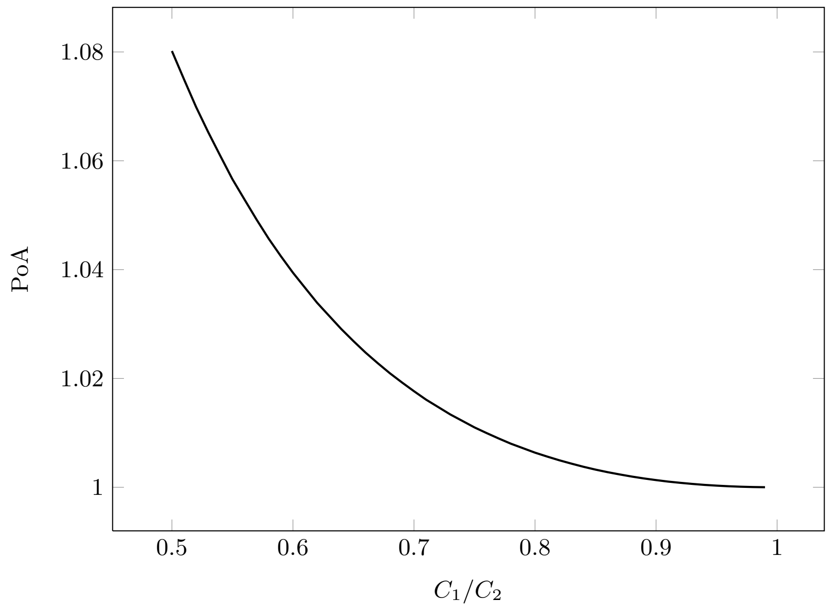


FIGURE 3.7: PoA without loyalty effect

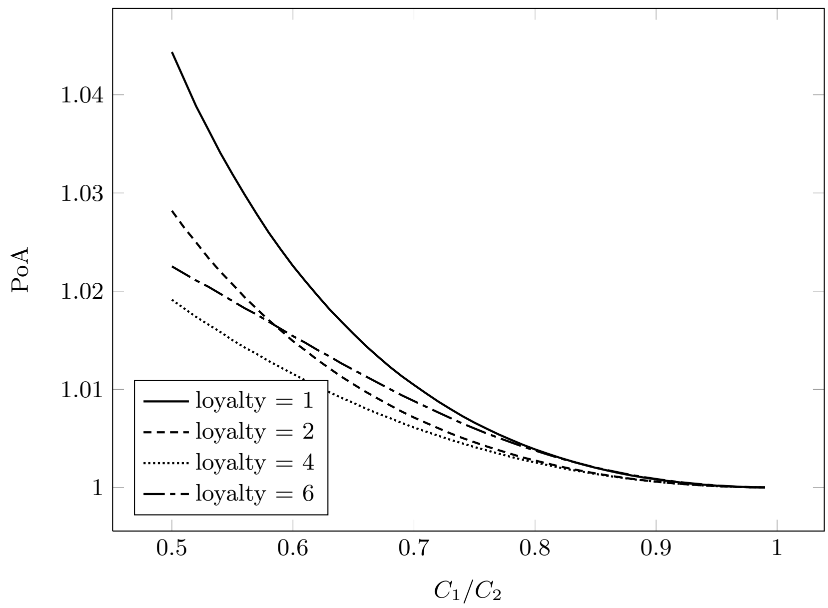


FIGURE 3.8: PoA with loyalty effect

are computed and distributed by a third-part entity, possibly using feedbacks from users' experienced QoS. We also took into consideration that users may be reluctant to switch to another access point, even if it proposes better QoS. We model this type of user behavior through a loyalty value, which drastically impacts the considered model. We have investigated the model dynamics, and we proved that the numbers of users in networks oscillate around their expectation values. Despite the fact that networks parameters never converge due to the realistic assumption of users arrivals being random, we managed to provide good estimates for them through an analytical expression.

We also studied the efficiency of the steady state of the proposed rating-based scheme. For this purpose we introduced the total users welfare function, we found its optimal value and then calculated the Price of Anarchy for different loyalty values. From the numerical study we observed that for the case when technologies used by providers are homogeneous (access points could serve almost the same number of users) the PoA is very close to one and thus the proposed scheme leads to an efficient situation.

The main disadvantage comes from the fact that this system needs a central controller, which gathers users feedback. However, we could assume that there is a special web service, which gathers QoS feedback automatically, and then users in real time could observe the rating of available networks.

Those results can be used to forecast providers' revenues, the handover frequency, and the energy consumption as we will show in Chapter 5. It gives an interesting insight on how this type of system can behave. Due to the middle controller the system is self-regulative, which potentially can reduce the harm providers make to users with their selfish revenue-maximizing pricing decisions. The historical impact allows to avoid ping-pong effects, which can take place in a crowded area, and gives an idea about how the rating computation can be organized in this kind of system.

However, before being implemented, the system needs a more careful parametrization. Due to Price of Anarchy analysis we found there is some inefficiency regarding social welfare of users, especially for the cases when the system is heterogeneous. In fact, it may be possible to introduce in the rating update mechanism some bias, which will allow to reach the social optimum. From the other point of view these ratings will not be truthful and also can be criticized by providers.

The model can be extended in several directions. It would first be interesting to consider different coverage areas for both networks, so that only some fraction of users would have a choice to make. Second, the mobility of users, moving from one area to another, would be worth considering. Finally, we intend to model not only two cells, but two cellular networks covering a wide area, with possibly different cell dimensions for each (representative of the different technologies considered).

# Chapter 4

# Focusing on equilibrium situations for the user game: non-atomic models

We expect that one of the major objectives in future generations of mobile networks would be to find a convenient solution for the vertical handover (switching between networks implementing different technologies) decision, for both the mobile users and the providers. Indeed, each user being able to select at any time its most suitable wireless network, i.e., to be *always-best-connected* [1] could cause the overload of some technologies and the under-utilization of others. This is due to user selfishness: users ignore the negative consequences of their actions on others when making their choices, which can lead to an ineffective situation. In order to cope with that problem and profit from the diversity of technologies, operators have to improve the current resource management technologies.

A number of recent papers in the transportation science literature addressed that same problem (see [34, 48, 58]). Those works discuss the introduction of some incentive tools, interpreted as taxes, which could influence user choices towards a more efficient situation. In this chapter, we focus on applying that idea to influence user's choice between several wireless heterogeneous networks. Due to the specificity of the wireless framework, our problem can be modeled as a routing game simpler than the general ones studied in [34, 48, 58], which allows us to reach analytical results.

We consider that users select their access network based on a combination of the tax imposed on each network and the QoS provided, where QoS is the (congestion-sensitive) latency. The problem is described as a non-cooperative game [59], where the mobile users are the players, and their strategies are the network they choose. For our analytical study, we assume that the number of users is large enough, so that the game is non-atomic [60], i.e. the individual actions of a player have no influence on the QoS of the others. Note that the final choice of which access network to use is left to the mobile user, thus avoiding the heavy computations and one-to-one signaling of a centrally-decided association scheme.

The network selection model proposed in Chapter 4 is too complex, and hard for analytical study, thus in this chapter we model users decisions in a more simple way: the users are assumed to be aware of the current QoS-level of each network connection and to select a network based on a trade-off between QoS and price. Also we consider that users are heterogeneous in their price perception, which makes the model more realistic. Also, if in Chapter 4 we were focused on the network selection problem mainly, here we already tackle the resource management problem of a single provider.

## 4.1 Model description

The network topology we consider here is close to the one considered in Chapter 3: there are $n$ wireless access networks owned by the same operator, who aims to achieve an efficient use of his access points. The coverage areas of all networks coincide as on Figure 4.1. We assume that users seek an Internet connection through one of the available networks, and their choices depend on the values of the taxes fixed by the operator and the QoS (here, the congestion-dependent latency) they experience. Note that the term "tax" used here rather in the sense of price or monetary cost. We decided to stick to this term mainly due to the fact that this is the common way to call the monetary cost in the routing game literature.

### 4.1.1 Mathematical formulation

We identify all parameters related to a specific network $i$ through the use of the lower index $_i$, for $1 \leq i \leq n$. Each network $i$ has a QoS-related cost function $\ell_i(f_i)$ that is the

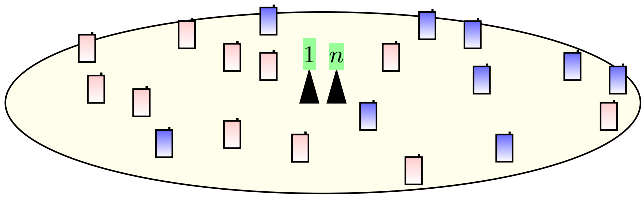


FIGURE 4.1: Network topology: $n$ networks cover the same area with two classes of users

latency function, where $f_i$ is the flow (cumulated throughput) on network $i$. All networks are owned by the same provider, which is aiming to minimize some cost function and could influence users behavior by charging a tax $\tau_i$ on each network $i$.

We consider $m$ classes of users, implying that users from the same class have the same price sensitivity value. We write all the parameters related to class $j$ with the upper index $^j$, $1 \leq j \leq m$; users in class $j$ have tax sensitivity $\alpha^j \geq 0$ and the total demand from class-$j$ users is denoted by $d^j$, so that $\sum_{j=1}^{m} d^j = D$.

The cost perceived by a class-$j$ user connected to network $i$ is a weighted sum of the congestion-sensitive cost (the latency) and the monetary cost (the tax) on that network:

$$C_i^j(f) = \ell_i(f_i) + \alpha^j \tau_i. \tag{4.1}$$

Assuming that only radio links incur QoS-related costs (i.e., latency), the setting described above could be seen as a routing problem, with a common source for all users, represented by the common coverage area of the networks, and one common destination (the Internet). Each user forwards his flow through one of $n$ routes, which are the $n$ networks, with a routing cost equal to the cost in (4.1), as depicted in Figure 4.2. When users selected their route, their interactions form a *noncooperative routing game*.

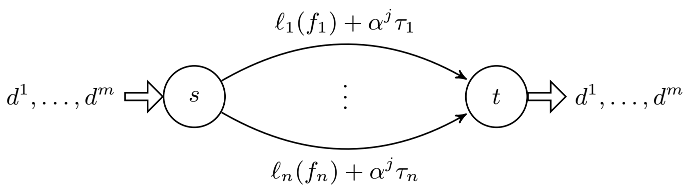


FIGURE 4.2: Logic representation of the network selection problem as a routing problem: the perceived cost on each route $i$ depends on the load $f_i$ and the tax $\tau_i$, but also on the user type $j$ through the sensitivity $\alpha^j$

To simplify notations, we assume without loss of generality that:

**Assumption 1.** *The users classes and networks are numbered such that:*

1. $c_1 \geq c_2 \geq \ldots \geq c_n$
2. $\alpha^1 < \alpha^2 < \ldots < \alpha^m$

The delay on each network $i$ is assumed to increase with the network load $f_i$, through the delay function $\ell_i$ described below.

**Assumption 2.** *The delay of a network carrying some flow level $f_i$ is assumed to be given by the mean sojourn time in an M/M/1 queue:*

$$\ell_i(f_i) = \begin{cases} (c_i - f_i)^{-1} & \text{if } f_i < c_i, \\ \infty & \text{if } f_i \geq c_i. \end{cases} \tag{4.2}$$

The units used need to be clarified: modelling the packets as clients of an M/M/1 queue, the average sojourn time should be the one in (4.2), but multiplied by the packet size in the network. Assuming that the packet size is the same on both networks, we remove that multiplicative constant without loss of generality, leading to an interpretation of the tax $\tau_i$ as the price charged per packet sent on network $i$.

### 4.1.2 Social cost

We assume that the provider owning all considered networks is interested in minimizing the social cost (or total cost) expressed as:

$$C(f) = \sum_{i=1}^{n} f_i \ell_i(f_i), \tag{4.3}$$

where $f = (f_1, \ldots, f_n)$ is the flow distribution vector, with $\sum_{i=1}^{n} f_i = D$. That cost corresponds to the aggregated latencies undergone by users and is the total cost classically considered in routing games [61, 62].

To minimize this cost function provider may to apply taxes, and the problem of their computation is tackled in the rest of the Chapter.

### 4.1.3 The case of several providers

In this study we consider that all networks are owned and controlled by the same entity, that we call the provider. The objective for the provider here is to make the best use of the network resource, in the sense of the aggregated user cost of Equation (4.3). Hence the provider is not directly driven by revenue, the taxes imposed on network are only used as incentives to reach the best flow repartition.

Considering several providers managing the different networks would totally change the paradigm, since those providers would compete to attract customers and make revenue, and would use taxes for that purpose. We would then have a non-cooperative game played among providers deciding their tax levels, and anticipating user reactions when making those decisions. Such situations of competing providers have been studied in [63] with cost functions similar to ours, but with few positive analytical results: even the existence of a Nash equilibrium of the tax-setting game is not guaranteed. However, if such an equilibrium exists, it can reasonably be expected to benefit to users (a general property of competition) with respect to a case where a single entity controls all networks and sets prices to maximize revenue (not the case treated here).

The case when several providers perfectly cooperate to optimize network usage would be equivalent to the one-provider case. However there are some in-between situations, where providers may partially compete and cooperate: for example they may have roaming agreements, or may have to share the capacity of their access networks. Those aspects are partially treated in [64] but deserve more attention.

## 4.2 User equilibrium and optimal situations

In this section we define the user equilibrium of the routing game, and compare the equilibrium without taxes to an optimal situation from the point of view of social cost (4.3).

### 4.2.1 User equilibrium

In order to model user behavior, we follow a common assumption of users being selfish, in the sense that each user routes his flow to the network which minimizes his individual cost given in (4.1). We assume that the number of users is large enough, and therefore each user is non-atomic [60], i.e. his individual action has no influence on the QoS of others. The cost functions given by (4.1) define a game between users, where the steady situation (or users equilibrium) follows Wardrop's principle [65]:

– *At equilibrium for each source-destination pair the travel costs on all the routes actually used are equal, or less than the travel costs on all non used routes.*

A flow repartition satisfying this principle is called a *Wardrop equilibrium* among users. It is actually the non-atomic version of the more general concept of *Nash equilibrium* [59].

Now we propose the Wardrop equilibrium definition for our model:

**Definition 4.1.** A Wardrop equilibrium is a flow repartition $f = (f_i^j)_{1\leq i\leq n, 1\leq j\leq m}$ , such that

$$\begin{cases} f_i^j \geq 0 & \forall i, j \\ d^j = \sum_{i=1}^n f_i^j & \forall j \end{cases}$$

and such that

$$\forall i, i', j \qquad f_i^j > 0 \Rightarrow \ell_i(f_i) + \alpha^j \tau_i \leq \ell_{i'}(f_{i'}) + \alpha^j \tau_{i'}, \tag{4.4}$$

with $f_i = \sum_{j=1}^m f_i^j$. The quantity $f_i^j$ represents the flow from class-$j$ users that is routed through network $i$ (recall that $d^j$ is the total flow of class-$j$ users).

In other words, at a Wardrop equilibrium, the cost of each used route is lower (for the users taking that route) than the cost of any other.

### 4.2.2 User equilibrium without taxes

Consider the case when the provider does not charge taxes for using his networks (or equivalently all taxes are the same), and thus users make their choices without any intervention from the provider. Then the flows at a Wardrop equilibrium have the form stated in the following proposition.

**Proposition 4.2.** *Under Assumptions 1 and 2, at a Wardrop equilibrium $f^{WE}$ with no taxes being applied, we have:*

$$f_i^{WE} = \begin{cases} \frac{D - \sum_{q=1}^{t} c_q + tc_i}{t} & \text{if } i \leq t, \\ 0 & \text{otherwise,} \end{cases} \tag{4.5}$$

*where $1 \leq t \leq n$ is the maximum index for which*

$$D - \sum_{i=1}^{t} c_i + tc_t > 0, \tag{4.6}$$

*and represents the number of used networks.*

The proof comes quite directly from Definition 4.1, since without taxes all users should perceive the same cost on all used routes.

Proposition 4.2 provides a way to compute the equilibrium flows (in a time linear in the number $n$ of flows).

**Example 4.1.** *In the case of two networks and two users classes, under Assumptions 1 and 2 the flows in Wardrop equilibrium are*

$$f^{WE} = \begin{cases} (D, 0) \text{ if } D \leq c_1 - c_2, \\ (\frac{D+c_1-c_2}{2}, \frac{D+c_2-c_1}{2}) \text{ otherwise.} \end{cases} \tag{4.7}$$

### 4.2.3 Optimal situation

In this section we investigate the optimum situation, which we later intend to reach by introducing appropriate taxes. An optimal flow assignment $f^{\text{opt}} = (f_1^{\text{opt}}, \ldots, f_n^{\text{opt}})$ which minimizes social cost (4.3) is the solution of the following mathematical program:

$$\min_{f_1,\ldots,f_n} \sum_{i=1}^{n} f_i \ell_i(f_i) \tag{4.8}$$

$$\text{s.t.} \begin{cases} \sum_{i=1}^{n} f_i = D \\ f_i \geq 0, \text{ for } i = 1, \ldots, n \end{cases} \tag{4.9}$$

Note that this problem does not distinguish among user classes, it only involves aggregate flows on each network. With the specific latency functions (4.2) we can express the optimal flows analytically.

**Proposition 4.3.** *Optimal flows $(f_i^{opt})_{1\leq i\leq n}$ minimizing (4.3) are unique and given by:*

$$f_i^{opt} = \begin{cases} c_i - \frac{\sqrt{c_i}(\sum_{j=1}^k c_j - D)}{\sum_{j=1}^k \sqrt{c_j}} & \text{if } i \leq k, \\ 0 & \text{otherwise}, \end{cases} \tag{4.10}$$

*where $1 \leq k \leq n$ is the maximum index for which*

$$c_i - \frac{\sqrt{c_i}(\sum_{j=1}^k c_j - D)}{\sum_{j=1}^k \sqrt{c_j}} \geq 0. \tag{4.11}$$

*Proof.* We apply the following result from [61]:

**Lemma 4.4** (Beckmann et al., 1956)**.** *For any non-atomic routing game with latency functions $(\ell_i)$, the optimal flows minimizing social cost (4.3) correspond to the Wardrop equilibrium flows of a modified game where latency functions are*

$$\bar{\ell}_i(f_i) = \ell_i(f_i) + f_i \ell_i'(f_i). \tag{4.12}$$

Therefore, applying the equilibrium conditions (4.4) there exists $H > 0$ such that for all $i, 1 \leq i \leq n$:

$$\begin{cases} f_i^{\text{opt}} > 0 \Rightarrow \ell_i(f_i^{\text{opt}}) + f_i^{\text{opt}} \ell_i'(f_i^{\text{opt}}) = H, \\ f_i^{\text{opt}} = 0 \Rightarrow \ell_i(f_i^{\text{opt}}) + f_i^{\text{opt}} \ell_i'(f_i^{\text{opt}}) = \ell_i(0) \geq H. \end{cases} \tag{4.13}$$

With our latency functions (4.2), we immediately remark that

$$f_i^{\text{opt}} > 0 \Leftrightarrow \frac{1}{c_i} < H, \tag{4.14}$$

thus from Assumption 1 there exists $k$ (the number of used networks at the optimal situation) such that $(f_i^{\text{opt}} > 0 \Leftrightarrow i \leq k)$. From (4.13) we get

$$f_i^{\text{opt}} = c_i - \frac{\sqrt{c_i}}{\sqrt{H}}, i = 1, \ldots, k, \tag{4.15}$$

and the condition $\sum_{i=1}^{k} f_i^{\text{opt}} = D$ yields $H = \frac{(\sum_{i=1}^{k}\sqrt{c_i})^2}{(\sum_{i=1}^{k} c_i - D)^2}$. Plugging that last expression into (4.15) gives (4.5), while plugging it into (4.14) leads to the characterization (4.11) for $k$.

□

Similarly to Proposition 4.2 for equilibrium flows, Proposition 4.3 implicitly defines a linear-time algorithm to compute optimal (i.e., globally cost-minimizing) flows. Note that to compute optimal (as well as equilibrium) flows we only need to know the network capacities $(c_i)_{1\leq i\leq n}$ and the total demand $D$, that do not depend on any characteristics of user classes.

**Example 4.2.** *In the case of two networks and two users classes, the optimal flows are given by the following equation:*

$$f^{opt} = (f_1^{opt}, f_2^{opt}) = \begin{cases} (D, 0) & \text{if } D \leq c_1 - \sqrt{c_1c_2}, \\ \left(\frac{(D-c_2)\sqrt{c_1}+c_1\sqrt{c_2}}{\sqrt{c_1}+\sqrt{c_2}}, \frac{(D-c_1)\sqrt{c_2}+c_2\sqrt{c_1}}{\sqrt{c_1}+\sqrt{c_2}}\right) & oth., \end{cases} \tag{4.16}$$

*with the corresponding total cost*

$$C^{opt} = \begin{cases} \frac{D}{c_1-D} & \text{if } D \leq c_2 - \sqrt{c_1c_2}, \\ \frac{2D-c_1-c_2+2\sqrt{c_2c_1}}{c_1+c_2-D} & \text{otherwise} . \end{cases} \tag{4.17}$$

## 4.3 Eliciting optimal user-network associations with taxes

To reduce the total cost the provider has to give an incentive to some users to switch networks, so as to provide higher QoS to the majority of users and lower QoS to some others, instead of providing the same QoS to everyone (what we get at the Wardrop equilibrium without taxes). Here the provider introduces special taxes, such that the flow assignment in the Wardrop equilibrium induced by these taxes is the optimum flow assignment. Previous works (see [48]) ensure that those taxes exist, and the following lemma will help to compute them.

**Lemma 4.5.** *Under Assumptions 1 and 2, optimal taxes are such that* $\tau_1 \geq \tau_2 \geq \ldots \geq \tau_k$*, where* $k$ *is the number of networks used (i.e., networks with positive flows) at the optimal situation. For networks* $i > k$*, it is sufficient to have* $\tau_i \geq \tau_k$*.*

*Proof.* Let us first consider used networks, i.e. networks $1, \ldots, k$. From Lemma 4.4, for $i, i' \leq k$ we have

$$\frac{c_i}{(c_i - f_i^{\text{opt}})^2} = \frac{c_{i'}}{(c_{i'} - f_{i'}^{\text{opt}})^2} := K^2 \tag{4.18}$$

for some constant $K$.

Suppose that $\tau_i < \tau_{i+1}$ for some $i < k$, and that those taxes lead to an equilibrium coinciding with the optimal situation. Then for a class of users $j$ choosing network $i+1$, we have from the equilibrium conditions

$$\ell_{i+1}(f_{i+1}^{\text{opt}}) + \alpha^j \tau_{i+1} \leq \ell_i(f_i^{\text{opt}}) + \alpha^j \tau_i,$$

hence $\ell(f_{i+1}^{\text{opt}}) < \ell(f_i^{\text{opt}})$.

But $\ell_i(f_i^{\text{opt}}) = 1/(c_i - f_i^{\text{opt}}) = K/\sqrt{c_i}$ from (4.18), therefore since $c_i \geq c_{i+1}$ we have $\ell(f_{i+1}^{\text{opt}}) \geq \ell(f_i^{\text{opt}})$, a contradiction.

Now, we consider networks $k+1, \ldots, n$, which do not carry any flow in the optimal situation: no user should prefer one of those networks to their current one. In particular, denoting by $j$ a class sending flow to network $k$ under optimal taxes, we must have

$$\ell_i(0) + \alpha^j \tau_i \geq \ell_k(f_k^{\text{opt}}) + \alpha^j \tau_k, \ \forall i = k+1, \ldots, n,$$

thus

$$\tau_i \geq \frac{\ell_k(f_k^{\text{opt}}) - \ell_i(0)}{\alpha^j} + \tau_k, \ \forall i = k+1, \ldots, n. \tag{4.19}$$

But from (4.13) we have $\ell_k(f_k^{\text{opt}}) - \ell_i(0) \leq 0$, therefore taking $\tau_i \geq \tau_k$ is sufficient to ensure that (4.19) holds, i.e., that networks $i = k+1, \ldots, n$ are not chosen by users. □

Now we provide a method to calculate the optimal taxes:

**Proposition 4.6.** *Under Assumptions 1, the following taxes are optimal:*

$$\tau_{i+1} = \tau_i + \frac{\ell_i(f_i^{opt}) - \ell_{i+1}(f_{i+1}^{opt})}{\alpha^{s_i}}, \tag{4.20}$$

*for $i = 1, \ldots, n-1$, with $\tau_1$ taken arbitrarily, and with*

$$s_i := \min \left\{ j : \sum_{r=1}^{i} f_r^{opt} \leq \sum_{q=1}^{j} d^q \right\}. \tag{4.21}$$

*For networks used at the optimal situation (networks with $f_i^{opt} > 0$), the index $s_i$ represents the class with maximum sensitivity among those sending flow to network $i$.*

*Proof.* For a network $i$ with positive optimal flow, we define $\alpha_i^{\max}$ and $\alpha_i^{\min}$ as respectively the maximum and minimum sensitivities among classes sending some flow to network $i$ (i.e., classes $j$ such that $f_i^j > 0$). Then the Wardrop equilibrium conditions for classes choosing networks $i$ and $i+1$ (both with positive optimal flows) yield

$$\alpha_i^{\max}(\tau_i - \tau_{i+1}) \leq \ell_{i+1}(f_{i+1}^{\mathrm{opt}}) - \ell_i(f_i^{\mathrm{opt}}) \leq \alpha_{i+1}^{\min}(\tau_i - \tau_{i+1})$$

Since $\tau_i \geq \tau_{i+1}$ from Lemma 4.5, we obtain $\alpha_i^{\max} \leq \alpha_{i+1}^{\min}$ .

• If $\alpha_i^{\max} = \alpha_{i+1}^{\min}$ then a class of users, denoted by $j'$, is indifferent between both networks. From the Wardrop equilibrium conditions we have:

$$\ell_i(f_i) + \alpha^{j'}\tau_i = \ell_{i+1}(f_{i+1}) + \alpha^{j'}\tau_{i+1}. \tag{4.22}$$

From this we derive (4.25), with $j'$ satisfying (4.21).

• If $\alpha_i^{\max} < \alpha_{i+1}^{\min}$, then this corresponds to a rare case, when two neighbor classes are perfectly divided, and there is no class whose users are indifferent between both networks. One more time using the Wardrop equilibrium conditions we write:

$$\begin{cases} \ell_i(f_i) + \alpha_i^{\max}\tau_i \leq \ell_{i+1}(f_{i+1}) + \alpha_i^{\max}\tau_{i+1} \\ \ell_i(f_i) + \alpha_{i+1}^{\min}\tau_i \geq \ell_{i+1}(f_{i+1}) + \alpha_{i+1}^{\min}\tau_{i+1}. \end{cases} \tag{4.23}$$

These two inequalities imply that

$$\tau_i + \frac{\ell_i(f_i) - \ell_{i+1}(f_{i+1})}{\alpha_i^{\max}} \leq \tau_{i+1} \leq \tau_i + \frac{\ell_i(f_i) - \ell_{i+1}(f_{i+1})}{\alpha_{i+1}^{\min}}.$$

So, in this particular case a whole range of taxes for network $i+1$ induce an optimal division of users. Note that our proposition in Equation (4.25) falls in that range.

For networks with empty flows in the optimal situation, our proposition is still valid. Indeed, since taxes decrease with the network index, the class $m$ with the highest sensitivity to price is the first class which would be interested in connecting to these empty networks. It is easy to see that the taxes defined by (4.25) will prevent them from doing this. If $k$ is the maximum index of a network with non-empty flow in optimal situation, then from the Wardrop equilibrium conditions we should have:

$$\ell_k(f_k^{\text{opt}}) + \alpha^m \tau_k \leq \ell_i(0) + \alpha^m \tau_i \qquad \forall i > k, \tag{4.24}$$

which is verified with the tax defined by (4.25). □

**Example 4.3.** *For the case of two networks and two users classes, the tax should be applied only on network* 1*. Under Assumptions 1 and 2, for given values of network capacities* $(c_i)_{i=1,2}$*, demands* $D = d^1 + d^2 < c_1 + c_2$*, and sensitivities* $(\alpha^j)_{j\in\{1,2\}}$*, an optimal tax* $\tau_1$ *to apply to network* 1 *when* $D > c_1 - \sqrt{c_1 c_2}$ *is given by*

$$\tau_1 = \begin{cases} \frac{c_1 - c_2}{\alpha^2 \sqrt{c_1 c_2}(c_2 + c_1 - D)} & \textit{if } d^1 \leq f_1^{opt}, \\ \frac{c_1 - c_2}{\alpha^1 \sqrt{c_1 c_2}(c_2 + c_1 - D)} & \textit{otherwise}. \end{cases} \tag{4.25}$$

*When* $D \leq c_1 - \sqrt{c_1 c_2}$*, no tax is necessary.*

Like the two previous propositions in the paper, Proposition 4.6 implicitly defines an algorithm to compute optimal taxes: Proposition 4.3 should first be applied to obtain optimal flows, then (4.21) provides the value of $s_i$ for each network $i$ to be inserted into (4.25) so as to get the tax value.

The freedom to arbitrary choose $\tau_1$ gives us an interesting feature: the provider could regulate his total revenue by adjusting appropriately $\tau_1$ without any harm to the social cost. For example, $\tau_1$ could be set (to a negative value) such that the total revenue is null.

The intuition behind Proposition 4.6 is illustrated in Figure 4.3. We already know

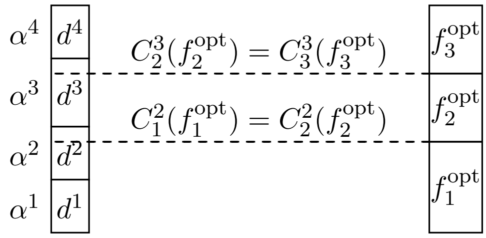


FIGURE 4.3: Example of user distribution among networks with optimal taxes for the case $m = 4, n = 3$: class-1 (resp. class-4) users all attach to network 1 (resp. 3), while class-2 (resp. class-3) users are split among networks 1 and 2 (resp. 2 and 3).

from Lemma 4.5 that the bigger tax should be charged on networks with lower indexes (bigger capacities). This in turn means that the "richest" users are connected to them (the smaller their sensitivity values). Thus, the least price-sensitive users will choose network 1. On the example on Figure 4.3, the total flow of class-1 users is not enough to ensure an optimal flow $f_1^{\text{opt}}$ in network 1. So, the following (by sensitivity value) class should fulfill the optimal flow in network 1. The total flow of classes 1 and 2 is bigger than the optimal flow $f_1^{\text{opt}}$, so we have to split users from class 2. Here we should use the Wardrop equilibrium conditions to find an expression for $\tau_2$ depending on $\tau_1$, this condition meaning that users of class 2 are indifferent between networks 1 and 2. In general, the only computational difficulty is to find a class with users indifferent between two networks with consecutive indices. In the proposed example, it is class 2 for networks 1 and 2, and class 3 for networks 2 and 3.

### 4.3.1 Effect of optimal taxes on perceived QoS

As intended, our approach allows to separate delay-sensitive and delay-insensitive requests, as illustrated on Figure 4.4, where the average latencies experienced for two classes of users are plotted when the proportion of class-2 users vary, and compared to the Wardrop Equilibrium case (where both classes have the same latency). Note that delay-sensitive class-1 users benefit from the best quality in average (lower latency), and this happens at the expense of class-2 users, which suffer higher delay but are less sensitive to it.

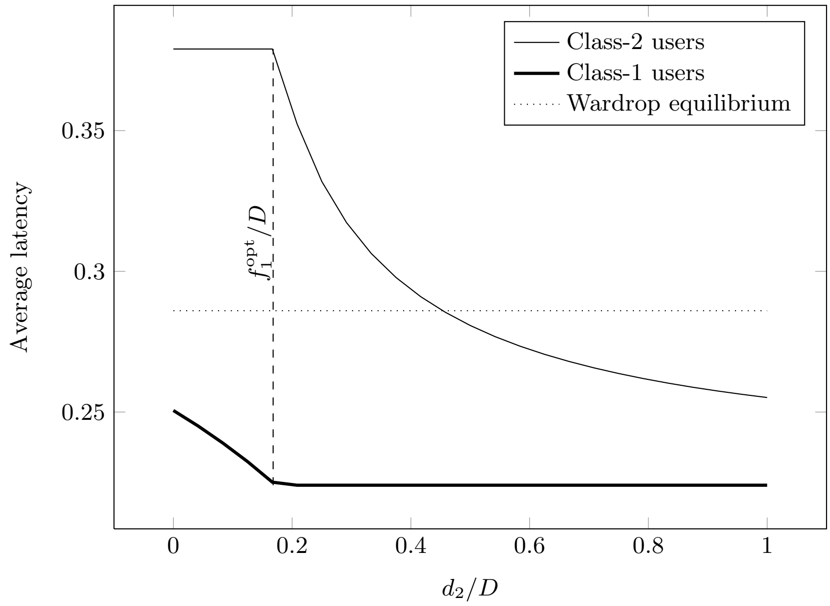


FIGURE 4.4: Average latency of both classes at the Wardrop equilibrium and for the optimal situation. Parameters: $D = 8$Mbit/s, $c_1 = 11$Mbit/s, $c_2 = 4$Mbit/s.

### 4.3.2 Information needed to compute the optimal taxes

In what follows we discuss the values needed for the tax computation, and the possible ways to measure them:

– The capacities $(c_i)_{i=1,\dots,n}$ of the access networks are obviously known by the network owner.

– The total demand $D$ varies over time (we nevertheless assume here that the demand variations are on a larger time scale than the time needed to compute and apply the taxes). Therefore, for the tax calculation the operator needs only to measure the total throughput on the gateway with the core network, which can be done quite easily, for example using SNMP (Simple Network Management Protocol) statistics.

– The optimal traffic flow $f_i^{\mathrm{opt}}$ that should use network $i$ is quite simple to compute by the operator, from the previously mentioned values: only the capacities $(c_i)_{i=1,\dots,n}$ of both networks and the overall network demand $D$ are needed.

– The tax sensitivities $(\alpha^j)_{j\in\{1,\dots,m\}}$ quantify the relative importance of taxation and QoS. These parameters depend on the applications type and the access network performance, and we assume them to be known from statistical observations.

– The total demand $d^j$ of users of class $j$ is the hardest to measure in practice, since the network owner can not determine the exact number of users from each class being connected to his networks at a particular moment of time. Note that our scheme does not require a user to declare his type when opening a connection, since incentives would then be needed to ensure user truthfulness. However, we recall that the exact values of $d^j$ are not needed: we need to know how they relate with optimal flows $f_i^{opt}$, and thus the taxes could be compute with a small error using approximate values of $d^j$.

An approximation we propose is to use the average load of users of class $j$, $j \in \{1, \ldots, m\}$, which is much easier to determine, as an estimator of $d^j$. We can indeed assume that the arrival process of class $j$ members and the time they spend in average in the network are known stochastically, and calculate the average class $j$ load. The impact of such an approximation will be evaluated through simulations in the simple case of two user classes in the next section.

## 4.4 Efficiency analysis

In this section we present some analytical investigations about the efficiency of our taxation method. As an efficiency measure we use the Price of Anarchy (PoA), which is the ratio between the total cost value achieved from the selfish users behavior and the minimum total cost value that could be reached by coordinating users [34]. This value is larger or equal to one. The larger the PoA, the less efficient the selfish users behavior, while if the PoA equals one, then selfish user behavior leads to an optimal situation and no intervention is needed. Recall that the taxes computed in Proposition 4.6 drive the system to an optimal situation, i.e., to a situation with PoA equal to one.

### 4.4.1 Influence of heterogeneity on the PoA

At first, we provide the PoA values while varying the heterogeneity among networks, which comes from the different capacities. For simplicity, we consider capacities of the form $c_i = c_0 w^{i-1}$ for $i = 1, \ldots, n$, where we call $w \in (0, 1]$ the homogeneity value. On Figure 4.9 we plot the PoA for different values of the total user demand $D$, with $c_0$ such that the total capacity of the system equals 10 [Mbit/s]. We observe more heterogeneous systems lead to a larger worst-case PoA (higher inefficiency due to user selfishness). It is especially clear when total demand is close to the total capacity value (i.e, the system is congested), but for very heterogeneous systems the PoA is quite high even for small demand values, thus the introduction of taxes would lead to significant performance gains.

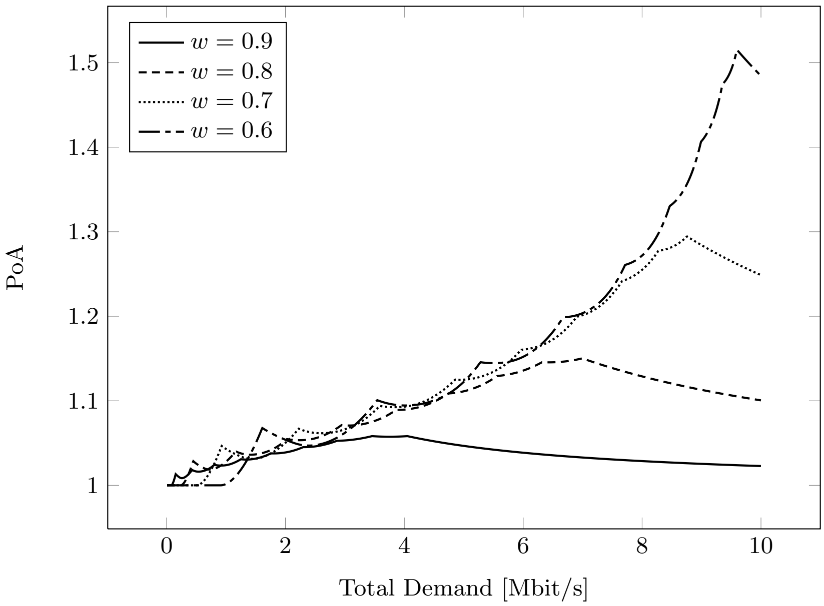


FIGURE 4.5: PoA versus total demand $D$ with $n = 10$ and total capacity equal to 10 [Mbit/s].

### 4.4.2 Some economic interpretations of the PoA

Finally, we present two counterparts for the Price of Anarchy in our model. For simplicity, we consider only a case with two networks in which $c_1 = 4$ [Mbit/s] and $c_2 = 11$ [Mbit/s]. First, Figure 4.6 shows how many more users the operator could serve if using network resources in an optimal way for the same total cost, compared to the case when he does not influence users behavior. In a somehow similar way, Figure 4.7 indicates the capacity (or investment) reduction that would lead to an unchanged total cost, just because of effective resource management. These two values are comparable to the Price of Anarchy, but have the advantage of being convertible into monetary gains, probably more appealing to network providers. These figures have to be understood as follows. Consider a system with relative load equal to 0.7 (dotted curve) and a PoA of 1.02: Figure 4.6 show that if we optimize resource usage (e.g., through optimal taxes), we could have 2% more users in our system without increasing the total cost. The analogical explanation works for Figure 4.7: in the same situation, if we introduce optimal taxes, we can decrease our system's capacity by 2% without changing the overall cost perceived by users.

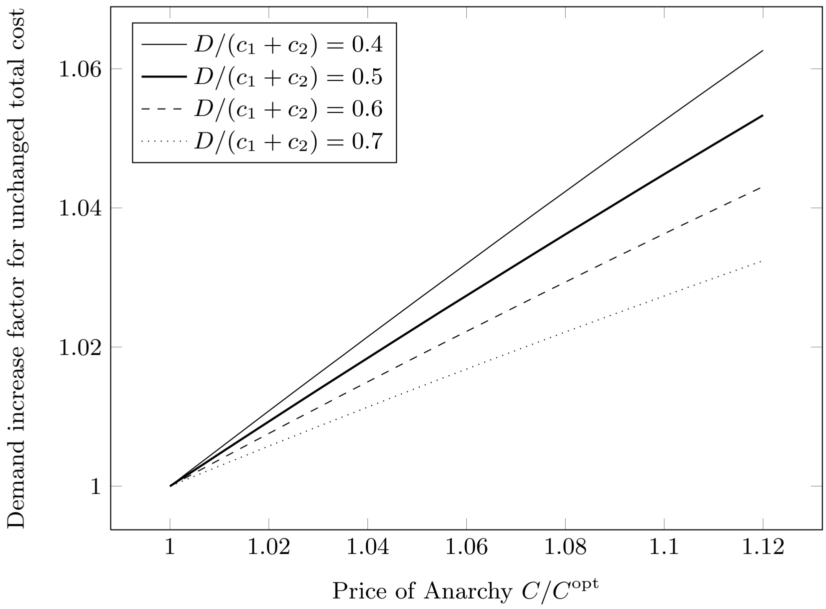


FIGURE 4.6: Demand gain versus PoA, for different demand levels in the case of two networks.

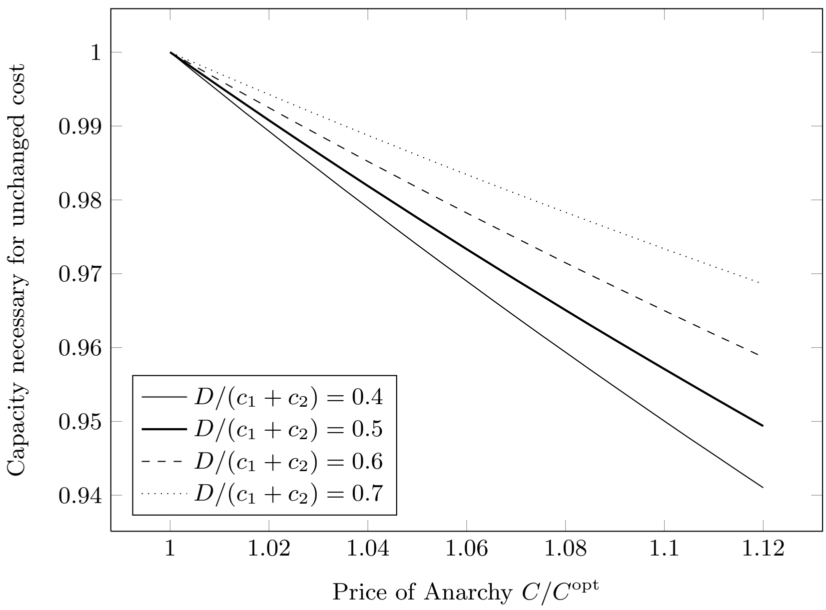


FIGURE 4.7: Capacity gain versus PoA, for different demand levels in the case of two networks.

## 4.5 Simulation Scenario

This section complements the mathematical results, by providing a simulation model to evaluate the performance of our tax mechanism in a wireless network where users dynamically enter and leave the system. For simplicity of both presentation and computation, we provide only results for a simple case with two access points and two users classes.

### 4.5.1 Simulation model and scenarios

We consider a simple scenario where the operator owns two access points, with respective WiFi implementations IEEE 802.11b ($c_1 = 11$Mbit/s) and IEEE 802.11g ($c_2 = 4$Mbit/s). We assume mobile users of class $i \in \{1, 2\}$ join the system according to a Poisson process with parameter $\lambda_i$. We further assume that the classes correspond to different services (traffic with the same properties):

– Delay-sensitive (real-time) video conversation call for users of class 1, with individual throughput $\epsilon_1 = 0.184$Mbit/s.

– Streaming audio (non-real time: music or radio, for example) for users of class 2, with individual throughput $\epsilon_2 = 0.064$Mbit/s.

Those definitions are compliant with our model convention, where type-1 users are more delay-sensitive than type-2, thus $\alpha_2 > \alpha_1$.

Note that each user has a non-zero individual throughput, hence the game is not perfectly non-atomic. Nevertheless, the individual throughput values are small with regard to the network capacities ($c_1 = 11$Mbit/s, $c_2 = 4$Mbit/s), so that the impact on QoS of individual choices remain small (unless networks are extremely loaded).

Each user is connected to the network for a duration modeled as a random variable, following an exponential distribution with parameter $\mu_i$. The average listening time of class-1 users is therefore $\frac{1}{\mu_1}$ (seconds), and the average video conversation time of class-2 users is $\frac{1}{\mu_2}$ (seconds). Users choose an access network upon their arrival, selecting the cheapest one in the sense of their cost (4.1) . We investigate two settings: in the first setting, users remain attached to the same network for the whole duration of their connection (no handovers), even if QoS conditions vary. In the second setting, vertical handovers between networks can occur.

Note that under our assumptions, the process describing the number of users of each class in each network is a continuous-time Markov chain, which we study through simulations due to the excessively large number of states. Note that the latency function we consider (Equation (4.2)) are only defined when demand is below capacity. We tackle this problem by dropping the arriving users for which there is no sufficient available capacity on any network. The resulting blocking rate is measured in our simulations.

We investigate three different scenarios for each aforementioned simulation setting. In the first scenario, the tax is not applied at all - the users act without any intervention from the provider's side. In the second scenario, the operator is willing to apply the optimal tax expressed, but is not able to measure the exact value of $d_1$. In that case the tax expression is chosen based on the *average load* of class-1 users ($\hat{d}_1 = \frac{\epsilon_1 \lambda_1}{\mu_1}$) in the network: the arrival process $\lambda_1$ of users 1 and the duration of time $\frac{1}{\mu_1}$ a user spends in the network are supposed to be known by the operator. The third scenario, called the optimum situation, assumes that the operator is able to determine precisely the load $d_1$ of class-1 users, and thus to apply the exact optimal tax. Recall that for the scenarios involving taxes, the tax is applied only when the network load $D$ exceeds $c_1 - \sqrt{c_1 c_2}$, i.e. when selfish user behavior does not lead to an optimal situation.

### 4.5.2 Simulation results

This section presents the results obtained with the simulations scenarios described above, for the parameter values $\frac{1}{\mu_1} = 2.5$ minutes, $\frac{1}{\mu_2} = 4$ minutes, and the tax sensitivities parameters $\alpha_1 = 1$ and $\alpha_2 = 2$ (cost units per (dollar per packet)).

In particular, we analyze the (average) Price of Anarchy, that is the ratio between the total cost value achieved from the selfish users behavior and the optimum total cost value [34]. This metric helps us investigate the efficiency of tax application for different flow conditions. There we compare both settings: when the vertical handover could take place and when it is forbidden.

Let us first observe some simulation trajectories, with the arrival rates $\lambda_1 = 4.5$ (arrivals/minute), $\lambda_2 = 3$ (arrivals/minute). The evolution of the total network load for one simulation is shown on Figure 4.8, with the horizontal line corresponding to the threshold value of total load $c_1 - \sqrt{c_1 c_2}$, above which taxes are needed to improve the QoS cost $C$.

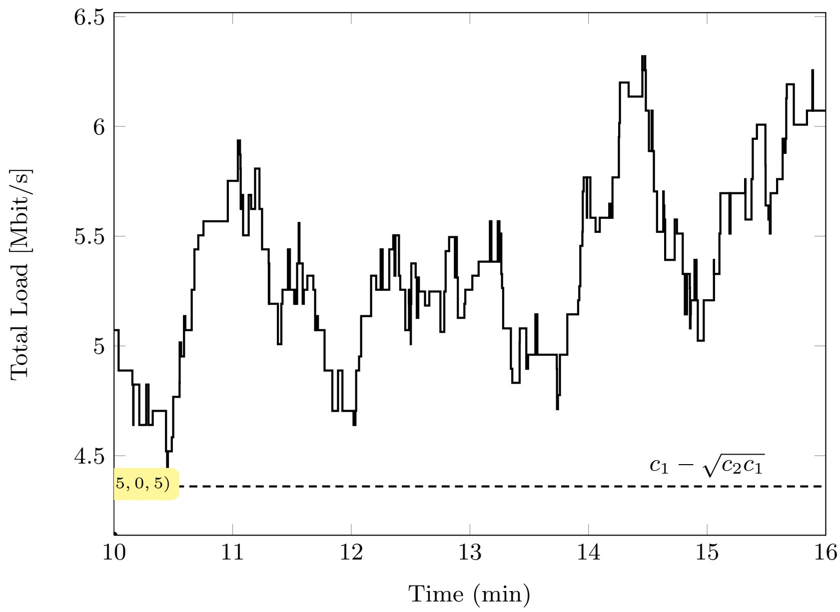


FIGURE 4.8: Total load versus time

Figure 4.9 displays the corresponding evolution over time of the Price of Anarchy for those three scenarios.

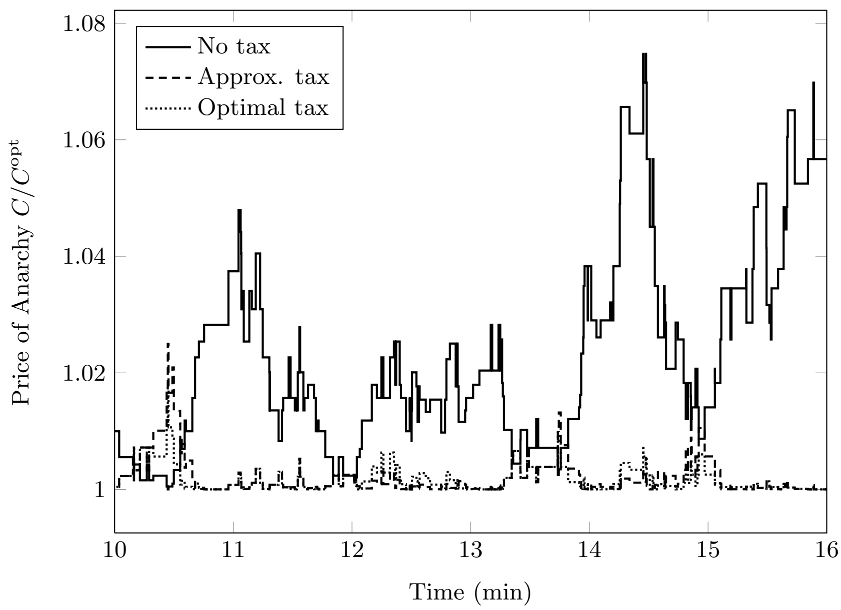


FIGURE 4.9: Price of anarchy versus time, without vertical handovers.

We notice from Figure 4.9 shows that taxation can yield significant performance gains, even if vertical handovers are not allowed (i.e., users do not constantly adapt to the changes in QoS conditions). Interestingly, we remark that the optimal tax does not always imply the lowest total cost: the total cost with that tax being sometimes even higher than without any tax. We have to recall here that there are several differences between our mathematical model and the simulation model considered in this section. Notably, we do not allow here users to switch networks, which can lead to the following situation. Consider some moment of time when the total flow suddenly falls below the $c_1 - \sqrt{c_1 c_2}$ threshold; the optimal flow in the network 2 then equals zero (see (4.16)). But in general more users (among those still in the system) had chosen network 2 when a tax was previously applied on network 1, hence the no-tax case leads temporarily to a situation closer to the optimal one. In other words, our simulation system without vertical handovers shows some inertia: the flow distribution cannot instantly change when QoS conditions evolve. This situation occurs in Figure 4.9 around $t = 10$ minutes for example, and similar cases (when demand suddenly drops and inertia impacts the outcome) occur around time $t = 14$ and $t = 15$ minutes even if demand remains above $c_1 - \sqrt{c_1 c_2}$.

Those phenomena being highlighted on one trajectory, we now turn our attention to their statistical impact, through extensively many simulations of the same scenarios. The results of these repeated simulations are presented on Figures 4.10-4.13, when the total average load $\frac{\bar{D}}{c_1+c_2}$ varies, with $\bar{D} = \frac{\lambda_1}{\mu_1}\epsilon_1 + \frac{\lambda_2}{\mu_2}\epsilon_2$. For comparison aims, we consider two different ratios between the arrival rates of users of both classes, namely we used $\lambda_1/\lambda_2 = 0.6$ and $\lambda_1/\lambda_2 = 1.5$.

We first notice from Figures 4.10-4.13 that the no-tax curve has a form similar to the one predicted by the theoretical study in [66]. On those figures, we also depicted the demand thresholds corresponding to some blocking rates values (proportion of users rejected due to lack of capacity). Since wireless systems are designed to have low blocking rates (below 1%), those values show that the range in which we expect some performance gain lies between 0 and 0.8.

We remark that when $\lambda_1/\lambda_2 = 0.6$, the curves corresponding to the optimal tax and the approximate tax appear to be very close to each other. It comes from the fact that the flow from delay-sensitive class-1 users is relatively small, thus in the majority of cases the optimal tax has the same value as the approximate one.

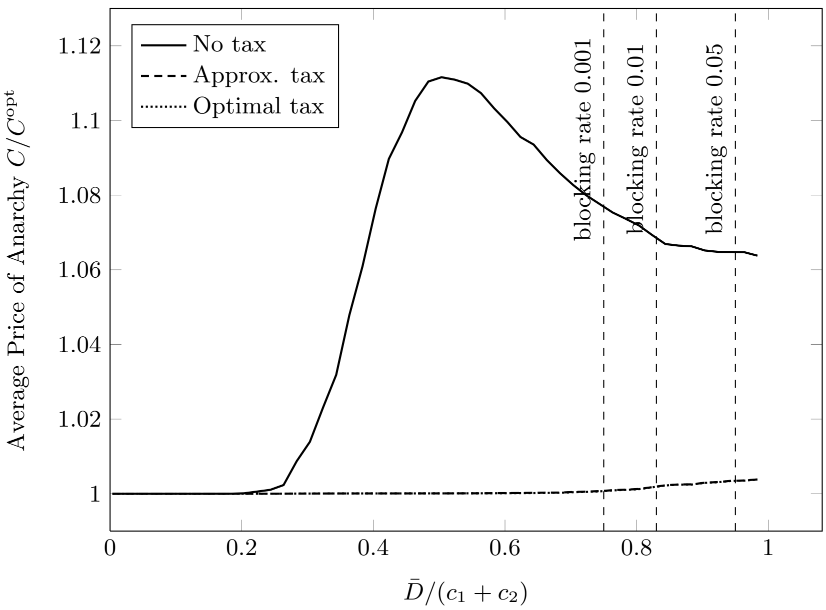


FIGURE 4.10: Average PoA versus load with $\lambda_1/\lambda_2 = 0.6$ with vertical handovers.

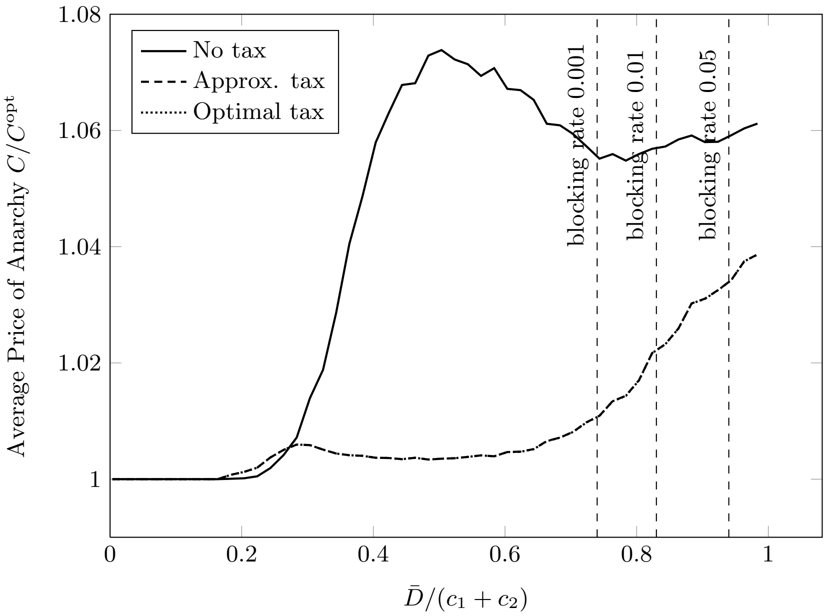


FIGURE 4.11: Average PoA versus load with $\lambda_1/\lambda_2 = 0.6$ without vertical handovers.

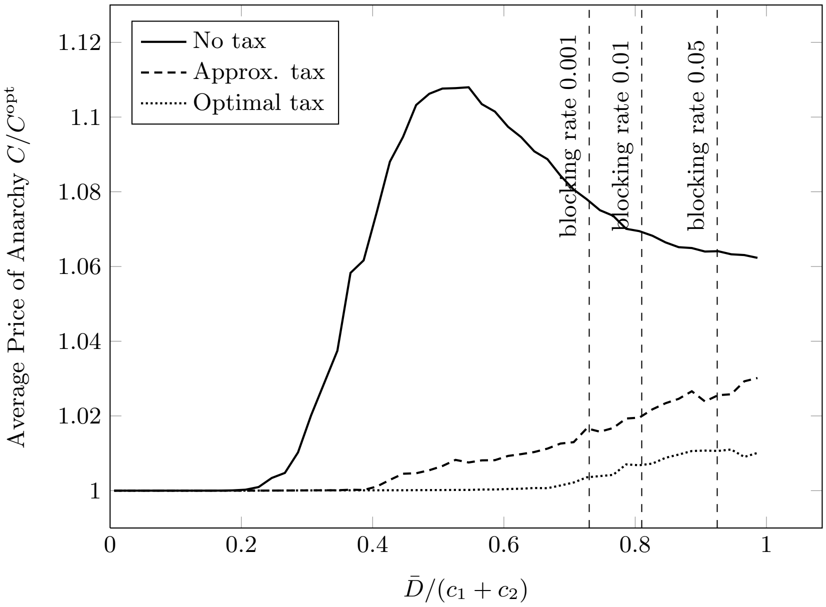


FIGURE 4.12: Average PoA versus load with $\lambda_1/\lambda_2 = 1.5$ with vertical handovers.

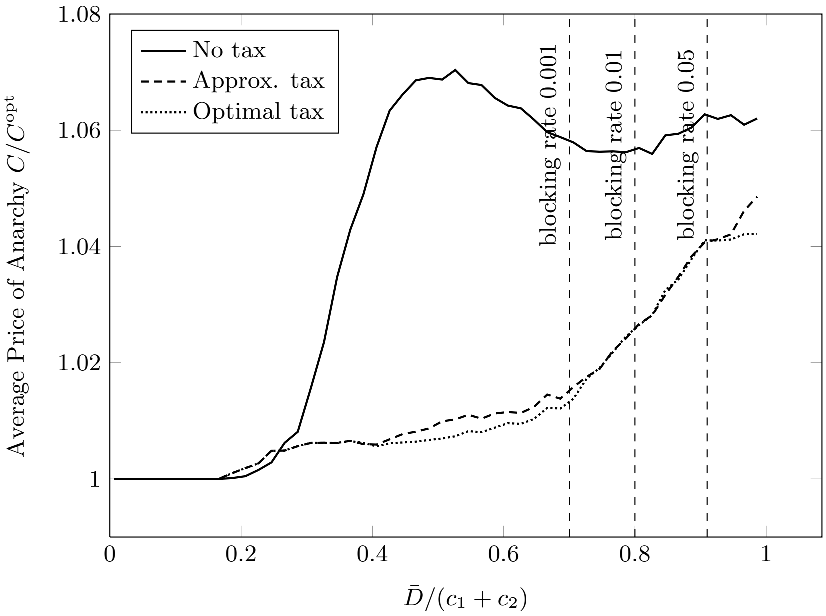


FIGURE 4.13: Average PoA versus load with $\lambda_1/\lambda_2 = 1.5$ without vertical handovers.

When vertical handovers are permitted, we observe that the PoA of the system applying optimal taxes is very close to the optimal situation - the PoA does not rise above 1.01, for both considered ratios between the arrival rates. For the approximate tax the PoA can reach 1.03, which is still significantly lower than the PoA of the no tax case (that goes up to 1.10). We also observe a significative influence on efficiency, of the presence of vertical handovers. As mentioned before, prohibiting handovers prevents the system from balancing rapidly the load among networks, implying larger costs.

A curious phenomenon worth mentioning from Figures 4.11 and 4.13 is the small range of average total load for which the average PoA of the no-tax case is lower than for the case with taxes (load values between 0.2 and 0.3). This implies that the cases explained similarly on the single trajectory before (Figure 4.9) are not so rare in that case. Indeed, as a result of total load being low, at very few moments of time the load goes above $c_1 - \sqrt{c_1 c_2}$, which causes the taxes introduction, deterring new entrants to use network 1. But this situation does not hold for a long time: quite soon the load goes below the threshold value, and because of switches being forbidden, the flow in the first network stays bigger than zero, causing inefficiency. Nevertheless, this inefficiency range remains small, and the PoA difference is limited, so this does not question the gain of our incentive mechanism. In our simulations, the taxation approach appears to be most effective for average total loads above 30% (for the considered simulation parameters) of the total capacity, and the highest efficiency gain is reached around loads corresponding to the 50% of the total capacity.

Finally, we present now two counterparts for the Price of Anarchy in our model. First Figure 4.6 shows how many more users the operator could serve if using network resources in an optimal way for the same total cost, compared to the case when he does not influence users behavior. In a somehow similar way, Figure 4.7 indicates the capacity (or investment) reduction that would lead to an unchanged total cost, just because of effective resource management. These two values are comparable to the Price of Anarchy, but have the advantage of being convertible into monetary gains, probably more appealing to network providers. As an example, we can see on Figure 4.10 that for an average network load of 0.5, the PoA is around 1.12. From Figures 4.6-4.7, we deduce that the system could handle about 5% more demand, or a capacity reduction of about 5%, for the same cost perceived by users.

## 4.6 Summary

In this chapter we have considered the inefficiency of selfish user behavior in heterogeneous wireless systems. Using the theoretical results of [34], which prove the existence of optimal taxes for our scenario, we derive an analytical expression of the optimal incentive (tax) for the case when the number of access points and user classes are arbitrary. We have showed that the "cost" of inefficiency of users allocation can have monetary equivalents.

We tried the proposed taxation policy in a realistic scenario, where not all information about users is available. In this case the provider has to compute an approximate tax, based on statistical information from his previous experience/history. We found that in the simple case of two access points being collocated, the gap between the performance of the optimal tax and the approximate one is relatively small, which supports the application of the proposed model in practice.

Our taxation algorithm can be applied to crowded access points with big capacity, where the non-atomicity assumption will make sense. Based on statistical observation the provider may compute the taxes which to minimize the revenue variation, while still making users to distribut in an optimal way. Moreover, if provider will still observe deviation from the optimal distribution, he may introduce corrections in taxes in order to achieve the desirable users allocation.

Our model relies on some strong assumptions, one of which is the simple network topology–all networks being supposed to have the same coverage area. Note that the model is easy to extend to a more realistic setting, where coverage areas have only partial overlapping. For this case we could predict to have decreased PoA comparing to what we observed in the current study.

Additionally, the non-atomicity assumption significantly simplifies the analysis, however its validity becomes questionable if we consider small-cell networks with only a few users and bandwidth-consuming applications. Extending our work to the atomic case would thus be of high interest; in such a case the decisions made by users could involve attaching simultaneously to several networks and splitting the flows among them (benefiting from protocols such as MultiPath TCP).

Finally, our work did not consider the practical implementation aspects of our mechanism. Those of course need to be examined for our mechanism to be applicable. In particular, measuring precisely the congestion level at the access point, and transmitting this information to users so that they make their decisions, warrants specific investigations. Among the possible tools that can be used for the latter task, one can evoke the 802.21 standard [67] and the Generic Access Network techniques for the management of cross-technology handovers and the information diffusion to users.

# Chapter 5

# The higher level: competition among providers

In this Chapter we discuss the competition which arise between Internet access providers in wireless networks. Two main scenarios are covered: in the first part we study the case, when access points, belonging to different providers, cover the same area (or have overlapping) and compete for static users, and in the second part we look at the case, when access points do not have any overlapping area, and the competition arises due to the high users mobility.

For the first scenario we use the model studied in Chapter 3. We apply the prediction of load distribution (resulting from the competition between users) in order to estimate providers revenues. These revenues, which depend on providers prices, generate a simultaneous game.

For the second scenario we consider a vehicular network, where mobile users owning the necessary equipment are passing providers' access points in a sequence, and for each access point they make a decision whether to connect to it or not, given the price charged by provider. We start by description of a basic model, where users keep their pricing preferences unchanged and access points are already located on a highway. We further extend this model, by an assumption that mobile users do not pay the full price they could afford when they see the first access point. Through this assumption we aim to model a type of users, which change their pricing constraints when get some additional information. Finally, we take into consideration a negative externalities, which competing Internet access providers could pose on clients of each other, specifically QoS degradation due to interference. By this we include one more parameter in providers competition, which is the distance between the access points.

## 5.1 Providers competition: static vs mobile users

Internet access providers interaction plays a crucial role in determining the satisfaction level of users in terms of price and QoS. But the type of users willing to establish an Internet connection in its turn also impacts the way competition between providers is organized. The two users types we consider in this Chapter are :

1. *Static users*, which do not move, or their movement distance is negligible comparing to the size of access points' coverage areas
2. *Mobile users*, which do move from one access point to another, following their own aims or trying to reach a more suitable access point (as in [53])

We differentiate these two types of users, because it influences whether the competition between different access points arise or not. On Figure 5.1 we depicted topologies, where competition arise between providers for different users types. Obviously, when users are static and access points do not overlap - there is no direct competition between providers, because their pricing policies influence only users in their own coverage area, and these users can not migrate to other access point. In this case the decision of users is binary one: they simply choose whether to pay to the only access point they see or not. The situation changes, when access points overlap: then, the users in "competition zone" do have a choice between two access points, and thus the pricing policy of one provider influences the revenue of the other one, which leads to providers competition

With mobile users the model is more complicated: they could move in random directions and with random speeds, and thus we have to make a stochastical analysis in order to obtain an expectation of providers revenues. In what follows, we do not consider that users intentionally move from one access point to another: actually we assume that they are unaware about others access point location. For our study we consider a vehicular network model, since it allows us to make several simplifying assumptions about users mobility patterns.

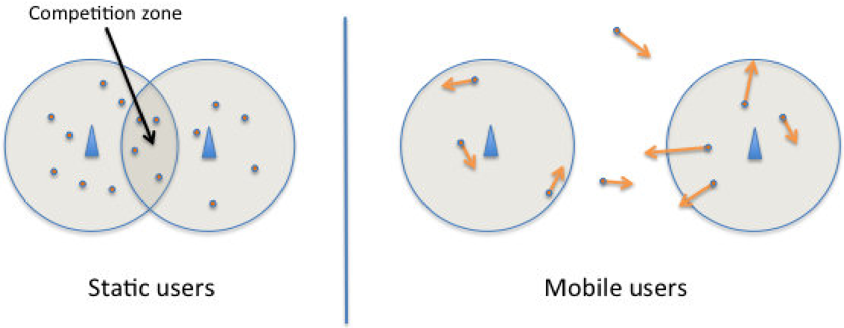


FIGURE 5.1: Static vs mobile users

## 5.2 Providers competition with static users

For the static users case we consider the model, described in Chapter 3. We study the higher-level of our game, that consists in provider(s) setting the prices $P_1$ and $P_2$ aiming to revenue maximization. Knowing the prices and access points ratings (which depends on latency users experienced in past), users make network selection decision. Since users arrivals and departure processes are random, we can only estimate the loads on each access point.

### 5.2.1 Model

We use Equations (3.5) and (3.7), which give a relation between the price profile $(P_1, P_2)$ and the average number of users on each network when the loyalty effect takes place:

$$
\begin{aligned}
n_i^* = n_i^* q &\left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*) - Q_i(n_i^*)(1+\alpha)} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*) - Q_i(n_i^*)(1+\alpha)} e^{-a(P_{\bar{i}} - P_i)}}\right] \\
&+ \left(\frac{\lambda}{1-q} - n_i^*\right) q \left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*)(1+\alpha) - Q_i(n_i^*)} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*)(1+\alpha) - Q_i(n_i^*)} e^{-a(P_{\bar{i}} - P_i)}}\right] \\
&+ \lambda \left[1 + K_i \ln \frac{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*) - Q_i(n_i^*)} e^{-b(P_{\bar{i}} - P_i)}}{1 + e^{Q_{\bar{i}}(n_{\bar{i}}^*) - Q_i(n_i^*)} e^{-a(P_{\bar{i}} - P_i)}}\right]
\end{aligned}
$$

First of all, we introduce an elastic (i.e., price-sensitive) demand, that prevents providers from charging the maximum possible price to maximize revenue. We assume here that the average number of user arrivals per time period depends on providers' prices as follows:

$$\lambda(P_1, P_2) = \lambda_{\max}\left(1 - \frac{P_1 + P_2}{P}\right), \tag{5.1}$$

where $P$ represents a price above which no one wants to use the network services, and $\lambda_{\max}$ is the number of users that would use the system if services were free. Note that the demand in (5.1) can be derived from classical linear demand functions, often used in the literature [68]: there could be two potential sources of demand, of the form $\lambda_1 = A - \eta P_1 + \alpha P_2$ and $\lambda_2 = B - \eta P_2 + \alpha P_1$, where $\eta$ (resp. $\alpha$) represent the direct (resp. indirect) effect of the price of an operator (resp., its competitor). Aggregating those demands, to consider that users enter the game based on those and then select a network, we obtain the form given in (5.1).

We now investigate how the prices $P_1$ and $P_2$ are fixed, depending on the relation between the network owners.

### 5.2.2 The noncooperative case: price competition

We first consider the situation where both networks are controlled by different entities (operators), that do not collaborate. The operators then play a pricing game to attract customers, but still making revenue. Their strategic choice is then driven by the maximization of their payoff.

The analysis of the two-player noncooperative game is then performed numerically: we look for a Nash equilibrium [27] $(P_1^*, P_2^*)$ as a price profile such that $P_i^*$ is the best that operator $i$ can play when its competitor sets $P_{\bar{i}}^*$ so as to maximize its revenue.

Figure 5.2 plots the best-response prices of both operators, for the parameter values given in Subsection 3.2.4 with loyalty effect, except that we consider the elastic demand case with $\lambda_{\max} = 200$ and $P = 20$, and we take price sensitivity values distributed over the interval $[a, b] = [0, 0.5]$. We observe that the game has a unique Nash equilibrium, an observation we also made for the other parameter values tried. Interestingly, remark in Figure 5.2 that best-response prices are not necessarily monotonous in the price of the competitor.

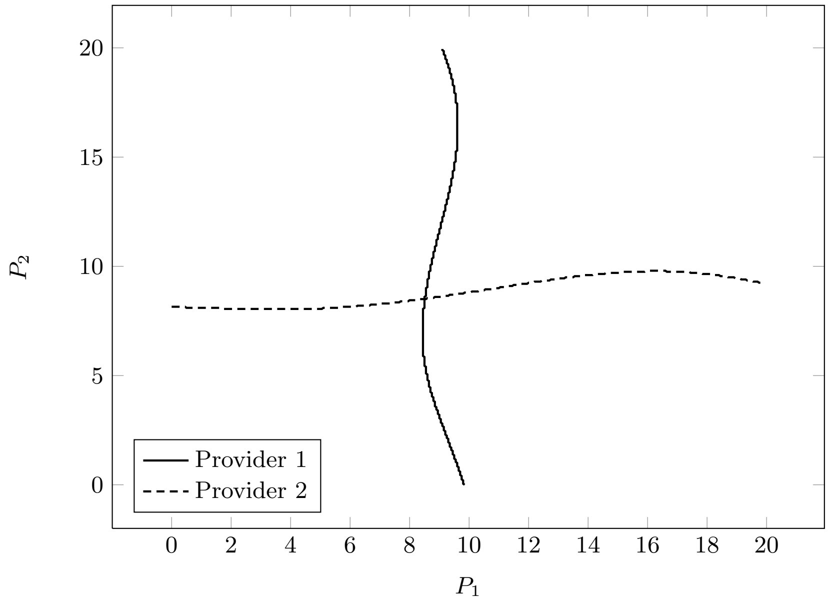


FIGURE 5.2: Best-response prices.

### 5.2.3 The cooperative case: a monopoly situation

We will also consider the situation where both networks are owned by the same entity (that then acts as a monopolist) fixing prices to maximize the global revenue $P_1 n_1^* + P_2 n_2^*$.

Equivalently, the same outcome is reached when two operators control one network each, but decide to collude and set prices to maximize the sum of their revenues, possibly through some agreements regarding the sharing of the benefits of collusion.

In what follows we compare the competitive and cooperative (monopoly) situations, in terms of different performance criteria. The parameters taken for the numerical results shown here are those of Subsection 3.2.4.When the ratio $C_2/C_1$ varies, we actually fix $C_1$ to 1000, and have $C_2$ vary from 100 to 1000.

### 5.2.4 Network prices

With a loyalty coefficient $\alpha$ varying from 0 to 6, we did not find any significant changes in equilibrium prices for both the monopoly and competition case. Figure 5.3 plots the equilibrium prices depending on the heterogeneity of the network (expressed by the ratio $C_2/C_1$). Here we observe that when heterogeneity decreases (i.e., $C_2/C_1$ gets closer to 1), prices for both settings converge to different values, and price in the competition case is lower than with a monopoly. In the competition situation, both providers tend to monotonically decrease their prices when $C_2$ increases. The price decrease for network 1 is obvious, because if the competitor increases the quality of its product, it then has to decrease price. For the second provider it is different: with the rise of the capacity of its network, operator is interested in attracting more users in the system, which it does by decreasing its price.

On the other hand, a monopolist is interested in charging a small price for the services in the network with bad capacity, and a high price for the network with better capacity, because in this case, a larger number of clients is attracted to the system (because of the total demand (5.1), that depends on the average of both prices, hence the low $P_2$), and because of congestion many of them will choose the largest (least congestion-sensitive) network, thus increasing the total revenue with a quite high $P_1$.

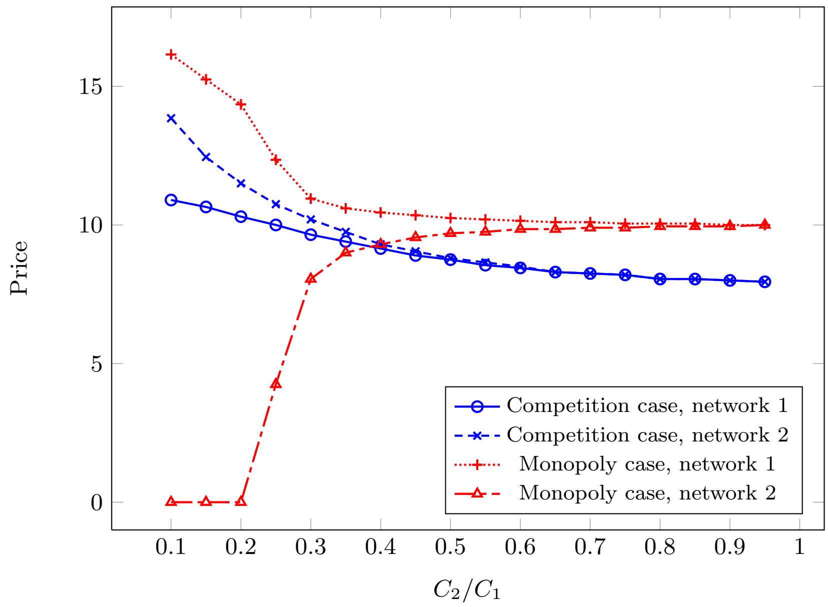


FIGURE 5.3: Equilibrium prices in each network, *versus* capacity heterogeneity $C_2/C_1$.

### 5.2.5 Number of users in each network

Very small changes of in the number of users in each network were noticed when the loyalty coefficient varies, which is consistent with the results of the previous section. In the same vein, the total number of users in the competition case appears to be bigger than in the monopoly case.

Figure 5.4 highlights the influence of the network heterogeneity on the user repartition among networks. As expected, in all cases the largest network attracts more users. In

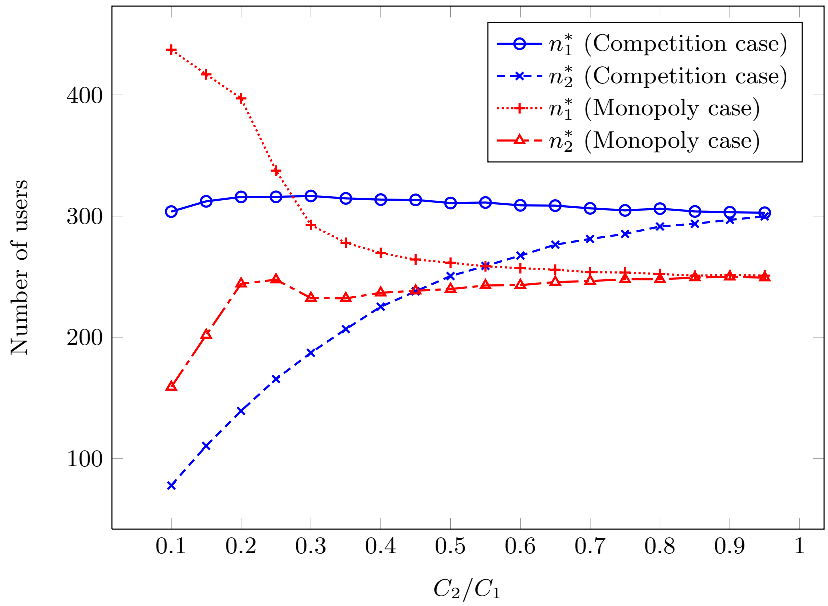


FIGURE 5.4: Equilibrium number of users in each network, *versus* capacity heterogeneity $C_2/C_1$.

accordance with Figure 5.3, when $C_2/C_1$ is close to 1, the total number of users in the competitive case is higher than in the monopoly case for each network.

### 5.2.6 Distribution of user sensitivities to prices among networks

It is interesting to see how a user's sensitivity to prices influences her network choices. This is illustrated in Figure 5.5, where the average sensitivity to price of users selecting each network is plotted.

We observe that the monopoly leads to a strong discrimination of users according to their price sensitivity: when the system is very heterogeneous ($C_2/C_1$ small), only users with a very low price sensitivity choose network 1 (that is the most expensive one but also the one with the best QoS). Note that the tendency is inverted for the competition case (network 1 tends to be chosen by less price-sensitive users than network 2), but the difference is much smaller. This can also be an argument in favor of the competition situation: the monopoly may lead to strong inequalities among users, where only "rich" users will benefit from a very good QoS.

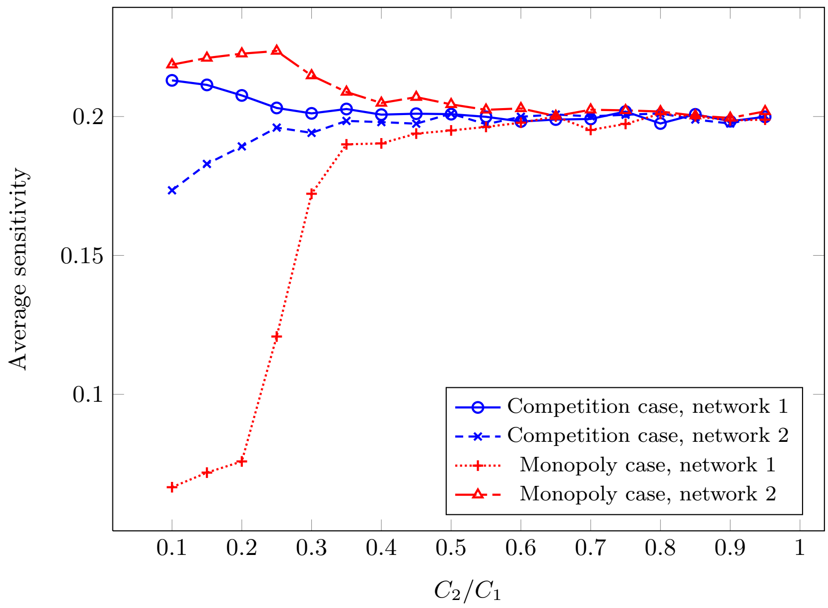


FIGURE 5.5: Average user sensitivities of users in each network, *versus* capacity heterogeneity $C_2/C_1$.

### 5.2.7 Energy consumption

Finally, we focus on the energetic performance of the competitive *versus* monopolistic situations. Figure 5.6 displays the average user's energy consumption (AEC) dynamics depending on the loyalty effect parameter $\alpha$. We compute the AEC as the average

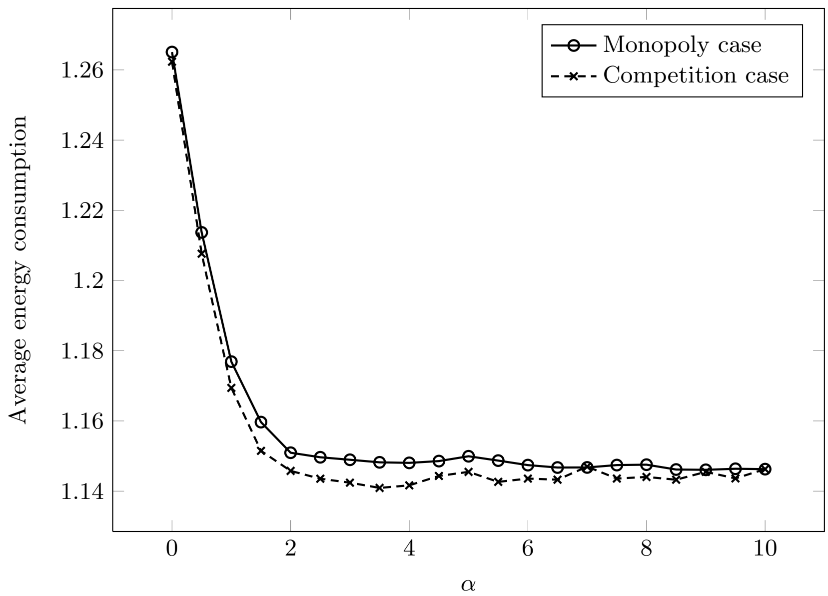


FIGURE 5.6: Energy consumption for different cases

value of $e_{i(j)} + v\mathbb{1}_{\{j \in N^{t-1}_{\bar{i}(j)}\}}$, where $e_i$ is the energy that a user consumes per time slot when connected to network $i$, $i(j)$ is the network chosen by $j$ at the current time slot, and $v$ is the energy cost of a handover, that takes place if the user was attached to the other network at the previous time slot. The following values have been considered here: $e_1 = 1, e_2 = 1.3, \lambda_{\max} = 200, P = 20, q = 0.2, v = 0.3$.As expected, for both cases the AEC value decreases when the loyalty effect becomes more significant, mainly because of the decrease in the number of handovers. We also notice a slightly smaller energy consumption in the competitive case with respect to the monopolistic one, but with small differences (less than 1%).

## 5.3 Providers competition with highly mobile users

For the mobile users scenario we decided to study a simplified model of a vehicular highway network. In the analysis of this type of networks, we can restrict our study only to two direction of users movement. Additionally we assume that all users move with the same speed, which also could be justified: there is a speed limit almost on every highway (with exception such as German autobahns) and the actual speeds of cars vary in some range below this limit value. These two assumption allow us to simplify a lot the general mobile users model and make an analytical traction possible.

### 5.3.1 Vehicular networks background

The constant increase in the number of cars traveling along the roads worldwide calls for effective means to improve the road safety and the efficiency of the overall transportation infrastructure. To this end, the research community, the industries and the governments all over the world are investing much of their efforts and money on the development of integrated Intelligent Transportation Systems (ITS) based on wireless communication networks allowing vehicles, equipment on the road, service centers and intelligent sensors to exchange information in a prompt and cost effective way. In this scenario, vehicles are geared with wireless communication hardware, often referred to as On Board Units (OBUs), to support communication with other vehicles (Vehicle-to-Vehicle, V2V) and with road infrastructure (Vehicle-to-Infrastructure, V2I). In this last case, the devices composing the roadside infrastructure are often called RoadSide Units (RSUs).

A broad classification of the applications which are enabled by vehicular networks can be found in [69] where a distinction is made between applications targeting safety, transport efficiency, and information/entertainment. Safety applications include, as an example, collision warning services, transport efficiency application may include lane merging assistance, and navigation services, whereas information/entertainment application range from file sharing among vehicles to Internet access on the move.

In this work, we focus on the vehicle-to-infrastructure (V2I) communication paradigm for VANETs to support content distribution to moving vehicles. Namely, we consider the case where multiple content providers coexist and compete in a given geographical area. Each content provider owns a physical infrastructure of RSUs which she uses to *sell* contents to moving vehicles. Content provider/RSU owners compete by adapting their pricing strategies with the selfish objective to maximize their own revenues. In such a scenario, we ask ourselves the following simple question: if competing providers wish to select the pricing strategy in order to provide or collect data to/from passing vehicles, what kind of strategies should they follow? The answer is far from being trivial as it predictably depends on several factors including the vehicles' willingness to pay, the traffic densities, the configuration of the physical networks of RSUs, and the strategic interaction among the content providers.

The design of efficient V2I and V2V networks has already attracted much attention within the research community. Most of the work generally targets the design and optimization of communication protocols to be used in vehicular networks. As an example, the optimization of V2I segment is targeted in [70] where the focus in on uplink and downlink packet scheduling techniques. Along the same lines, Yang *et al.* study in [71] the applicability and performance of IEEE 802.16 for the communication between groups of vehicles and an RSU.

V2V communications are addressed in [72–74]. In [72] a Medium Access Control (MAC) protocol is proposed to support reliable communication among vehicles. The work in [74] proposes a protocol framework to support the dissemination of warning messages in V2V, whereas the use of V2V communications to support proactive data monitoring in urban environments in studied in [73].

In the field of V2I networks, besides the work on protocol design/optimization, it is worth mentioning the research field targeting the optimal design of the roadside infrastructure. In this case, the goal is to optimize the deployment of the RSUs with respect to specific objectives which are generally related to the coverage ratio of vehicles. Trullols *et al.* [75] propose three formulations for the the deployment problem as a Maximum Coverage Problem, Knapsack Problem, and Maximum Coverage with Time Threshold Problem, respectively; heuristics based on local-search and greedy approaches are then introduce to get suboptimal solutions. Along the same lines, Cavalcante *et al.* [76] focus on the Maximum Coverage with Time Threshold Problem and propose a genetic algorithm to solve it. Yan *et al.*. [77] study the very same RSU deployment problem in case the candidate sites for deployment are limited to the intersections between crossing roads. The interested reader may refer to [78] and references therein for a more comprehensive description on the general problem of RSU deployment. Different from the aforementioned work which assumes one central entity to optimize the RSU deployment, [79] studies the competitive scenario where different network operators compete in the deployment of their respective RSUs by resorting to a non-cooperative game. Spatial positioning games are also proposed in [56] for generic wireless access networks.

Game theory has been used to evaluate the strategic interaction between the different agents in vehicular networks [80]. In [81], the authors introduce a stochastic game among OBUs which compete to get service from shared RSUs. Nyiato *et al.* propose in [82] a hierarchical game framework to capture the competition of different actors; besides OBUs and RSUs, the concept of Transit Service Provider is used to model an entity which manages groups of vehicles and is in charge of minimizing the total cost to support streaming application to its vehicles while meeting the application QoS requirement. The available bandwidth at each RSU can be split in reserved bandwidth and on-demand bandwidth. OBUs make short-term decisions between on-demand and reserved bandwidth (if available), TSPs decides what kind of bandwidth split to purchase from different RSUs along the road, whereas Network Service Providers owning RSUs set their price for on-demand bandwidth to maximize their revenues. Differently, in [83] a coalition formation game among RSU is analyzed, with the aim of better exploiting V2V communications for data dissemination.

The matter of pricing in generic wireless access networks is largely debated in the literature. Reference [55] provides a nice overview on pricing problems in wireless networks, and further analyze a specific case where two wireless Internet service providers compete on prices, one owning a WiMAX-based infrastructure and the other running a WiFi-based infrastructure. Differently from previously mentioned literature, in this work we focus on price competition between network operators for V2I networks, which is, to the best of our knowledge, a novel issue. Even if V2I networks bear some similarities with generic wireless access networks, there are distinctive features which make the pricing problem worth analyzing; in generic wireless access networks, the network operator competition is generally on the "common" users, that is, those users which fall in the coverage area of the competing network providers. In other words, there is actually a competition only if the coverage areas of the network providers (partially) overlap as in [55]. Users themselves tend to choose the network operator which maximizes some quality measure as in [12]. On the other side, in V2I networks competition may arise due to vehicles mobility even if the coverage areas of competing RSUs are not overlapping, since if a RSU does not serve a moving vehicle in its own coverage range, the very same user can be served later by competing operators.

### 5.3.2 Basic model

#### 5.3.2.1 Usage scenario

We consider two Internet access providers (labeled by 1 and 2), competing to attract users on a stretch of a highway. They offer the possibility to access the Internet through Road Side Units, which allows cheaper or better QoS than the other available cellular networks. (Note that we ignore vehicle-to-vehicle communications in this paper.) We assume that each provider has already deployed one RSU –on different locations along the road–, and that both RSU are identical; we denote their individual goodput (or capacity) by $c$. (Note that this model easily extends to the case when providers own disjoint "connectivity regions", each one made of several RSUs and with total service capacity $c$.)

Since both providers' RSU are at different locations, vehicles taking the road in one direction first enter the coverage area of Provider 1's RSU, while those traveling in the opposite direction first see Provider 2. We denote by $\rho_j$, $j = 1, 2$ the average number of commuters per time unit that first enter Provider $j$'s coverage area; they will cross the competitor's coverage area afterwards (since we are considering only one road).

Each user wants to download data files, for an average volume per user (assumed independent of the travel direction) normalized to 1 without loss of generality; the potential demand (in volume) from users seeing Provider $j$ first thus also equals $\rho_j$. In this paper, we treat those average loads as static values, i.e. we do not model the time variations of the load. Moreover, we assume that the coverage area size of RSUs and the vehicles' speed do not constrain the transfers: if a RSU's capacity exceeds its (average) load, all requests are successfully served.

Each provider $j = 1, 2$ chooses the (flat-rate) price $p_j$ to charge for the connection service. To model heterogeneity among users, we assume that only a proportion $w(p)$ of users accept to pay a unit price $p$ for the service (this being independent of the download volume). As a result, if Provider $j$ sets his price to $p_j$, the users who first enter Provider $j$'s service area generate a demand (again, per time unit, and treated as static) of $w(p_j)\rho_j$. Note that we are assuming here that users do not try to anticipate the price set by the next provider: when a user first sees an RSU access offer, she responds to it as if there were no other RSU afterwards.

Figure 5.7 summarizes that scenario in terms of demand flows. The total potential demand (volume per time unit) $\rho_j$ from users seeing Provider $j$ can be decomposed into:

1. users accepting the price $p_j$ and being served by Provider $j$;
2. users accepting the price $p_j$ and being rejected due to the RSU capacity limit (forming a spillover flow $\rho_j^{\mathrm{sp}}$ heading to the competitor's RSU);
3. and users refusing the price $p_j$ (forming a flow $\rho_j^{\mathrm{ref}}$ heading to the competitor's RSU).

The two latter flows then enter the coverage area of the competing provider, where they can be served or not. In the latter case, we denote the corresponding (unserved) demand by $\rho_j^{us}$. Note that we assume users keep the same willingness-to-pay for the service when they enter the second RSU coverage area.

#### 5.3.2.2 Mathematical formulation

We now give analytical expressions for the different demand components, using the RSU capacity $c$ and the willingness-to-pay function $w(\cdot)$. In the whole paper, $w(\cdot)$ is assumed continuous and non-increasing, and such that $w(0) = 1$ and $w(p_{\max}) = 0$ for some

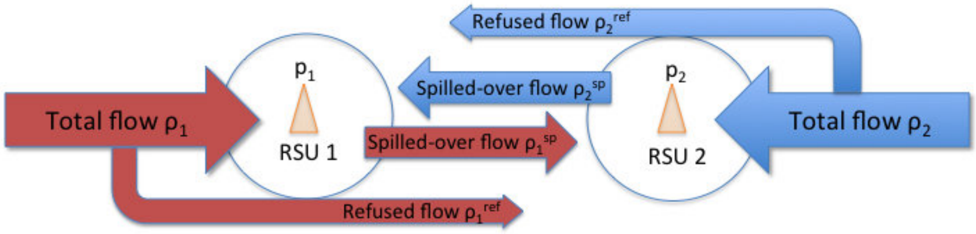


FIGURE 5.7: Flows involved in the model: among the total potential demand $\rho_j$ seeing Provider $j$ first, we distinguish $\rho_j^{\mathrm{sp}}$ (demand from users agreeing to pay $p_j$, but not served by this provider), $\rho_j^{\mathrm{ref}}$ (demand from users refusing to pay $p_j$).

$p_{\max} > 0$. If the quality of the alternative cellular access (say, 4G) is sufficient, the price $p_{\max}$ may be interpreted as the unit price for that cellular service: above $p_{\max}$, users have no interest to use an RSU-based access.

The demand submitted to Provider $j$ comes from three different types of users:

1. those seeing Provider $j$ first, and accepting to pay the proposed price $p_j$, hence issuing a total demand
$$w(p_j)\rho_j;$$
2. those seeing Provider $k \neq j$ (the competing provider) first, who refused to pay $p_k$ but would accept the price $p_j$, forming a total demand level (smaller than $\rho_k^{\mathrm{ref}}$, and null when $p_k \leq p_j$)
$$\rho_k[w(p_j) - w(p_k)]^+,$$
where $x^+ := \max(0, x)$ for $x \in \mathbb{R}$.
3. and those seeing Provider $k$ first, who agreed to pay $p_k$ but were rejected because of Provider $k$'s limited capacity, and who also agree to pay $p_j$, for a total demand
$$\min\left(1, \frac{w(p_j)}{w(p_k)}\right)\rho_k^{\mathrm{sp}},$$
where $\rho_k^{\mathrm{sp}}$ is the part of the demand $w(p_k)\rho_k$ that is spilled over by Provider $k$.

The total demand $\rho_j^{\mathrm{T}}(p_j, p_k)$ for Provider $j$ then equals the sum of the aforementioned components:

$$\rho_j^{\mathrm{T}}(p_j, p_k) := \\ w(p_j)\rho_j + \rho_k[w(p_j) - w(p_k)]^+ + \min\left(1, \tfrac{w(p_j)}{w(p_k)}\right)\rho_k^{\mathrm{sp}}$$

Note the dependance in both prices, although for simplicity we will sometimes just write $\rho_j^{\mathrm{T}}$ when there is no ambiguity.

When the total demand at an RSU exceeds its capacity, some requests are rejected: we assume the RSU serves users up to its capacity level, and the rejected requests are selected randomly among all requests. This leads to an identical probability of success $P_j$ for each request submitted to Provider $j$, that is simply given by

$$P_j = \min\left(1, \frac{c}{\rho_j^{\mathrm{T}}}\right) \tag{5.2}$$

so that the served traffic at RSU $j$ equals $\rho_j^{\mathrm{T}} P_j = \min(c, \rho_j^{\mathrm{T}})$. Again, the probability $P_j$ depends on the price vector $(p_i, p_j)$. The corresponding revenue of Provider $j$ is then

$$R_j = p_j \min[c, \rho_j^{\mathrm{T}}(p_j, p_k)]. \tag{5.3}$$

The traffic $\rho_j^{\mathrm{sp}}$ spilled over by Provider $j$ (and that will then enter the competitor's coverage area) also depends on both prices through the probability $P_j$, and equals

$$\rho_j^{\mathrm{sp}} = w(p_j)\rho_j(1 - P_j), \tag{5.4}$$

with

$$P_j = \min\left(1, \frac{c}{w(p_j)\rho_j + [w(p_j) - w(p_k)]^+\rho_k + \min[1, \frac{w(p_j)}{w(p_k)}]\rho_k^{\mathrm{sp}}}\right). \tag{5.5}$$

Remark that for a given price configuration $(p_1, p_2)$, the success probabilities $P_1$ and $P_2$ are the solution of a fixed-point system, since the success probability $P_j$ of Provider $j$ depends on the spillover demand $\rho_k^{\mathrm{sp}}$ and thus on $P_k$, that itself depends on $\rho_j^{\mathrm{sp}}$ and hence on $P_j$. More specifically, assuming without loss of generality that $p_1 \geq p_2$, those success probabilities should satisfy

$$\begin{cases} P_1 = \min\left(1, \frac{c}{w(p_1)(\rho_1+\rho_2)-w(p_1)\rho_2 P_2}\right) \\ P_2 = \min\left(1, \frac{c}{w(p_2)(\rho_1+\rho_2)-w(p_1)\rho_1 P_1}\right). \end{cases} \tag{5.6}$$

**Proposition 5.1.** *For any price vector $(p_1, p_2)$, the system (5.18) has a unique solution $(P_1, P_2)$.*

*Proof.* We again assume without loss of generality that $p_1 \geq p_2$. Since the right-hand sides of the equations in (5.18) are continuous in $(P_1, P_2)$ and fall in the interval $[0, 1]$, Brouwer's fixed-point theorem [84] guarantees the existence of a solution to the system.

To establish uniqueness, remark that $P_2$ is uniquely defined by $P_1$ through the second equation in (5.18), so $(P_1, P_2)$ is unique if $P_1$ is unique. But $P_1$ is a solution in $[0, 1]$ of the fixed-point equation $x = g(x)$ with

$$g(x) := \min\left(1, \frac{1}{a + b - b\min\left(1, \frac{1}{a+b+\epsilon-ax}\right)}\right),$$

where $a = \frac{w(p_1)\rho_1}{c}$, $b = \frac{w(p_1)\rho_2}{c}$, and $\epsilon = \frac{(w(p_2)-w(p_1))(\rho_1+\rho_2)}{c}$ are all positive constants; we also assume $a > 0$ and $b > 0$ otherwise the problem is trivial. As a combination of two functions for the form $x \mapsto \min\left(1, \frac{1}{K_1-K_2 x}\right)$, $g$ is continuous, nondecreasing, strictly increasing only on an interval $[0, \bar{x}]$ (if any) –it is in addition convex on that interval–, and constant for $x \geq \bar{x}$ (note we can have $\bar{x} = 0$ or $\bar{x} \geq 1$).

Assume $g(x) = x$ has a solution $\tilde{x} \in (0, \bar{x}]$. Then $g$ is left-differentiable at $\tilde{x}$, and

$$g'(\tilde{x}) = \frac{\tilde{x}^2 ab}{(a + b + \epsilon - a\tilde{x})^2} \leq \frac{\tilde{x}^2 a}{(a + b + \epsilon - a\tilde{x})} \tag{5.7}$$

where we used the fact that $\tilde{x} \leq 1$ (as a fixed point of $g$). Moreover, since $\tilde{x}$ is in the domain where $g$ is strictly increasing we have $\eta := \frac{1}{a+b+\epsilon-a\tilde{x}} \leq 1$ on one hand, and $\tilde{x} = \frac{1}{a+b-b\eta}$ on the other side. Their combination yields $\tilde{x} \leq \frac{1}{a}$ and finally

$$g'(\tilde{x}) \leq \tilde{x} \leq 1.$$

Remark also that $g'(\tilde{x}) < 1$ if $\tilde{x} < 1$. We finally use the fact that $g(0) > 0$ to conclude that the curve $y = g(x)$ cannot meet the diagonal $y = x$ more than once: assume two intersection points $\tilde{x}_1 < \tilde{x}_2$, then $g'(\tilde{x}_1) < 1$ thus the curves cross at $\tilde{x}_1$, another intersection point $\tilde{x}$ would imply $g'(\tilde{x}_2) > 1$ (recall $g$ is convex when strictly increasing), a contradiction. Hence the uniqueness of the fixed point and of the solution to (5.18). □

We can also establish continuity properties for the solution of (5.18), which will be used in the remainder of this paper.

**Proposition 5.2.** *The success probability pair* $(P_1, P_2)$ *is continuous in the price profile* $(p_1, p_2)$.

*Proof.* For a given price profile $(p_1, p_2)$, the solution $(P_1, P_2)$ of (5.18) can also be seen as a solution of the minimization problem

$$\min_{(P_1,P_2)\in[0,1]^2} \left(P_1 - \min\left(1, \frac{c}{w(p_1)(\rho_1+\rho_2) - w(p_1)\rho_2 P_2}\right)\right)^2 + \left(P_2 - \min\left(1, \frac{c}{w(p_2)(\rho_1+\rho_2) - w(p_1)\rho_1 P_1}\right)\right)^2,$$

where the objective function is jointly continuous in $(P_1, P_2)$ and $(p_1, p_2)$. From the Theorem of the Maximum [85], the mapping of prices $(p_1, p_2)$ into the corresponding set of solutions $(P_1(p_1, p_2), P_2(p_1, p_2))$ is an upper hemicontinuous correspondence. From the uniqueness result above, that correspondence is single-valued and hence continuous. We therefore have continuity for $p_1 \geq p_2$ and for $p_2 \geq p_1$ (exchanging the roles of providers), hence continuity for all price profiles. □

### 5.3.3 Revenue-maximizing price for a provider

In this section we assume that provider $k$ has already chosen his price, while provider $j$ has to set his. We describe the revenue function of provider $j$ for different scenarios, and provide an example when the willingness-to-pay function is linear.

In this whole section, we only consider prices $p$ such that $w(p) > 0$, since a larger price would yield no revenue to the provider setting it.

We first establish a monotonicity result, that will be useful in the rest of the analysis.

**Lemma 5.3.** *The total demand* $\rho_j^T$ *of provider* $j$ *is a continuous function of his price* $p_j$*; that function is in addition non-increasing while provider* $j$ *is not saturated (i.e., while* $\rho_j^T < c$*).*

*Proof.* Recall that

$$\begin{aligned}\rho_j^{\mathrm{T}}(p_j,p_k) &= w(p_j)\rho_j+\rho_k[w(p_j)-w(p_k)]^+\\ &\qquad +\min\left(w(p_k),w(p_j)\right)\rho_k(1-P_k).\end{aligned}$$

The components of the first line are trivially continuous and non-increasing in $p_j$ with our assumptions on $w(\cdot)$.

The continuity of $\rho_j^{\mathrm{T}}(p_j,p_k)$ follows from the continuity of $P_k$ in the price vector $(p_j,p_k)$, established in the previous section. To establish monotonicity, we distinguish two cases.

- If $p_j \le p_k$, we show that the success probability $P_k$ is non-decreasing in $p_j$: applying System (5.18) (with $k=1$, $j=2$) we get that $P_k$ is the solution of the fixed-point equation $x=g(x)$, where the function $g$ can be written as

$$g(x)=\min\left(1,\frac{c}{w(p_k)\rho_k+w(p_k)\rho_j\left[1-\frac{c}{w(p_j)(\rho_j+\rho_k)-w(p_k)\rho_k x}\right]^+}\right).$$

We then remark that, all else being equal, $g(x)$ is non-decreasing in $p_j$, so the solution $P_k$ of the fixed-point equation $g(x)=x$ is also non-decreasing in $p_j$.

As a result, when $p_k \ge p_j$ the component $\min\left(w(p_k),w(p_j)\right)\rho_k(1-P_k)$ decreases with $p_j$, and so does $\rho_j^{\mathrm{T}}$.

- If $p_k < p_j$, then we have

$$\rho_j^{\mathrm{T}}(p_j,p_k)=w(p_j)\rho_j+w(p_j)\rho_k(1-P_k).$$

When $\rho_k^{\mathrm{T}} < c$, then $P_k = 1$ and $\rho_j^{\mathrm{T}}$ is non-increasing in $p_j$.

Now if $\rho_k^{\mathrm{T}} > c$ then from System (5.18) (this time with $k=2$, $j=1$), we have $w(p_k)(\rho_j+\rho_k)-w(p_j)\rho_j P_j > c$ and

$$\begin{aligned}\rho_j^{\mathrm{T}}(p_j,p_k) &= w(p_j)(\rho_j+\rho_k)\\ &\qquad +w(p_j)\rho_k\frac{c}{w(p_k)(\rho_j+\rho_k)-w(p_j)\rho_j P_j}.\end{aligned}$$

Assuming that provider $j$ is not saturated, $P_j = 1$ and thus $\rho_j^{\mathrm{T}} = f(w(p_j))$ with

$$f(x) := x(\rho_j + \rho_k) - x\rho_k \frac{c}{w(p_k)(\rho_j + \rho_k) - x\rho_j}.$$

But $f$ is a non-decreasing function of $x$ when $x \in [0, w(p_k)]$ and $w(p_k)(\rho_j+\rho_k)-x\rho_j > c$: differentiating we indeed get

$$\begin{aligned}
\frac{f'(x)}{\rho_j + \rho_k} &= 1 - \rho_k c \frac{w(p_k)}{(w(p_k)(\rho_j + \rho_k) - x\rho_j)^2} \\
&\geq 1 - \frac{\rho_k w(p_k)}{w(p_k)(\rho_j + \rho_k) - x\rho_j} \\
&\geq 1 - \frac{\rho_k w(p_k)}{w(p_k)(\rho_j + \rho_k) - w(p_k)\rho_j} \geq 0,
\end{aligned}$$

where we used $w(p_k)(\rho_j + \rho_k) - x\rho_j > c$ in the second line, and $x \leq w(p_k)$ in the last one. The non-increasingness of $\rho_j^{\mathrm{T}} = f(w(p_j))$ in $p_j$ then comes from that of $w(\cdot)$. $\square$

#### 5.3.3.1 Capacity saturation price

For further analysis, we define the *capacity saturation price* of a provider, that depends on the price of his competitor.

**Definition 5.4.** The capacity saturation price of provider $j$ is

$$p_j^{\mathrm{c}}(p_k) := \inf\{p \in [0, p_{\max}] : \rho_j^{\mathrm{T}}(p, p_k) < c\}.$$

Since $\rho_j^{\mathrm{T}}(p_{\max}, p_k) = 0$, for all $p_k$ we have $p_j^{\mathrm{c}}(p_k) < p_{\max}$.

Additionally, Lemma 1 implies that if $p_j^{\mathrm{c}} > 0$, then $\rho_j^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) = c$ and $p_j \leq p_j^{\mathrm{c}} \Rightarrow \rho_j^{\mathrm{T}} \geq c$.

We now provide analytical expressions for that price, in the case when $\rho_j^{\mathrm{T}}(p, p_k) \geq c$. In that case $\rho_j^{\mathrm{T}}(p_j^{\mathrm{c}}) = c$, hence $p_j^{\mathrm{c}}$ satisfies

$$\begin{cases} w(p_j)\rho_j + \rho_k[w(p_j) - w(p_k)]^+ + \min\left(1, \frac{w(p_j)}{w(p_k)}\right)\rho_k^{\mathrm{sp}} = c, \\ \rho_k^{\mathrm{sp}} = w(p_k)\rho_k \left[\frac{[w(p_k)-w(p_j)]^+\rho_j + w(p_k)\rho_k - c}{[w(p_k)-w(p_j)]^+\rho_j + w(p_k)\rho_k}\right]^+. \end{cases} \tag{5.8}$$

Let us define a generalized inverse of $w$, as

$$W(q) := \inf\{p \in [0, p_{\max}] : w(p) < q\}. \tag{5.9}$$

For $q \leq 1$, $W(q)$ is the maximum price that can be accepted by a proportion $q$ of users.

Then the capacity saturation price can be computed as follows. (The proof is omitted due to space constraints.)

– If $w(p_k) \leq \min[\frac{c}{\rho_j}, \frac{c}{\rho_k}]$, then $p_j^{\mathrm{c}} = W\left(\frac{c+w(p_k)\rho_k}{\rho_j+\rho_k}\right)$.

– If $\frac{c}{\rho_k} < w(p_k) \leq \frac{2c}{\rho_j+\rho_k}$, then $p_j^{\mathrm{c}} = W\left(\frac{2c}{\rho_j+\rho_k}\right)$.

– If $\frac{c}{\rho_j} < w(p_k) \leq \frac{2c}{\rho_j+\rho_k}$, then $p_j^{\mathrm{c}} = W\left(\frac{c}{\rho_j}\right)$.

– If $w(p_k) > \frac{2c}{\rho_j+\rho_k}$, then $p_j^{\mathrm{c}} = W(x)$, with $x$ the unique solution in $[0, w(p_k)]$ of

$$-x^2\rho_j + x\left(w(p_k)(\rho_j + \rho_k) - c\frac{\rho_k - \rho_j}{\rho_j + \rho_k}\right) - cw(p_k) = 0.$$

#### 5.3.3.2 Piece-wise expression of the revenue function

The revenue function of each provider $j$ is continuous in his price (from the continuity of $\rho_j^{\mathrm{T}}$ and of $P_j$), and can be expressed analytically on different segments.

1. When $p_j \leq p_j^{\mathrm{c}}(p_k)$ (or $\rho_j^{\mathrm{T}}(p_j) \geq c$ when $p_j^{\mathrm{c}}(p_k) > 0$), the RSU capacity of provider $j$ is saturated, and thus his revenue is simply

$$R_j = p_j c. \tag{5.10}$$

   This is the case in Figure 5.9 for prices $p_j$ below approximately 2.5. Figure 5.11 shows that for these prices, provider $j$ spills some flow over toward provider $k$.

   Above $p_j^{\mathrm{c}}$, provider $j$ is not saturated anymore. Then if the total demand $\rho_k(p_j^{\mathrm{c}}, p_k)$ of the competitor is strictly below $c$, we have a price range with no provider being saturated. In that case we have no spillover demand, and the revenue of provider $j$ is:

$$R_j = p_j\left(w(p_j)\rho_j + [w(p_j) - w(p_k)]^+\rho_k\right).$$

   If $p_j^{\mathrm{c}} < p_k$, then we remark that necessarily $\rho_k^{\mathrm{T}}(p_k, p_k) \leq c$, i.e., we meet the price of the opponent provider before he gets saturated. Indeed, at $(p_j^{\mathrm{c}}, p_k)$ provider $j$ does not spill traffic over to $k$, thus $\rho_k^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) = \rho_k w(p_k)$ (where we also used the fact that $w(p_k) \leq w(p_j^{\mathrm{c}})$). From the definition of $p_j^{\mathrm{c}}$, provider $j$ is not saturated at $(p_k, p_k)$, so that $\rho_k^{\mathrm{T}}(p_k, p_k) = \rho_k w(p_k) = \rho_k^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) \leq c$. Summarizing, we then have the two following segments.

2. If $p_j^{\mathrm{c}} < p_k$ and $\rho_k^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) \leq c$, then for $p_j \in [p_j^{\mathrm{c}}, p_k]$

$$R_j = p_j \left(w(p_j)(\rho_j + \rho_k) - w(p_k)\rho_k\right).$$

   Remark that this segment is empty if $p_j^{\mathrm{c}} \geq p_k$ or $\rho_k^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) \geq c$. Figure 5.11 illustrates that when $p_j$ is between approximately 2.5 and 4, provider $j$ serves his own traffic and the one from the competitor who refused the price $p_k$ but agrees to pay $p_j$.

3. If $\rho_k^{\mathrm{T}}(p_j^{\mathrm{c}}, p_k) \leq c$, then for $p_j \geq \max(p_j^{\mathrm{c}}, p_k)$ we have while provider $k$ remains unsaturated:

$$R_j = p_j w(p_j).$$

4. Now if $\rho_k(p_j^{\mathrm{c}}, p_k) > c$, then provider $k$ is saturated for $p_j \in [p_j^{\mathrm{c}}, p_{\max}]$ (which is easy to see since $j$ has no spillover traffic), and for $p_j \in [p_j^{\mathrm{c}}, p_k]$ we have

$$R_j = p_j \left(w(p_j)(\rho_j + \rho_k) - c\right).$$

   Remark that this segment appears only when both providers can be simultaneously saturated, a case not occurring in the example we display here.

5. There may be a price of provider $j$ larger than $p_k$, and above which the competitor gets saturated, so that provider $j$ may serve part of the traffic spilled over by $k$. In that case the revenue of provider $j$ is:

$$R_j = p_j \left(w(p_j)\rho_j + \frac{w(p_j)}{w(p_k)}\rho_k^{\mathrm{sp}}\right), \tag{5.11}$$

   where

$$\rho_k^{\mathrm{sp}} = w(p_k)\rho \frac{(w(p_k) - w(p_j))\rho_j + w(p_k)\rho_k + \rho_j^{\mathrm{sp}} - c}{(w(p_k) - w(p_j))\rho_j + w(p_k)\rho_k + \rho_j^{\mathrm{sp}}}. \tag{5.12}$$

   Figure 5.12 shows that provider $k$ gets saturated, and the spillover traffic is served partly by provider $j$ as illustrated in Figure 5.11.

Figure 5.8 illustrates those different zones for the special case $\rho_1 = \rho_2 = 11$, $c = 10$, and $w(p) = 1 - p/10$. Figure 5.9 shows the corresponding different segments for $R_j$

when $p_k = 4$, and Figure 5.10 for various prices $p_k$ of the competitor. We observe that a revenue-maximizing price can belong to different segments, depending on the competitor's price.

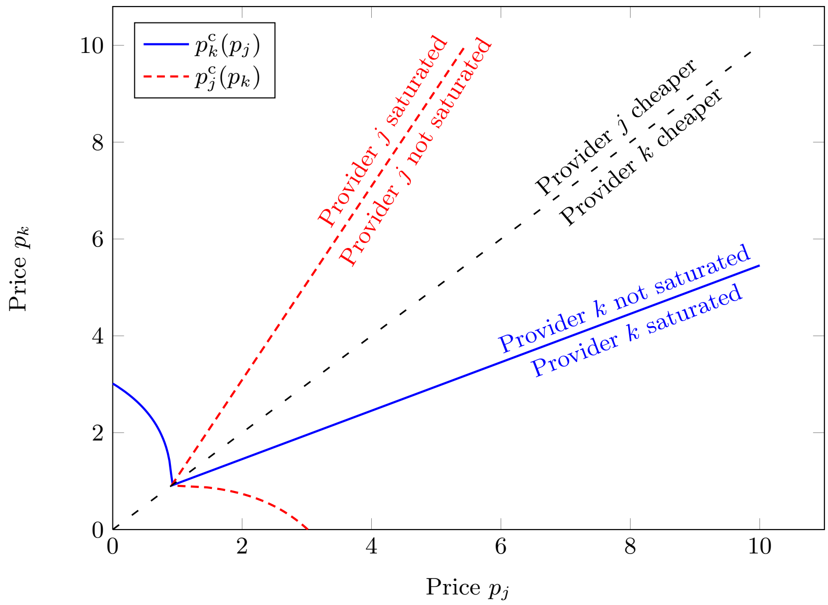


FIGURE 5.8: Capacity saturation prices, and the different zones where the expressions of revenues vary.

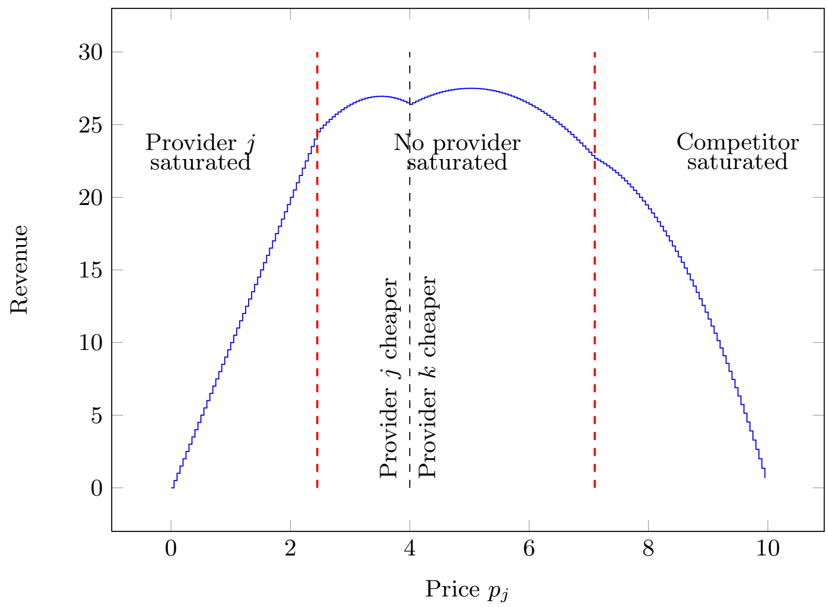


FIGURE 5.9: Revenue of provider $j$ as a function of his price $p_j$, for $p_k = 4$, $\rho_1 = \rho_2 = 11$, $c = 10$ and $w(p) = \frac{10-p}{10}$, illustrating the different segments.

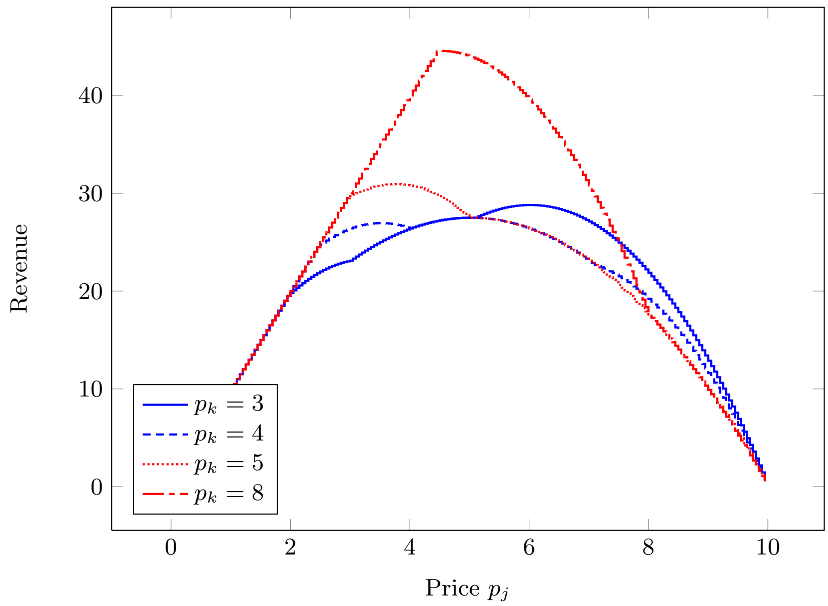


FIGURE 5.10: Revenue of provider $j$ vs his price for different $p_k$ values, when $w(p)$ is linear. The different segments correspond to the zones delimited in Figure 5.8 for each given $p_k$.

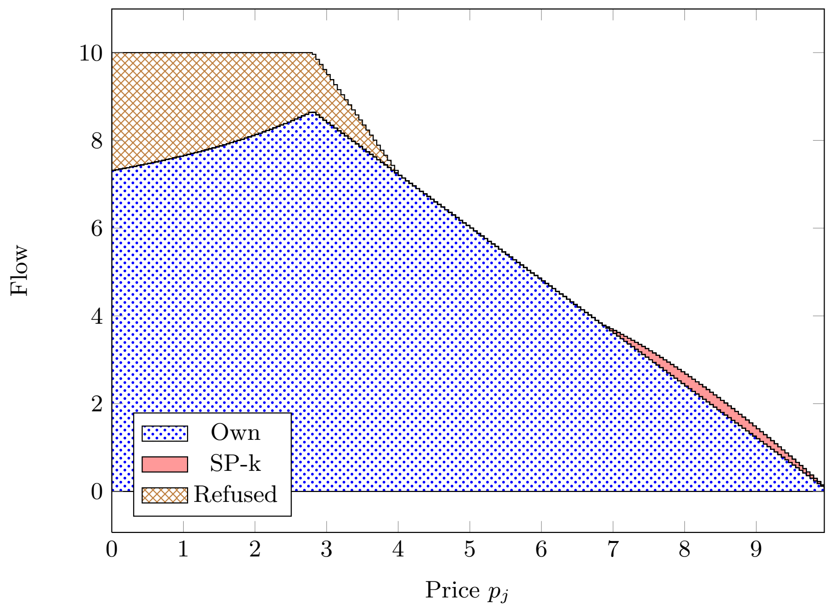


FIGURE 5.11: Flow served by provider $j$, for $p_k = 4$. "Own" denotes the part of original flow $\rho_j$ served by provider $j$, "SP-k" the part from users who agreed to pay $p_k$ but were unserved by $k$ due to capacity constraints, and "Refused" the part from users who refused to pay $p_k$

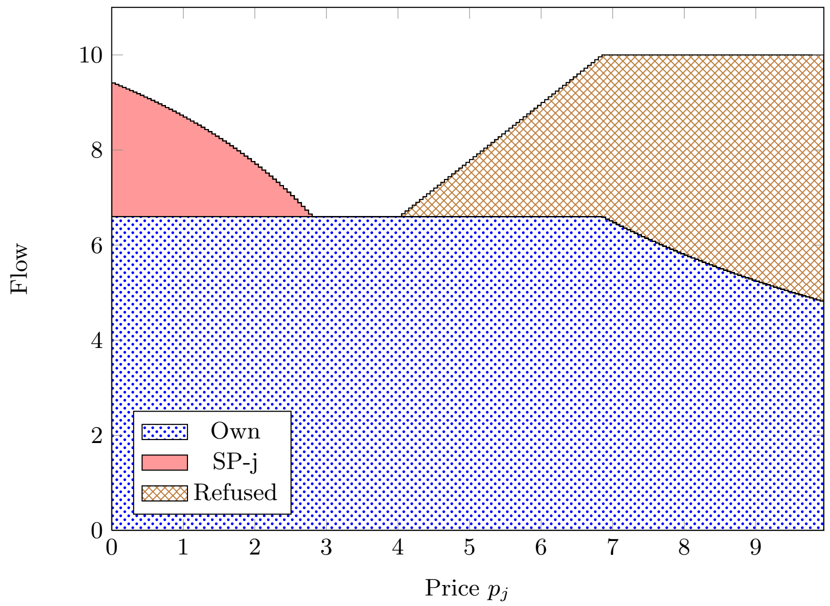


FIGURE 5.12: Flow served by provider $k$ for $p_k = 4$. "Own" denotes the part of the original flow $\rho_k$ served by provider $k$, "SP-j" the part from users agreeing to pay $p_j$ but unserved by $j$ due to capacity constraints, and "Refused" is the part of users refusing to pay $p_j$.

### 5.3.4 Providers pricing game

In this section we consider a non-cooperative game, where providers –the players– simultaneously choose their prices, trying to maximize their individual payoffs given by (5.3). Our aim is to find a Nash equilibrium (NE) of this game: a pair of prices $(\bar{p}_1, \bar{p}_2)$, such that no player could increase his revenue by unilaterally changing his price. Further, we investigate the situation where providers would decide to cooperate, trying to maximize the sum of their individual revenues (as a monopoly would do). We analyze how much the providers lose in terms of total revenue by refusing to cooperate.

Below is a more formal definition of the Nash equilibrium in the pricing game.

**Definition 5.5.** A pair of prices $(\bar{p}_1, \bar{p}_2)$ is a Nash equilibrium for the pricing game if

$$R_1(\bar{p}_1, \bar{p}_2) \geq R_1(p_1, \bar{p}_2) \text{ for all } p_1 \in (0, p_{\max}],$$
$$R_2(\bar{p}_1, \bar{p}_2) \geq R_2(\bar{p}_1, p_2) \text{ for all } p_2 \in (0, p_{\max}].$$

Nash equilibria can be interpreted as predictions for the outcome of the competition between selfish entities, assumed rational and taking decisions simultaneously.

#### 5.3.4.1 The case of large capacities

We first consider here that RSUs capacities are larger than the users flows ($c \geq \rho_j + \rho_k$). So, for any price pair RSUs capacities are not saturated and spillover traffic never appears.

Without loss of generality we consider that $\rho_1 = \gamma\rho_2 = \gamma\rho$, for $\gamma \in (0, 1]$. (The case $\gamma = 0$ is trivial and not considered here.) In all this subsection, we consider a linear willingness-to-pay function, i.e., $w(p) = 1 - p/p_{\max}$ for some $p_{\max} > 0$.

**Proposition 5.6.** *The pricing game has at maximum two equilibria:*

$$\begin{cases} \bar{p}_1 = \frac{p_{\max}(\gamma+1/2)}{2(1+\gamma)}, \\ \bar{p}_2 = \frac{p_{\max}}{2}, \end{cases}$$

*is an equilibrium for any* $\gamma \in (0, 1]$ *and*

$$\begin{cases} \bar{p}_1 = \frac{p_{\max}}{2}, \\ \bar{p}_2 = \frac{p_{\max}(1+1/2\gamma)}{2(1+\gamma)}, \end{cases}$$

*is an equilibrium only for* $\gamma \in [s, 1]$*, where* $s \approx 0.73$

*Proof.* To show that a pair of prices is a Nash equilibrium, we verify that no provider can increase his individual revenue by unilaterally changing his price. For the large capacities case, the revenue curves of both providers have only two different expressions, since RSUs are never saturated. Again, we consider two cases:

- First, when $p_1 \leq p_2$ we have to verify that the price profile

$$\begin{cases} \bar{p}_1 = \frac{p_{\max}(\gamma+1/2)}{2(1+\gamma)}, \\ \bar{p}_2 = \frac{p_{\max}}{2} \end{cases}$$

is an equilibrium.

For Provider 1, we thus have to check whether the revenue-maximizing price $p_1 = p_{\max}/2$ in the zone where $p_1 \leq p_2$ could be better for him, or equivalently, determine for which $\gamma$ we could have $R_1(p_1, \bar{p}_2) > R_1(\bar{p}_1, \bar{p}_2)$, in which case the proposed profile is not an equilibrium. Taking the mathematical expressions:

$$\begin{aligned}
R_1(\bar{p}_1, \bar{p}_2) &= \frac{p_{\max}\rho(\gamma+1/2)^2}{4(1+\gamma)}, \\
R_1(p_1, \bar{p}_2) &= 1/4\rho\gamma p_{\max}.
\end{aligned}$$

We then observe that for any $\gamma \geq 0$, we have $(\gamma+1/2)^2 \geq \gamma(1+\gamma)$ and thus $R_1(p_1, \bar{p}_2) \leq R_1(\bar{p}_1, \bar{p}_2)$. Hence Provider 1 cannot increase his revenue by unilaterally changing his price.

For Provider 2, we similarly have to check, whether the peak price $p_2 = \frac{p_{\max}+\bar{p}_1\gamma}{2(1+\gamma)}$ in the zone where $p_1 \leq p_2$ could be preferable. Let us first rewrite

$$p_2 = \frac{p_{\max} + \bar{p}_1\gamma}{2(1+\gamma)} = \frac{p_{\max}(\gamma^2 + 5/2\gamma + 2)}{4(1+\gamma)^2}.$$

The revenue corresponding to this price is

$$\begin{aligned}
R_2(\bar{p}_1, p_2) &= \frac{p_{\max}(\gamma^2 + 5/2\gamma + 2)}{4(1+\gamma)^2} \times \\
&\quad \Big((1+\gamma)\rho(1 - \frac{(\gamma^2 + 5/2\gamma + 2)}{4(1+\gamma)^2}) \\
&\quad -\gamma\rho(1 - \frac{\gamma + 1/2}{2(1-\gamma)})\Big) \\
&= \frac{p_{\max}\rho(\gamma^2 + 5/2\gamma + 2)^2}{16(1+\gamma)^3},
\end{aligned}$$

while with $\bar{p}_2$ Provider 2 gets a revenue

$$R_2(\bar{p}_1, \bar{p}_2) = 1/4\rho p_{\max}. \tag{5.13}$$

Therefore the condition $R_2(\bar{p}_1, p_2) > R_2(\bar{p}_1, \bar{p}_2)$ is equivalent to

$$\begin{aligned}
(\gamma^2 + 5/2\gamma + 2)^2 &> 4(1+\gamma)^3 \\
\Leftrightarrow \gamma^4 + \gamma^3 - 7/4\gamma^2 - 2\gamma &> 0.
\end{aligned}$$

For $\gamma \in [0, 1]$ that last condition is never satisfied (the function being nonpositive), and thus Provider 2 cannot increase his revenue by unilaterally changing his price.

As a result, the proposed price profile $(\bar{p}_1, \bar{p}_2)$ is indeed a Nash equilibrium for all $\gamma \in (0, 1]$.

• We now turn our attention to the case when $p_1 > p_2$. As before, we have to verify whether

$$\begin{cases} \bar{p}_1 = \frac{p_{\max}}{2}, \\ \bar{p}_2 = \frac{p_{\max}(1+1/2\gamma)}{2(1+\gamma)}, \end{cases}$$

is a Nash equilibrium. Let us study whether Provider 1 has an incentive to change his price to $p_1 = \frac{\gamma p_{max}+\bar{p}_2}{2(1+\gamma})$ (his best price in the zone where he is cheaper than the competitor). Plugging the expression of $\bar{p}_2$ we have:

$$p_1 = \frac{2\gamma^2 + 5/2\gamma + 1}{4(1+\gamma)^2},$$

which would earn Provider 1 a revenue

$$\begin{aligned} R_1(p_1, \bar{p}_2) &= p_{\max}\rho\frac{2\gamma^2 + 5/2\gamma + 1}{4(1+\gamma)^2} \cdot \\ &\quad \Big(\frac{2\gamma^2 + 11/2\gamma + 3}{4(1+\gamma)} - \frac{3/2\gamma + 1}{2(1+\gamma)}\Big) \\ &= \frac{\rho p_{\max}(2\gamma^2 + 5/2\gamma + 1)^2}{16(1+\gamma)^3}, \end{aligned}$$

while under the proposed price profile he gets

$$R_1(\bar{p}_1, \bar{p}_2) = 1/4\rho p_{\max}\gamma.$$

Then we have $R_1(p_1, \bar{p}_2) > R_1(\bar{p}_1, \bar{p}_2)$ if and only if

$$\begin{aligned} (2\gamma^2 + 5/2\gamma + 1)^2 &> 4\gamma(1+\gamma)^3 \\ \Leftrightarrow 2\gamma^3 + 7/4\gamma^2 - \gamma - 1 &< 0. \end{aligned}$$

The polynomial expression above has a unique root $s \approx 0.73$: thus, for $\gamma < s$ Provider 1 could increase his revenue by changing his price, and $(\bar{p}_1, \bar{p}_2)$ is not an equilibrium. On the other hand, for $\gamma > s$, the price $\bar{p}_1$ is a best-response of Provider 1 to $\bar{p}_2$.

Finally, for Provider 2 we follow the same logic, investigating whether taking the optimal price above the price of Provider 1 could lead to a revenue increase:

$$\begin{aligned} R_2(\bar{p}_1, p_2) &= 1/4\rho p_{\max}, \\ R_2(\bar{p}_1, \bar{p}_2) &= \frac{p_{\max}\rho(1+1/2\gamma)^2}{4(1+\gamma)}. \end{aligned}$$

Observing that $(1+1/2\gamma)^2 > 1+\gamma$ for all $\gamma$, we deduce that $R_2(\bar{p}_1, p_2) \leq R_2(\bar{p}_1, \bar{p}_2)$, hence Provider 2 cannot increase his revenue by unilaterally changing his price.

*Thus an equilibrium with $p_1 > p_2$ exists only for $\gamma \in [s, 1]$.* □

Summarizing, we have:

1. When $p_1 \leq p_2$, the provider revenue functions are

$$\begin{cases} R_1 = p_1(w(p_1)\rho(1+\gamma) - w(p_2)\rho), \\ R_2 = p_2 w(p_2)\rho. \end{cases}$$

with an equilibrium pair of prices $\begin{cases} \bar{p}_1 = \frac{p_{\max}(\gamma+1/2)}{2(1+\gamma)}, \\ \bar{p}_2 = \frac{p_{\max}}{2}. \end{cases}$

The corresponding total revenue is

$$R = R_1 + R_2 = \frac{p_{\max}\rho(\gamma^2 + 2\gamma + 5/4)}{4(1+\gamma)}.$$

2. When $p_1 > p_2$, the revenue functions are:

$$\begin{cases} R_1 = p_1 w(p_1)\gamma\rho, \\ R_2 = p_2(w(p_2)\rho(1+\gamma) - w(p_1)\gamma\rho), \end{cases}$$

giving the equilibrium existing only for $\gamma \in [s, 1]$, where $s \approx 0.73$: $\begin{cases} \bar{p}_1 = \frac{p_{\max}}{2}, \\ \bar{p}_2 = \frac{p_{\max}(1+1/2\gamma)}{2(1+\gamma)}, \end{cases}$

with the corresponding total revenue

$$R = \frac{p_{\max}\rho(5/4\gamma^2 + 2\gamma + 1)}{4(1+\gamma)}.$$

We now compare the minimum total revenue in the duopoly case with the revenue a monopolist would obtain, to evaluate the cost of competition. Following the literature on the Price of Anarchy [86], we use the ratio between the total revenue in the worst-case Nash equilibrium and the monopoly total revenue as the cost measure.

It is easy to check, that the second Nash equilibria highlighted before –corresponding to the case $p_1 > p_2$– gives a lower total revenue if it exists.

**Proposition 5.7.** *The cost of competition is:*

$$\begin{cases} \frac{4(1+\gamma^3)}{(3+4\gamma)(\gamma^2+2\gamma+5/4)} & \textit{if } \gamma \in (0,s), \\ \frac{4(1+\gamma^3)}{(3+4\gamma)(5/4\gamma^2+2\gamma+1)} & \textit{if } \gamma \in [s,1]. \end{cases} \tag{5.14}$$

*Proof.* We first derive an expression for the optimal (for providers) revenue value, which is the maximum possible sum of their revenues (that we can reach by collaborating). We use a linear expression for the willingness-to-pay function $w(p) = \frac{p_{\max}-p}{p_{\max}}$ As previously, we consider two cases:

- First, when $p_1 \leq p_2$ the total revenue is

$$\begin{aligned} R &= R_1 + R_2 \\ &= p_1\Big(\frac{p_{\max}-p_1}{p_{\max}}(1+\gamma)\rho - \frac{p_{\max}-p_2}{p_{\max}}\rho\Big) + p_2\rho\frac{p_{\max}-p_2}{p_{\max}} \\ &= \frac{\rho}{p_{\max}}(p_1\gamma p_{\max} - p_1^2(1+\gamma) + p_1p_2 + p_{\max}p_2 - p_2^2). \end{aligned}$$

The necessary extremum condition are :

$$\begin{cases} \frac{\partial R}{\partial p_1} &= \frac{\rho}{p_{\max}}(\gamma p_{\max} - 2p_1(1+\gamma) + p_2) = 0, \\ \frac{\partial R}{\partial p_2} &= \frac{\rho}{p_{\max}}(p_1 + p_{\max} - p_2) = 0. \end{cases}$$

Therefore the prices maximizing the total revenue should satisfy

$$\begin{cases} p_1^{\text{opt}} &= \frac{\gamma p_{\max}+p_2^{\text{opt}}}{2(1+\gamma)}, \\ p_2^{\text{opt}} &= \frac{p_{\max}+p_1^{\text{opt}}}{2}. \end{cases}$$

Solving that system gives

$$\begin{cases} p_1^{\text{opt}} = \frac{p_{\max}(2\gamma+1)}{3+4\gamma}, \\ p_2^{\text{opt}} = \frac{p_{\max}(3\gamma+2)}{3+4\gamma}. \end{cases} \tag{5.15}$$

It is easy to verify that the sufficient conditions for this pair to be a maximum are also satisfied. The maximum total revenue with $p_1 \leq p_2$ is therefore

$$R = \frac{p_{\max}\rho(\gamma+1)^2}{3+4\gamma}.$$

- Let us now consider the case $p_1 > p_2$. The total revenue is:

$$\begin{aligned} R &= R_1 + R_2 \\ &= \frac{\rho}{p_{\max}}(-\gamma p_1^2 + p_2 p_{\max} - (1+\gamma)p_2^2 + p_1 p_2 \gamma + p_1 \gamma p_{\max}). \end{aligned}$$

The necessary extremum conditions are :

$$\begin{cases} \frac{\partial R}{\partial p_1} = \frac{\rho}{p_{\max}}(\gamma p_{\max} - 2p_1\gamma + p_2\gamma) = 0, \\ \frac{\partial R}{\partial p_2} = \frac{\rho}{p_{\max}}(p_1\gamma + p_{\max} - p_2 2(1+\gamma)) = 0, \end{cases}$$

leading to the system

$$\begin{cases} p_1^{\text{opt}} = \frac{p_{\max}+p_2^{\text{opt}}}{2}, \\ p_2^{\text{opt}} = \frac{p_{\max}+p_1^{\text{opt}}\gamma}{2(1+\gamma)}, \end{cases}$$

from which we get

$$\begin{cases} p_1^{\text{opt}} = \frac{p_{\max}(2\gamma+3)}{4+3\gamma}, \\ p_2^{\text{opt}} = \frac{p_{\max}(\gamma+2)}{4+3\gamma}. \end{cases}$$

Again, the sufficient maximality conditions are satisfied; the total revenue is:

$$R = \frac{p_{\max}\rho(\gamma+1)^2}{4+3\gamma}.$$

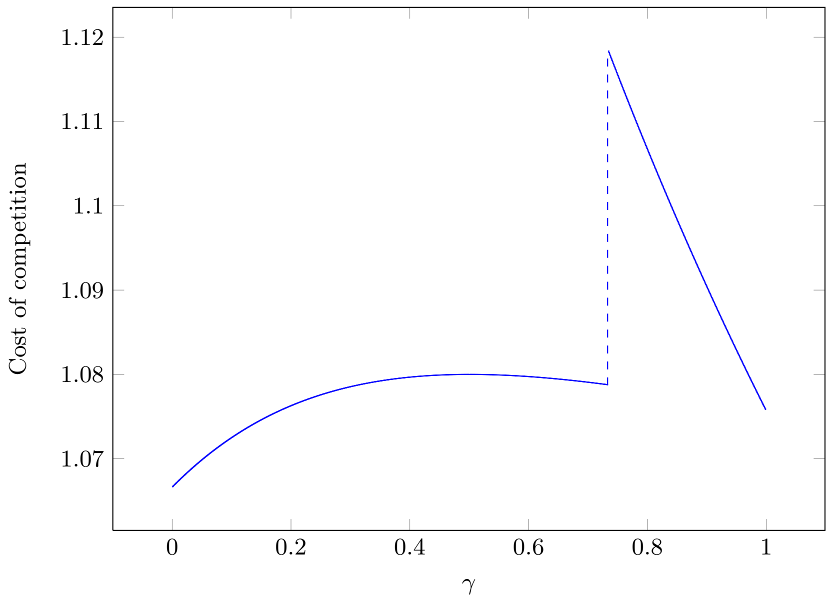


FIGURE 5.13: Cost of competition for large capacities in the heterogeneous-flows case ($\gamma$ small corresponds to high flow heterogeneity). The cost of competition function is discontinuous (because the least efficient equilibrium does not exist for all $\gamma$) and reaches its maximum at $\gamma = s$.

Comparing both cases, we find that for $\gamma \in (0, 1]$ the total revenue with $p_2 \geq p_1$ is larger than (or equal to) in the other case, meaning that the price profile (5.15) maximizes total revenue.

Then, we divide that revenue by the minimum equilibrium revenue (i.e., we compute the Price of Anarchy 5.14 for the game played among providers); it is easy to remark that the equilibrium yielding the smallest total revenue is the one with $p_1 > p_2$ (which exists only for $\gamma \in [s, 1]$, otherwise there is only one equilibrium).

$\square$

Remark that if we consider only the best-case Nash equilibrium (under a Price of Stability logic), then the first expression above applies for $\gamma \in [0, 1]$. Figure 5.13 shows the cost of competition of (5.14), that is maximum for $\gamma = s$, i.e., when the second candidate becomes actually an equilibrium.

#### 5.3.4.2 Homogeneous flows and arbitrary capacities

With arbitrary capacities, the model becomes intractable analytically. We treat here the special case when user flows are homogeneous, i.e., $\rho_1 = \rho_2 := \rho$. In that case, we can prove necessary conditions for a price profile to be an equilibrium.

**Proposition 5.8.** *If* $(\bar{p}_j, \bar{p}_k)$ *is an equilibrium, then*

$$\bar{p}_j > p_j^{\mathrm{c}}(\bar{p}_k),$$
$$\bar{p}_k > p_k^{\mathrm{c}}(\bar{p}_j).$$

*Proof.* We first prove that if at least one provider –say $j$– charges a price lower than or equal to his capacity saturation price, then the price profile is not an equilibrium. Assume that $(\bar{p}_j, \bar{p}_k)$ is an equilibrium, with $\bar{p}_j < p_j^{\mathrm{c}}(\bar{p}_k)$: then provider $j$ is saturated and gets revenue $\bar{p}_j c$. But deviating to $p_j^{\mathrm{c}}(\bar{p}_k)$ would improve his revenue to $R_j = p_j^{\mathrm{c}}(\bar{p}_k)c$, a contradiction.

Now we prove that there is no equilibrium where at least one provider charges his exact capacity saturation price. Again we assume that $(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j))$ is an equilibrium. From the result above we necessarily have $\bar{p}_j \geq p_j^{\mathrm{c}}(p_k^{\mathrm{c}}(\bar{p}_j))$, hence $\rho_j^{\mathrm{sp}} = 0$.

• We first show that $\bar{p}_j \geq p_k^{\mathrm{c}}(\bar{p}_j)$: if it were not the case ($\bar{p}_j < p_k^{\mathrm{c}}$, omitting writing $\bar{p}_j$ in the saturation price of $k$) then $w(\bar{p}_j) \geq w(p_k^{\mathrm{c}}(\bar{p}_j))$, yielding

$$\rho_k^{\mathrm{T}}(\bar{p}_j, p_k^{\mathrm{c}}) = w(p_k^{\mathrm{c}})\rho = c,$$
$$\rho_j^{\mathrm{T}}(\bar{p}_j, p_k^{\mathrm{c}}) = 2w(\bar{p}_j)\rho - w(p_k^{\mathrm{c}})\rho = 2w(\bar{p}_j)\rho - c \leq c.$$

This implies $w(\bar{p}_j) \leq w(p_k^{\mathrm{c}})$, therefore $w(\bar{p}_j) = w(p_k^{\mathrm{c}})$ thus $\rho_j^{\mathrm{T}}(\bar{p}_j, p_k^{\mathrm{c}}) = c$, yielding $\bar{p}_j \leq p_j^{\mathrm{c}}(p_k^{\mathrm{c}}(\bar{p}_j))$. Since the opposite inequality also holds we have $\bar{p}_j = p_j^{\mathrm{c}}(p_k^{\mathrm{c}}(\bar{p}_j))$, i.e., each provider charges his saturation price. We then deduce that they are equal, because they are both maximum prices such that $w(p)\rho = c$, which contradicts our assumption that $\bar{p}_j < p_k^{\mathrm{c}}(\bar{p}_j)$.

- Therefore $\bar{p}_j \geq p_k^{\mathrm{c}}(\bar{p}_j)$. Consider some $p > \bar{p}_j$; we have

$$R_j(p, p_k^{\mathrm{c}}(\bar{p}_j)) \quad = \quad pw(\bar{p}_j)\rho\Big(2 - \frac{c}{2w(p_k^{\mathrm{c}})\rho - w(\bar{p}_j)\rho}\Big).$$

We now prove that this revenue, as a function of $p$, has a positive right-derivative at $p = \bar{p}_j$. Differentiating, we get

$$R_j'(p, p_k^{\mathrm{c}}(\bar{p}_j)) = (pw'(p)\rho + w(p)\rho)\Big(2 - \frac{c}{2w(p_k^{\mathrm{c}})\rho - w(p)\rho}\Big) \\ -pw(p)\rho\frac{cw'(p)\rho}{(2w(p_k^{\mathrm{c}})\rho - w(p)\rho)^2}.$$

At $(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j))$ the flow of provider $k$ equals $c$:

$$\rho_k^{\mathrm{T}}(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j)) = w(p_k^{\mathrm{c}})\rho + (w(p_k^{\mathrm{c}})\rho - w(\bar{p}_j))\rho) = c,$$

which implies

$$R_j'(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j)) = w(\bar{p}_j)\rho + \rho\bar{p}_j w'(\bar{p}_j)(1 - w(\bar{p}_j)\rho/c). \tag{5.16}$$

If $w'(\bar{p}_j) = 0$, $R_j'(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j))$ is strictly positive. We now show it is also the case if $w'(\bar{p}_j) < 0$.

– First, if $\bar{p}_j > p_j^{\mathrm{c}}(p_k^{\mathrm{c}}(\bar{p}_j))$, then

$$R'{}_j(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j)) = w(\bar{p}_j)\rho + \rho\bar{p}_j w'(\bar{p}_j)(1 - w(\bar{p}_j)\rho/c) \\ > w(\bar{p}_j)\rho + \rho\bar{p}_j w'(\bar{p}_j).$$

  But as an equilibrium price, $\bar{p}_j$ should maximize the revenue of provider $j$ over $(p_j^{\mathrm{c}}(p_k^{\mathrm{c}}(\bar{p}_j)), p_{\max})$, and thus $\bar{p}_j$ should make the derivative of $R_j = pw(p)\rho$ equal to zero, giving $w(\bar{p}_j)\rho + \rho\bar{p}_j w'(\bar{p}_j) = 0$, which implies $R'{}_j(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j)) > 0$ .

– Second, if $\bar{p}_j = p_k^{\mathrm{c}}(\bar{p}_j)$, then $w(\bar{p}_j)\rho = c$ and the revenue function derivative in (5.16) is equal to $c > 0$.

Thus by increasing his price Provider $j$ could increase his revenue, therefore $(\bar{p}_j, p_k^{\mathrm{c}}(\bar{p}_j))$ is not an equilibrium. □

Formally, in order to show that a pair of prices is an equilibrium, we have to compare the revenue they yield with the maximum revenues in all other zones (as defined in Subsection 5.3.3.2) for each provider. Proposition 5.8 reduces this search, to zones where both prices are strictly above capacity saturation prices. It can be easily checked that situations where providers charge equal prices cannot be equilibria. Therefore the equilibrium candidates remaining can be characterized by

$$\begin{cases} p_1 \neq p_2, \\ p_1 > p_1^{\mathrm{c}}(p_2); p_2 > p_2^{\mathrm{c}}(p_1), \\ R_1'(p_1, p_2) = 0; R_2'(p_1, p_2) = 0, \end{cases}$$

To show that such pairs are indeed Nash equilibria, we have to compare the revenue they give with the maximum revenue in other segments.

For a linear willingness-to-pay function, the system above only leaves two candidates

$$(\bar{p}_1, \bar{p}_2) \in \{(1/2p_{\max}, 3/8p_{\max}), (3/8p_{\max}, 1/2p_{\max})\} \tag{5.17}$$

Numerically, we found that these two pairs of prices are indeed equilibria only when $\rho/c \leq t$, with $t \approx 1.23$. Figure 5.14 shows the best-response curves when $\rho/c = t$.

#### 5.3.4.3 The cost of ignoring competition

In our scenario, providers may not be aware of the presence of each other (especially if they are located far from each other), and thus do not play a noncooperative game on prices. In that case each provider would treat users seeing him first the same way as he treats those coming from the competitor's direction. We estimate here the cost of this ignorance in terms of revenue loss.

Assume that each provider $j$ believes his total flow to be $\rho_1 + \rho_2$ independently of $p_j$, and therefore simply selects his price so as to maximize $p_j \min(c, (\rho_1 + \rho_2)w(p_j))$, leading to a situation where $p_1 = p_2 = \arg\max_p p \min(c, (\rho_1 + \rho_2)w(p))$. Let us consider a situation where each provider believes he is the only one serving users, so that he can set his price to the monopoly price ($p_{\max}/2$ for linear willingness-to-pay functions, if capacities are sufficiently large). Then in practice each provider will serve only some of the users seeing

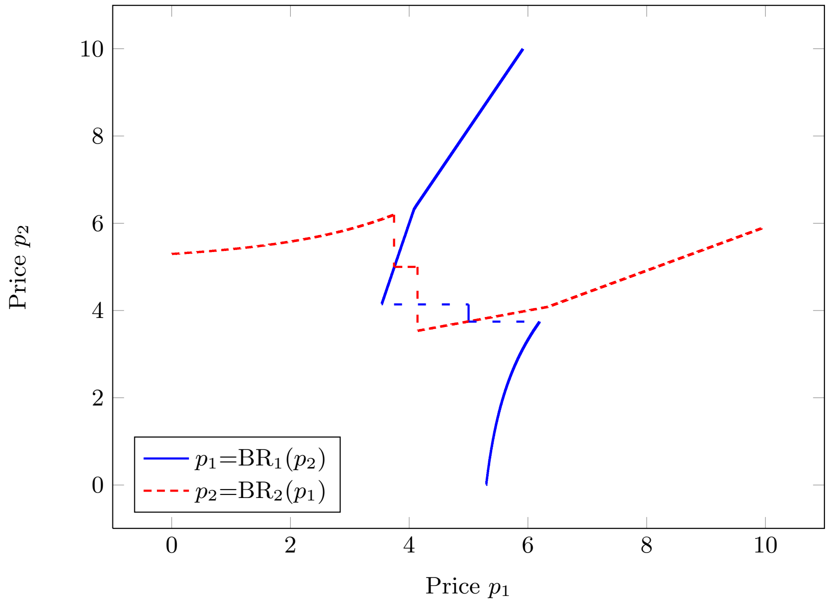


FIGURE 5.14: Best-response curves in the general case, when $p_{\max} = 10$, for the maximum value of $\rho/c$ such that an equilibrium exists. At the equilibrium point $(3.75, 5)$, the best-response function of Provider 2 is discontinuous: that provider is indifferent between the maximum in the segment where Provider 1 is saturated ($\approx 6.2$) and the maximum in the segment where provider 1 is not saturated (3.75).

him first. But from such a situation, one provider could lower his price to serve some of the traffic that refused to pay the price of the competitor, and increase his revenue. We compute here the amount of extra revenue that a provider could get by making this price move.

When $p_j \leq p_k$, the revenue of Provider $j$ with symmetric flows ($\rho_1 = \rho_2 = \rho$) is

$$R_j = p_j(2w(p_j)\rho - w(p_k)\rho).$$

The optimal price when the willingness-to-pay function $w(\cdot)$ is linear equals $3/8p_{\max}$, leading to $R_j = 3/8p_{\max}(5/4\rho - 1/2\rho) = 9/32p_{\max}\rho$, while the revenue was $R_j = 1/4p_{\max}\rho$ initially. Hence Provider $j$ can improve his revenue by a factor $1/8 \approx 12\%$.

Let us consider again the large capacity case, but heterogeneous flows. Assume $\rho_1 = \gamma_\rho < \rho_2 = \rho$. When Provider 1 sets his price below $p_2 = p_{\max}/2$, his revenue is

$$\begin{aligned} R_1 &= p_{\max}\frac{\gamma + 1/2}{2(1+\gamma)}(\frac{\gamma + 3/2}{2(1+\gamma)}(1+\gamma)\rho - 1/2\rho) \\ &= \frac{p_{\max}(\gamma + 1/2)^2\rho}{4(1+\gamma)}, \end{aligned}$$

to be compared to $R_1 = 1/4\rho\gamma p_{\max}$ when ignoring Provider 2 (i.e., when taking $p_1 = p_{\max}/2$). The ratio between the two revenues equals $\frac{(\gamma+1/2)^2}{\gamma(1+\gamma)}$ and is maximized for $\gamma = 1/2$, in which case it equals 4/3.

### 5.3.5 Users, varying their willingness-to-pay

With respect to our basic model, we consider here that users may change their price acceptance threshold after meeting one provider and having either refused its price or been rejected due to capacity limits. Several interpretations can explain this kind of behavior:

– If the user's request was rejected due to congestion, this signal of resource scarcity may increase the user's willingness-to-pay.

– Alternatively, users may know that there are several RSUs on the highway they are using, and hence may "take a bet" for the first RSU they meet, by being more demanding than they could really afford. The logic in this case is that probably the next RSUs are cheaper. As more RSUs are crossed, the risk raises to find no other RSU (or only more expensive ones) before some delay limit, hence a higher price acceptance threshold after passing each RSU.

This willingness-to-pay change impacts two components of the total available demand at a provider–refused and spilled-over users from the competitor–, making them more valuable for the provider (who may extract more revenue from those users).

We consider a simple acceptance threshold change, of a multiplicative form:

– if a user refused to pay the price of the first RSU met, his price acceptance threshold is multiplied by $\alpha > 1$;

– if a user accepted the price of an RSU but his request was rejected due to congestion, his price acceptance threshold is multiplied by $\beta > 1$.

Note that if all users simultaneously accept to pay a price $k$ times bigger than before, then the proportion of users accepting to pay price $p$ is changed from $w(p)$ to $w(\frac{p}{k})$.

We now decompose formally the components of the user flows reaching Provider $j$ and accepting to pay his price $p_j$:

1. those seeing Provider $j$ first, thus issuing a total demand (since they accept to pay $p_j$)

$$w(p_j)\rho_j;$$

2. those seeing Provider $k \neq j$ (the competing provider) first, who refused to pay $p_k$ but would accept the price $p_j$ (possibly due to the acceptance threshold increase), forming a total demand level (smaller than $\rho_k^{\mathrm{ref}}$, and null when $p_k \leq p_j/\alpha$)

$$\rho_k[w(p_j/\alpha) - w(p_k)]^+,$$

where $x^+ := \max(0, x)$ for $x \in \mathbb{R}$;

3. and those seeing Provider $k$ first, who agreed to pay $p_k$ but were rejected because of Provider $k$'s limited capacity, and who also agree to pay $p_j$, for a total demand

$$\min\left(1, \frac{w(p_j/\beta)}{w(p_k)}\right)\rho_k^{\mathrm{sp}},$$

where $\rho_k^{\mathrm{sp}}$ is the part of the demand $w(p_k)\rho_k$ that is spilled-over by Provider $k$.

The total demand $\rho_j^{\mathrm{T}}(p_j, p_k)$ for Provider $j$ then equals the sum of the aforementioned components:

$$\begin{aligned}\rho_j^{\mathrm{T}}(p_j, p_k) :=& \\ & w(p_j)\rho_j + \rho_k[w(p_j/\alpha) - w(p_k)]^+ + \min\left(1, \tfrac{w(p_j/\beta)}{w(p_k)}\right)\rho_k^{\mathrm{sp}}\end{aligned}$$

#### 5.3.5.1 Rejected users and uniqueness of flows

Analogically to the basic model formula (5.5), the success probability equals

$$P_j = \min\left(1, \frac{c}{w(p_j)\rho_j + [w(p_j/\alpha) - w(p_k)]^+\rho_k + \min[1, \frac{w(p_j/\beta)}{w(p_k)}]\rho_k^{\mathrm{sp}}}\right).$$

To simplify a bit the analysis, we assume in the following that $\alpha = \beta$, i.e., users that are not served increase their acceptance threshold price by the same factor, whether they had accepted or refused the price of the first RSU they met. Such an assumption is realistic, if the price variation is interpreted as a response to the decreasing likelihood of finding another (cheap) RSU. If $p_1/\alpha > p_2$, then those success probabilities should satisfy

$$\begin{cases} P_1 = \min\left(1, \frac{c}{w(p_1)\rho_1 + w(p_1/\alpha)\rho_2 - w(p_1/\alpha)\rho_2 P_2}\right) \\ P_2 = \min\left(1, \frac{c}{w(p_2)\rho_2 + w(p_2/\alpha)\rho_1 - w(p_1)\rho_1 P_1}\right). \end{cases} \tag{5.18}$$

We obtain similar equations when $p_1 < p_2/\alpha$, by switching the roles of Providers 1 and 2. Finally, if $p_2/\alpha \leq p_1 \leq p_2\alpha$ then

$$\begin{cases} P_1 = \min\left(1, \frac{c}{w(p_1)\rho_1 + w(p_1/\alpha)\rho_2 - w(p_2)\rho_2 P_2}\right) \\ P_2 = \min\left(1, \frac{c}{w(p_2)\rho_2 + w(p_2/\alpha)\rho_1 - w(p_1)\rho_1 P_1}\right). \end{cases} \tag{5.19}$$

And now we have to prove extension of Proposition 5.1

**Proposition 5.9.** *For any price vector* $(p_1, p_2)$*, the system* (5.18) *has a unique solution* $(P_1, P_2)$*.*

*Proof.* The proof is analogical to the one in the basic model. For the details see Appendix A.1. □

The proof of Proposition 5.2 stating that success probability pair is continuous in the price profile $p_1, p_2$ for our case is straightforward and similar to the one in basic model.

#### 5.3.5.2 Piece-wise expression of the revenue function

In this section, we study the situation when provider $k$ has fixed his price $p_k$, and provider $j$ wants to maximize his revenue by setting appropriately his price $p_j$.

The Lemma 5.3 stating that the total demand $\rho_j^{\mathrm{T}}$ of provider $j$ is a continuous function of his price $p_j$ is non-increasing while provider $j$ is not saturated, is still valid in our extended model, the proof could be found in Appendix A.2.

The revenue function piece-wise expression has more cases, comparing to the basic model:

1. When $\rho_j^{\mathrm{T}}(p_j) \geq c$ (or $p_j \leq p_j^{\mathrm{c}}(p_k)$ when $p_j^{\mathrm{c}}(p_k) > 0$), the RSU capacity of provider $j$ is saturated, and thus his total load is simply

$$\rho_j^T = c,$$

the revenue then equals

$$R_j = p_j c.$$

The corresponding segment of the revenue curve is the linear part on Figure 5.16, and corresponds in Figure 5.15 to prices on the left of the capacity saturation curve of provider $j$.

2. If $p_j < p_k/\alpha$, then provider $k$ cannot attract users having refused the price of provider $j$:
$$\rho_j^T = w(p_j)\rho_j + w(p_j/\alpha)\rho_k - w(p_k)\rho_k + \rho_k^{\mathrm{sp}},$$
with
$$\rho_k^{\mathrm{sp}} = \Big[w(p_k)\rho_k - c\Big]^+.$$
   (a) If $p_k < p_k^c$, then the capacity of provider $k$ is saturated and
$$R_j = p_j\big(w(p_j)\rho_j + w(p_j/\alpha)\rho_k - c\big),$$
   (b) Otherwise, provider $k$ is not saturated and
$$R_j = p_j\big(w(p_j)\rho_j + w(p_j/\alpha)\rho_k - w(p_k)\rho_k\big).$$
Only case 2b occurs on the example of Figures 5.15-5.16.

3. If $p_k/\alpha \le p_j \le p_k\alpha$, then both providers are able to serve the refused traffic of each other:
$$\rho_j^T = w(p_j)\rho_j + w(p_j/\alpha)\rho_k - w(p_k)\rho_k + \rho_k^{\mathrm{sp}},$$
with
$$\rho_k^{\mathrm{sp}} = \left[w(p_k)\rho_k \frac{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j - c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j}\right]^+$$
   (a) If $p_k < p_k^c$, then the capacity of provider $k$ is saturated and he gains
$$R_j = p_j\big(w(p_j)\rho_j + w(p_j/\alpha)\rho_k - \frac{c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j}\big),$$
   (b) Otherwise, provider $k$ is not saturated and his revenue is
$$R_j = p_j\big(w(p_j)\rho_j + w(p_j/\alpha)\rho_k - w(p_k)\rho_k\big).$$

Figures 5.15-5.16 illustrate both cases, with the only remark that on Figure 5.16, cases 2b and 3b constitute one segment of the revenue curve (indeed, the expressions of the revenue function are identical in both cases).

4. If $p_j > p_k\alpha$, then the total load of provider $j$ is

$$\rho_j^T = w(p_j)\rho_j + \frac{w(p_j/\alpha)}{w(p_k)}\rho_k^{\text{sp}},$$

where

$$\rho_k^{\text{sp}} = \left[w(p_k)\rho_k \frac{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j - c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j}\right]^+.$$

   (a) If $p_k < p_k^c$, then the capacity of provider $k$ is saturated and his revenue is

$$\begin{aligned} R_j &= p_j\Big(w(p_j)\rho_j + w(p_j/\alpha)\rho_k \cdot \\ &\quad \frac{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j - c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j}\Big), \end{aligned}$$

   (b) Otherwise, provider $k$ is not saturated and his revenue is simply

$$R_j = p_j w(p_j)\rho_j.$$

We could observe both cases on the example on Figures 5.15-5.16.

Due to the complex form of the revenue function, computing the optimal price as a response to the price of the opponent leads to considering many subcases and hence appears analytically intractable. However, it is quite easy to compute it numerically.

#### 5.3.5.3 Large capacities assumption

In what follows, we assume that **RSU capacities exceed the total user flow** (i.e., $c \geq \rho_j + \rho_k$). In particular, for any price profile RSU capacities are not saturated, and there is no spillover traffic.

This assumption is not necessarily restrictive; indeed in the basic model we have established that at an equilibrium (if any) of the pricing game, no provider is saturated.

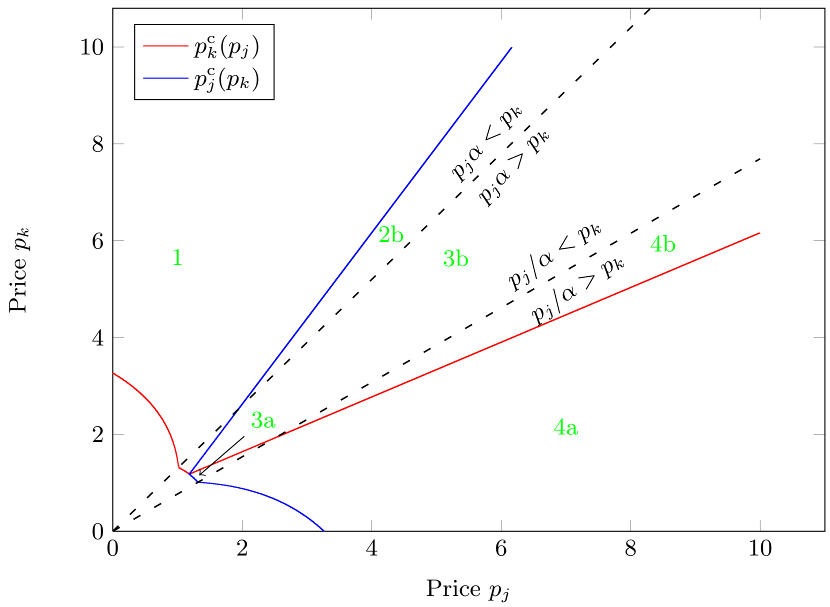


FIGURE 5.15: Capacity saturation prices and the different prices areas they form for $\alpha = 1.3$, $p_k = 4$, $c = 10$, $\rho_1 = \rho_2 = 11$, and for a linear willingness-to-pay function $w(p) = [1 - p/10]^+$

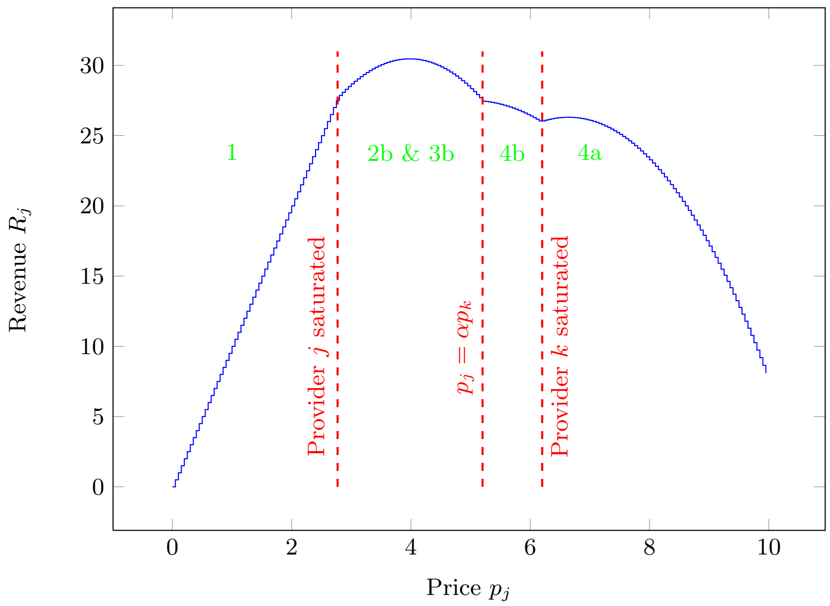


FIGURE 5.16: Revenue of provider $j$ when $p_k = 4$, $c = 10$, $\rho_1 = \rho_2 = 11$ and $\alpha = 1.3$, and for a linear willingness-to-pay function $w(p) = [1 - p/10]^+$

For the case of homogeneous users flows, if there is an equilibrium in the general case - it is identical to the one we have in the scenario when spilled-over flows do not exist, which is the case of the large capacities. Thus, in the large capacities case we have all the equilibria we may have in the general case, with the only difference that in the general case the equilibria may not exist.

#### 5.3.5.4 Providers competition

The revenue expressions are again defined by segments (only two now, because of the large-capacity assumption):

$$R_j = \begin{cases} p_j\Big(w(p_j)\rho_j + w(\frac{p_j}{\alpha})\rho_k - w(p_k)\rho_k\Big) & \text{if } p_j \leq p_k\alpha, \\ p_j w(p_j)\rho_j & \text{otherwise.} \end{cases}$$

In the rest of this section, we derive analytical expressions for the particular case of a linear willingness-to-pay function, of the form $w(p) = [1 - p/p_{\max}]^+$ for some constant $p_{\max}$.

We are interested in obtaining the best response function $\mathrm{BR}_j(p_k)$ of each provider $j$, that is the function indicating the optimal price to set as a response to the competitor's price $p_k$. For the best response function of provider $j$ we isolate only two candidate values from the revenue piecewise expressions above:

1. On the segment $[0, p_k\alpha]$, the best response of Provider $j$ is

$$\mathrm{BR}_j^a = \min\left(p_k\alpha, \frac{p_{\max}\rho_j + p_k\rho_k}{2\rho_j + 2\rho_k/\alpha}\right).$$

   which is strictly below $p_k\alpha$ if $p_k > \frac{p_{\max}\rho_j}{2\rho_j\alpha+\rho_k}$.

2. On the segment $[p_k\alpha, \infty)$, Provider $j$ maximizes his revenue with

$$\mathrm{BR}_j^b = \max\left(p_k\alpha, p_{\max}/2\right),$$

   which is strictly larger than $p_k\alpha$ if $p_k < \frac{p_{\max}}{2\alpha}$.

Now remark that $\frac{p_{\max}\rho_j}{2\rho_j\alpha+\rho_k} < \frac{p_{\max}}{2\alpha}$, hence because of the continuity of the revenue function:

– if $p_k < \frac{p_{\max}\rho_j}{2\rho_j\alpha+\rho_k}$ the best response is $\mathrm{BR}_j = p_{\max}/2$;

– if $p_k > \frac{p_{\max}}{2\alpha}$ the best response is $\mathrm{BR}_j = \frac{p_{\max}\rho_j+p_k\rho_k}{2\rho_j+2\rho_k/\alpha}$;

– for $\frac{p_{\max}\rho_j}{2\rho_j\alpha+\rho_k} \leq p_k \leq \frac{p_{\max}}{2\alpha}$, we have to compare the two best-response candidates above, which we do now in the case of symmetric flows.

**Proposition 5.10.** *Assume user flows are homogeneous, i.e., $\rho_1 = \rho_2 = \rho$, and consider a linear willingness-to-pay function $w(p) = [1 - p/p_{\max}]^+$. Then the best-response of Provider $j$ is*

$$BR_j = \begin{cases} \frac{p_{\max}+p_k}{2+2/\alpha} & \textit{if } p_k \geq p_{\max}(\sqrt{1+\frac{1}{\alpha}}-1) \\ \frac{p_{\max}}{2} & \textit{otherwise.} \end{cases}$$

*Proof.* Let us focus on the region where $\frac{p_{\max}\rho_j}{2\rho_j\alpha+\rho_k} \leq p_k \leq \frac{p_{\max}}{2\alpha}$. In that region,

$$R_j(\mathrm{BR}_j^b) = \frac{p_{\max}}{4}\rho$$

and

$$\begin{aligned} R_j(\mathrm{BR}_j^a) &= \frac{p_{\max}+p_k}{2+2/\alpha}\rho\Big[1 - \frac{1+\frac{p_k}{p_{\max}}}{2+2/\alpha} - \frac{1+\frac{p_k}{p_{\max}}}{2\alpha+2} + \frac{p_k}{p_{\max}}\Big] \\ &= \frac{p_{\max}+p_k}{\alpha(2+2/\alpha)^2}\rho\Big[\alpha+1+\alpha\frac{p_k}{p_{\max}} + \frac{p_k}{p_{\max}}\Big]. \end{aligned}$$

The difference $R_j(\mathrm{BR}_j^a) - R_j(\mathrm{BR}_j^b)$ has the same sign as

$$p_k^2\frac{1}{p_{\max}} + 2p_k - \frac{p_{\max}}{\alpha},$$

which is positive iff $p_k \geq p_{\max}(\sqrt{1+\frac{1}{\alpha}}-1)$. Finally we check that for all $\alpha$,

$$1/(2\alpha+1) < \sqrt{1+\frac{1}{\alpha}} - 1 < 1/(2\alpha),$$

which concludes the proof. □

At a Nash equilibrium $(p_1^*, p_2^*)$, each provider is playing a best-response to the price set by the competitor. But we remark that we cannot have an equilibrium of the form $(\mathrm{BR}_1^b, \mathrm{BR}_2^b)$, since this would imply that $p_1 \geq \alpha p_2$ and $p_2 \geq \alpha p_1$. As a result, only two types of equilibrium can occur:

– A *symmetric* Nash equilibrium, of the form $(\mathrm{BR}_1^a, \mathrm{BR}_2^a)$, leading to

$$p_1^* = p_2^* = \frac{p_{\max}(2\frac{\rho_j^2}{\alpha} + \rho_k^2 + 2\rho_k\rho_j)}{4(\rho_k + \frac{\rho_j}{\alpha})(\rho_j + \frac{\rho_k}{\alpha}) - \rho_j\rho_k}; \tag{5.20}$$

– an *asymmetric* Nash equilibrium, with one provider (say, Provider $j$) playing $\mathrm{BR}_j^a$ and the other one playing $\mathrm{BR}_k^b$, leading to

$$\begin{cases} p_j^* = \frac{p_{\max}(\rho_j+\rho_k/2)}{2\rho_j+2\rho_k/\alpha} \\ p_k^* = p_{\max}/2 \end{cases} \tag{5.21}$$

Considering again the homogeneous flow case, we determine the conditions on $\alpha$ for those price profiles to be Nash equilibria.

1. From Proposition 5.10, the symmetric equilibrium described in (5.20) exists only when
$$p_1^* \geq p_{\max}(\sqrt{1+\frac{1}{\alpha}} - 1),$$
i.e. when $\frac{2/\alpha+3}{4(1+1/\alpha)^2-1} \geq \sqrt{1+\frac{1}{\alpha}} - 1$, which holds if and only if $\alpha \geq \sqrt{\frac{4}{3}}$.
2. For the asymmetric equilibrium described in (5.21), the conditions of existence are:
$$\begin{cases} p_{\max}/2 & \geq & p_{\max}(\sqrt{1+\frac{1}{\alpha}} - 1), \\ \frac{3p_{\max}/2}{2+2/\alpha} & \leq & p_{\max}(\sqrt{1+\frac{1}{\alpha}} - 1). \end{cases}$$
The first condition is always satisfied (recall that $\alpha \geq 1$), while the second one holds if and only if $\alpha \leq s$, where $s \approx 1.0766$.

Table 5.1 summarizes the equilibrium outcomes we can expect from the pricing game, depending on the value of $\alpha$.

Two sets of best responses curves are shown on Figure 5.17, for different $\alpha$ values illustrating the different types of equilibria. We observe that the prices in the symmetric equilibrium are lower than prices in asymmetric ones, which means that users accepting to pay more (through a larger $\alpha$) may lead to a situation where providers charge lower prices, a counterintuitive phenomenon

Figure 5.18 shows the corresponding equilibrium prices and the average price payed by users depending on $\alpha$ and Figure 5.19 plots the equilibrium revenue of both providers.

| | |
|---|---|
| $\alpha \in [1, s]$ | 2 equilibria: $\begin{cases} p_j^* = 3p_{\max}/(4+4/\alpha) \\ p_k^* = p_{\max}/2 \end{cases}$ and $\begin{cases} p_j^* = p_{\max}/2 \\ p_k^* = 3p_{\max}/(4+4/\alpha) \end{cases}$ |
| $\alpha \in (s, \sqrt{\frac{4}{3}})$ | No equilibrium |
| $\alpha \geq \sqrt{\frac{4}{3}}$ | 1 equilibrium: $p_1^* = p_2^* = p_{\max}\frac{2/\alpha+3}{4(1+1/\alpha)^2-1}$ |

TABLE 5.1: Nash equilibria of the pricing game, with homogeneous flows and a linear willingness-to-pay function.

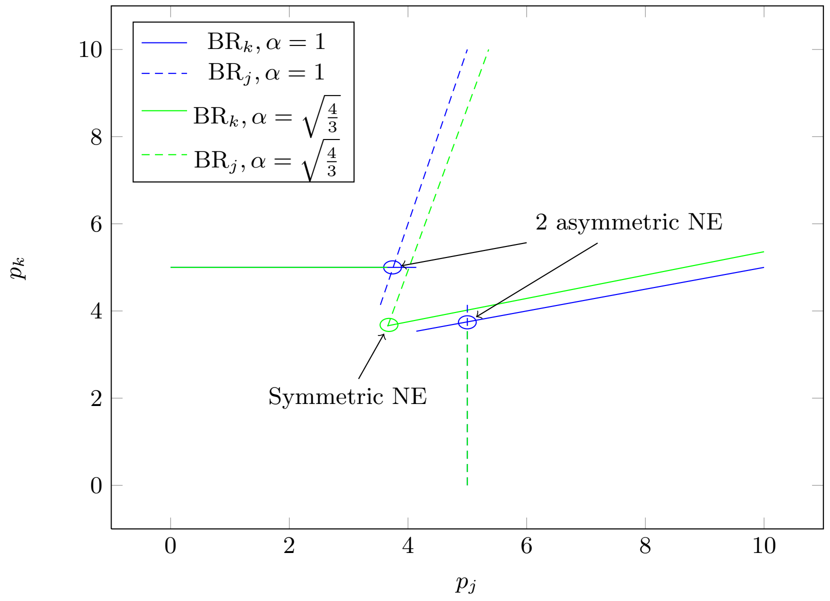


FIGURE 5.17: Best responses curves for $\rho_1 = \rho_2 = 11$, for various $\alpha$

These figures confirm that for some values of $\alpha$, providers decrease their prices with respect to the reference case $\alpha = 1$, resulting in a decrease of their total revenue.

#### 5.3.5.5 Providers cooperation

For comparison purposes we can assume that both providers agreed to cooperate, trying to maximize the sum of their revenue. We again assume users flows to be homogeneous, i.e., $\rho_1 = \rho_2 = \rho$.

To find optimal prices, we again have to consider two cases:

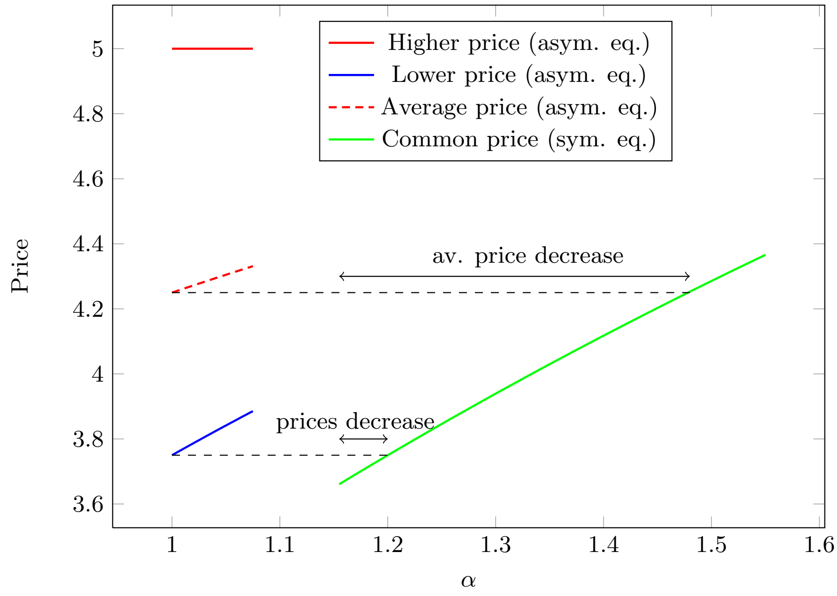


FIGURE 5.18: Prices and the average price payed by users values among all users at equilibrium for $\rho_1 = \rho_2 = 11$. Note that for the symmetric equilibrium the average price is the (common) price charged by providers

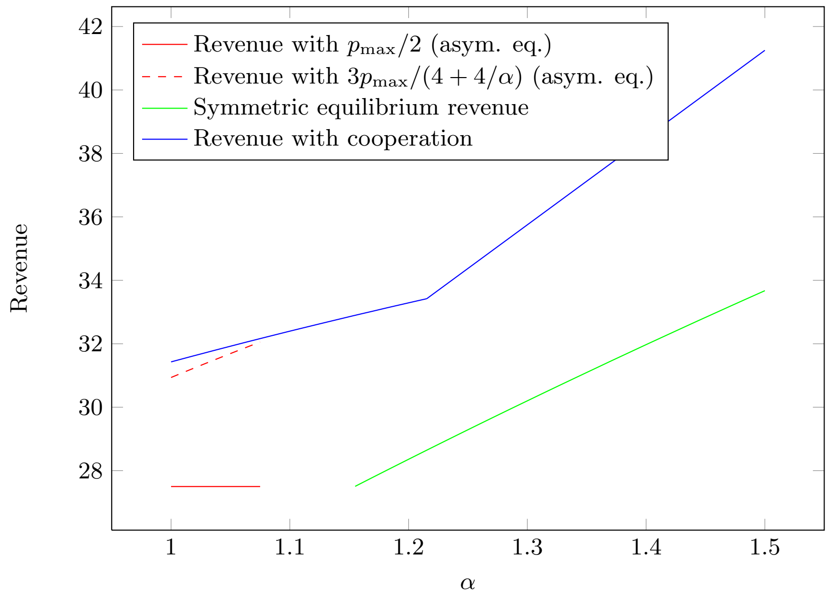


FIGURE 5.19: Providers revenue in cooperative and competitive equilibrium cases for $\rho_1 = \rho_2 = 11$

1. First, if $p_j \le \frac{p_k}{\alpha}$ The total revenue is

$$R^T = p_j\Big(w(p_j)\rho + w(\frac{p_j}{\alpha})\rho - w(p_k)\rho\Big) + p_k w(p_k)\rho.$$

Further, we take partial derivatives:

$$\begin{aligned}\frac{\partial R^T}{\partial p_j} &= \rho\Big(1 - \tfrac{p_j}{p_{\max}}(2+2/\alpha) + \tfrac{p_k}{p_{\max}}\Big) = 0,\\ \frac{\partial R^T}{\partial p_k} &= \rho\Big(1 + \tfrac{p_j}{p_{\max}} - \tfrac{2p_k}{p_{\max}}\Big) = 0,\end{aligned}$$

and finally get expressions for optimal prices:

$$\begin{cases}\bar{p}_j = & \frac{3p_{\max}}{3+4/\alpha},\\ \bar{p}_k = & \frac{(3+2/\alpha)p_{\max}}{3+4/\alpha}.\end{cases}$$

After substituting this optimal prices in total revenue expression we obtain:

$$\bar{R}'^T = \frac{p_{\max}\rho(9\alpha + 15 + 4/\alpha)}{\alpha(3+4/\alpha)^2}.$$

2. If $\frac{p_k}{\alpha} < p_j < p_k\alpha$, the total revenue is:

$$\begin{aligned}R^T = p_j&\Big(w(p_j)\rho + w(\frac{p_j}{\alpha})\rho - w(p_k)\rho\Big) +\\ &p_k\Big(w(p_k)\rho + w(\frac{p_k}{\alpha})\rho - w(p_j)\rho\Big).\end{aligned}$$

Further, we take partial derivatives:

$$\begin{aligned}\frac{\partial R^T}{\partial p_j} &= \rho\Big(1 - \tfrac{p_j}{p_{\max}}(2+2/\alpha) + \tfrac{2p_k}{p_{\max}}\Big) = 0,\\ \frac{\partial R^T}{\partial p_k} &= \rho\Big(1 - \tfrac{p_k}{p_{\max}}(2+2/\alpha) + \tfrac{2p_j}{p_{\max}}\Big) = 0,\end{aligned}$$

and the optimal prices are:

$$\bar{p_j} = \bar{p}_k = \frac{p_{\max}\alpha}{2}.$$

Putting them in total revenue expression gives:

$$\bar{R''}^T = \frac{p_{\max}\alpha\rho}{2}.$$

Now we have to decide, which revenue expression gives higher value depending on $\alpha$. Let us obtain a condition on $\bar{R'}^T \geq \bar{R''}^T$:

$$\frac{p_{\max}\rho(9\alpha + 15 + 4/\alpha)}{\alpha(3 + 4/\alpha)^2} \geq \frac{p_{\max}\alpha\rho}{2},$$

and after some manipulations we obtain:

$$9\alpha^3 + 6\alpha^2 - 14\alpha - 8 < 0,$$

and taking into account that $\alpha \geq 1$, we have only root. Than we could write:

$$\begin{cases} \alpha \in [1, \lambda] & R^T = \frac{p_{\max}\rho(9\alpha+15+4/\alpha)}{\alpha(3+4/\alpha)^2}, \\ \alpha > \lambda & R^T = \frac{p_{\max}\alpha\rho}{2}. \end{cases}$$

where $\lambda \approx 1.215$.

Assuming equal share of cooperative revenue, we plotted individual revenue of both providers in competition and cooperation cases on Figure 5.19.

#### 5.3.5.6 Users surplus

In this section we evaluate equilibria we got in previous section from the point of view of the users surplus, which is the difference the between what the users wanted to pay, and what they actually payed in the case of large capacities. In our scenario, in which users willingness-to-pay varies, the users, which accept to pay the price of the second provider met, actually pays more than they wanted to pay originally and thus in this case the users surplus may be negative.

If we consider just one direction of flow $\rho_j$ and denote by $p_j$ the price of the first provider this flow meets, and by $p_k$ the price of the second one, then we have positive part of

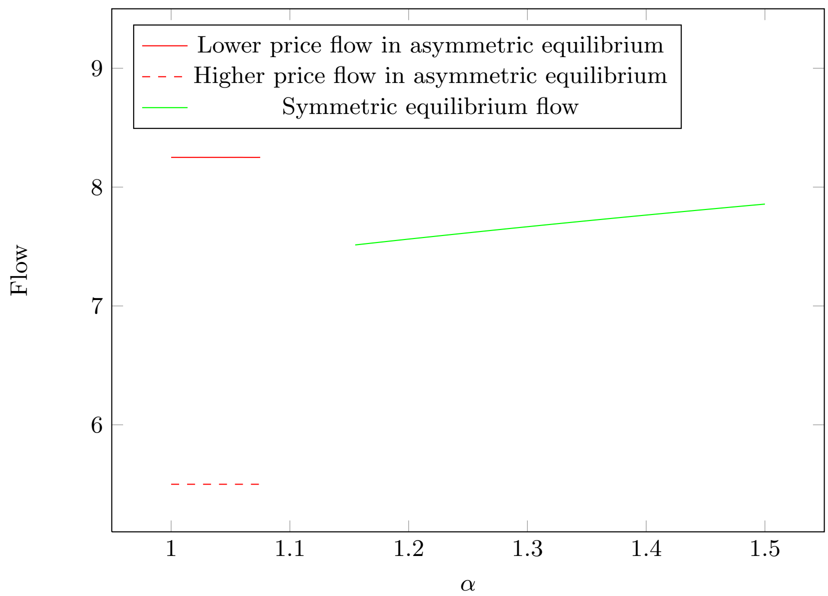


FIGURE 5.20: Flows served by providers in equilibriua depending on $\alpha$ for $\rho_1 = \rho_2 = 11$

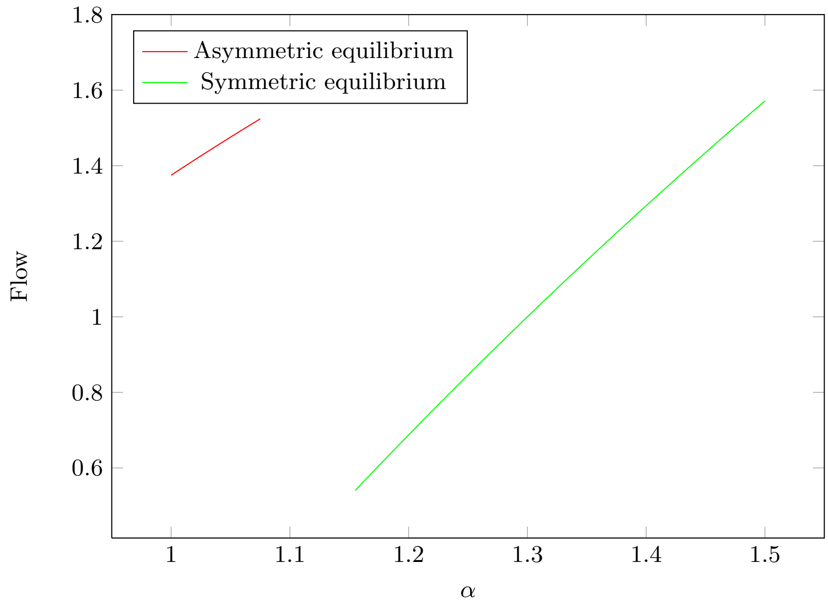


FIGURE 5.21: Flows of users payed second price (refused users accepted to pay the price) in equilibrium depending on $\alpha$ for $\rho_1 = \rho_2 = 11$

users surplus as follows:

$$US_j^+ = \int_{p_j}^{p_{\max}} w(p)\rho \mathrm{d}p + \int_{p_k}^{p_j} [w(p) - w(p_j)]^+ \rho \mathrm{d}p,$$

which includes surplus from the served at first operator $j$ flow and from the refused flow, served at operator $k$. The negative part of users surplus is:

$$\begin{aligned} US_j^- = & \left[w(p_k/\alpha) - \max(w(p_j), w(p_k))\right]^+ (p_k - p_k/\alpha)\rho - \\ & \int_{p_k/\alpha}^{\min(p_j,p_k)} \left[w(p) - \max(w(p_j), w(p_k))\right]^+ \rho \mathrm{d}p, \end{aligned}$$

which is the case when users from refused traffic of provider $j$ accepted to pay price of provider $k$, which is higher than their original price they accepted to pay. Notice, that expression of $US_j^-$ is general enough to present both cases when $p_j > p_k$ and $p_j < p_k$.

Figures 5.23 - 5.24 illustrate the logic behind computation of users surplus for two cases of relation between competing providers prices. Red square denotes the negative part of users surplus, when they pay more, than wanted, and yellow zones denote positive parts of users surplus.

In the case of linear willingness-to-pay function they transform to:

$$US_j^+ = (p_{\max} - p_j) w(p_j) \frac{\rho}{2} + (w(p_k) - w(p_j))[p_j - p_k]^+ \frac{\rho}{2},$$

and

$$\begin{aligned} US_j^- = & \frac{\rho}{2}(w(p_k/\alpha) - w(p_k))(p_k - p_k/\alpha) - \\ & \frac{\rho}{2}(w(p_j) - w(p_k))[p_k - p_j]^+ \end{aligned}$$

and the total user surplus is

$$\begin{aligned} US = & US_j^+ + US_k^+ - \\ & US_j^- - US_k^-. \end{aligned}$$

Figure 5.22 shows total users surplus for different $\alpha$ values for large capacities case in the similar settings as above. We could see that it is consistent with what we observed about average price payed by user: quite big range of values of $\alpha$ leads to increase in users surplus, which means that accepting to pay more leaded to the situation when overall users pay less.

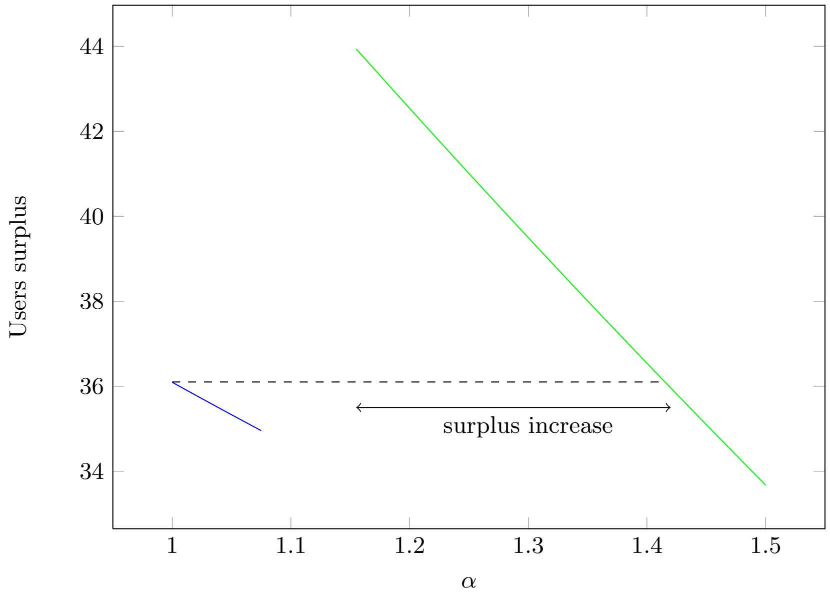


FIGURE 5.22: Users surplus in equilibrium vs $\alpha$ for $\rho_1 = \rho_2 = 11$

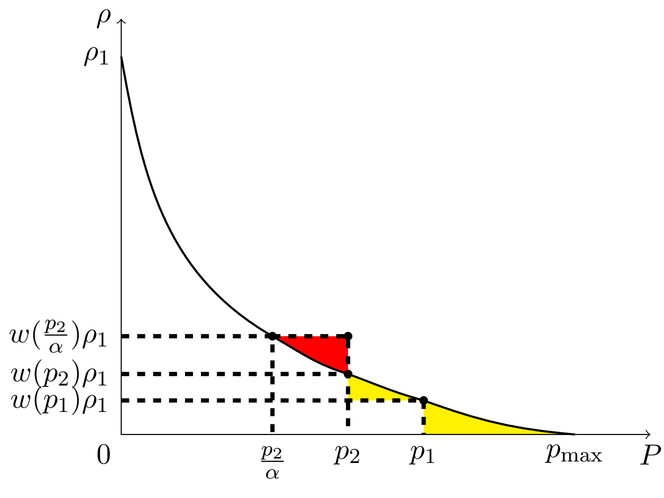


FIGURE 5.23: Users surplus of $\rho_1$ flow when $p_1 > p_2$

#### 5.3.5.7 Heterogeneous willingness-to-pay variations

In this section we assume that user pricing preferences change differently for both flow directions. Some users may for example move toward a city and thus expect to meet quite a lot APs, while the users moving in the opposite direction are risking not to meet

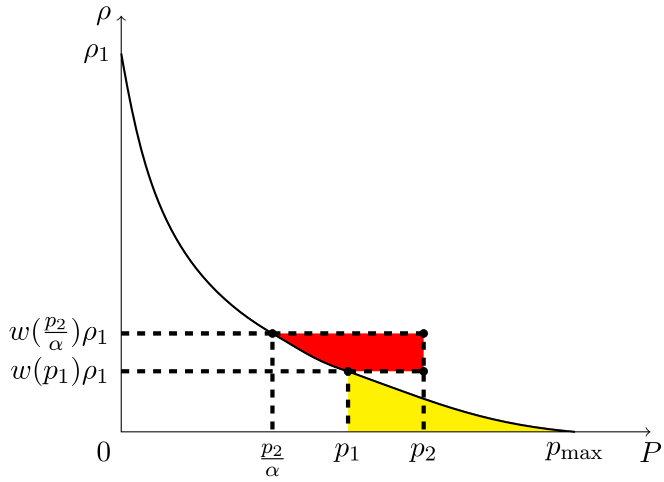


FIGURE 5.24: Users surplus of $\rho_1$ flow when $p_2 > p_1$

any APs in the nearest future. The former may not increase much their willingness-to-pay, while the latter have higher risks to fail to establish Internet connection, and thus are more flexible in price perception.

Let us consider without loss of generality that $\alpha_j = h\alpha_k = h\alpha$, for some $h \geq 1$.

Similarly to the case when $\alpha$ was common to both flow directions, we consider three cases:

1. If $p_j < \frac{p_k}{\alpha}$, then

$$\begin{cases} R_j = p_j\Big(w(p_j)\rho_j + w(\frac{p_j}{\alpha h})\rho_k - w(p_k)\rho_k\Big), \\ R_k = p_k w(p_k)\rho_k \end{cases}$$

and for a linear $w(p)$

$$\begin{cases} \mathrm{BR}_j^a = \frac{p_{\max}\rho_j + p_k\rho_k}{2\rho_j + \frac{2\rho_k}{\alpha h}}, \\ \mathrm{BR}_k^b = p_{\max}/2. \end{cases}$$

and

$$\mathrm{BR}_j^a(\mathrm{BR}_k^b) = \frac{p_{\max}(\rho_j + 1/2\rho_k)}{2\rho_j + \frac{2\rho_k}{\alpha h}}.$$

This is valid for

$$\alpha \leq \frac{\rho_j + \sqrt{{\rho_j}^2 + 4\rho_k/h(\rho_j + 1/2\rho_k)}}{2\rho_j + \rho_k},$$

which in homogeneous case

$$\alpha \leq \frac{1 + \sqrt{1 + 6/h}}{3}.$$

2. If $\frac{p_k}{\alpha} \leq p_j \leq p_k \alpha h$, then

$$\begin{cases} R_j = p_j \Big( w(p_j)\rho_j + w(\frac{p_j}{\alpha h})\rho_k - w(p_k)\rho_k \Big), \\ R_k = p_k \Big( w(p_k)\rho_k + w(\frac{p_k}{\alpha})\rho_j - w(p_j)\rho_j \Big) \end{cases}$$

and for a linear $w(p)$

$$\begin{cases} \mathrm{BR}_j^a = \frac{p_{\max}\rho_j + p_k\rho_k}{2\rho_j + \frac{2\rho_k}{\alpha h}}, \\ \mathrm{BR}_k^a = \frac{p_{\max}\rho_k + p_j\rho_j}{2\rho_k + \frac{2\rho_j}{\alpha}}, . \end{cases}$$

and

$$\begin{cases} \mathrm{BR}_j^a(\mathrm{BR}_k^a) = \frac{p_{\max}(2\rho_j\rho_k + \frac{2\rho_j^2}{\alpha} + \rho_k^2)}{(2\rho_j + \frac{2\rho_k}{\alpha h})(2\rho_k + \frac{2\rho_j}{\alpha}) - \rho_j\rho_k}, \\ \mathrm{BR}_k^a(\mathrm{BR}_j^a) = \frac{p_{\max}(2\rho_j\rho_k + \frac{2\rho_k^2}{\alpha h} + \rho_j^2)}{(2\rho_k + \frac{2\rho_j}{\alpha})(2\rho_j + \frac{2\rho_k}{\alpha h}) - \rho_j\rho_k}. \end{cases}$$

For this equilibrium the condition $\frac{p_k}{\alpha} \leq p_j \leq p_k \alpha h$ is true only if

$$\begin{cases} \alpha \geq \frac{-\rho_j(\rho_j - 2\rho_k) + \sqrt{{\rho_j}^2(\rho_j - 2\rho_k)^2 + 8{\rho_k}^3/h(\rho_k + 2\rho_j)}}{2\rho_k(\rho_k + 2\rho_j)}, \\ \alpha \geq \frac{-\rho_k(\rho_k - 2\rho_j) + \sqrt{{\rho_k}^2(\rho_k - 2\rho_j)^2 + 8{\rho_j}^3 h(\rho_j + 2\rho_k)}}{2h\rho_j(\rho_j + 2\rho_k)}, \end{cases}$$

and in homogeneous flows case:

$$\begin{cases} \alpha \geq \frac{1 + \sqrt{1 + 24/h}}{6}. \end{cases}$$

3. If $p_j > p_k \alpha h$, then

$$\begin{cases} R_j = p_j w(p_j)\rho_j, \\ R_k = p_k \Big( w(p_k)\rho_k + w(\frac{p_k}{\alpha})\rho_j - w(p_j)\rho_j \Big) \end{cases}$$

and for a linear $w(p)$

$$\begin{cases} \mathrm{BR}_j^b = p_{\max}/2, \\ \mathrm{BR}_k^a = \frac{p_{\max}\rho_k + p_j\rho_j}{2\rho_k + \frac{2\rho_j}{\alpha}}, . \end{cases}$$

and

$$\mathrm{BR}_k^b(\mathrm{BR}_j^a) = \frac{p_{\max}(\rho_k + 1/2\rho_j)}{2\rho_k + \frac{2\rho_j}{\alpha}},$$

with a condition on $\alpha$

$$\alpha < \frac{\rho_k + \sqrt{{\rho_k}^2 + 4\rho_j h(\rho_k + 1/2\rho_j)}}{2h(\rho_k + 1/2\rho_j)},$$

and in homogeneous flows case:

$$\alpha < \frac{1 + \sqrt{1 + 6h}}{3h}.$$

What is different in this new scenario is that we have three types of equilibrium now $(\mathrm{BR}_j^a, \mathrm{BR}_k^b)$, and $(\mathrm{BR}_j^a, \mathrm{BR}_k^b)$ are not identical anymore). With homogeneous users flows we have the following conditions:

1. $(\mathrm{BR}_j^a, \mathrm{BR}_k^b)$ is an equilibrium when

$$\begin{cases} \mathrm{BR}_j^a(\mathrm{BR}_k^b) < p_{\max}(\sqrt{1 + \frac{1}{\alpha h}} - 1), \\ \mathrm{BR}_k^b(\mathrm{BR}_j^a) \geq p_{\max}(\sqrt{1 + \frac{1}{\alpha}} - 1), \\ \alpha < \frac{1+\sqrt{1+6/h}}{3}. \end{cases}$$

   or $\alpha < \min\{s/h, \frac{1+\sqrt{1+6/h}}{3}\}$.

2. $(\mathrm{BR}_j^a, \mathrm{BR}_k^a)$ is an equilibrium when

$$\begin{cases} \mathrm{BR}_j^a(\mathrm{BR}_k^a) \geq p_{\max}(\sqrt{1 + \frac{1}{\alpha h}} - 1), \\ \mathrm{BR}_k^a(\mathrm{BR}_j^a) \geq p_{\max}(\sqrt{1 + \frac{1}{\alpha}} - 1), \\ \alpha \geq \frac{1+\sqrt{1+24/h}}{6}. \end{cases}$$

This set of inequalities is not solvable for $\alpha h$, but for each concrete $h$ value we can find numerically a condition on $\alpha$ for the conditions to hold. This dependence is presented on Figure 5.25

3. $(\mathrm{BR}_j^b, \mathrm{BR}_k^a)$ is an equilibrium when

$$\begin{cases} \mathrm{BR}_j^b(\mathrm{BR}_k^a) \geq p_{\max}(\sqrt{1+\frac{1}{\alpha h}}-1), \\ \mathrm{BR}_k^a(\mathrm{BR}_j^b) < p_{\max}(\sqrt{1+\frac{1}{\alpha}}-1), \\ \alpha < \frac{1+\sqrt{1+6h}}{3h}, \end{cases}$$

or $\alpha < \min\{s, \frac{1+\sqrt{1+6h}}{3h}\}$.

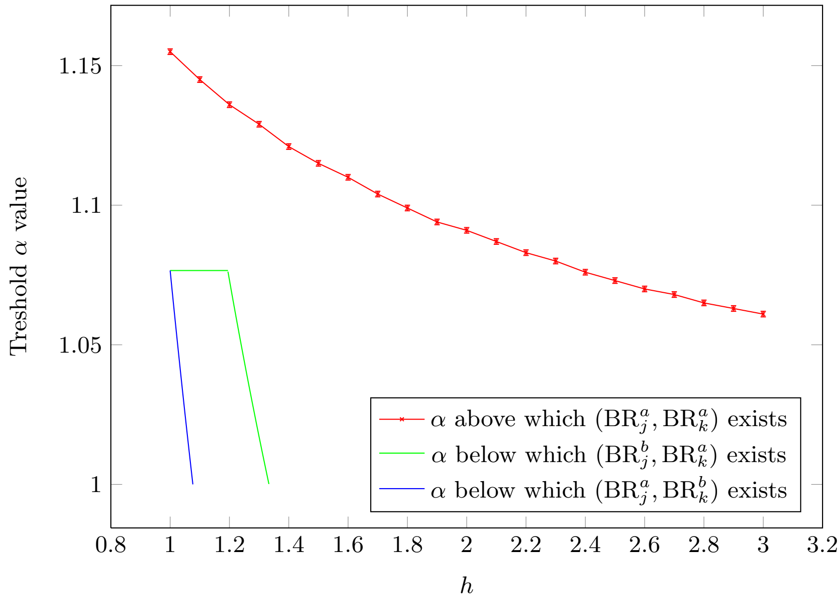


FIGURE 5.25: Threshold $\alpha$ values for different $h$

Figure 5.25 shows threshold $\alpha$ values for different $h$, showing whether there exists a particular type of equilibria. The figure suggests that there is no pair of $\alpha$ and $h$ such that all three types of equilibria exist.

#### 5.3.5.8 Various WTP functions

Due to complexity of the model it is hard to analytically show, that the phenomenon, when increase of users willingness-to-pay may cause prices decrease is still valid for general willingness-to-pay function (almost all our results so far concerned linear willingness-to-pay function only). Note, that it is possible to prove, that in the case of large capacities at high $\alpha$ values we indeed have at least one symmetric equilibrium, but we could say nothing about its quality.

Thus, in current section we present numerical results for equilibrium prices for several willingness-to-pay functions. We were interested in finding a minimum willingness-to-pay variation value $\bar{\alpha}$ at which symmetric equilibrium appears and to compare prices in this equilibrium with those we have in the case of $\alpha = 1$.

TABLE 5.2: Equilibrium prices decrease for different willingness-to-pay functions. The results are obtained numerically with the step of 0.01

| $w(p)$ | Equilibrium prices, $\alpha = 1$ | Equilibrium prices, $\alpha = \bar{\alpha}$ | $\bar{\alpha}$ |
|---|---|---|---|
| Linear | $(3.75, 5.0)$ | $(3.68, 3.68)$ | 1.16 |
| Square | $(2.35, 3.33)$ | $(2.27, 2.27)$ | 1.2 |
| Power Law $(5, 2.2)$ | $(1.35, 1.92)$ | $(1.32, 1.32)$ | 1.17 |
| Exponential | $(0.65, 1.0)$ | $(0.59, 0.59)$ | 1.25 |

We are considering the following functions:

– Linear: $w(p) = 1 - \frac{p}{p_{\max}}$

– Square: $w(p) = (1 - \frac{p}{p_{\max}})^2$

– Power Law $(C, n)$: $w(p) = \frac{C}{C+p^n}$

– Exponential: $w(p) = \frac{1}{e^p}$

Table 5.2 shows the prices which providers charge in competitive equilibrium in the case when there is no variation ($\alpha = 1$) and when the variation leaded to symmetric equilibrium. We observe that for these willingness-to-pay functions, which follow our convexity and monotonicity assumptions, we still have the prices decrease after some $\alpha$. From this results we may suspect that it is somehow more general result and is not caused by simple linear willingness-to-pay function. However, this general result seems to us to be relatively hard to prove and is matter for further research.

### 5.3.6 Optimal RSUs location

In this subsection we study a scenario, when RSUs belonging to competing Internet access providers operate on the same frequency and thus could interfere. Note, that we do not consider that users may change their willingness-to-pay; in what follows we study the scenario, when users keep their pricing preferences unchanged.

We further assume that capacity of closely situated RSUs depends on distance $d \in [0, D]$ between them. In what follows we assume that all users move with the same constant speed. We could describe signal to interference and noise ratio (SINR) for user at distance $x$ from RSU he is connected to by the following function:

$$SINR = \frac{P(x)}{N + P(d-x)},$$

where $P(x)$ is the power of incoming transmission of RSU located at distance $x$, and $d$ is the distance between RSUs. As in Altman's paper [56] let $P(x) = (1 - x^2)^{-\alpha/2}$ for some $\alpha \geq 1$. Each users is connected to a RSU inside its diameter $L$. Thus, the average goodput they experience could be defined as:

$$c(d) = \int_{-L/2}^{L/2} \frac{w}{L} \log_2(1 + \frac{(1+x^2)^{-\alpha/2}}{N + (1 + (d-x)^2)^{-\alpha/2}})\mathrm{d}x, \tag{5.22}$$

where $w$ is allocated bandwidth. If provider wants to ensure that all connected users finish files download, the total flow per second, served by provider $\rho_j^T$ should not exceed this goodput value $c(d)$. In what follows, when we write $c(d)$ we imply this capacity function.

#### 5.3.6.1 The optimal distance

We assume that Provider 1 has already fixed his price $p_1$ and location, and only Provider 2 makes decision simultaneously about price $p_2 \in [0, p_{max}]$ and distance $d \in [0, D]$ from RSU 1 position. We are interested in getting some insight on which distance Provider 2 has to choose to maximize his revenue.

Further, we denote:

– by $p^{opt}(d)$ the optimal price of Provider 2 for distance $d$.

– by $d^{sp} := \sup\{d \in [0, D] : \rho_1^{sp}(d, p^{opt}(d)) > 0\}$ the maximum distance at which Provider 1 has positive spillover traffic when Provider 2 charges the optimal price. If it is undefined, we assume it to be equal to zero.

– by $d^c := \sup\{d \in [0, D] : p^{opt}(d) = p_2^c(d)\}$ the maximum distance at which the optimal price of Provider 2 equals his capacity saturation price $p_2^c$, i.e., when the optimal strategy of Provider 2 is to charge a price, such that all his capacity is used. If this distance is undefined, we assume it to be equal to zero.

To get some analytical results about the optimal distance and price for Provider 2, we introduce several lemmas. The first lemma gives us a useful result about monotonicity of the revenue function of Provider 2 in distance $d$ in the case when Provider 1 has a positive spillover traffic:

**Lemma 5.11.** *If for some $d^2 > 0$ and for $p = p^{opt}(d^2)$, spillover traffic of Provider 1 is positive $\rho_1^{sp}(d^2, p) > 0$, then for any $d^c \leq d^1 < d^2$, $R_2(d^1, p) > R_2(d^2, p)$.*

*Proof.* Let us consider both revenue functions. The flows are homogeneous, and thus $p > p_1$, because $p$ is optimal price for provider 2 and thus he has no spillover for $d^1$ and moreover for $d^2$.

$$R_2(d^1, p) = w(p)\rho(2 - \frac{c(d^1)w(p)\rho}{2w(p_1)\rho - w(p)\rho}),$$
$$R_2(d^2, p) = w(p)\rho(2 - \frac{c(d^2)w(p)\rho}{2w(p_1)\rho - w(p)\rho}),$$

from what is it easy to see since $c(d^2) > c(d^1)$ that $R_2(d^1, p) > R_2(d^2, p)$ □

The following lemma states that for every distance smaller than $d^{sp}$, Provider 1 is saturated when Provider 2 charges the optimal price.

**Lemma 5.12.** *In homogeneous flows case for $d \in [0, d^{sp}]$ when Provider 2 charges the optimal price, Provider 1 has positive spilled-over traffic $\rho_1^{sp}(p^{opt}(d)) > 0$.*

*Proof.* See Appendix A.3. □

And the last lemma will help us further to identify the optimal distance:

**Lemma 5.13.** *If $d^{sp} > d^c$, then for $d \in [d^c, d^{sp}]$ revenue of Provider 2 $R_2(d, p^{opt}(d))$ is decreasing function of $d$.*

*Proof.* See Appendix A.4. □

For simplicity we introduce the following assumption:

**Assumption 3.** *In what follows we assume that capacity of RSUs degrade significantly in the case of collocation, or* $c(0) \ll \rho$.

Taking into account Assumption 3 we can determine the optimal distance:

**Proposition 5.14.** *In homogeneous flows case we determine the optimal distance* $d^{opt}$ *for Provider* 2 *as follows:*

1. *If* $d^{sp} = 0$*, then* $d^{opt}$ *is any* $d \in [d^c, D]$.
2. *If* $d^{sp} > d^c \geq 0$*, then*

$$d^{opt} = \bar{d}$$

3. *If* $d^c \geq d^{sp} > 0$*, then we have several candidates for the optimal distance:*

$$d^{opt} = \begin{cases} \bar{d} \text{ or} \\ \forall\, d \;\in [d^c, D], \end{cases} \tag{5.23}$$

*where*

$$\bar{d} = \arg \max_{d \in [0, \min(d^c, d^{sp})]} c(d) p^c(d).$$

*Proof.* See Appendix A.5. □

Taking into account the result of Proposition 5.14 the following Assumption can simplify the further analysis.

**Assumption 4.** *In what follows we assume that if there is a set of optimal distances, Provider* 2 *will prefer the smallest one.*

Assumptions 3 - 4 and Proposition 5.14 allow us decrease complexity of the model: in what follows we consider that at optimal situation Provider 2 **always** charges **capacity saturation** price.

Figure 5.26 shows for different capacity values flows of both providers in the case when Provider 2 charges the optimal price and $p_1$ is fixed. On this Figure we could clearly see two special points: the maximum capacity at which Provider 2 is fulfilled (saturated capacity) which corresponds to distance $d^c$ and the maximum capacity at which Provider 1 has spillover traffic (when demand flow exceeds capacity) which happens at distance $d^{sp}$.

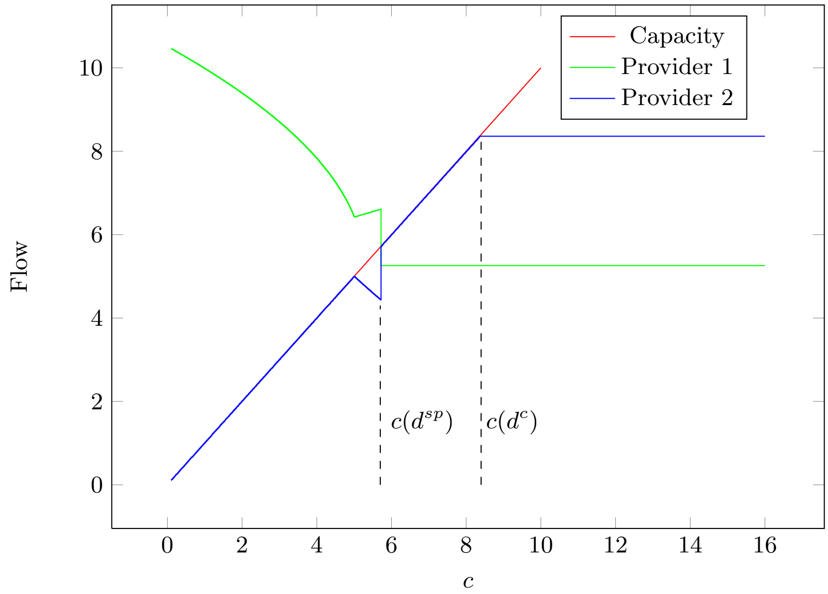


FIGURE 5.26: Flow of providers with optimal price of Provider 2 depending on capacity when $\rho = 10$, $p_1 = 5.23$, $p_{max} = 10$ and linear $w(p)$

#### 5.3.6.2 Optimal strategy of Provider 2

Now we study a scenario, when price $p_1$ of Provider 1 is fixed, the willingness-to-pay function is linear (for general $w(p)$ similar results could be obtained numerically) and Provider 2 tries to find his best response in terms of capacity (distance) and price, taking into account Assumptions 3-4.

Since we assume that Provider 2 always has his capacity saturated, he has two opportunities: 1) charge a higher than $p_1$ price and cause spillover from Provider 1 and 2) to charge a lower price in order to serve refused traffic of Provider 1 (and thus spillover of Provider 1 is zero). Thus we write:

– If $p_2 > p_1$ then Provider 2 at the optimal price causes spillover traffic of Provider 1 to appear (otherwise such a price is not optimal). We know that capacity saturation price when opponent has spillover is:

$$\bar{p}_2^c = \Big(1 - w(p_1) + \sqrt{\frac{w(p_1)}{\rho}(w(p_1)\rho - \bar{c}^{opt})}\Big)p_{max},$$

and taking into account that price and distance chosen by Provider 2 should maximize $p_2^c \cdot c(d)$, we get:

$$\bar{c}^{opt} = \min\Big[C, \frac{\rho}{\frac{9}{2}w(p_1)}\Big(-1 + 2w(p_1) + 2w^2(p_1) + \sqrt{1 - 4w(p_1) + 9w^2(p_1) - 10w^3(p_1) + 4w^4(p_1)}\Big)\Big], \tag{5.24}$$

where $C = c(D)$ is the maximum possible capacity and $w(p_1)\rho \geq \bar{c}^{opt}$.

– If $p_2 \leq p_1$, then Provider 2 charges the capacity saturation price:

$$\underline{p}_2^c = (1 - \frac{\underline{c}^{opt} + w(p_1)\rho}{2\rho})p_{max},$$

with optimal distance

$$\underline{c}^{opt} = \min[C, \rho - \frac{w(p_1)\rho}{2}],$$

with a condition $w(p_1)\rho \leq \underline{c}^{opt}$ or $w(p_1) \leq \frac{2}{3}$.

We found that for situation when

$$\max(\bar{c}^{opt}(p_1), \underline{c}^{opt}(p_1)) < C, \;\; \forall\, p_1 \in [0, p_{max}] \tag{5.25}$$

functions $\bar{p}_2^c \cdot \bar{c}^{opt}$ and $\underline{p}_2^c \cdot \underline{c}^{opt}$ are monotonously increasing and decreasing in $p_1$, respectively, and thus there is only one price $p^t$ at which Provider 2 is indifferent between both opportunities.

Figure 5.27 illustrates demand flows (of users accepting to pay providers prices) for the case when Provider 2 chooses optimal distance and price with price of Provider 1 $p_1$ varying. We see, that Provider 2 flow is always equals capacity, while the flow of Provider 1 is strictly below capacity when his price is high, and above capacity when his price is low enough, to provoke Provider 2 to set spillover-causing price.

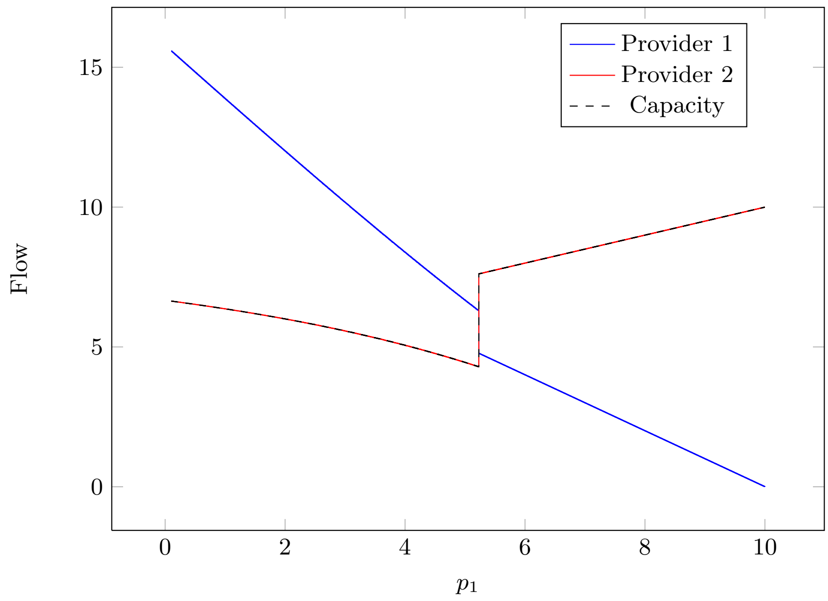


FIGURE 5.27: Demand flows (flow of users accepted to pay) when Provider 2 charges optimal price and chooses optimal distance vs $p_1$ with $\rho = 10$, $C = 10$, $p_{max} = 10$ and linear $w(p)$

#### 5.3.6.3 Simultaneous game

From the basic model we know that for the simultaneous providers game, in equilibrium both RSUs capacities are not fulfilled, i.e., $p_1 > p^c(p_2), p_2 > p^c(p_1)$. From this fact we deduce that the only one type of equilibrium is possible. Indeed, from all possible optimal distances and prices, only one strategy profile leads to zero spilled-over flows and thus is acceptable for both providers:

$$p_1^{opt}(d), p_2^{opt}(d), d,$$

such that $d \in (\max(d^c(p_1^{opt}), d^{sp}(p_1^{opt})), D]$. When such kind of triple of values does not exist - there is no equilibrium in simultaneous game.

We remind that in Assumption 3 we agreed that if Provider 2 has a choice between several distances which give the same optimal revenue, he will choose the smallest. Under this assumption, equilibrium in simultaneous game never exists (taking into account that $d^c > 0$), since Provider 2 always charge his capacity saturation price. But if we release this assumption, we may notice that the only possible equilibrium is the one, when Provider 2 does not want to decrease RSUs capacity, which corresponds to the scenario when there is no interference between two RSUs (the basic model).

#### 5.3.6.4 Leader-Follower game

Here we discuss the case when Provider 1 makes his price decision first, and then Provider 2 chooses the optimal distance (and thus capacity) and price.

From the previous section we deduce that when Provider 1 charges a high enough price, Provider 2 does not cause him to spill over, and when he charges a lower price, some part of his flow would be spilled-over due to the capacity constraints imposed by Provider 1.

Thus, when Provider 1 has no spillover, his revenue equals

$$p_1 w(p_1)\rho.$$

However, when the competitor cause him to spillover, the revenue changes to

$$p_1 \bar{c}^{opt},$$

where $\bar{c}^{opt}$ is determined by equation (5.25).

What is more profitable for Provider 1 - to have higher price without being forced to spill-over or to charge a lower price and loose some customers due to interference posed by competitor? The answer on this question is given on Figures 5.28-5.29 where the prices and demand flows in equilibrium are depicted for different users flows values $\rho$. We notice that when the users flow $\rho$ is lower than some threshold value $\rho^T$ (which could be computed numerically), Provider 1's best strategy is to charge high price and to avoid having spillover traffic, while after this threshold value the flow is so big, that the maximum revenue is gained when Provider 1 has positive spillover.

Putting $p_1^t$ value (which we find numerically) in (5.25) we get a threshold value of $\rho^t$, below which optimal prices of both providers are independent of $\rho$:

$$\max\left(\underline{c}^{opt}(\rho^t, p_1^t), \bar{c}^{opt}(\rho^t, p_1^t)\right) = C.$$

This threshold value is depicted on Figure 5.28.

A curious phenomena we notice is that for relatively small flow values ($\rho \in [0, \approx 10]$) Provider 1 tends to charge higher price comparing to the situation when he is the only provider on the highway. Actually, Provider 1 has to do so, in order to not be caused to spill-over. This is something different from what we observe in other types of competition, where introduction of a new player leads to prices decrease.

Figure 5.30 shows providers revenues in equilibrium depending on users flow value $\rho$. We observe, that in the case of leader follower game with interference, Provider 2 **always** gains higher revenue. If we remove capacity term from consideration and assume that RSUs operate on different frequencies (thus do not interfere), we will observe that provider, that makes his turn first always has an advantage. He will charge lower price in order to serve refused traffic coming from the opponent and this allows him to gain higher than competitor's revenue. Thus, the possibility to harm opponent's capacity drastically changes the rules of the competition.

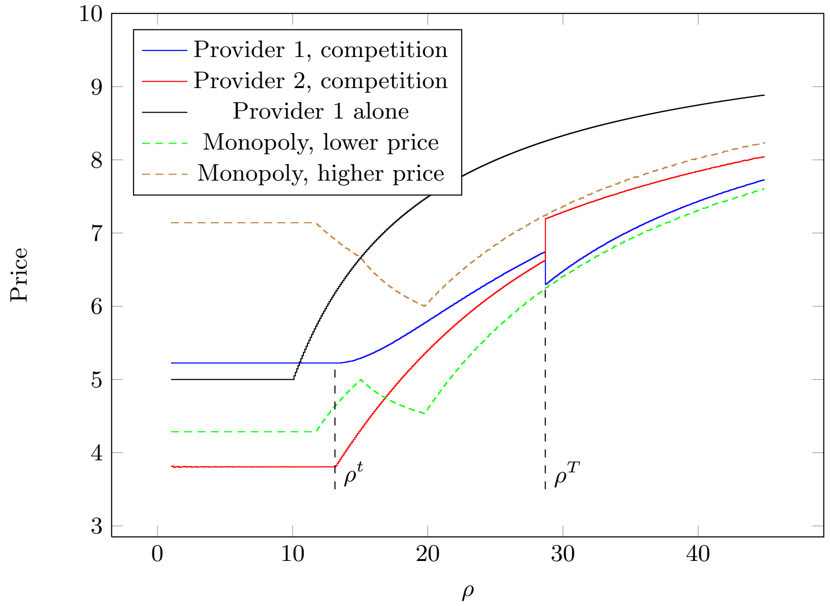


FIGURE 5.28: Optimal prices in leader follower game for different flows values with $C = 10$, $p_{max} = 10$ and linear $w(p)$

#### 5.3.6.5 Monopoly case

In the monopoly case, when both RSUs belong to one provider, that provider will prefer to locate his access points as far as not to harm their capacities, so we further assume that RSUs are situated at maximum distance $D$ from each other.

Figure 5.29, illustrating flows in optimal situation, contains an interesting phenomena: monopolist is interested in causing one of his RSUs to spill-over. Despite that it looks as a contra productive strategy, logically it makes sense.

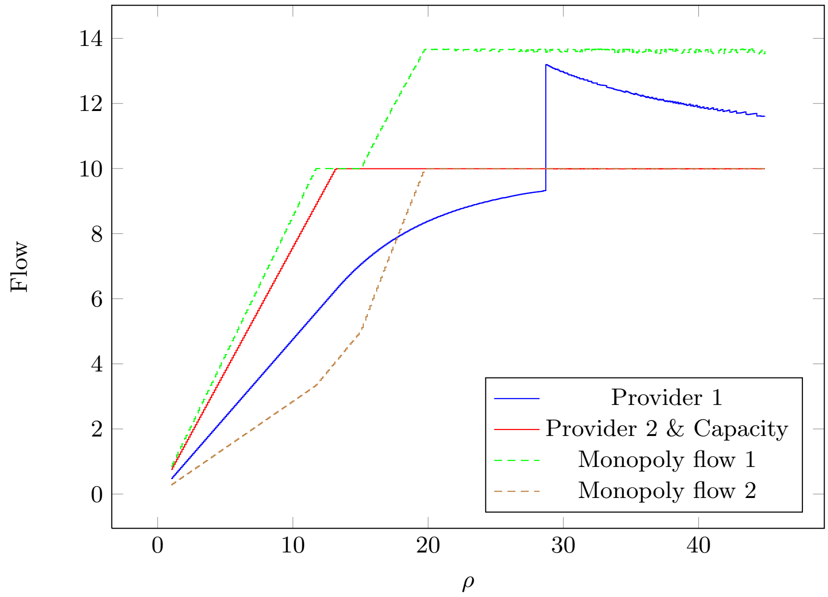


FIGURE 5.29: Optimal demand flows in leader follower game for different $\rho$ with $C = 10$, $p_{max} = 10$ and linear $w(p)$

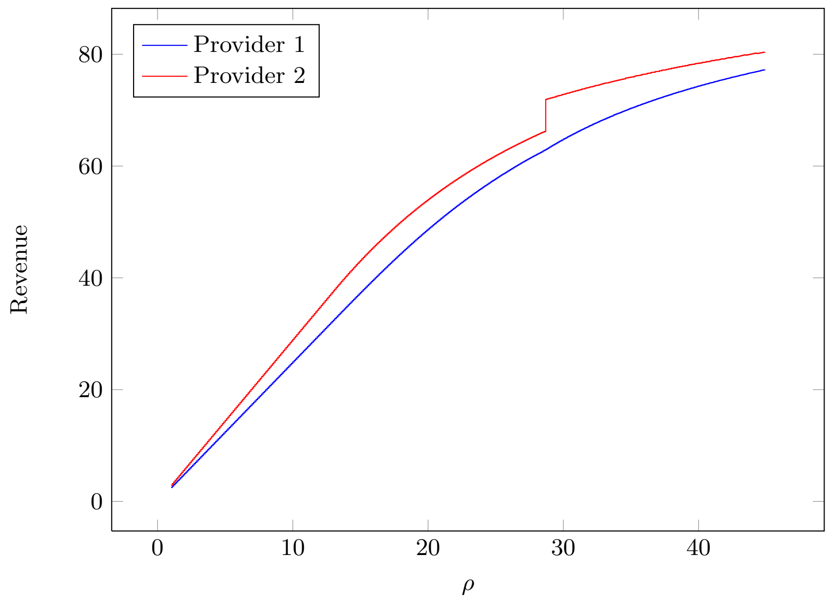


FIGURE 5.30: Providers revenue in equilibrium in leader follower game for different flows with $C = 10$, $p_{max} = 10$ and linear $w(p)$

Let us consider such a big $\rho$, that optimal prices of monopolist are not higher than corresponding capacity saturation prices. Note, that such $\rho$ exists, because if both optimal prices are higher than capacity saturation prices, then they do not depend on $\rho$ (since there is no spilled-over traffic), while capacity saturation prices are increasing and limiting to $p_{max}$ on infinity.

From the basic model we know, that if both access points charge capacity saturation prices, then one of providers could increase his price, in order to cause spilled-over traffic from opponent and thus increase his own revenue, without harming the revenue of competitor (the competitor serves the same traffic for the same price). The same strategy is adopted by the monopolist: he prefers one of his RSU to have higher price, in order to maximize the revenue. We propose the following interpretation: the monopolist wants to efficiently use willingness-to-pay function by making users which could afford higher expenses to pay more. The monopolist causes one of his RSUs to spill-over, in order to make some of users with high willingness-to-pay to come to the second RSU which is more expensive. Other way to tract it is as en effort to separate users by their willingness-to-pay.

#### 5.3.6.6 Users Surplus and Social Welfare

We measure the users welfare in terms of users surplus, which is the sum of differences between what each user wanted to pay and what he actually pays. In both cases of competition and monopoly, only one provider has spillover. Let us for simplicity assume that $p_1 < p_2$. Then the total users surplus in the case of linear willingness-to-pay function is:

$$
\begin{aligned}
US = \quad & P_1 w(p_1)\rho\frac{(p_{max}-p_1)}{2} + \\
& w(p_2)\rho\frac{(p_{max}-p_2)}{2} + \\
& P_1(w(p_1)-w(p_2))\rho\frac{(p_2-p_1)}{2} + \\
& \frac{w(p_2)}{w(p_1)}\rho_1^{sp}\frac{(p_{max}-p_2)}{2},
\end{aligned}
$$

where $P_1$ is probability for user to be served defined in (5.18).

Figure 5.31 shows users surplus variation depending on flow values for monopoly and competition cases. Surprisingly, for some users flow $\rho$ values monopoly situation brings higher users surplus than competition between providers. If we look on Figures 5.29 and 5.28 we may notice that for these $\rho$ values the total flow served in monopoly case is higher than in competition, while the average prices are quite close. The monopolist tends to use his capacities fully, while in competition Provider 1 still prefers to have no spillover traffic.

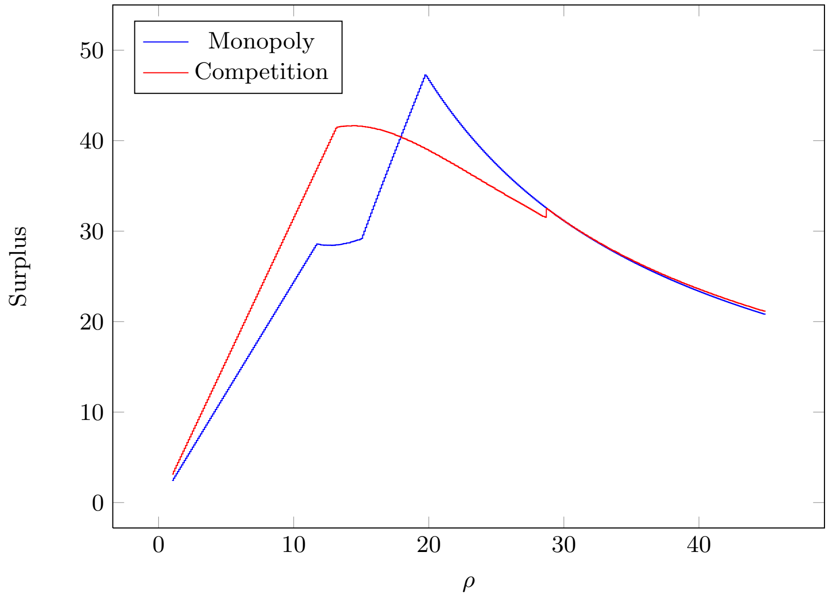


FIGURE 5.31: Users surplus in monopoly and competition scenarios for different users flows values $\rho$ with $C = 10$, $p_{max} = 10$ and linear $w(p)$

## 5.4 Summary

In this Chapter we studied competition of Internet access providers and its influence on the user welfare. We analyzed two different types of models: in the first one the users assumed to be static, while in the second one we assume users to be moving.

For static users we considered the same model as in Chapter 3, where the rating-based game between users was studied. We used the demand estimation in order to predict the revenues of providers, based on the prices they charge. We considered both the competition case and the monopolistic scenario, where both access points belong to one provider. Those cases have been compared in terms of prices, network usage, and energy consumption, highlighting some interesting results like the fact that monopolistic situations are likely to lead to very unfair outcomes.

For the mobile users case we analyzed the price competition between roadside units operators which are providing wireless Internet access to moving vehicles. In contrast to the static users case, in the reference scenario competition arises due to vehicles mobility even if the roadside units do not have overlapping coverage areas. The strategic interaction between two roadside units operators is analyzed through game theoretic tools. Namely, the analysis of the best-response function having fixed the competitor's price sheds light on interesting and counterintuitive behaviors in the systems which lead one roadside unit operator to increase her price to cause traffic spillover from the competitor's side. The results from the best-response analysis are then leveraged to characterize the simultaneous competitive game between the two roadside units operators in terms of equilibrium existence and optimality.

We further extended this basic model of roadside units operators competition in two ways. At first, we studied a specific scenario when users may change their pricing preferences. Several reasons might cause this change in preferences: the users could experience a fear to be unserved, and thus accept to pay more, or try to risk and see if the next access point they meet will be cheaper and thus not pay the price they could really afford to the first provider met. We studied the optimal behavior of a provider, given the opponent's price fixed. We managed to study the competitive game between providers trying to maximize their revenues, and compared it to the cooperative scenario, where providers agree on prices and share the total revenue.

We found that in the framework of our model the perturbations of users willingness-to-pay drastically impacts competition of providers: in the case when users willingness-to-pay is modeled by the linear function we showed that releasing pricing constraints by users could make providers lower their prices and lead to significant losses in terms of revenue for one provider. Further we showed that releasing pricing constraints may also reduce the average price payed by users as well as total users surplus, measured as the difference between what users wanted to pay and what they actually have payed. Numerically, we experimented with different types of willingness-to-pay functions and observed the similar phenomenon, which allows us to suspect that this paradoxical situation has a general nature and thus needs further study.

The second extension covers the case when roadside units operate on the same frequency and thus interfere with each other. In this scenario we studied the problem of the optimal location of access points and discovered that for a monopolist it may be profitable to charge prices with a bigger gap, thus segmenting users by their willingness-to-pay. We observed that in the leader-follower game when users demand is low, the provider which takes his turn first is interested to charge of higher price in order to avoid provoking his opponent to decrease capacity and to cause spillover. Thus a paradoxical situation, when competition makes one of the agents to increase his price, may occur. Finally, we showed that interference could be used as a powerful tool to increase one's revenue, while harming that of the opponent. We also discovered non-trivial optimal pricing strategy for a monopolist, owning both access points and thus trying to maximize the total revenue he gets.

Our results reveal how an Internet access provider should behave, in different scenarios, and how his behavior influences users. We found that the model where mobile users are considered significantly differs from the static cases; several paradoxical situations occur when users are moving. Also, by our study we provided an insight on how emerging provider has to behave in order to have an advantage over the already operating one. This kind of competition may seem unrealistic, but it indeed can influence providers pricing policies: the provider owning already deployed RSU may charge a higher price at first (in order to not provoke emerging provider to harm his capacity), and then to deviate to his real optimal strategy, which we studied in the scenario where interference is not taken into account.

These results can be used as well by government regulator, which gives an permision for operating on a highway. Knowing that monopolist -owning several access points- may cause spilled over traffic on some of his RSU for selfish revenue maximization (which means that some users may end up unserved), the regulator may pose some additional requirements on him in order to maximize the number of users which successfully establish an Internet connection.

# Chapter 6

# Conclusion and perspectives

## 6.1 Thesis summary

In this thesis we have studied the user allocation problem in heterogeneous wireless networks. We fragmentized this problem into three interconnected topics and tackled them separately. This fragmentation is based on the time scale at which the problem is considered.

The smallest time scale we considered corresponds to the decisions made by users by themselves: we focused on how users select among several connection alternatives and what kind of information about these alternatives is given prior to that choice. In the situation of static users this decision is just a choice between a number of available networks. In Chapter 3 we studied a system with a third-party entity, which is responsible for gathering users feedback on the QoS they experienced during their connection, and propagating this information as network ratings back to users. Users, when choosing which network to connect to, observe these ratings to make their choice based on a trade-off between the QoS they expect and the price to pay. We modeled dynamical users arrivals through a Poisson process and by introducing some assumptions estimate the demand distribution, that providers can expect, given their prices. This type of system is interesting because the choice is simplified for users: if in the general case they know nothing about the network they decide to connect to, in the rating-based system they know what to expect. Also, the system is self-regulating, and we found that the equilibrium demand distribution in this system is not far from the optimal situation (with the best total QoS). Our study gives an insight on how rating-based systems could evolve. We showed that even when users arrivals and departures are modeled as stochastic processes (and thus demand distribution varies over time) it is possible to have good estimations of the user behavior outcome.

Further, in Chapter 4 we considered a simplified model of user choice. First, users are assumed to be non-atomic, meaning that their individual decisions have a negligible influence on the other users' welfare. We assumed that all users possess information about the QoS level in observed networks and that their choices are trade-offs between the QoS of the network and the price they have to pay. However, users differ in the way they perceive the latency of their connection (or equally the price they have to pay); we model this diversity with different users classes, each one having its own price sensitivity value. The competition between users in the described model is convenient to model as a routing game, which is known to have an equilibrium.

In this setting we studied the higher time scale problem: how users distribute in the system given that every individual behaves selfishly and how we could incentivize them to "cooperate" in order to achieve the optimal social welfare. We defined the social welfare as the sum of latencies that all users in the system experience, and we considered taxes which the access points owner charges as an incentive for users. It is well known that selfish individuals behavior may lead to inefficient outcomes, and this is true for the model under consideration. In our analysis we considered a general scenario where the number of access points and users classes are arbitrary, and proposed analytical expressions of the optimal taxes, as well as an algorithm to compute them.

For illustration purposes we developed further a basic version of the model, with only two users classes and two access points. We showed that the proposed taxation scheme performs well even in the realistic scenario when users arrivals and departures are random (our initial model consider user demands to be static). Also we proposed a novel interpretation of the PoA measure in terms of inefficiently used capacity, and in terms of demand that could be served without any additional cost if the provider charges the optimal taxes. These interpretations allow providers to estimate in economic terms their potential losses due to inefficient resource management.

The largest time scale corresponds to the competition among Internet access providers. We focus on this topic in Chapter 5, where two cases are tackled separately. In the first case we extend the model proposed in Chapter 3, by allowing the two providers to choose access prices which influence the available demand (demand is elastic). Providers set their prices to maximize their revenues. We studied the simultaneous game in the case when provider infrastructures differ: we assumed that the serving capacity of one provider is bigger, and then compared to the situation when only one provider owns all the infrastructure.

In the second case we studied the Internet access providers competition in a vehicular network. We considered a simple case with only two road side units owned by two providers, but this model could be generalized. The users demand flows in two directions, and thus there are users which see one provider prior to another, and their decisions are whether to pay the price they observe or not. The providers, knowing this, have to take into account that there are two flows of users: those who haven't seen any provider before, and those who have seen a competitor and either refused to pay his price (that means that price they want to pay is low) or were rejected due to capacity constraints. We studied the competition as a simultaneous game, described its equilibria, and found that the first provider to charge a low price always makes more revenue than his competitor. Though our model of Internet access providers competition is fairly simple, it shows important differences with the general competition scenarios. We found an interesting strategy which one provider can adopt to maximize his revenue: he can decide to charge a high price, such that the competitor will be caused to spillover some users with high willingness-to-pay values.

Further, we studied the case when users may change their pricing preferences after passing the first provider they met, assuming they release their pricing constraints, i.e, they accept to pay more. We found an interesting phenomenon: with respect to the case when users do not vary their pricing preferences the providers pricing game may end up with an equilibrium where both players charge lower prices and one provider experiences revenue losses.

Finally we considered the case when road side units operate on the same frequency and thus interfere to clients of each other (if we consider only downlink traffic). We studied the problem of optimal access point location, and found several interesting strategies for an emerging provider through a leader-follower game. We also discovered that for a monopolist provider owning two RSUs, it may be profitable to charge one of his RSU a high price to cause the second RSU to spillover. By this the monopolist can gain more revenue, exploiting at maximum the users willingness-to-pay. We also observed that when users demand is low, the provider makes sets his price first is interested to charge a higher price in order to avoid provoking his opponent to decrease capacity and to cause spillover. This leads to a paradoxical situation, when competition leads to higher prices that the monopoly. To sum up, we showed that interference can be used as a powerful tool by providers, allowing to increase their revenue, while harming their of opponent.

## 6.2 Contributions with regard to the main research challenges

In the current thesis we tried to tackle some of the research challenges mentioned in the end of Chapter 2. The first rating-based network selection problem is aimed to answer the question which type of information may be available to users, and what will happen when the information is not full or not up-to-date. We studied the system, where ratings are refreshed periodically with some time gap and also includes information about previous quality of an access point, and we found that with this mechanism we still can predict the outcome of users behavior.

In Chapter 4 we studied the resource management problem, where we found a simple algorithm (can make real-time optimization fast) which determines the taxes, that minimize the total latency experienced by users. In this model, the lowest tax can be chosen arbitrary, which allows to control the total revenue level of provider (we do not consider elastic demand there). However, the users welfare (which also depends on the tax the users pay) obviously may be harmed by the proposed taxation mechanism. Further, we managed to investigate the situation, when the provider does not have all the information about users. For this we considered a simple example, where the provider is not aware about users classes separation; in this case the proposed taxation mechanism lead to a situation fairly close to the optimal one.

Finally, in Chapter 5 we considered a vehicular network, where all users are mobile. In the general case mobile users study is quite complex, but our model allows to simplify this issue. Also, at an extension of the basic model, we introduce an interference phenomena, which is one more step forward the realistic system description.

## 6.3 Prospective research directions

Usually, when we discuss the network selection problem, we have to strictly define which information is known by users and by which means. The two most realistic network selection models, where some QoS parameters of networks are known by users are based on 1) rating and 2) probing. The former model was described in Chapter 3, and the probing based approach was proposed in [6]. The authors considered different probing schemes, which should be adopted by all users, and for each scheme they studied a corresponding game. The chosen scheme defines information which is known by users. This model seems to be quite interesting, and one of the follows-up for it could consist in considering the case where users selfishly select their probing schemes. That means that users select how many networks they want to probe, and, obviously, a higher probing number leads to overall QoS degradation.

The rating-based model is also a good candidate for further research. We showed that it is easy to predict the users demand distribution given providers pricing, and further we investigated how providers compete in this setting. But in this model we did not take into account that users may experience different QoS in the same network conditions. For example, we did not consider that the latency degrades due to path loss. Also, we considered only two networks with overlapping coverage areas, and it is interesting to study more general topologies (where the rating could play as less important role).

We have discovered several features of vehicular network providers competition, which shows that this type of competition differs from what we observe in the case of static users. If in the static users case, users select the network to connect to, in vehicular networks the decision is rather binary: users decide whether or not to connect to the currently observed access point at the proposed price conditions. We focus more on competition among providers which aim to maximize their revenues, but we found there an interesting feature: if all users decide to pay a higher price for the second access point, than they initially wanted to pay, then in average the price they will pay may decrease. This result inspires one more research direction: it is interesting to study how the variation of individual users willingness-to-pay values (separately) may influence the total population welfare. This can be considered as a game, where each user decides based on the service evaluation, how much he can risk on refusing a price which he actually is able to pay.

Other possible follow-up for the model with vehicular networks providers competition is to consider cellular networks as one more alternative for mobile users. In this case users will make a trade-off between cost and quality of connection (assuming that the data rate in the cellular network is lower). There can be several interesting strategies for users, which will influence providers competition: if the users are unaware of the number of RSUs on the road, they may decide whether they can tolerate a delay or they need to fulfill their requests immediately, which also dictates which type of network connection they will prefer.

# Appendix A

# Chapter 5 Proofs

## A.1 Proof of Proposition 5.9

We first assume that $p_1/\alpha \geq p_2$. Since the right-hand sides of the equations in (5.18) are continuous in $(P_1, P_2)$ and fall in the interval $[0,1]$, Brouwer's fixed-point theorem [84] guarantees the existence of a solution to the system.

To establish uniqueness, remark that $P_2$ is uniquely defined by $P_1$ through the second equation in (5.18), so $(P_1, P_2)$ is unique if $P_1$ is unique. But $P_1$ is a solution in $[0,1]$ of the fixed-point equation $x = g(x)$ with

$$g(x) := \min\left(1, \frac{1}{a + b - b\min\left(1, \frac{1}{a+b+\epsilon-ax}\right)}\right),$$

where $a = \frac{w(p_1)\rho_1}{c}$, $b = \frac{w(p_1/\alpha)\rho_2}{c}$, and $\epsilon = \frac{(w(p_2/\alpha)-w(p_1))\rho_1+(w(p_2)-w(p_1/\alpha))\rho_2}{c}$ are all positive constants; we also assume $a > 0$ and $b > 0$ otherwise the problem is trivial. As a combination of two functions for the form $x \mapsto \min\left(1, \frac{1}{K_1 - K_2 x}\right)$, $g$ is continuous, nondecreasing, strictly increasing only on an interval $[0, \bar{x}]$ (if any) –it is in addition convex on that interval–, and constant for $x \geq \bar{x}$ (note we can have $\bar{x} = 0$ or $\bar{x} \geq 1$).

Assume $g(x) = x$ has a solution $\tilde{x} \in (0, \bar{x}]$. Then $g$ is left-differentiable at $\tilde{x}$, and

$$g'(\tilde{x}) = \frac{\tilde{x}^2 ab}{(a + b + \epsilon - a\tilde{x})^2} \leq \frac{\tilde{x}^2 a}{(a + b + \epsilon - a\tilde{x})} \tag{A.1}$$

where we used the fact that $\tilde{x} \le 1$ (as a fixed point of $g$). Moreover, since $\tilde{x}$ is in the domain where $g$ is strictly increasing we have $\eta := \frac{1}{a+b+\epsilon-a\tilde{x}} \le 1$ on one hand, and $\tilde{x} = \frac{1}{a+b-b\eta}$ on the other side. Their combination yields $\tilde{x} \le \frac{1}{a}$ and finally

$$g'(\tilde{x}) \le \tilde{x} \le 1.$$

Remark also that $g'(\tilde{x}) < 1$ if $\tilde{x} < 1$. We finally use the fact that $g(0) > 0$ to conclude that the curve $y = g(x)$ cannot meet the diagonal $y = x$ more than once: assume two intersection points $\tilde{x}_1 < \tilde{x}_2$, then $g'(\tilde{x}_1) < 1$ thus the curves cross at $\tilde{x}_1$, another intersection point $\tilde{x}$ would imply $g'(\tilde{x}_2) > 1$ (recall $g$ is convex when strictly increasing), a contradiction. Hence the uniqueness of the fixed point and of the solution to (5.18).

By symmetry, we have the same kind of results when $p_2/\alpha \ge p_1$.

Finally, we can also prove existence and uniqueness of a solution of system (5.19), when $p_2/\alpha \le p_1 \le p_2\alpha$. Here we have

$$g(x) := \min\left(1, \frac{1}{a+b-d\min\left(1, \frac{1}{d+a+\epsilon-ax}\right)}\right),$$

where $a = \frac{w(p_1)\rho_1}{c}$, $b = \frac{w(p_1/\alpha)\rho_2}{c}$, $d = \frac{w(p_2)\rho_2}{c}$ and $\epsilon = \frac{w(p_2/\alpha)\rho_1 - w(p_1)\rho_1}{c}$ are all positive constants; we again assume $a > 0$ and $b > 0$ otherwise the problem is trivial.

Differentiating $g$ at $\tilde{x}$, we get

$$g'(\tilde{x}) = \frac{\tilde{x}^2 ad}{(a+d+\epsilon-a\tilde{x})^2} \le \frac{\tilde{x}^2 a}{(a+d+\epsilon-a\tilde{x})}, \tag{A.2}$$

and the rest is similar to the case when $p_1/\alpha \ge p_2$.

## A.2 Proof of Lemma 5.3 for the varying WTP case

Recall that

$$\begin{aligned} \rho_j^{\mathrm{T}}(p_j, p_k) \quad &= \quad w(p_j)\rho_j + \rho_k[w(p_j/\alpha) - w(p_k)]^+ \\ &\qquad + \min\left(w(p_k), w(p_j/\alpha)\right)\rho_k(1-P_k). \end{aligned}$$

The components of the first line are trivially continuous and non-increasing in $p_j$ with our assumptions on $w(\cdot)$.

The continuity of $\rho_j^{\mathrm{T}}(p_j, p_k)$ follows from the continuity of $P_k$ in the price vector $(p_j, p_k)$, established in the previous section.

To establish the monotonicity result, we distinguish three cases.

- If $p_k < p_j/\alpha$, then we have

$$\rho_j^{\mathrm{T}}(p_j, p_k) = w(p_j)\rho_j + w(p_j/\alpha)\rho_k(1 - P_k).$$

When $\rho_k^{\mathrm{T}} < c$, then $P_k = 1$ and $\rho_j^{\mathrm{T}}$ is non-increasing in $p_j$.

Now if $\rho_k^{\mathrm{T}} > c$ then from System (5.18) (this time with $k = 2$, $j = 1$), we have $w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j P_j > c$ and

$$\begin{aligned}
\rho_j^{\mathrm{T}}(p_j, p_k) \quad &= \quad w(p_j)\rho_j + w(p_j/\alpha)\rho_k \\
&\quad -w(p_j/\alpha)\rho_k \frac{c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j P_j}.
\end{aligned}$$

Assuming that provider $j$ is not saturated, $P_j = 1$. Then

$$\begin{aligned}
\rho'^{T}_j(p_j, p_k) \quad &= \quad w'(p_j)\rho_j + \frac{w'(p_j/\alpha)\rho_k}{\alpha} \\
&\quad -\frac{w'(p_j/\alpha)\rho_k}{\alpha} \frac{c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j} \\
&\quad +w(p_j/\alpha)\rho_k \frac{cw'(p_j)\rho_j}{(w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j)^2} \\
&< \quad w'(p_j)\rho_j + \frac{w'(p_j/\alpha)\rho_k}{\alpha} \\
&\quad -\frac{w'(p_j/\alpha)\rho_k}{\alpha} \\
&\quad +w(p_j/\alpha)\rho_k \frac{cw'(p_j)\rho_j}{(w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j)^2} \\
&\leq \quad 0,
\end{aligned}$$

where the last inequality comes from the nonincreasingness of $w(\cdot)$.

- If $p_j/\alpha \leq p_k \leq p_j\alpha$ then

$$\rho_j^{T} = w(p_j)\rho_j + w(p_j/\alpha)\rho_k - \frac{cw(p_k)\rho_k}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j P_j}$$

Assuming that provider $j$ is not saturated and then $P_j = 1$ we can differentiate in $p_j$:

$$\begin{aligned}\frac{\mathrm{d}\rho_j^T}{\mathrm{d}p_j} &= w'(p_j)\rho_j + w'(p_j/\alpha)\frac{\rho_k}{\alpha} \\ &+ \frac{cw(p_k)w'(p_j)\rho_j\rho_k}{(w(p_k)\rho_k + w(p_k/\alpha)\rho_j - w(p_j)\rho_j)^2} \leq 0,\end{aligned}$$

where $w'$ is the derivative of $w$, and the last inequality comes from the fact that $w'(\cdot) \leq 0$.

• If $p_k > p_j\alpha$, we show that the success probability $P_k$ is non-decreasing in $p_j$: applying System (5.18) (with $k = 1$, $j = 2$) we get that $P_k$ is the solution of the fixed-point equation $x = g(x)$, where the function $g$ can be written as

$$g(x) = \min\left(1, \frac{c}{w(p_k)\rho_k + w(p_k/\alpha)\rho_j\left[1 - \frac{c}{w(p_j)\rho_j + w(p_j/\alpha)\rho_k - w(p_k)\rho_k x}\right]^+}\right).$$

We then remark that, all else being equal, $g(x)$ is non-decreasing in $p_j$, so the solution $P_k$ of the fixed-point equation $g(x) = x$ is also non-decreasing in $p_j$.

As a result, when $p_k \geq p_j/\alpha$ the component $\min\left(w(p_k), w(p_j/\alpha)\right)\rho_k(1 - P_k)$ decreases with $p_j$, and so does $\rho_j^{\mathrm{T}}$.

## A.3 Proof of Lemma 5.12

Assume it is not true and there is some $\bar{d} \in [0, d^{sp}]$, such that $\rho_1^{sp}(p^{opt}(\bar{d})) = 0$. Further we assume that in $d^{sp}$ we have strictly $\rho_1^{sp}(p^{opt}(\bar{d})) > 0$, otherwise we consider some $d^{sp} - \epsilon$, at which by definition of $d^{sp}$ spillover of provider 1 should be greater than zero. The optimal price at distance $d^{sp}$ we denote by $\hat{p} = p^{opt}(d^{sp})$, and from the fact that at $d^{sp}$ Provider 1 has spillover, we deduce that $\hat{p} \geq p_1$.

Note that $\rho_2^T(p^{opt}(d)) \leq c(d)$ for all $d \in [0, D]$ (otherwise the price is not optimal). Also we could deduce that $p^{opt}(d^{sp}) > p_1$, because at $(d^{sp}, p^{opt}(d^{sp}))$ provider 1 has spillover and provider 2 does not.

For this particular $\bar{d}$, consider two cases. First, if $\bar{p} = p^{opt}(\bar{d}) < p_1$. Then,

$$R_2(\bar{d}, \bar{p}) = \bar{p}\Big[2w(\bar{p})\rho - w(p_1)\rho\Big],$$

and because $\bar{p}$ is optimal price, we should have $R_2(\bar{d}, \bar{p}) \geq R_2(\bar{d}, \hat{p})$).

Note that $\rho_2^T(\bar{d}, \hat{p}) < c(d)$, otherwise $\bar{p}$ would't be optimal price: indeed, we have to recall that $\bar{p} < p_1 < \hat{p}$

Then, from Lemma 5.11 we know that $R_2(\bar{d}, \hat{p}) > R_2(d^{sp}, \hat{p})$. Further we could deduce that $R_2(\bar{d}, \bar{p}) \leq R_2(d^{sp}, \bar{p})$, because at $d^{sp}$ we have additional traffic coming from spillover of Provider 1, which is not the case for $\bar{d}$. Thus we have a contradiction, because

$$R_2(d^{sp}, \bar{p}) \geq R_2(\bar{d}, \bar{p}) \geq R_2(\bar{d}, \hat{p}) > R_2(d^{sp}, \hat{p}),$$

which is not possible because $\hat{p} = p^{opt}(d^{sp})$.

The same is true for the case if $\bar{p} = p^{opt}(\bar{d}) \geq p_1$ with revenue function

$$\begin{aligned} R_2(\bar{d}, \hat{p}) \leq R_2(\bar{d}, \bar{p}) = \bar{p}(w(\bar{p})\rho) = \\ \bar{p}w(\bar{p})\rho = R_2(d^{sp}, \bar{p}). \end{aligned}$$

and we know from Lemma 5.11 that $R_2(\bar{d}, \hat{p})) > R_2(d^{sp}, \hat{p}))$, from which we again receive contradiction.

## A.4 Proof of Lemma 5.13

Let us consider $\bar{d} < \hat{d}$, and $\bar{d}, \hat{d} \in [d^c, d^{sp}]$.

Let us assume contrary to lemma's statement that:

$$R_2(\bar{d}, \bar{p}) \geq R_2(\hat{d}, \hat{p}),$$

where $\bar{p} = p^{opt}(\bar{d})$ and $\hat{p} = p^{opt}(\hat{d})$. Now if we take a look on the revenue function of Provider 2 (taking into account Lemma 5.12) for some $d \in (d^c, d^{sp}]$ at fixed price, e.g. $\hat{p}$:

$$R_2(d, \hat{p}) = \hat{p}w(\hat{p})\rho\Big[2 - \frac{c(d)}{2w(p_1)\rho - w(\hat{p})\rho}\Big],$$

we could notice that it is decreases with $d$ due to increasing $c(d)$. Thus we could deduce that

$$R_2(\bar{d}, \hat{p}) > R_2(\hat{d}, \hat{p}) > R_2(\bar{d}, \bar{p}),$$

which contradicts to the fact that $\bar{p} = p^{opt}(\bar{d})$.

## A.5 Proof of Proposition 5.14

For the first case with $d^{sp} = 0$, when Provider 2 decreases his distance below $d^c$, he obviously decreases the maximum revenue he gets, since for any distance Provider 1 has no spilled-over traffic.

For the case when $d^{sp} > d^c \geq 0$, from Lemma 5.13 we deduce that the optimal distance for Provider 2 belongs to $[0, d^c]$, i.e, at optimal distance Provider 2 has his capacity saturated. This implicitly means that in situation when $d^{sp} > d^c \geq 0$, the optimal distance for Provider 2 is

$$\arg \max_{d \in [0, d^c]} c(d) p^c(d),$$

When $d^c \geq d^{sp} > 0$ the only result we get is that the optimal distance does not belong to the range $(d^{sp}, d^c)$. Since $d^{sp} \leq d^c$, all distances from $[d^c, D]$ are equivalent from the point of view of optimal revenue (optimal price does not depend on capacity, as well as revenue function). The optimal distance could belong to this range and thus could be expressed as

$$\arg \max_{d \in [0, d^{sp}]} c(d) p^c(d),$$

or to $[0, d^{sp}]$. Numerically we can find the threshold price of Provider 1, which defines the optimal distance Provider 2 have to choose in this case, and it does not depend on users flow value $\rho$.

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## Résumé

Dans ce travail de thèse, nous étudions le problème de la répartition des utilisateurs dans les réseaux sans fil hétérogènes. Nous avons séparé ce problème en trois thèmes interconnectés que nous avons abordés séparément. Cette fragmentation est fondée sur l'échelle de temps à laquelle est considéré le problème.

Nous étudions d'abord un système où une entité tierce est chargée de recueillir les informations fournies par les utilisateurs sur la qualité de service dont ils ont bénéficié lors de leur connexion, et de propager cette information sous forme d'un score aux autres utilisateurs qui auront à effectuer un choix. Ainsi, les utilisateurs suivants pourront baser leur choix de point d'accès sur un compromis entre la qualité de service qu'ils peuvent attendre (estimée par les scores reçus) et le prix qu'ils auront à payer.

Nous étudions ensuite un modèle simplifié de choix des utilisateurs, qui nous permet une étude analytique complète. Les utilisateurs sont supposés être non-atomiques, c'est-à-dire que leurs décisions individuelles ont une influence négligeable sur la qualité perçue par les autres. On peut alors influencer, via des prix, ces décisions de façon à minimiser le coût total perçu par les utilisateurs.

Enfin, nous analysons la concurrence entre fournisseurs d'accès dans les réseaux véhiculaires. Nous considérons un cas simple avec seulement deux points d'accès, appartenant à deux fournisseurs, disposés le long d'une route. Les utilisateurs se déplacent dans les deux sens, et voient un fournisseur avant l'autre; ils doivent alors décider d'accepter ou non de payer le prix observé.



**Mots-clés** : Théorie des jeux, Réseau véhiculaire, Jeu de routage, Réseau sans fil



## Abstract

Almost all modern mobile devices are equipped with a number of various wireless interfaces simultaneously, so that each user is free to select between several types of wireless networks. This opportunity raises a number of challenges, since in general selfish choices do not lead to a globally efficient repartition of users over networks. In order to study this problem, we split the general users allocation subject into three subtopics.

At first, we study how the users are making network selection decision, which information is available for them and by which means. We develop a model, where users decision are lead by ratings of available networks and prices they have to pay.

At the second step we already study the outcome of selfish users behavior, which we found to be inefficient. We decide to introduce a specific taxation policy, which takes into account the users diversity in price (or QoS) perception, and lead to an optimal situation, when the total QoS experienced by users is maximized.

The last subtopic covers the problem of providers interaction, which has a crucial impact on users welfare. We study both models with static and mobile users, and for the former case we propose a novel model of Internet access providers competition in the vehicular networks.



**Keywords** : Game theory, Wireless network, Routing game, Vehicular network



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