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Representing 3D models for alignment and recognition

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Abstract

Thanks to the success of 3D reconstruction algorithms and the development of online tools for computer-aided design (CAD) the number of publicly available 3D models has grown significantly in recent years, and will continue to do so. This thesis investigates representations of 3D models for 3D shape matching, instance-level 2D-3D alignment, and category-level 2D-3D recognition.

The geometry of a 3D shape can be represented almost completely by the eigen-functions and eigen-values of the Laplace-Beltrami operator on the shape. We use this mathematically elegant representation to characterize points on the shape, with a new notion of scale. This *3D point signature* can be interpreted in the framework of quantum mechanics and we call it the *Wave Kernel Signature* (WKS). We show that it has advantages with respect to the previous state-of-the-art shape descriptors, and can be used for 3D shape matching, segmentation and recognition.

A key element for understanding images is the ability to align an object depicted in an image to its given 3D model. We tackle this *instance level 2D-3D alignment* problem for arbitrary 2D depictions including drawings, paintings, and historical photographs. This is a tremendously difficult task as the appearance and scene structure in the 2D depictions can be very different from the appearance and geometry of the 3D model, e.g., due to the specific rendering style, drawing error, age, lighting or change of seasons. We represent the 3D model of an entire architectural site by a set of visual parts learned from rendered views of the site. We then develop a procedure to match those scene parts that we call *3D discriminative visual elements* to the 2D depiction of the architectural site. We validate our method on a newly collected dataset of non-photographic and historical depictions of three architectural sites.

We extend this approach to describe not only a single architectural site but an entire object category, represented by a large collection of 3D CAD models. We develop a *category-level 2D-3D alignment* method that not only detects objects in cluttered images but also identifies their approximate style and viewpoint. We evaluate our approach both qual-

itatively and quantitatively on a subset of the challenging Pascal VOC 2012 images of the "chair" category using a reference library of 1394 CAD models downloaded from the Internet.

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Contents

1 Introduction **11**

1.1 Goals . . . 11

1.2 Motivation . . . 13

1.3 Challenges . . . 14

1.3.1 3D local descriptors . . . 15

1.3.2 Instance-level 2D-3D alignment . . . 16

1.3.3 Category-level 2D-3D object recognition . . . 18

1.4 Contributions . . . 20

1.4.1 Wave Kernel Signature . . . 20

1.4.2 3D discriminative visual elements . . . 21

1.5 Thesis outline . . . 21

1.6 Publications . . . 23

2 Background **25**

2.1 3D shape analysis . . . 25

2.1.1 From the ideal shapes to discrete 3D models . . . 26

2.1.2 3D point descriptors . . . 28

2.1.3 3D shape alignment methods . . . 33

2.2 Instance-level 2D-3D alignment . . . 36

2.2.1 Contour-based methods 36
2.2.2 Local features for alignment 40
2.2.3 Global features for alignment 46
2.2.4 Relationship to our method 47
2.3 Category-level 2D-3D alignment 47
2.3.1 2D methods 48
2.3.2 3D methods 52
2.3.3 Relationship to our method 53

3 Wave Kernel Signature 55

3.1 Introduction 55
3.1.1 Motivation 56
3.1.2 From Quantum Mechanics to shape analysis 57
3.1.3 Spectral Methods for shape analysis 59
3.2 The Wave Kernel Signature 65
3.2.1 From heat diffusion to Quantum Mechanics 65
3.2.2 Schrödinger equation on a surface 67
3.2.3 A spectral signature for shapes 68
3.2.4 Global vs. local WKS 70
3.3 Mathematical Analysis of the WKS 71
3.3.1 Stability analysis 71
3.3.2 Spectral analysis 75
3.3.3 Invariance and discrimination 76
3.4 Experimental Results 77
3.4.1 Qualitative analysis 77

3.4.2 Quantitative evaluation . . . 81
3.5 Applications . . . 87
3.6 Conclusion . . . 90

4 Painting-to-3D Alignment 91

4.1 Introduction . . . 91
4.1.1 Motivation . . . 93
4.1.2 From locally invariant to discriminatively trained features . . . 93
4.1.3 Overview . . . 94
4.2 3D discriminative visual elements . . . 96
4.2.1 Learning 3D discriminative visual elements . . . 97
4.2.2 Matching as classification . . . 98
4.3 Discriminative visual elements for painting-to-3D alignment . . . 100
4.3.1 View selection and representation . . . 100
4.3.2 Least squares model for visual element selection and matching . . . 102
4.3.3 Calibrated discriminative matching . . . 107
4.3.4 Filtering elements unstable across viewpoint . . . 108
4.3.5 Robust matching . . . 111
4.3.6 Recovering viewpoint . . . 111
4.3.7 Summary . . . 112
4.4 Results and validation . . . 113
4.4.1 Dataset for painting-to-3D alignment . . . 113
4.4.2 Qualitative results . . . 114
4.4.3 Quantitative evaluation . . . 117
4.4.4 Algorithm analysis . . . 124

4.5 Conclusion . . . 129

5 Seeing 3D Chairs 131

5.1 Introduction . . . 131

5.1.1 Motivation . . . 133

5.1.2 From instance-level to category-level alignment . . . 134

5.1.3 Approach Overview . . . 135

5.2 Discriminative visual elements for category-level 3D-2D alignment . . . 136

5.2.1 Representing a 3D shape collection . . . 136

5.2.2 Calibrating visual element detectors . . . 140

5.2.3 Matching spatial configurations of visual elements . . . 141

5.3 Experiments and results . . . 143

5.3.1 Large dataset of 3D chairs . . . 145

5.3.2 Qualitative results . . . 145

5.3.3 Quantitative evaluation . . . 147

5.3.4 Algorithm analysis . . . 150

5.4 Conclusion . . . 154

6 Discussion 156

6.1 Contributions . . . 156

6.2 Future work . . . 157

6.2.1 Anisotropic Laplace-Beltrami operators . . . 157

6.2.2 Object compositing . . . 158

6.2.3 Use of 3D shape collection analysis . . . 159

6.2.4 Synthetic data for deep convolutional network training . . . 159

6.2.5 Exemplar based approach with CNN features . . . 159

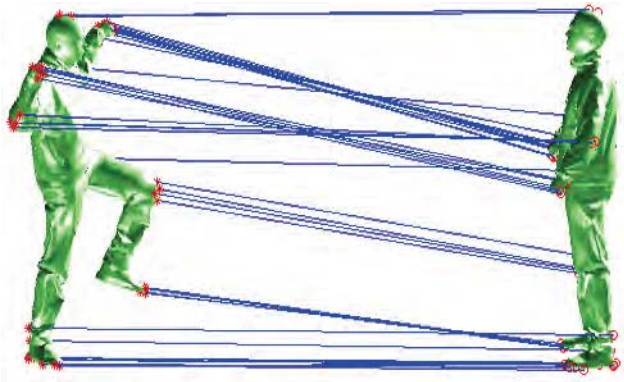
Chapter 1

Introduction

1.1 Goals

The goal of this thesis is to develop representations of 3D models for (i) alignment with other 3D models, (ii) alignment with an image containing the same object instance and (iii) alignment with an image containing an object from the same category. Those three tasks are illustrated in figure 1.1. What is a "good" representation will depend on the task :

- **Matching, segmentation and recognition of 3D shapes:** Shape matching aims at computing correspondences between two similar 3D shapes. Shape segmentation attempts to partition the shape into a set of meaningful regions by analyzing the single shape or a collection of 3D shapes. Finally, shape recognition typically classifies a given shape into categories defined by examples of other shapes. The work presented in chapter 3 aims at improving the 3D point descriptors that are used for those tasks by modeling and optimizing the descriptor variability. An example of shape matching is shown on figure 1.1a.



(a) *Matching between two 3D shapes using our Wave Kernel Signature.*



(b) *Examples of non-photographic depictions of Notre Dame in Paris correctly aligned to its 3D model (shown in the top left inset) and visualized in Google Earth [56]*



(c) *Approximate models correctly recovered by our algorithm and overlapped with the input image (left) in their recovered viewpoint (right).*

Figure 1.1: *The three different 3D model alignment tasks addressed in this thesis.*

- **Instance-level 2D-3D alignment across depiction style:** while computer vision has mainly focused on analyzing photographs, we aim at understanding historical and non-photographic imagery. Given the 3D model of an architectural site and its 2D depiction, we wish to recover the viewpoint of the image with respect to the 3D model. To apply this idea on a large scale, our goal is to develop an automatic method that is robust to the style variations, to the errors in the depictions and to the variable quality of the 3D model. Examples of paintings aligned by our algorithm with a 3D model of Notre-Dame in Paris are shown on figure 1.1b.

- **Object category-level recognition by 2D-3D alignment:** we want to go beyond instance-level alignment, which requires knowing in advance the 3D models of the object instances present in the image, and develop category-level 2D-3D alignment. We assume that the object categories are represented by large collections of 3D CAD models. Our goal is to take as input an unseen image and to output not only the categories of the objects that are present but also approximate 3D models, correctly aligned with the input depiction, as shown in figure 1.1c.

1.2 Motivation

Automatic, high quality, large scale 3D reconstructions are one of the major successes of computer vision. It is now possible to easily scan an object with a smart phone [104, 172], capture a living room with a Kinect [91, 131, 159], or visit a virtual city on Google Earth [56]. Computer-aided design has also evolved to the point that public or commercial libraries of millions of 3D models of objects are available [157, 175]. This

growing amount of data, of which some examples are shown in figure 1.2, requires new tools but also is an opportunity to develop new applications. Example applications include:

- **Browsing and parametrization of large shape collections.** Creating a new 3D model requires time and expert knowledge. Instead of always creating new models, the designer could browse, search, and manipulate in an intuitive way existing 3D models in the collection.

- **Browsing historical data.** Imagine a computer could automatically recover the viewpoint of all existing historical imagery. This could change the way archivists, architects or historians access and browse archives of historical images. The users could browse the images intuitively and to compare depictions from similar places at different times.

- **Smart image editing.** Imagine a computer could identify objects in an input 2D image and automatically recover their 3D models. It would be possible to use the recovered 3D model to edit the image by manipulating objects in 3D. Currently, this editing requires manual annotation [100].

- **Robotic manipulation.** For a robot to manipulate an object, it needs to know not only in which direction the object is, but also to have access to its 3D model including, for example, unseen parts.

1.3 Challenges

While there are several exciting applications for 3D alignment, finding good representations and matching algorithms is very difficult.



(a) *Interactive reconstruction with a cell phone [172]*



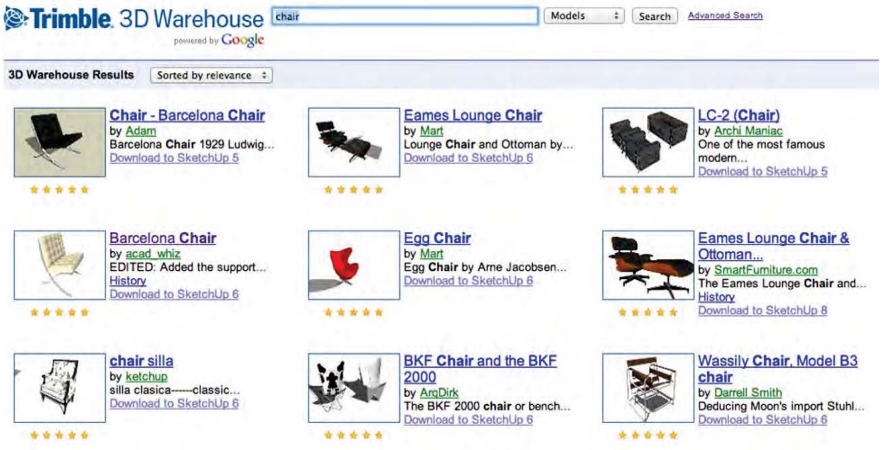
(b) *Reconstruction of a room with Kinect fusion [91]*



(c) *High-resolution reconstruction by Acute3D [5]*



(d) *Berkeley campus on Google Earth [56]*



(e) *Search for 3D models of chairs on Trimble 3D Warehouse [175]*

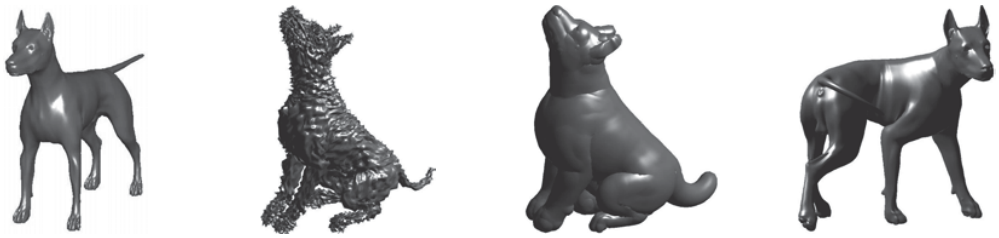
Figure 1.2: *3D models are becoming common and easy to acquire.*

1.3.1 3D local descriptors

Defining purely geometric descriptors for 3D shapes is a difficult and open problem. An good descriptor for a point on a 3D shape would:

- be **discriminative** to distinguish different points on the shape as well as different shapes from each other.

- be **robust** to perturbations such as near-isometric deformations, noise or topo-



(a) *Reference shape* **(b)** *Noise* **(c)** *Local dilatation* **(d)** *Near-isometric*

Figure 1.3: *Challenges of 3D shape alignment. Examples of shape perturbations from the TOSCA dataset [50].*

logical changes. Examples of such perturbations are shown in figure 1.3. This is needed for or example to match two models of a person in different poses or to align models of different quadrupeds.

- include informations from all **scales** of the shape to allow recognition and matching of parts of the shape, for example to recognize a handle within a 3D model of a complete door.

Those characteristics are hard to achieve and are often conflicting. We focus particularly on the trade-off between robustness and discriminative power, while developing a new intuition about scale.

1.3.2 Instance-level 2D-3D alignment

Viewpoint, illumination and occlusion. First, the space of possible viewpoints of the same 3D model is huge, especially for a full architectural site, and the appearance of the 3D model can change significantly with the viewpoint as shown in figure 1.4a. Second, the illumination conditions, for example related to the season and the time of the day also change significantly the appearance of the scene as shown in figure 1.4b and 1.4c. Third, the appearance of the 3D model itself is highly specific, ranging from simplified CAD models to highly detailed models obtained from recent multi-view



(a) *Viewpoint changes*



(b) *Illumination changes*



(c) *Season changes*



(d) *Model specific appearance*



(e) *Clutter and occlusion*



(f) *Historical and non-photographic imagery*

Figure 1.4: *Challenges of 2D-3D instance alignment.*



Figure 1.5: *Matching local features, such as SIFT [121] often works well for two similar photographs (left), but fails between two very different images such as a photograph and a watercolor (right). This figure shows the most confident SIFT matches between the top image and the two bottom images in terms of the first to second nearest neighbor ratio [121].*

stereo algorithms, as can be seen in figure 1.4d. Finally, in many cases, occlusions and clutter change the appearance of an image as shown figure 1.4e.

Depiction style and drawing errors. Non-realistic depictions such as paintings, drawings and engravings are even harder to work with. They have very particular depiction style and often (sometime intentional) drawing errors. In some cases, the appearance of the place may have changed over time, because of construction or aging. Examples of such effects are shown in figure 1.4f. For these reasons, local descriptors, such as SIFT [121], traditionally used for instance-level alignment, often fail for non-realistic depictions as shown in figure 1.5.

1.3.3 Category-level 2D-3D object recognition

The difficulties discussed above for instance-level alignment also apply to category-level alignment. An example can be seen in figure 1.6 where recognizing objects, such as chairs, is difficult because of the occlusions, shadows, clutter and the different



Figure 1.6: *Aligning 3D models to objects, such as chairs in this image is difficult because of occlusion, clutter, the different viewpoints and illumination effects. Note that this image was computer generated.*



Figure 1.7: *Intra-class variation is one of the main challenges of category-level recognition. This figure shows a small set of chairs with different appearance, topology and parts.*

viewpoints.

Intra-class variation. To recognize not only a given instance but any instance from a given object category, we must also deal with the intra-class variation. The different instances belonging to the same category can have different textures, more or less parts parts or even a completely different topology, as shown in figure 1.7 for the "chair"

category.

1.4 Contributions

This section presents the two main contributions of this thesis. More details about the technical contributions are given in section 6.1.

1.4.1 Wave Kernel Signature

One of the basic tasks in shape processing is to establish correspondences between two shapes (cf. figure 1.1a). This can be achieved by associating to each point of the two shapes a descriptor with two (conflicting) characteristics: being (i) discriminative and (ii) invariant to deformations. In chapter 3, we analyze the influence of variations in the metric on the eigen-values of the Laplace-Beltrami operator. This leads to the definition of a descriptor that provides an optimal trade-off between discriminativeness and invariance. We show that this descriptor, that we call the *Wave Kernel Signature* (WKS), can be naturally interpreted in the framework of Quantum Mechanics as the average probability of finding a particle of a given energy distribution freely diffusing on the shape at the specific point it describes. We compare this descriptor to the *Heat Kernel Signature* (HKS), which has a similar formulation and show the difference in the notion of scale they imply. We evaluate our descriptor on the standard SHREC 2010 benchmark [33] and show that for some tasks the WKS improves over the HKS, which is the state-of-the-art shape descriptor. We also show that the WKS can be used for shape segmentation and retrieval.

1.4.2 3D discriminative visual elements

We bring to 3D the notion of discriminative visual elements and introduce *3D discriminative visual elements* to represent 3D models for 2D-3D alignment and recognition. These are visual parts extracted from rendered views of a 3D model and are associated to a calibrated 2D sliding window detector and a 3D location, orientation and scale on the model. The extracted parts summarize the 3D model in a way that is suitable for part-based matching to 2D depictions. The two most important technical points in the definition of the discriminative visual elements are: (i) their selection, based on a discriminative cost function and (ii) the definition and calibration of the associated detector. We use the visual elements to address two challenges. In chapter 4 we use 3D discriminative visual elements to solve a difficult instance alignment problem: aligning non-photographic depictions to 3D models of architectural sites. In chapter 5 we show that the extracted elements can also be used to describe object categories defined by collections of 3D models. In detail, we show that 3D discriminative visual elements can be used to to detect and recognize a new instance of an object category in real world photographs, and to provide an approximate viewpoint and 3D model of the detected instance.

1.5 Thesis outline

In chapter 2 we review methods in 3D shape analysis, instance-level alignment and category-level recognition that are most related to this thesis. In detail, we give an overview of 3D shape representation and alignment methods, 2D-3D contour-based and local feature-based alignment, and category-level recognition in images using 2D and 3D methods.

In chapter 3 we introduce a novel geometric descriptor for 3D shapes, the Wave Kernel Signature and explain how it relates to existing descriptors. In particular, we develop a model of shape perturbations that shows that it achieves an optimal trade-off between robustness and discriminability and we present the notion of scale separation to which it is associated. We experimentally compare its performance to other descriptors, explain why it improves on current state-of-the-art results and show that it can be used for shape matching, segmentation and recognition.

In chapter 4 we present and analyze a new method for registering non-photographic depictions of an architectural site with its 3D model. We introduce a new representation of the 3D model formed by visually informative parts that are learned from rendered views of the model, together with a robust matching method to detect the parts in 2D depictions despite changes in the depiction style. We analyze both contributions separately and compare our full method with different alternatives that we designed based on state-of-the-art algorithms. For this evaluation we introduce a new dataset of non-photographic and historical depictions and run an extensive user study.

Finally, in chapter 5 we present a method to perform category-level object recognition by 2D-3D alignment. The shape representation introduced in chapter 4 is extended to represent not only a single 3D model but an entire object category. In particular, we introduce a new efficient calibration of part detector scores based only on negative data. We evaluate our method on the standard PASCAL VOC dataset for object category detection. Our method provides more information about an image content than standard recognition methods since it goes beyond predicting a name for an object or its approximate bounding-box, but also provides an approximate 3D model aligned

with the input image.

1.6 Publications

The idea of the Wave Kernel signature presented in chapter 3 was published in 2011 in an ECCV workshop, 4DMOD [21], and the applications of this descriptor to segmentation and recognition the same year in DAGM/GCPR [20]. The first work [21] has already been cited more than 90 times and an extended version is in submission to PAMI. The painting-to-3d alignment work presented in chapter 4 was released as a technical report in 2013, published in TOG and presented at Siggraph in 2014 [17]. A shorter version was published as an invited paper in RFIA 2014 [19] and it lead to an invited presentation in the "Registration of Very Large Images" workshop at CVPR 2014. An extension to geo-localization is going to appear as a book chapter [18]. Finally, the work on object category recognition from 3D shape collections presented in chapter 5 was published at CVPR 2014 [14].

The code corresponding to those projects and the publications are publicly available [2, 3, 4].

I have also published several papers that go beyond the scope of this thesis. The work of my Master degree on the relationship between dense camera calibration and bundle adjustment was published in ICCV 2011 [13] and was included by Bastian Goldlücke in a paper published in IJCV 2014 [77]. The work I did during an internship at Adobe on detail manipulation and style transfer was released as a technical report in 2011 [16] and published in TOG and presented at Siggraph in 2014 [15]. Finally, an idea about the use of anisotropy for shape analysis, briefly presented in 6.2.1 was developed by

Mathieu Andreux and published in NORDIA, an ECCV workshop, in 2014 [10].

Chapter 2

Background

This thesis builds on ideas from what have traditionally been separate sub-fields of computer vision, namely 3D shape analysis, instance-level alignment and category-level recognition in images. In this chapter, we give an overview of the classical methods that are most relevant to this dissertation. Each following chapter of this thesis contains more information about the novelty of the described method.

2.1 3D shape analysis

In this section, we will explain how the work presented in chapter 3 relates to the more general problematic of 3D shape analysis and especially shape alignment methods. We will first explain how 3D shapes can be represented in computers, then present the main local descriptors that have been designed to describe these shapes, and finally summarize the different competing approaches for shape alignment.

2.1.1 From the ideal shapes to discrete 3D models

The first question that arises when working with shapes is: how to model them? We have an intuitive notion of what the shape of an object is, but it is not straightforward to formalize. When working with a computer it is also necessary to define a discrete version of this intuition to represent the shape by a (digital) 3D model. In this section, we present some of the main competing shape representations. ace from a discrete point set), but their discussion is out of the scoop of this thesis.

2.1.1.1 Volumetric representation

One possible way to think about a shape is as a volume in the 3D space. The natural way to define a discrete representation based on this intuition is to discretize the space in a regular grid of voxels, and for each voxel to store if the central point is inside or outside the volume [60]. The problem with this representation is that it is very expensive in terms of memory consumption. The required memory is cubic with respect to the resolution of the 3D model: to represent a shape in a cube of 1000x1000x1000 voxels, it is necessary to store one billion numbers. However, at the cost of a less intuitive representation, the same volume can be represented more efficiently using octrees [92].

This representation is often used in practice because several optimization algorithms are naturally formulated in the voxel space. This is in particular the case for some dense 3D reconstruction methods, of which an overview is available in [155]. The main reason is that by relaxing the values associated to each voxel to $[0; 1]$ instead of $\{0, 1\}$ it is often possible to formulate the problem as convex optimization that can be solved efficiently. This relaxation also makes topology changes straightforward to handle.

2.1.1.2 Point Clouds

Opposite to the intuition that a shape is a volume, used in dense representations, is the idea that a shape is a collection of the points on its surface. Based on this idea, a discrete representation of the shape can be computed by sampling a finite set of 3D points on this surface. This set of points is sometimes augmented by the collection of normals to the surface at each point. This is the natural representation when 3D measurements are provided by a laser scans, which provide the position of a sparse set of points, and also for many feature based 3D reconstruction algorithms (e.g. [70, 6]) which outputs are a set of points on the object surface. The main limitations of this representation are that: (i) it depends on the sampling of the points and (ii) it does not provide the surface of the object. For this reasons point clouds are often meshed.

2.1.1.3 Meshes

Meshes and, in particular, triangular meshes are probably today the most common representation of shapes. Rather than specifying for each 3D point if it is inside or outside the shape, or to simply list a set of points on the surface, a mesh represents the shape by an approximation of its surface, which is often modeled as a 2-dimensional manifold in $\mathbb{R}^3$. Concretely, a mesh is a set of vertices (3D points) and faces (sets of coplanar vertices). Texture can be associated to a mesh by providing a color for each vertex, or by mapping each face to an image. Models reconstructed automatically from multiple images typically have one color per vertex, while models designed manually typically associate an image to each face.

Most of the work presented in this thesis has been done with 3D shapes represented as meshes, but could easily be adapted to use point clouds or volumetric representations.

2.1.2 3D point descriptors

None of the shape representations described in section 2.1 is invariant to translation, rotation or scaling. Given a point on a shape it is very difficult to match it, for example in a rotated version of the same shape. For this reason local shape descriptors invariant to rigid transformations have been developed. They can be understood as embeddings of the shape in another space, potentially high dimensional. To cope with non-rigid transformations, more elaborate descriptors have been designed to be robust to limited non-isometric deformations.

In this thesis, we focus on the use of point signatures for matching and alignment, but they can have different applications. In fact, the very idea of using a point signature for 3D shapes has been introduced in [43] to use putative matches for recognition. An alternative way to perform recognition with local features is to accumulate them into global shape descriptor [12, 34, 71, 135]. Local descriptors have also been used for shape segmentation [20, 145, 148]. Indeed a simple clustering such as K-means can be meaningful in the descriptors space.

The main local shape descriptors can be separated in two categories. We first introduce descriptors that accumulate local information about the shape in an histogram, and then present spectral descriptors which utilize the eigen-decomposition of a differential operator on the shape.

2.1.2.1 Descriptors based on local histograms

Those descriptors were developed first, following the success of similar methods in 2D image analysis (see sections 2.2.2 and 2.3.1.2).

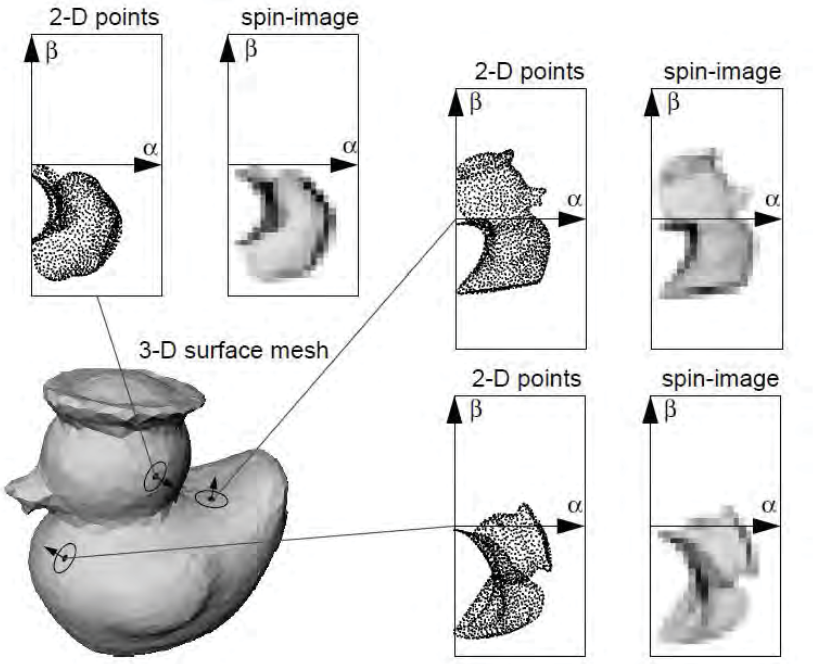
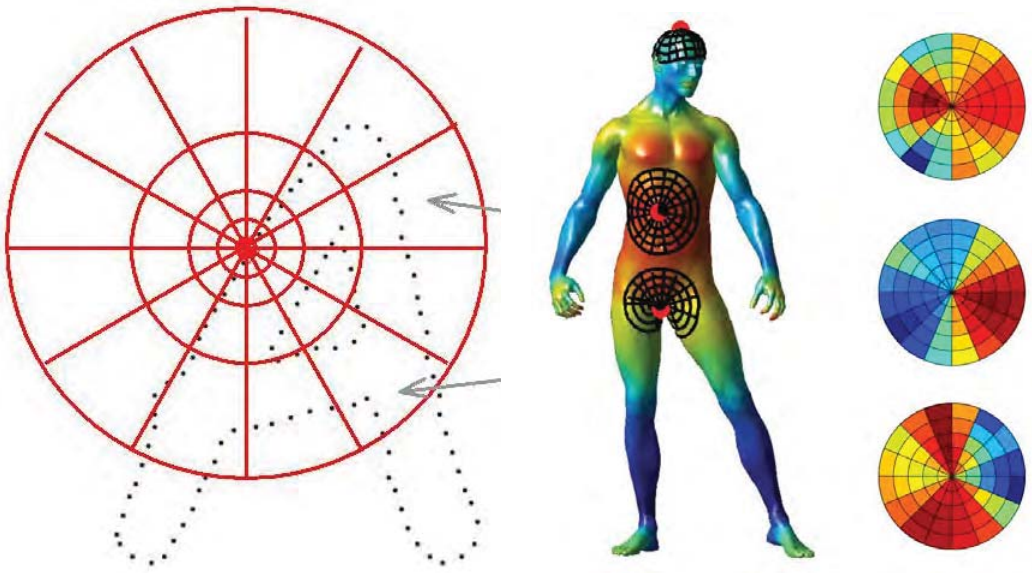


Figure 2.1: *Spin Image [94] defines a local image for each point of a shape based on cylindric coordinates. Figure from [94]*

Spin image The idea of spin images [9, 94, 95] is to associate to each point of a shape an image that describes the local context of this point in the shape. Techniques of image matching can then be used to match descriptors and thus perform surface matching. The input necessary to compute spin images is a point cloud together with the normals associated to each point. For each point of the cloud a cylindrical coordinate system is defined with the described point at the center and its normal as the cylinder axis. This definition is ambiguous only for the angular coordinate, and the two distance coordinates are well defined. A 2D histogram of those coordinates for all the points of the shape is computed and used as a descriptor, as illustrated figure 2.1.

Shape context Shape context was introduced in [28] to describe lines in an image. Similar to spin images, it stores for each point the distribution of the relative positions of other points. In shape context the histogram is done in a log-polar way which



(a) *Original shape-context for 1D surfaces. Figure from [28]*

(b) *Shape context for 2D surfaces using geodesic distances. Figure from [103]*

Figure 2.2: *Shape context accumulates information in a log-polar way.*

presents several advantages, including giving more importance to close-by points and transforming rotations and scaling of the shape into a translation of the descriptor. This is visualized figure 2.2a. The original article [28] also shows 3D object recognition results using shape context on a set of views of the 3D model. The idea of shape context was extended in a more principled way to shapes in 3D by [103, 106], as illustrated in figure 2.2b.

Shape HOG: The idea of shape HOGs [183] is similar to shape context in the sense that it computes histograms in log-polar coordinates, but it aims at describing a texture on a shape rather than the shape itself. As a consequence, it stores histograms of the dominant gradient orientations of the projected texture for each bin instead of the density of points.

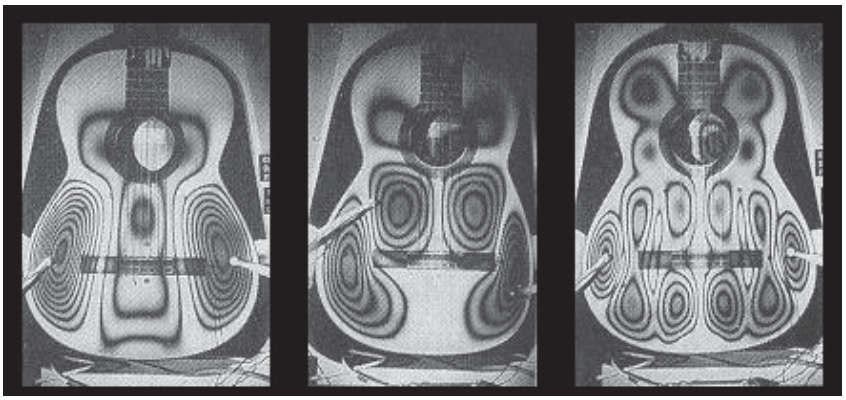


Figure 2.3: *The eigen-functions of the Laplace-Beltrami operator on a shape correspond to its vibration modes. The vibration modes are closely related to the shape, as visualized here for 553, 731 and 1174Hz [66].*

2.1.2.2 Spectral signatures

In most cases, one can recover all the intrinsic information about a shape using the eigen-values and eigen-functions of the Laplace-Beltrami operator on the shape [97]. These eigen-functions are the generalization of the Fourier basis on general manifolds. They are closely related to several physical phenomena, including vibration modes, which are shown figure 3.2. The two most important spectral signatures, the Global Point Signature (GPS) and Heat Kernel Signature (HKS) are presented in this section. The Wave kernel Signature (WKS), presented in chapter 3 falls into the same category of spectral descriptors. Section 3.1.3 presents in more detail the mathematical aspects of spectral shape analysis.

Global Point signature (GPS): The first spectral point signature was developed by Rustamov [148]. To each point of a shape, the *Global Point Signature* associates a vector. Its kth component is the value of the kth eigen-function at the described point divided by the square root of the norm of kth eigen-value. This division gives more importance to the eigen-vectors associated to the low frequencies. The main drawback of the GPS is that if a shape is slightly modified, the order of the eigen-functions may

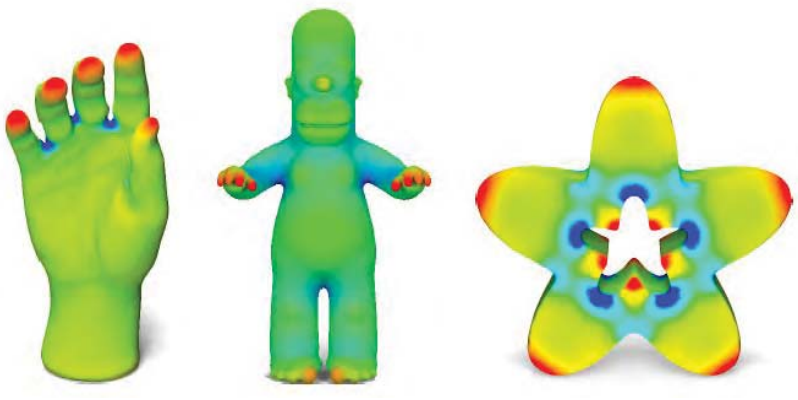


Figure 2.4: *The Heat Kernel Signatures [168] stores for each point on the shape how much of the heat deposed initially exactly at the point remains after a time t. This figure shows the HKS on several shape for small time. One can see that area with high positive curvature remain warm when area with very negative curvature are colder.*

be changed, resulting in two completely different signatures.

Heat Kernel Signature (HKS): The *Heat Kernel Signature* [168] is widely considered the state of the art shape signature. Similar to the GPS it is defined using the eigen-decomposition of the Laplace-Beltrami operator on the shape. However the HKS does not separate the eigen-functions but combines them in a way that naturally arises from the analysis of heat diffusion on the shape surface. A particular value of the HKS for several shapes is visualized on figure 2.4. More technical details about the HKS are given in 3.1.3.2. There have been several extensions of the HKS. In particular, Bronstein et al. [37] modified HKS to be scale invariant and Raviv et al. [142] considered the heat diffusion in the shape volume rather than on its surface, defining a volumetric HKS.

Wave Kernel Signature (WKS): The *Heat Kernel Signature* presented in chapter 3 extends the idea of combining the eigen-function introduced in HKS. Using a perturbation analysis of the eigen-values of the Laplace-Beltrami operator, it combines them in a way that is optimal under some hypothesis. The WKS improves the precision of

the HKS for matching and additionally has a natural interpretation in the framework of Quantum Mechanics.

2.1.3 3D shape alignment methods

3D shape matching is the problem of finding a point to point correspondence between two different shapes. There are two main approaches to tackle this problem. The one most related to this thesis is to define for each point a descriptor that will be discriminative but also robust to some transformations of the shape and then find a matching that preserves L^2 distance between the descriptors. Another popular strategy is to minimize the distortion induced by the mapping. This section gives an overview of those approaches. For a more detailed survey the reader can refer to [171] and [173].

2.1.3.1 Metric approaches

Iterative closest point methods were the first introduced to solve rigid 3D shape matching [29, 41]. They were later extended to cope with some non-rigid deformations [8] by iteratively rigidly aligning the shapes and deforming them using a non-rigid parametric transformation. This idea however can only work with limited deformations in terms of Euclidean distance in the 3D space.

For shape matching, the intrinsic properties of the shape, such as geodesic distances, are more meaningful. Indeed they are mostly preserved under usual deformations. For example, the geodesic distance between the two hands of a human body will remain approximately the same even if their distance in the 3D space changes a lot. This leads to the idea of viewing the shapes as metric spaces and the problem of aligning them as finding an isometry that minimize the distance between those spaces. If the distance

used is the Euclidean distance in the ambient 3D space the natural distance between the shapes is the Hausdorff distance in the Euclidean 3D space and the standard ICP algorithm can be used to find a locally optimal alignment. However other distances such as the geodesic distance are more meaningful.

Using the geodesic or diffusion distance on the shapes makes the problem of finding an optimal isometry much harder. The first challenge is to find a metric space in which the two shapes can be meaningfully compared. Elad et al. [57] embed the shapes in a nearly isometric way in a finite dimension Euclidean space using multidimensional scaling (MDS) [46]. They then perform ICP in this new space and thus recover correspondences between the initial shapes. However, in [57] the metric space toward which the embedding is done and in which the Hausdorff distance is minimized is selected in an arbitrary way. Memoli and Sapiro [125] solve this problem by applying to 3D shape analysis the ideas of the Gromov-Hausdorff distance [78]. They compare the shapes using their isometric embedding in a metric space that minimizes the Hausdorff distance between them. This idea was further developed for shape matching using geodesic [36] and diffusion distances [35].

Windheuser et al. [177] use a related approach and find a deformation minimizing the elastic energy cost of the deformation rather than the Gromov-Hausdorff distance. Their formulation has the advantage that it leads to a binary linear program that can be efficiently solved. Example of their results can be found in figure 2.5a



(a) *Shape matching minimizing the elastic energy of the deformation. Figure from [177]*



(b) *Shape matching with functional maps enforcing HKS and WKS consistency. Figure from [136]*

Figure 2.5: *Shape matching by minimizing the deformation (left) and enforcing descriptor consistency (right)*

2.1.3.2 Feature-based approaches

Inspired by the success of feature based methods in 2D alignment, many papers have used local descriptors to match shapes. For example Gelfand et al.[72] use putative correspondences of points with very discriminative features to propose candidate rigid transformations and each transformation is then evaluated. The best transformation is then used as an initialization to an ICP algorithm. Similarly Brown and Rusinkiewicz [38] use local feature correspondences to initiate their non-rigid ICP.

An elegant way to enforce descriptor consistency in shape matching is to use the framework of *functional maps* developed in [136]. The general idea is to formulate the problem of shape alignment as the problem of finding correspondences between the functions on the shapes. This transforms the descriptor preservation into a linear constraint that naturally fits into optimization. Figure 2.5b shows an example of matching with functional maps. Note that functional maps used the Wave Kernel Signature introduced in chapter 3 as point descriptor.

Other examples of feature-based 3D alignment include [88, 101, 102, 119, 137].

All those feature-based approaches rely on the quality of the descriptor used, that must be both robust to perturbations and discriminative between points. For this reason, a high quality descriptor such as the Wave Kernel Signature introduced in chapter 3 is important to all those methods.

2.2 Instance-level 2D-3D alignment

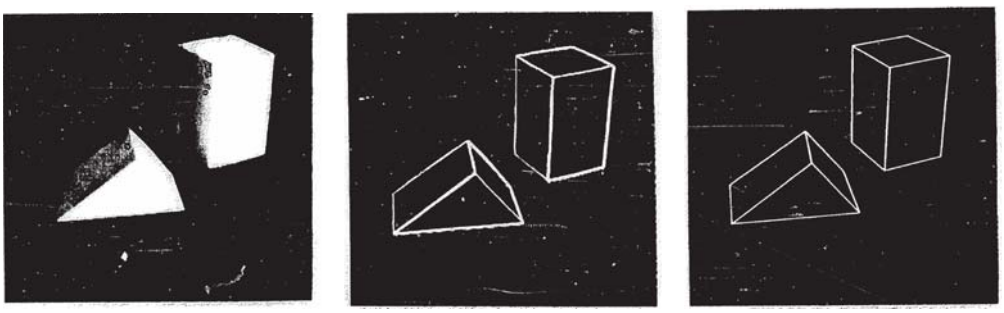
Object instance-level alignment is the problem of recovering a given object instance in a test image together with its pose with respect to the reference representation.. The reference representation can be either a 3D model or an image. It is a difficult problem due to the variations in the object appearance, induced by the viewpoint, the illumination and partial occlusions (see section 1.3.2).

In this section, we present the most important methods to solve this problem. We begin with methods based on contours, that were very popular in the early days of computer vision. We then present local feature based methods, which are often the most effective. Finally we present some global features that were designed to address the sensitivity of local features to non-linear effects such as illumination effects, season changes and depiction style.

2.2.1 Contour-based methods

2.2.1.1 Classical methods

Since its very beginning Computer Vision aligns 3D models to images. Indeed Roberts in the abstract of his PhD in 1963 [144] explains that his ultimate goal is "to make it possible for a computer to reconstruct and display a three-dimensional array of solid objects from a single photograph". Because this objective is too complex, he restricts



(a) *Input image* **(b)** *differential image* **(c)** *reconstructed 3D solids*

Figure 2.6: *Object instance- level alignment by Lawrence Roberts 1963 [144]*

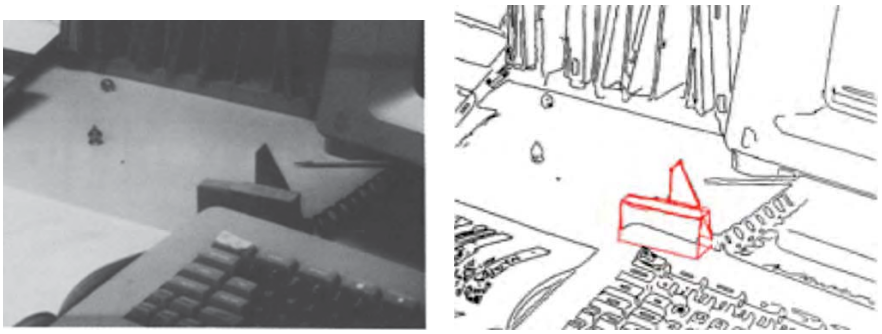
himself to cases of objects which have a "known three-dimensional solid", thus being the first to consider the 2D-3D instance alignment problem.

His work as most of the works until the nineties [129] relies on object contours. This idea implies two main difficulties:

- to detect edges reliably.
- to aggregate the information from several edges to align the 3D model.

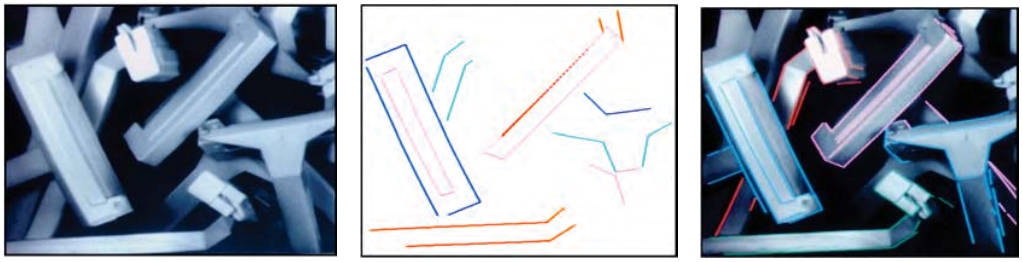
The problem of reliably detecting edges in cluttered scene has lead to a wide variety of methods (eg. [40]) but is very difficult and is still open. This is one of the main reasons why most modern alignment methods avoid explicit detection of contours by using keypoints (see section 2.2.2) or dense representations.

To aggregate information from different edges, several methods have been developed. In [144] Roberts uses the hypothesis of a block world to recover polygons from sets of lines. In [89] Huttenlocher and Ullman use an hypothesis-test paradigm. They consider the 3D model of the object known. They begin by using the edges to define keypoints in the image based on edges corners and inflexions. They then iterate the following procedure: (i) they use three putative correspondences between the image points and points from the 3D model to hypothesize a pose using a weak perspective projection



(a) *Input image* **(b)** *aligned 3D instance*

Figure 2.7: *Instance alignment using minimal keypoint correspondence in [89]*



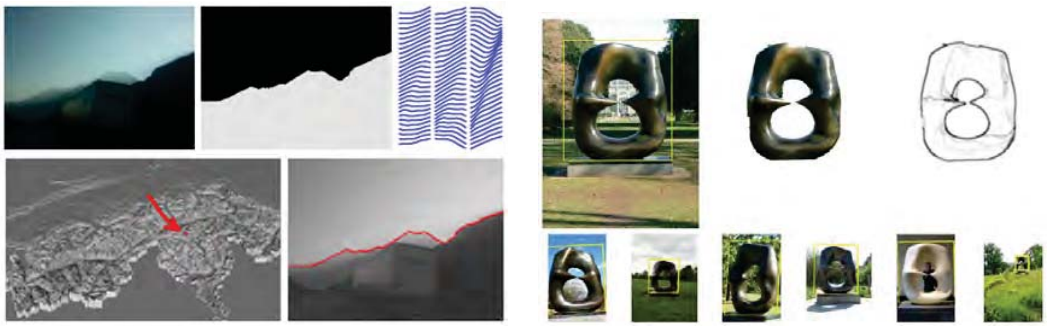
(a) *Input image* **(b)** *perceptual grouping* **(c)** *Results*

Figure 2.8: *Instance alignment using perceptual grouping by David Lowe [122]. Figure from [122].*

model (i.e. a projective model with a single scale factor); (ii) they render the 3D model from the hypothesized viewpoint and compare it to the image. They keep the proposed detection if it is coherent with the image. The use of a set of keypoints to describe the object makes this method invariant to some degree of occlusion, as shown in figure 2.7. Another strategy inspired by the human ability to intuitively detect groups of edges in an image, has been developed by David Lowe in [122]. The method uses the idea of line grouping to hypothesize a smaller number of possible correspondences between the image and the model. According to Lowe, the conditions that must be satisfied for perceptual grouping are:

1. having some invariance with respect to the viewpoint

2. being unlikely in random arrangement to allow detection.



(a) *Baatz et al. [22]* **(b)** *Arandjelovic and Zisserman [11]*

Figure 2.9: *Recent use of contour based alignment methods for geo-localization (left) and sculpture recognition (right).*

Those two points are very related to the ones we will use to define discriminative visual elements in chapter 4. Results of this method are shown in figure 2.8.

2.2.1.2 Modern developments

While contour-based methods have been replaced for many applications and, in particular for alignment by feature-based methods, the recent literature provides a few interesting examples where contours can be efficiently used.

First, contour-based methods perform well when the object contours can be reliably extracted both from the 2D image and the 3D model. A recent example illustrated in figure 2.9a shows that it is possible to geo-localize photographs using semi-automatically extracted skylines matched to clean contours obtained from rendered views of digital elevation models [22, 23].

Second, contour based alignment can produce state of the art results when the absence of discriminative structures leads to the failure of feature based methods. This is the case for smooth objects which is addressed in [11] and illustrated figure 2.9b. To solve

the problem of extracting edges from real photographs, the authors present a solution which trains a classifier that classifies super-pixels either as sculpture or not-sculpture.

Finally, contours have been successfully used to refine initial alignments provided by features. For example Lim et al. [117] use contours to refine the pose estimation of non-textured objects. More related to our work, Russell et al. [147] use contours to refine the alignment between a painting and a 3D mesh reconstructed from photographs. However, those methods require a good initialization with a close-by viewpoint.

2.2.2 Local features for alignment

Local feature descriptors summarize local image informations in regions that were previously detected by a specific feature detector. A large variety of detectors and descriptors exist. Selecting and describing in a robust way a set of local features in an image has many applications. Examples include large scale 3D reconstruction and exploration [6, 165, 166], image mosaicing [169], visual search [164], visual localization [153], and camera tracking [26] to list but a few. Local features can also be used for alignment as we will see in this section and for category-level recognition (see 2.3.1.1).

Local features were designed to tackle the problem of finding image to image correspondences. However, their use can be extended to finding 2D-3D correspondences and perform 2D-3D instance-level alignment [146] and retrieval [45, 140]. Large 3D scenes, such as a portion of a city [115], can be represented as a 3D point cloud where each 3D point can be associated with local features that were used to reconstruct it [151]. 2D-3D correspondences are obtained by matching the features extracted from a test

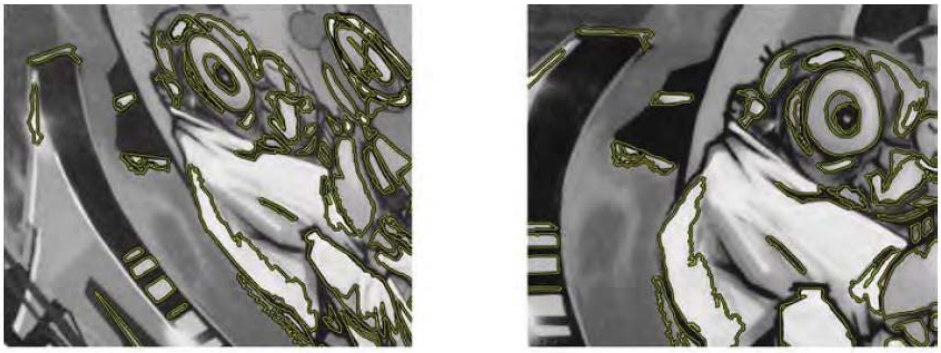
image with the features associated to the 3D model. Standard camera resectioning can then be used to recover the camera of the test photograph [83]. In this section, we first introduce the main feature detectors, then feature descriptors and finally the key steps for robust alignment.

2.2.2.1 Local region detection

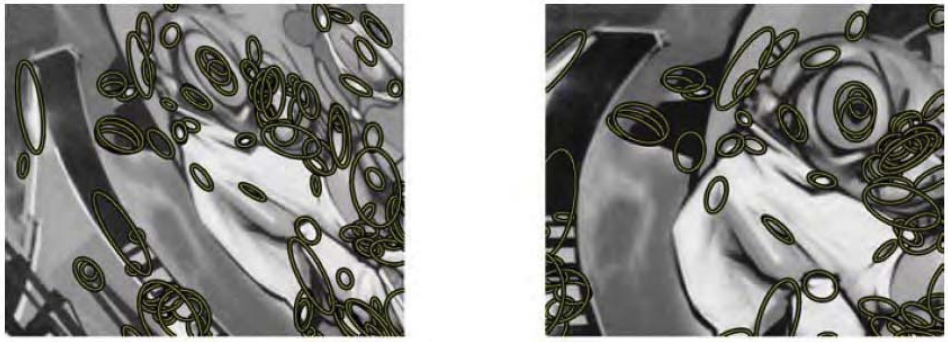
The main interest for using feature detectors for alignment is to transform the problem of finding dense correspondences between two images (or an image and a 3D model) into the problem of finding correspondences between two sparse sets of features. The features must be repeatable, i.e. if a region is selected in an image, the projection of its pre-image in another image must also be detected. For this reason, those regions are often called co-variant with respect to a family of image transformations [128].

In general, the choice of the co-variance of the detectors corresponds to a trade-off between invariance and discriminability. Increasing the invariance of the detectors can lead to performance losses if it is not necessary. For example, affine region detectors work better when there is strong perspective distortion, but they are otherwise outperformed by circular detectors, invariant only to similarities. The region detectors that are of most interest to us are affine-covariant detectors, because affine transformations often approximate well the changes induced in an image by limited 3D transformations. One could use any kind of regions (see figure 2.10a), but ellipsoids (see figure 2.10b) are the most natural shapes for affine covariant regions and are used in most applications.

A good overview and evaluation of affine covariant feature detectors is available in [128]. In this section, we discuss two of the most important detectors, Harris corners



(a) *MSER [124] can define complex regions*



(b) *The Harris-Affine detector [126, 128] defines ellipsoids*

Figure 2.10: *The idea of detecting stable region across different viewpoint and imaging conditions is very general, however, most applications consider elliptic regions [128].*

and detectors based on the Hessian. Both of them rely on finding extrema of a function in the image. Lindeberg [118] provided a framework to consider extrema across scales as well as across space. This framework is usually used for those two detectors.

Harris corners: The eigen-vectors of the auto-correlation matrix (or the second moment matrix) of the intensity of the image correspond to the directions in which the image varies the most and the least. The norm of gradient in those two directions is the square root of the two corresponding eigen-values. The Harris corner detector [82] detects points in an image based on the local maxima of sums and products of those eigen-values. They correspond to locations for which the image varies strongly in all directions.

Hessian-based detectors: Similarly, the eigen-vectors of the Hessian matrix correspond to the directions with the highest and smallest curvature. The eigen-values of the Hessian give the second derivative in both those directions. A region detector based on those eigen-values, either by taking their sum [121], which is also the Laplacian of the image, or their product [128], responds strongly to blobs and ridges. The Laplacian can be approximated by convolving the image with a difference of Gaussians (DoG). The DoG was the detector initially used to compute SIFT features [121]. It can be computed efficiently using image pyramids, and approximated even faster using integral images [174, 27].

2.2.2.2 Local region description

To be able to match the features from two images, it is necessary to describe them in a way that is robust to perturbations induced for example by illumination changes, noise, and small errors in the localization. Note that most of the invariance to viewpoint is not achieved by the descriptor itself, but by the detector. As for detectors, there exists a wide variety of descriptors. An overview and a comparison is available in [127].

Among local descriptors, some are the 2D counterpart of the descriptors presented in 2.1.2 for 3D shape matching. In particular, the idea of spin images [94] was adapted to 2D images in [109] and shape context [28] was initially developed for images, storing the distribution of the edges given by the Canny edge detector [40].

These two descriptors belong in fact to the wide class of image descriptors using histograms to describe the content of the image in the interest region. The SIFT descriptor [121], which is arguably the most successful local descriptor, belongs to this category.

The SIFT descriptor has been engineered to aggregate in an optimal way the gradient orientations, using 8 orientation bins in a 4x4 grid subdivision of the region. Two of its key ingredients are the normalization, which makes it invariant to affine illumination changes and the limitation of the influence of strong gradients, that gives some robustness to non-linear illumination effects. If the region is not a circle as in the original paper, but an ellipsoid as in most modern algorithms, it must be normalized before the description. Because of its success, several methods have tried to optimize further the SIFT descriptor, for example by making its computation faster using integral images [174, 27] or by reducing its dimension [99].

Another category of descriptors is based on computing local derivative, wavelet coefficients or in general linear filters at the interest point. It includes in particular steerable filters [67] and complex filters [152]. These methods have had success for texture classification and similar ideas are still used in this context [161], but they proved less robust than histogram-based methods for local region description.

Another interesting descriptor developed for texture classification called Local Binary Pattern (LBP) [132] builds histograms of the results of binary comparisons between pixels. Inspired by this idea, BRIEF [39] captures the local appearance in a way different from the two previous categories of descriptors. It simply stores the binary results of intensity comparisons between random (but fixed) pairs of points in the interest region.

One of the problems of the descriptors described above is that they remain sensitive to appearance variations. A greater invariance can be achieved by matching the geometry or symmetry pattern of local image features [44, 84, 158], rather than the local features

themselves. However, such patterns are hard to detect consistently between different views.

2.2.2.3 Robust alignment

Given two sets of image features, potential matches are given by considering for each region in the first image the one that has the closest descriptor in the second image. However, many of those candidate matches will be wrong. In this section, we address the problem of recovering the good matches and the corresponding deformation between the two images from these candidate correspondences. This is typically done [121] in two steps: first selecting the most confident matches; second computing the most confident transformation for those matches.

Confident matches selection: The most natural way to select the most confident matches would be to select those for which the descriptors are the closest. However, such a method would not work because some structures are much more likely than others and thus a descriptor can have many close descriptors which correspond to false matches. Lowe [121] suggests to use instead the ratio between the distances of the nearest and second nearest descriptor as an confidence score. This first-to-second nearest neighbor distance ratio test greatly helps discarding false matches, and proves useful in chapter 4.

Consistent transformation selection: Once the most confident matches are selected, the selection can be refined further by checking the consistency between the matches. In [121] each match defines a transformation and the Hough transform is

used to select the most consistent one. The most popular method to check the geometric consistency between matches in recent works is to perform RANSAC [64] on the putative matches. Both methods are designed to deal efficiently with the presence of outliers. The output of this last step is a transformation between the original 3D model (or the original image) and the instance visible in the image.

2.2.3 Global features for alignment

The main limitation of local feature matching is its sensitivity to changes in appearance, e.g. due to illumination, seasons, and depiction style (see figures 1.4 and 1.5). Global descriptors of images have been developed to cope with those difficulties in the case of scene recognition. They can also be applied to the problem of instance-level alignment by rendering a set of views of the model and comparing them to a test image.

GIST. Those most used global feature is the GIST descriptor [133]. It divides the image in typically 4x4 blocs and for each block stores the energy associated with different orientations (typically 8) at different scales (typically 3). It is designed to represent the shape of the scene and avoids looking at very local information. Thus it is robust to important changes in the local scene appearance such as those induced by the change of depiction style.

Exemplar-based methods. Exemplar-based methods apply to image matching the idea developed for category-level recognition presented in section 2.3.1.2. From a single positive example they learn a classifier [123]. If used with a descriptor of an image as input, they have been shown to recover images of the same instance despite changes in the depiction style [160]. This is mainly because the classifier focuses on the most discriminative parts, which are likely to be present in all depictions of the scene. More

details on these methods are given in section 4.1.2 and 4.2.2.

2D-3D alignment with global descriptors Global descriptors are more robust to the depiction style, but they do not handle well viewpoint changes. Thus, to align a 3D model with an image, one must compute a huge number of viewpoints of the 3D model, and compare each rendered view with the image as was done in [147] using the GIST descriptor.

2.2.4 Relationship to our method

The method described in chapter 4 propose an alternative approach to instance-level 2D-3D alignment. We build on the success of exemplar-based method to design a part-based representation of the 3D model. This part based method is much more robust to viewpoint changes than global methods. Moreover, it does not suffer from the sensitivity of contour-based method because it is based on a soft representation of the edges, namely HOG descriptors (see section 2.3.1.2). It also avoids the difficulty of reliably detecting interest regions by performing dense matching instead of feature-based matching, while keeping the robustness given by the RANSAC selection of inliers.

2.3 Category-level 2D-3D alignment

The goal of category-level 2D-3D alignment is both to recognize an object from a given category in a test image and to output a 3D model aligned with the image. Until recently this problem has received little attention. Category recognition was rather performed using purely 2D methods. However these 2D methods implicitly handle the fact that the object appearance varies with the viewpoint and they can be used

to recover the viewpoint if enough training data is available. For this reason, we will begin by reviewing 2D category-level recognition methods and then present the more recent work on 2D-3D category-level alignment.

2.3.1 2D methods

2.3.1.1 Bag of features

It is possible to describe the content of an image using the local features present in this image (introduced in section 2.2.2). This approach was introduced by Csurka et al. [48] and Opelt et al. [134]. A detailed evaluation of "Bag of features" methods is presented in [184] and shows that they are surprisingly robust to intra-class variation, which is one of the main difficulty of category-level recognition compared to instance-level recognition. The typical pipeline for a "Bag of features" recognition pipeline is as followed. First, local features such as affine SIFTs are extracted from all training images. They are then aggregated in an histogram defined using a codebook called *visual vocabulary* learned from all images. Finally a classifier, typically a linear SVM, is learned to differentiate between the histograms of the different classes. This approach has been extended to encode spatial information using a spatial pyramid in [110] and can be used a global descriptor (see section 2.2.3).

2.3.1.2 Single template method

Bag of features methods are based on the aggregation of local features. On the contrary one can represent an object by a single template or a small set of templates corresponding to the different possible viewpoints of the object. This idea was applied successfully in [49] to pedestrian detection, using *histograms of oriented gradients* or HOGs and a linear SVM classifier. The HOG descriptor is very similar to the SIFT

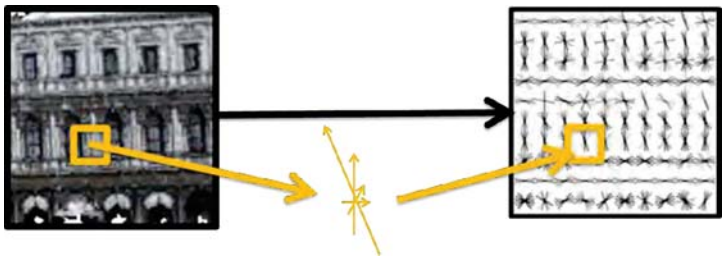


Figure 2.11: *The HOG descriptor [49] summarizes the local gradient orientations.*

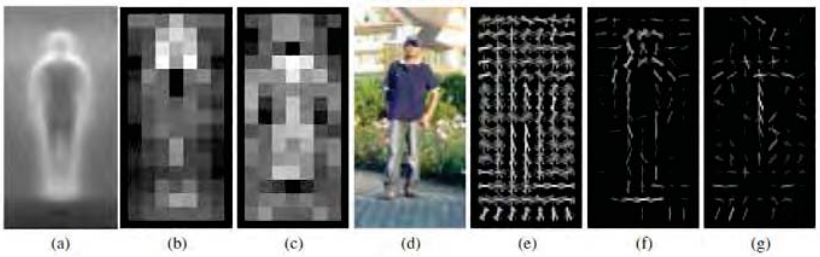


Figure 2.12: *The SVM in the HOG-SVM pipeline [49] emphasizes the important gradient orientations. (a) Average gradient for the pedestrian category. (b) and (c) Positive and negative SVM weights for each HOG cell. (d) Test image. (e) HOG descriptor of the test image. (f) and (g) HOG descriptor of the image weighted by the positive and negative SVM weights. Figure from [49].*

descriptor [121] but is designed to be computed densely over an image. Each HOG cell essentially represents in a robust way the dominant gradient orientations as visualized in figure 2.11. The SVM weights the contribution of the different cells and orientations, emphasizing the important gradients for the category as shown in figure 2.12.

2.3.1.3 Deformable parts model (DPM)

Fischler and Elschlager [65] introduced the idea of defining a *pictorial structure* to describe the arrangements of parts of an object, as shown for example on figure 2.13. This idea was revisited in [62] and made popular by Felzenszwalb et al. [61]. They train a latent-SVM model using HOG features to create a state-of-the-art method for object category detection. Deformable part models represent an object by a root HOG and parts represented by HOGs at twice the resolution of the root HOG. The detectors for the root and parts as well as their locations and weights in the final score are learned

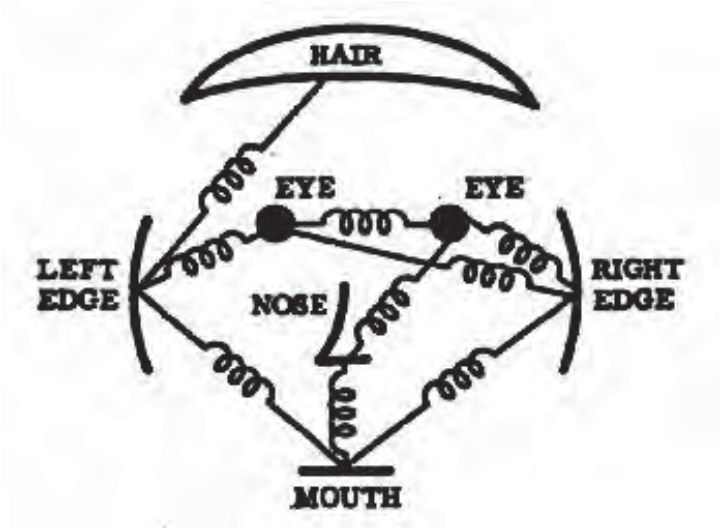


Figure 2.13: *The idea of pictorial structures [65] is to represent an object category by a set of parts with constraint relative locations. Figure from [65]*

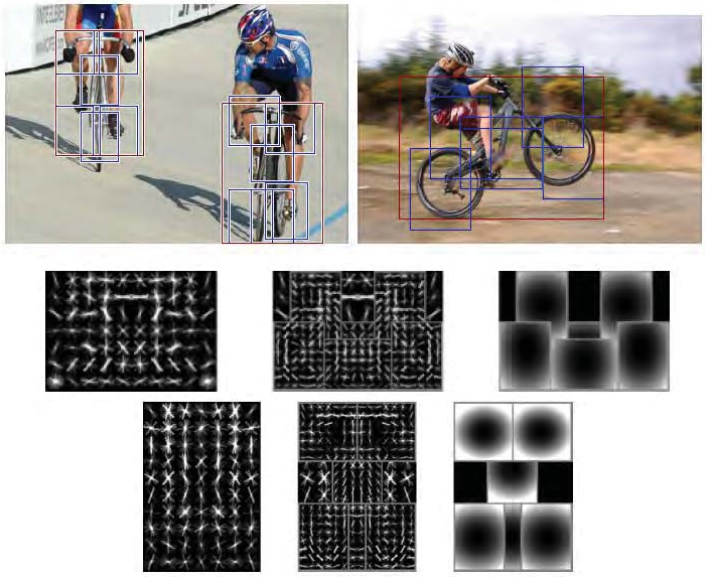


Figure 2.14: *Deformable part models capture jointly the appearance of object parts for several viewpoints. Figure from [61].*

jointly.

The 3D variations of the object appearance is handled by DPM by considering several models corresponding to different aspect ratios for each object category and by allowing the parts to move with respect to the root with a Gaussian deformation cost.

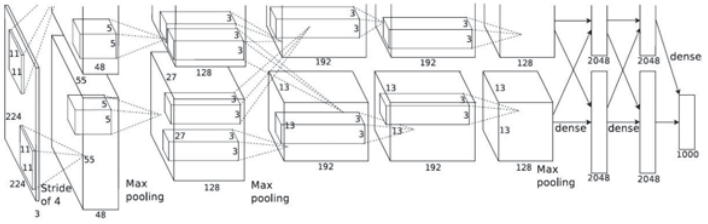


Figure 2.15: *Architecture of the convolutional neural network used by Krizhevsky et al. [107] for object classification. Figure from [107]*

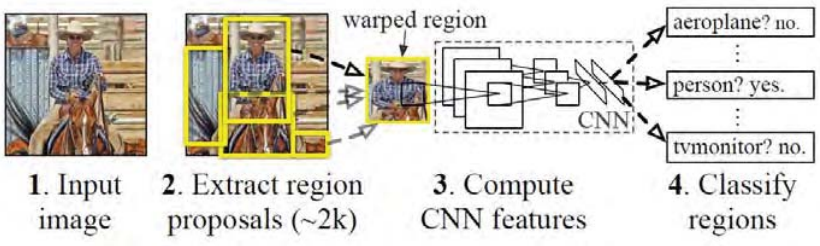


Figure 2.16: *Detection pipeline proposed by Girshick et al. [75]. Figure from [75].*

2.3.1.4 Convolutional neural networks

Recently, DPMs have been outperformed by deep learning methods. Convolutional neural networks [111] are formed by a succession of convolutions, rectifying non-linear units (ReLU), max poolings and local normalizations (see figure 2.15). They had already shown impressive and practical results on optical character recognition [112, 162], but until recently their performance for other vision task was limited by the available training data and computational power. The developpement of GPU computing and the appearance of the large scale ImageNet dataset [54] allowed Krizhevsky et al. [107] to develop a network architecture, shown in figure 2.15, that outperforms by a significant margin other methods for image classification. This method was extended into a state of the art method for object detection in [75] using a fine-tuned version of the initial network to classify warped candidate regions as shown figure 2.16. Interestingly, and related to the work on non-realistic depictions presented in chapter 4, CNNs have also been shown to perform well for object-category classification in paintings [47].

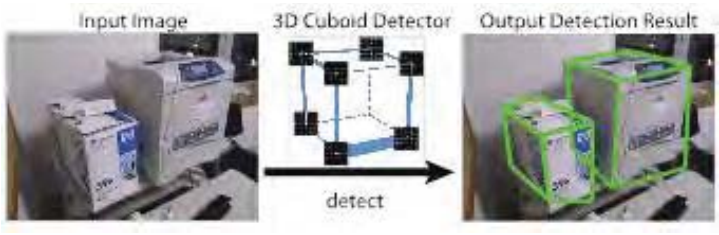


Figure 2.17: *Detection of block-like objects by Xiao et al. [180]. Figure from [180].*

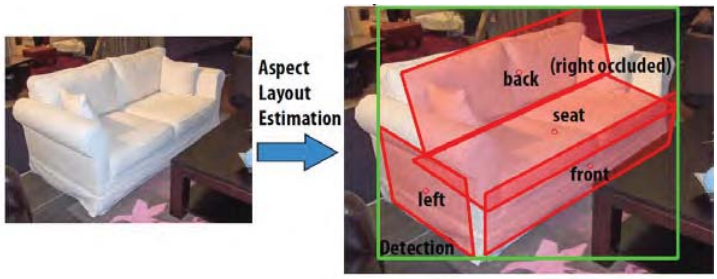


Figure 2.18: *Xiang et al. [179] approximate objects by set of planes. Figure from [179].*

2.3.2 3D methods

Recently several papers aimed at using 3D information to perform category level recognition and alignment. Two main research directions have been explored

The first way to tackle the problem is to make simplifying and non-realistic hypothesis about the 3D content of the world, revisiting the "block world" methods of the early days of computer vision. This has been explored for buildings in outdoor scenes in [79] and for box-like objects in indoor scenes in [63, 180]. An example of the results of those methods is shown figure 2.17. This approach was extended to more complex shapes than blocks either by designing manually object models using a combination of blocks [42, 53] or by learning an approximation of the layout of an object category using a set of planes [179] as shown figure 2.18. Using a non-parametric approach [150] showed promising results, but is designed to work in highly structured scenes.

Another way to approach the problem is to extend to 3D the ideas of the DPM [62],

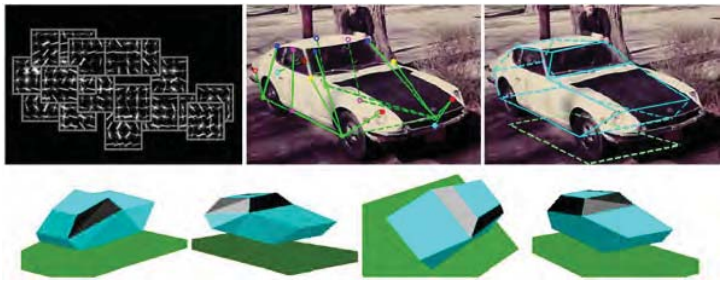


Figure 2.19: *The idea of extending DPMs to 3D, here in the framework of Hejrati and Ramanan [85]. Figure from [85].*

which have been very successful for 2D object recognition. This idea is visualized figure 2.19. For example [185] uses manually labelled points on 36 car models to learn a deformation model of the shape and the local appearance. Glasner et al. [76] have a slightly different approach and used patches from images of 22 registered models of cars to vote for the location and orientation. Other examples include [85, 138, 185]. More recently Yang et al. [182] attempted to extend CNN for viewpoint estimation. Those methods use a relatively small number of models and typically focus on objects with a simple geometry or a limited intra-class variation such as cars. They also often require manual annotation of 3D key-points that are used to represent the shape of the objects and train detectors.

2.3.3 Relationship to our method

In chapter 5 we present a data-driven part-based method to tackle the problem of 3D category-level recognition. Opposite to previous methods which typically use simple 3D models, we use the complex "chair" category that we represent using more than thousand 3D models collected from the Internet and more than 80 000 rendered views. While we use a star star-model similar to the DPM approach, we also use an exemplar-based method to avoid the need of large collections of annotated images to learn our model. Finally, our approach has the new advantage that given an input image it

returns a 3D model from our shape collection similar to the depicted object and aligned with the image.

Chapter 3

Wave Kernel Signature

3.1 Introduction

In this chapter, we are interested in developing a local descriptor that enables robust and accurate matching between two meshes representing the same 3D object. This local descriptor is illustrated figure 3.1. We place ourself in the context of the harmonic analysis of shapes introduced in 2.1.2.2 and design the Wave Kernel Signature (WKS). It aims at achieving an optimal trade-off between robustness and accuracy by studying the influence of perturbations in the metric on the eigen-values of the Laplace-Beltrami operator. The WKS has a natural interpretation in the framework of Quantum Mechanics. It represents the time-averaged probability of measuring a quantum mechanical particle at a specific location on the shape. By considering particles of varying energy distribution, the WKS encodes and separates information from the different Laplace eigenfrequencies. Both theoretically and in quantitative experiments we demonstrate that the WKS is substantially more discriminative and therefore leads to better feature matching than the commonly used Heat Kernel Signature (HKS). We also show that the WKS can be used for other applications such as shape segmentation

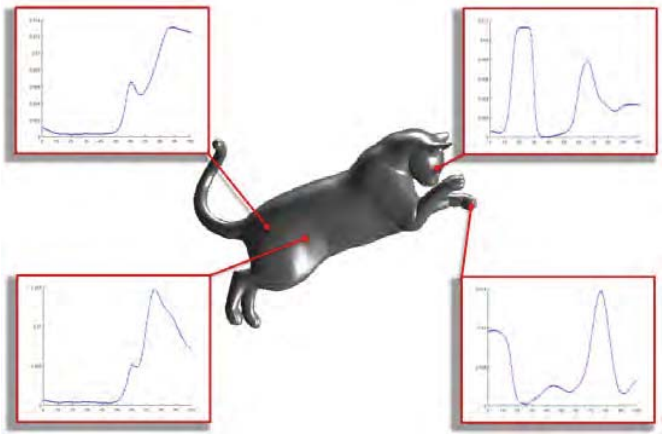


Figure 3.1: *Schrödinger's cat [154] and its Wave Kernel Signatures. Based on the Schrödinger equation each point on an object's surface is associated with a Wave Kernel Signature. Note that the signature captures shape variations in the environment of the considered point at various spatial scales: While the two points of the bottom are quite similar for large scales (small values of the energy), the two others are quite different.*

and recognition.

3.1.1 Motivation

The analysis of *shape* plays a central role in computer vision and beyond. To analyze and compare 3D shapes is by no means straight-forward. Even the computation of a distance between two given 3D shapes leads to NP-hard combinatorial problems [36, 125, 177] (see section 2.1.3.1).

As mentioned in section 2.2.2 local descriptors have drastically improved and simplified many computational challenges for images, in particular alignment problems. Algorithms for image matching, correspondence finding, camera pose estimation, tracking and recognition have substantially gained in speed and real-world performance by relying on local invariant descriptors such as SIFT [121] (see section 2.2.2). The enormous number of citations to these works indicates the practical relevance of such feature descriptors. In analogy, appropriate feature descriptors for 3D shapes will facilitate their analysis, improving and accelerating shape matching, shape recognition, shape

retrieval and shape decomposition.

As explained in 2.1.2 a shape descriptor assigns a signature to each point of a 3D shape characterizing this point with respect to the full shape. This signature should be invariant or robust to certain transformations and perturbations, yet it should be discriminative in the sense that corresponding points on the two shapes should be assigned similar signatures and points not in correspondence should have different signatures. As discussed in section 2.1.3.2, having a good feature descriptor for 3D shapes would simplify many challenges and in particular alignment.

In this chapter, we introduce a novel highly discriminant feature descriptor for 3D shape analysis which is based on a quantum mechanical treatment of shape.

3.1.2 From Quantum Mechanics to shape analysis

The mathematical models employed for the analysis of images and shapes are frequently inspired from physics. The curve evolutions employed for segmentation and tracking in [31], for example, were inspired by Newtonian equations of motion. The theory of Fourier analysis and frequency decompositions originated with Fourier's studies of heat propagation. The heat equation itself stands at the origin of nonlinear diffusion filtering [139, 176]. The process of heat diffusion also inspired the derivation of the *Heat Kernel Signature* (HKS) as a feature descriptor for 3D shapes [168]. The central physical analogy underlying this signature is to place a source of heat at a given point of the 3D shape and to study how the heat diffuses over time.

A mathematical framework which has attracted fairly little attention in the analysis of images or shapes is that of *Quantum Mechanics*. Although Quantum Mechanics revolutionized the mathematical modeling of microscopic physical phenomena at the beginning of the 20th century, the respective mathematical models have not been ap-

plied much in image or shape analysis. A notable exception is the use of Schrödinger's equation and complex diffusion for image denoising and image enhancement [74].

The theory of Quantum Mechanics has many facets and a complete review is beyond the scope of this thesis. For an excellent introduction the reader is referred to [149]. In this paper, we will show how a family of shape descriptors can be interpreted in the framework of Quantum Mechanics and introduce a new feature descriptor called the *Wave Kernel Signature* which exhibits several advantages over the frequently used Heat Kernel Signature. The central physical analogy we propose is that we place a quantum mechanical particle on a 3D shape and determine the time-averaged probability of observing this particle in a given location. The Wave Kernel Signature assigns to each point on the surface a vector containing the time-averaged probability for locating particles of different energy. While the process of heat diffusion underlying the HKS converges over time to a steady state, the evolution of quantum mechanical particles underlying the WKS is governed by Schrödinger's equation and does not converge to a steady state. Therefore time averaging provides a meaningful quantity. In fact, through the time-averaging the notion of time actually disappears such that the WKS is a signature associated with a stationary analysis of shape.

As we will show in section 3.1.3.3 and 3.3.2, the HKS and the underlying heat diffusion process tend to merge shape information on different spatial scales (or frequencies). In contrast, the WKS nicely separates shape information on different scales. Moreover, we demonstrate that the Wave Kernel Signature is superior to the Heat Kernel Signature and its variants as it allows a better discrimination of points on the shape and a more accurate localization of correspondences.

In the following we well give a review of the mathematical framework of spectral meth-

ods in the context of 3D shape analysis. We will then introduce a family of shape descriptors, the Wave Kernel Signature which generalizes the Heat Kernel Signature. Next, we derive a particularly well-suited choice of the WKS and we derive some of its mathematical properties. Finally we will show in an extensive experimental evaluation that the Wave Kernel Signature has a favorable performance compared to existing feature descriptions such as the Heat Kernel Signature and its derivatives.

3.1.3 Spectral Methods for shape analysis

Because of its interesting properties mentioned in 2.1.2.2, techniques relying on the spectral decomposition of the Laplace–Beltrami operator have become in recent years fundamental in data analysis [108] and more specifically in shape analysis [114]. In this section we will briefly review the mathematical background of the Laplace–Beltrami operator on surfaces and the relation of its spectral decomposition to classical Fourier analysis. Next, we will recall the definition and the main properties of the Heat Kernel Signature (HKS) [168]. Finally, we will perform a spectral analysis of HKS which indicates that despite its theoretical elegance, HKS does not organize the information on points in an optimal way.

3.1.3.1 The Laplace–Beltrami operator and PDEs on surfaces

Let $X \subset \mathbb{R}^3$ be a closed differentiable surface, and let g denote the Riemannian metric on X which is induced by the embedding in the Euclidean $\mathbb{R}^3$. The Laplace–Beltrami operator is a linear second-order differential operator

$$\Delta : L^2(X) \to L^2(X). \tag{3.1}$$

For a function $f \in \mathcal{C}^2(X;\mathbb{R})$, it is defined as

$$\Delta f = \operatorname{div}(\operatorname{grad} f). \tag{3.2}$$

In local coordinates x^1, x^2 with metric tensor $g = g_{ij}$, inverse metric tensor g^{ij}, and metric determinant $\det(g) = |g|$, the Laplace–Beltrami operator is given by

$$\Delta f = \frac{1}{\sqrt{|g|}} \frac{\partial}{\partial x^i} \left(\sqrt{|g|} g^{ij} \frac{\partial}{\partial x^j} f \right). \tag{3.3}$$

The Laplacian is a negative, self-adjoint operator on $L^2(X)$. If X is compact, Δ admits a discrete spectral decomposition. Let $0 = -E_0 > -E_1 \geq \ldots$ be the eigenvalues of Δ, denote by $\phi_0, \phi_1, \ldots$ the corresponding L^2-normalized eigenfunctions. In other words, the ϕ_k satisfy Helmholtz' equation

$$\Delta \phi_k = -E_k \phi_k. \tag{3.4}$$

Note that $E_0 = 0$ because the constant functions are solutions of the equation with eigenvalue 0 and that the ϕ_k are defined only up to an orthogonal transformation of the eigenspaces.

The functions ϕ_k are the natural generalization to surfaces of the classical Fourier basis $\sqrt{2}\sin(n\pi x), \sqrt{2}\cos(n\pi x)$ which solve Helmholtz' equation on the unit circle $\mathbb{S}^1$. In paricular, they can be thought of as vibration modes: The value of an eigenfunction ϕ_k in a point represents how much it moves in the vibration mode, and the corresponding eigenvalue is minus the square frequency of vibration. This (classical) mechanical interpretation of the eigenvectors gives a first intuition about them, in particular eigenvectors that vary fast spatially are also the ones with high frequency, as can be seen in



Figure 3.2: *First and 300th eigenfunction of the Laplace-Beltrami operator. The colors represent the values of the eigenfunctions, blue being the most negative and red the most positive value. The eigenfunctions can be thought of as vibration modes, with low frequencies having slow spatial variation, and high frequencies having fast spatial variation.*

Fig. 3.2. This connection between frequencies of the Laplace-Beltrami operator and spatial scales will be important for understanding the scale separation properties of respective 3D shape signatures.

The classical Fourier decomposition allows to develop any periodic signal f, that is any signal defined on $\mathbb{S}^1$ in its frequency components

$$f(x) = \sum_k a_k \sqrt{2}\cos(\pi k x) + b_k \sqrt{2}\sin(\pi k x), \tag{3.5}$$

where

$$\begin{aligned} a_k &= \int_{\mathbb{S}^1} f(x)\sqrt{2}\cos(\pi k x)dx, \\ b_k &= \int_{\mathbb{S}^1} f(x)\sqrt{2}\sin(\pi k x). \end{aligned} \tag{3.6}$$

Similarly, given a signal $f : X \to \mathbb{R}$ defined on X, we can decompose f into its frequency components

$$f = \sum_{k \geq 0} c_k \phi_k, \tag{3.7}$$

where $c_k = \int_X f \cdot \phi_k$.

Given a point $x \in X$, the Dirac delta function corresponding to x is the distribution defined by

$$\delta_x : \mathcal{C}^\infty(X) \to \mathbb{R}, \quad f \mapsto f(x). \tag{3.8}$$

The point x is uniquely determined by its delta function. As a generalized signal on X, δ_x can be decomposed into its frequency components. The Fourier coefficients of δ_x are given by

$$c_k = \int_X \delta_x(y)\phi_k(y)dy = \phi_k(x). \tag{3.9}$$

With this in mind, it is tempting to define a feature descriptor for points on surfaces simply by concatenating the Fourier coefficients c_k into a vector indexed over k. Up to a multiplicative factor in each component, this is exactly the idea underlying the Global Point Signature (GPS) defined by Rustamov [148]. Indeed, this signature characterizes points uniquely up to isometry. Unfortunately, the coefficients c_k depend on the choice of the sign of the Laplace eigenfunctions ϕ_k. In the case of repeated eigenvalues, the situation becomes even worse because the coefficients are only determined up to an orthogonal transformation of the eigenspace. As a consequence, a different choice of basis functions may lead to an entirely different signature.

3.1.3.2 The Heat Kernel Signature

The HKS overcomes these difficulties in a very elegant way. As indicated by the name, it relies on the diffusion of heat on the surface. Given an initial heat distribution $u_0 : X \to \mathbb{R}$, the heat distribution $u(\cdot, t) : X \to \mathbb{R}$ for times $t > 0$ solves the *heat equation*

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = \Delta u(x,t), \\ \\ u(x,0) = u_0(x). \end{cases} \tag{3.10}$$

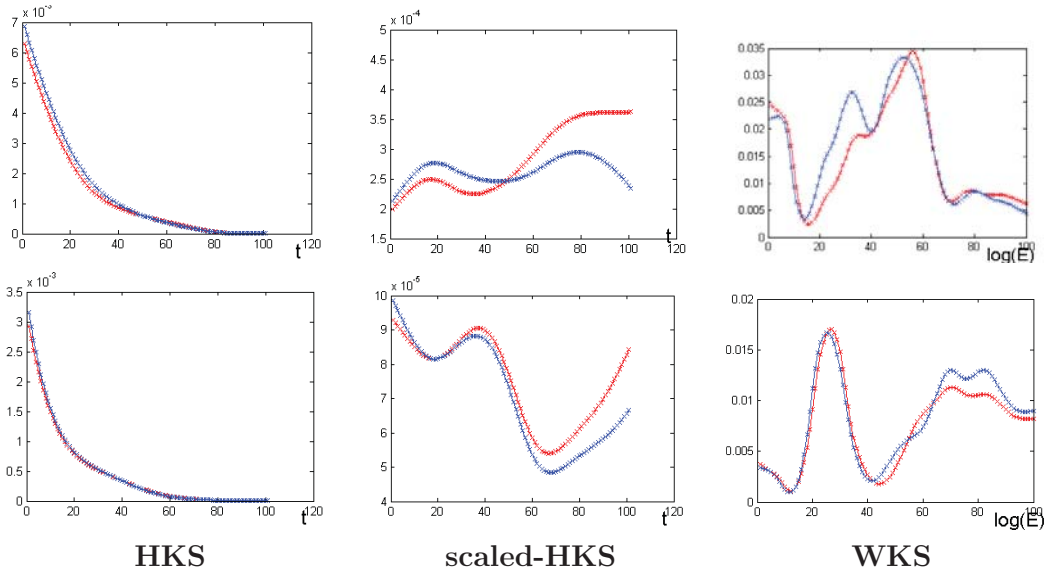


Figure 3.3: *Comparison of the Heat Kernel Signature (first column), the scaled Heat Kernel Signature (second column) and the Wave Kernel Signature (third column) for two different points (first and second line) and two different shapes (red and blue). Note that while remaining robust to deformations the WKS captures more information including shape differences at finer scales.*

The fundamental solution to the heat equation is given by the *heat kernel* $K : X \times X \times \mathbb{R}_{>0} \to \mathbb{R}$ defined by

$$K(x, y, t) = \sum_{k \geq 0} e^{-E_k t} \phi_k(x) \phi_k(y). \tag{3.11}$$

Thus, equation (3.10) is solved by

$$u(x, t) = \int_X K(x, y, t) u_0(y) dy. \tag{3.12}$$

The *Heat Kernel Signature* at a point $x \in X$ is based on the following physical experiment: Assume that initially at time $t_0 = 0$ an infinite amount of heat is placed at x and that there is no heat on $X \setminus \{x\}$, corresponding to a Dirac delta function δ_x as initial heat distribution. Let this initial distribution diffuse and define the HKS at x and time $t > 0$ as the amount of heat remaining in x at time t. According to the

above, the HKS can be computed as

$$\begin{aligned} \mathrm{HKS}(x,t) &= \int_X \left(\sum_{k\geq 0} e^{-E_k t} \phi_k(x)\phi_k(y)\delta_x(y) \right) dy \\ &= \sum_{k\geq 0} e^{-E_k t} \phi_k(x)^2. \end{aligned} \tag{3.13}$$

Sun et al. [168] introduced this signature as a point descriptor and showed that points on a 3D shape are almost completely characterized by their HKS (see Section 3.3.3 for the precise statement in the analogous case of WKS). Moreover, being the result of a physical experiment, HKS does not depend on the signs or on the ordering of the Laplace eigenfunctions. This can also be read off directly from formula (3.13).

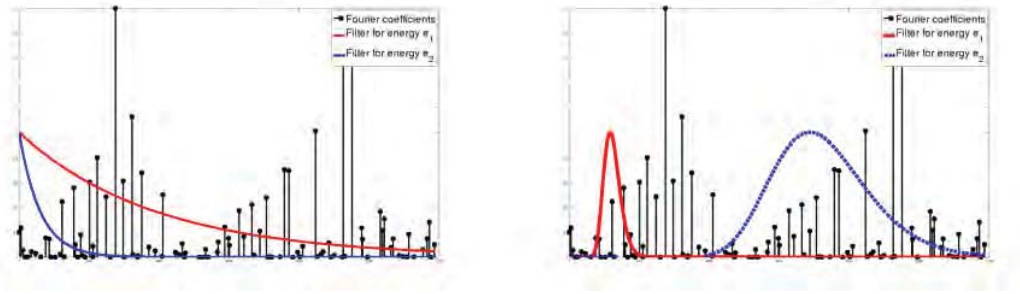
Sun et al. [168] noticed that the HKS tends to decrease exponentially, as can be seen in figure 3.3. Thus, they introduced a heuristic modification of the HKS to avoid this effect, that we will refer as sHKS:

$$s\,\mathrm{HKS}(x,t) = \frac{HKS(x,t)}{\int_X HKS(y,t)dy} \tag{3.14}$$

One of the main drawbacks of this descriptor is that it is not invariant to scale. For this reason Bronstein and Kokkinos [37] introduced the scale-invariant heat kernel signature (SI-HKS) which makes the HKS invariant to scale using logarithmic sampling and Fourier transform.

3.1.3.3 Limitations of the HKS

Despite its elegant physical derivation and its appealing theoretical properties, the HKS has a number of practical drawbacks. In order to discuss these, we come back to the signal-theoretic interpretation of $\phi_k(x)$ as the k-th Fourier coefficient of the delta function δ_x. From this point of view, $\mathrm{HKS}(x,t)$ is an expression in the squared Fourier



(a) *HKS* **(b)** *WKS*

Figure 3.4: *This figure shows the weight given by the* HKS *and* WKS *(blue and red lines) to the coefficients (black spikes) corresponding to the different energies (x axis). The weighting of* WKS *is more intuitive and discriminative since it aggregates information only from similar eigenvalues.*

coefficients of δ_x. More precisely, $(\mathrm{HKS}(x,t))_{t>0}$ can be seen as a collection of low-pass filters applied to δ_x: for large t, the high frequencies are suppressed – see Figure 3.4. This brings about the following disadvantages of the HKS:

- With increasing time t, the HKS mixes information from all scales. Separating shape differences on different spatial scales is not possible.

- Regardless of the choice of $t > 0$, $\mathrm{HKS}(x,t)$ is dominated by low frequency information, which corresponds to global properties of the shape.

3.2 The Wave Kernel Signature

3.2.1 From heat diffusion to Quantum Mechanics

The above limitation of the HKS is inherently tied to the process of heat diffusion, where for larger values of time t, the heat distribution invariably converges to an entirely uninformative constant temperature. Can we overcome this limitation of the

HKS by reverting to a different physical process? Can we rely on another mathematical equation whose solution would not be dominated by the global properties of the shape which are embedded in the eigenfunctions associated with the large frequencies?

We would like to adhere to the Laplace-Beltrami operator because of its favorable properties, its eigenfunctions being associated with frequencies of vibration [97] as visualized in figure 3.2. Yet, we would like a physical process which does not attenuate the high frequencies as the heat diffusion does.

A mathematical solution is to multiply the Laplace-Beltrami operator by the imaginary number 'i'. That way the eigenvalues of the operator are complex and the contribution of the different frequencies will not be attenuated over time. The resulting equation is a specific case of the *Schrödinger equation*, which describes the behavior of a quantum particle. The state of the quantum particle at time t is fully described by the complex function $\psi(\cdot, t)$, often referred to as the *wave function*. In particular, the squared norm $|\psi(x,t)|^2$ of the wave function corresponds to the probability density for detecting the particle at a location x at time t. The evolution of a particle described at $t = 0$ by the function ψ_0 is governed by the equation:

$$\begin{cases} i\,\dfrac{\partial}{\partial t}\psi(x,t) = H\psi(x,t) = \left(-\Delta_x + V(x)\right)\,\psi(x,t), \\ \\ \quad \psi(x,0) = \psi_0(x). \end{cases} \tag{3.15}$$

Here $H = -\Delta + V$ is called the Hamiltonian operator, and $V(x)$ is a real potential representing an external field – for example an electric field – acting on the particle. For simplicity, we assume this external field to be constant in time. In Section 3.2.4, we will discuss how to exploit such an external field in order to constrain particles to a certain spatial vicinity, thereby localizing the shape analysis and allowing a multiscale

analysis of shapes.

3.2.2 Schrödinger equation on a surface

To solve the Schrödinger equation we can exploit the fact that the Hamiltonian operator $H = -\Delta + V$ is compact and self-adjoint on the Hilbert space of complex L_2 functions on X. According to the spectral theorem we can therefore diagonalize it in an orthonormal basis. Let $(\phi_0, \phi_1, \ ...)$ denote a basis of eigenfunctions of the operator H, and let $(E_0, E_1, \ ...)$ denote the corresponding eigenvalues. We can further assume that $E_0 \leq E_1 \leq ....$ For simplicity we assume that there are no degeneracies, i.e. that the eigenvalues are distinct, which is the general case.

To solve the Schrödinger equation (3.15), we simply expand the solution $\psi(x,t)$ in the basis $(\phi_0, \phi_1, \ ...)$:

$$\psi(x,t) = \sum_{k=0}^{\infty} f_k(t)\, \phi_k(x), \tag{3.16}$$

where

$$f_k(t) = \int_X \bar{\phi_k}(x)\, \psi(x,t)\, dx$$

denotes the k-th frequency component at time t. Inserting this basis expansion in (3.15), we get:

$$\sum_{k=0}^{\infty} \frac{d}{dt} f_k(t)\, \phi_k(x) = \sum_{k=0}^{\infty} iE_k\, f_k(t)\, \phi_k(x) \tag{3.17}$$

Since $(\phi_0, \phi_1, \ ...)$ is an orthogonal basis, we can solve in each coordinate separately, and thus the solution is given by

$$\psi(x,t) = \sum_{k=0}^{\infty} f_k(0)\, e^{iE_k t} \phi_k(x) \tag{3.18}$$

The expected value of the Hamiltonian H provides the energy of a particle in a state

$\psi(x)$ by $\int_S \psi H \psi$. The only possible output in the measurement of the energy are the eigen-states of H, and the probability to find an energy E_k (assuming that all the eigen-energies are distinct) is given by $\int_S \phi_k H \psi$. Thus the ϕ_k are also known as energy eigen-states, since a particle in the state ϕ_k will always be measured to have an energy E_k.

3.2.3 A spectral signature for shapes

The remaining task is to derive a signature which characterizes points on the 3D shape based on the physical process of quantum particle evolution described by equation (3.15). This task can be decomposed into two aspects:

- Choose an initial distribution of energy.
- Choose a quantity to consider/measure.

For the HKS, the signature of a point was determined by using the evolution of the temperature on a point with a Dirac distribution in this point at $t = 0$. The intuition was that for small time, the solution was influenced mainly by the local shape, and that the signature grew more global with time. Nevertheless, the study of the equation showed that a transparent interpretation in terms of scale is not obvious given that different frequencies are increasingly mixed and the small eigenvalues dominate. In practice researchers compensated this dominance of small eigen-values at larger times by introducing a logarithmic sampling of time.

In the following, we therefore suggest to parameterize the shape descriptor not as a function of time, as done in the HKS, but as a function of the energy of the particle. This will give rise to a new feature descriptor which we call the *Wave Kernel Signature* (WKS). While it is also derived from a physical process, the WKS is a time-independent

signature. More specifically denote the energy distribution of the quantum mechanical particle by $f_k(0) = f_E(E_k)$ (E will be the parameter of the distribution, see Section 3.3.1 for the exact choice of this distribution).

Definition 1. *The* Wave Kernel Signature *(*WKS*) associated with a given point $x \in X$ is the time-averaged probability of detecting a particle of a certain energy distribution f_E at that point. Mathematically, it can be computed as:*

$$\begin{aligned}
\mathrm{WKS}(x,E) &= \lim_{T\to\infty} \frac{1}{T}\int_0^T |\psi(x,t)|^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_0^T \left|\sum_{k=0}^{\infty} f_E(E_k)\phi_k(x)e^{iE_k t}\right|^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_0^T \sum_{k,l=0}^{\infty} f_E(E_k)\phi_k(x)\overline{f_E(E_l)\phi_l(x)}e^{iE_k t}e^{-iE_l t}dt \\
&= \sum_{k,l=0}^{\infty} f_E(E_k)\phi_k(x)\overline{f_E(E_l)\phi_l(x)} \lim_{T\to\infty} \frac{1}{T}\int_0^T e^{iE_k t}e^{-iE_l t}dt
\end{aligned} \tag{3.19}$$

As a result, we obtain:

$$\mathrm{WKS}(x,E) = \sum_{k=0}^{\infty} |f_E(E_k)|^2 |\phi_k(x)|^2. \tag{3.20}$$

The last equality holds because limits can be exchanged. While respective limits do not generally commute, they do in our case for a large range of functions f_E (including Gaussians and exponentials) since as shown in [86]:

$$||\phi_k||_\infty \leq C E_k^{\frac{1}{4}} \tag{3.21}$$

The above definition of the Wave Kernel Signature is very general: the function f_E can be chosen in many ways leading to different kinds of signatures. In this chapter we use a general stability analysis of the shape as detailed in section 3.3 to derive a particular

f_E. Our result is also physically meaningful since it corresponds to a particle which energy has been measures with a log-normal error. Bronstein [120] later suggested this function could be learned to optimize some task on a specific dataset. 3.3.1.

Proposition 1. *The family of descriptors introduced in Definition 1 is a generalization of the Heat Kernel Signature.*

Proof. For the choice of an exponential energy distribution

$$f_E(E_k) = e^{-\sqrt{E_k}t},$$

the WKS in (3.20) is equivalent to the HKS in (3.13). □

3.2.4 Global vs. local WKS

In our original article [21], the Schrödinger equation was used without potential, and thus the signature of each point depended on the entire shape. Adding a spatial potential V forces particles with low energy to stay in some part of the shape. As a consequence, one can design a shape signature which will only be affected by a predefined local neighborhood. For example, to describe a point x the following potential can be used:

$$V_x(y) = \begin{cases} 0 & \text{if } d(x,y) \leq d_0 \\ \infty & \text{if } d(x,y) > d_0 \end{cases} \tag{3.22}$$

where d is the geodesic distance between two points of the shape and d_0 is a reference distance. The corresponding local descriptor will no longer be scale invariant (since a reference distance will be introduced in the potential), but it will be more suitable for *partial* shape matching where local parts should be identified as *similar* even if their relation to the remainder of the shape is very different. The appropriate scale can

be chosen using some property of the shape. Alternatively one can sample numerous scales in a logarithmic manner leading to a full multi-scale description of the shape. An example of such a local descriptor is given in 3.4.1.2. In the rest of this chapter, we focus on the case of a constant potential.

3.3 Mathematical Analysis of the WKS

In this section we will present a mathematical analysis of the Wave Kernel Signature introduced above. In particular, we will perform a theoretical stability analysis which justifies the choice of the log-normal distribution for the energy levels in the definition of WKS. Next, we will argue that in contrast to the HKS, the WKS ensures a clear separation of scales. Finally, we will summarize the favorable properties of WKS concerning its invariance to non-rigid deformations as well as its discriminative power.

3.3.1 Stability analysis

In this subsection, we derive an adapted energy distribution f_E for a particle on the surface with expected energy level E. By better understanding how the spectral decomposition of the shape varies with small deformations, we will be able to design a descriptor that will be both informative and robust to small non-isometric perturbations of the considered surface.

Assume that a surface X is slightly deformed in a non-isometric way. Mathematically we can interpret such a deformation as a perturbation $g(\varepsilon)$ of the metric $g = g(0)$ on X for a real parameter ε with $|\varepsilon|$ small. Assume that the deformation is regular in the sense that $g(\varepsilon) = g(0) + \varepsilon g^1 + \varepsilon^2 g^2 + \dots$ and the corresponding Laplace–Beltrami operators $\Delta(\varepsilon) = \Delta(0) + \varepsilon\Delta_1 + \varepsilon^2\Delta_2 + \dots$ depend analytically on ε (compare also [143, Def. 3]).

For simplicity, we assume that the Laplace–Beltrami operator $\Delta(0)$ corresponding to $g(0)$ has no repeated eigenvalues. By [143, prop. 2], for each eigenvalue $-E_k$ of $\Delta(0)$, there exists an analytic family $E_k(\varepsilon)$ with $E_k(0) = E_k$ and $-E_k(\varepsilon)$ in the spectrum of $\Delta(\varepsilon)$.

Theorem 1. *Denote by $C = ||g^1||_{g(0)}$ the first order norm of the metric deformation, where the space of symmetric tensors $TX^* \otimes TX^*$ is endowed with the norm induced by $g(0)$. Then for $|\varepsilon| > 0$ sufficiently small we have*

$$|E_k(\varepsilon) - E_k| \leq CE_k \cdot |\varepsilon| + \mathcal{O}(\varepsilon^2).$$

Proof. Denote by $\phi_k(\varepsilon)$ the normalized eigenfunctions to the eigenvalue $E_k(\varepsilon)$. By [143], we have

$$\phi_k(\varepsilon) = \phi_k(0) + \mathcal{O}(\varepsilon). \tag{3.23}$$

Let $a_{k,i}(\varepsilon) = \langle \phi_k(\varepsilon), \phi_i(0) \rangle_{L^2(X)}$. Then

$$\phi_k(\varepsilon) = \sum_i a_{k,i}(\varepsilon)\phi_i(0) \tag{3.24}$$

and by (3.23) we can infer that

$$a_{k,k} = \mathcal{O}(1),\ a_{k,i} = \mathcal{O}(\varepsilon) \text{ for } i \neq k. \tag{3.25}$$

Now, we plug this into the eigenvalue equation

$$\Delta(\varepsilon)\phi_k(\varepsilon) = E_k(\varepsilon)\phi_k(\varepsilon). \tag{3.26}$$

On the left hand side of (3.26) we get

$$\begin{aligned}
&\Big(\Delta(0) + \varepsilon\Delta^1 + \mathcal{O}(\varepsilon^2)\Big)\Big(\sum_i a_{k,i}(\varepsilon)\phi_i(0)\Big) \\
&= E_k(0) \cdot a_{k,k}(\varepsilon)\phi_k(0) \\
&\quad + \varepsilon a_{k,k}(\varepsilon)\Delta^1\left(\phi_k(0)\right) + \sum_{i\neq k} a_{k,i}(\varepsilon)E_i(0)\phi_i(0) \\
&\quad + \mathcal{O}(\varepsilon^2).
\end{aligned} \tag{3.27}$$

The right hand side of (3.26) is equal to

$$E_k(\varepsilon) \cdot \Big(a_{k,k}(\varepsilon)\phi_k(0) + \sum_{i\neq k} a_{k,i}(\varepsilon)\phi_i(0)\Big). \tag{3.28}$$

Now we take on both sides the $L^2(X)$-scalar product with $\phi_k(0)$ to obtain

$$\begin{aligned}
a_{k,k}(\varepsilon)E_k(0) + \varepsilon a_{k,k}(\varepsilon)\int_X \phi_k(0)\Delta^1\left(\phi_k(0)\right) + \mathcal{O}(\varepsilon^2) \\
= E_k(\varepsilon)a_{k,k}(\varepsilon).
\end{aligned} \tag{3.29}$$

Since $a_{k,k}(\varepsilon) \neq 0$ for $|\varepsilon|$ sufficiently small, we get

$$E_k(\varepsilon) = E_k(0) + \varepsilon\int_X \phi_k(0)\Delta^1\left(\phi_k(0)\right) + \mathcal{O}(\varepsilon^2). \tag{3.30}$$

Thus, we have to show that

$$\left|\varepsilon\int_X \phi_k(0)\Delta^1(\phi_k(0))\right| \leq CE_k\varepsilon + \mathcal{O}(\varepsilon^2). \tag{3.31}$$

Since $\varepsilon\Delta^1 = \Delta(0) - \Delta(\varepsilon) + \mathcal{O}(\varepsilon^2)$, we get

$$\begin{aligned} & \left|\varepsilon \int_X \phi_k(0)\Delta^1\big(\phi_k(0)\big)\right| \\ = & \left| \int_X \phi_k(0)\big(\Delta(0) - \Delta(\varepsilon)\big)\big(\phi_k(0)\big)\right| + \mathcal{O}(\varepsilon^2) \\ \leq & \int_X \left|g(0)\big(d\phi_k(0), d\phi_k(0)\big) - g(\varepsilon)\big(d\phi_k(0), d\phi_k(0)\big)\right| \\ & + \mathcal{O}(\varepsilon^2) \end{aligned} \tag{3.32}$$

where we used Gauss' theorem on surfaces. Now, $g(0) - g(\varepsilon) = \varepsilon g^1 + \mathcal{O}(\varepsilon^2)$. On the other hand, for all $v \in \mathbb{R}^n, A \in \mathbb{R}^{n,n}$ we have $\big|\langle v, Av\rangle\big| \leq \|A\|_F \cdot \langle v, v\rangle$, where $\|\cdot\|_F$ denotes the Frobenius norm. This implies that $\big|g^1(\alpha,\alpha)\big| \leq C \cdot g(0)\big(\alpha,\alpha\big)$ for all 1-forms α on X. Thus,

$$\begin{aligned} & \left|\varepsilon \int_X \phi_k(0)\Delta^1\big(\phi_k(0)\big)\right| \\ \leq & \int_X \left|\varepsilon g^1\big(d\phi_k(0), d\phi_k(0)\big)\right| + \mathcal{O}(\varepsilon^2) \\ \leq & \,|\varepsilon| \cdot C \cdot \int_X g\big(d\phi_k(0), d\phi_k(0)\big) + \mathcal{O}(\varepsilon^2) \\ = & \,|\varepsilon| \cdot CE_k + \mathcal{O}(\varepsilon^2). \end{aligned} \tag{3.33}$$

This concludes the proof. □

The theorem implies that there exist c_k with $|c_k| \leq C$ such that

$$E_k(\varepsilon) = (1 + \varepsilon c_k)\, E_k + \mathcal{O}(\varepsilon^2).$$

This can be reformulated as

$$\log\left(\frac{E_k(\varepsilon)}{E_k}\right) = \log(1 + \varepsilon c_k + \mathcal{O}(\varepsilon^2)) = \varepsilon c_k + \mathcal{O}(\varepsilon^2). \tag{3.34}$$

For convenience, we model the c_k coefficients by independent normally distributed random variables:

$$\log(E_k(\varepsilon)) \sim \mathcal{N}(\log(E_k), \sigma). \tag{3.35}$$

Thus, in our model, the eigen-energies of an articulated shape X are log-normally distributed random variables. We design our descriptor to take into account this information without any assumption about the variations of the eigenfunctions.To achieve this, we choose f_E^2 as a log-normal distribution.

$$\boxed{\mathrm{WKS}(x, e) = \sum_{k=0}^{\infty} e^{-\frac{(e-\log(E_k))^2}{2\sigma^2}} |\phi_k(x)|^2} \tag{3.36}$$

In this equation the parameter σ allows an intuitive trade-off between discrimination ($\sigma \to 0$) and robustness to deformations ($\sigma \to \infty$).

Note that this formulation also provides a very meaningful interpretation in the Quantum Mechanics framework: suppose that the log-energy e of a particle is measured with a Gaussian error of variance σ, the average probability to measure it in x is $\mathrm{WKS}(x, e)$.

3.3.2 Spectral analysis

Our descriptor as defined in equation 3.36 performs an efficient spectral separation. In contrast to HKS a specific eigenvector influence only a few values in the descriptor and a value of the descriptor is influenced only by the eigenvectors corresponding to few eigenvalues as shown in Fig. 3.4. For this reason, the differences in the eigenvector

and eigenvalues that exist between two shapes do not cumulate in the descriptor as in the HKS as one can see in Fig. 3.3.

3.3.3 Invariance and discrimination

The Wave Kernel Signature has a number of favorable properties which make it very well suited for a variety of tasks in non-rigid shape analysis.

- WKS is *intrinsic*: if $T : X \to Y$ is a (rigid or non-rigid) isometry, then $\mathrm{WKS}(x, e) = \mathrm{WKS}(T(x), e)$ for all $x \in X, e \in \mathbb{R}$.

- WKS is *robust to shape perturbations*: shapes which are perceived as similar (even if they appear in different poses), differ by small non-isometric deformations. By the choice of the energy distributions in Section 3.3.1, WKS is stable under such deformations.

- WKS is *informative*: assume that two shapes are homeomorphic via a map $T : X \to Y$. Then T is an isometry if and only if $\mathrm{WKS}(x, e) = \mathrm{WKS}(T(x), e)$ for all $x \in X, e \in \mathbb{R}$. The proof is analogous to the proof of Theorem 1 in [168].

- WKS encodes information from *various spatial scales*: indeed, WKS is parametrized over the (logarithmic) energies of particles which are directly related to scales. Large energies correspond to highly oscillatory particles which are mostly influenced by local geometry whereas small energies correspond to properties induced by the global geometry. For a discussion on the separation of different scales, we refer to Section 3.3.2

3.4 Experimental Results

In this section we analyse the discriminability and robustness of the WKS for matching. It was indeed designed to achieve an optimal trade-off between those two desirable properties of shape descriptors. We compare the WKS with other state of the art spectral descriptors: the HKS, sHKS and SI-HKS introduced in 3.1.3.2. An analysis of how spectral descriptors relate to other kind of descriptors is out of the scope of this work. For a recent survey on shape descriptors, the reader can refer to [171].

In all our our experiments we computed descriptors of size 100 and used 300 eigenvalues. We used the shapes of the TOSCA dataset [50], which were computer generated and present a variety of perturbations.

3.4.1 Qualitative analysis

3.4.1.1 Comparison with HKS

To compare qualitatively the behavior of the WKS, HKS, and s HKS, we take a reference point at random in a shape, compute its descriptor, and plot (using a color code) the distances between the reference descriptor and the descriptors of all points of a deformed shape.

Figure 3.5 shows a typical example in which the spectral separation avoids the confusion induced by the change in a low frequency eigenvalue by using high frequency information.

In figure 3.6 we use this visualization to understand better how scales are treated in all three descriptors by looking at the distances associated to the full descriptors, to the 50 first and to the 50 last values of the descriptors. For the WKS, we can see that the

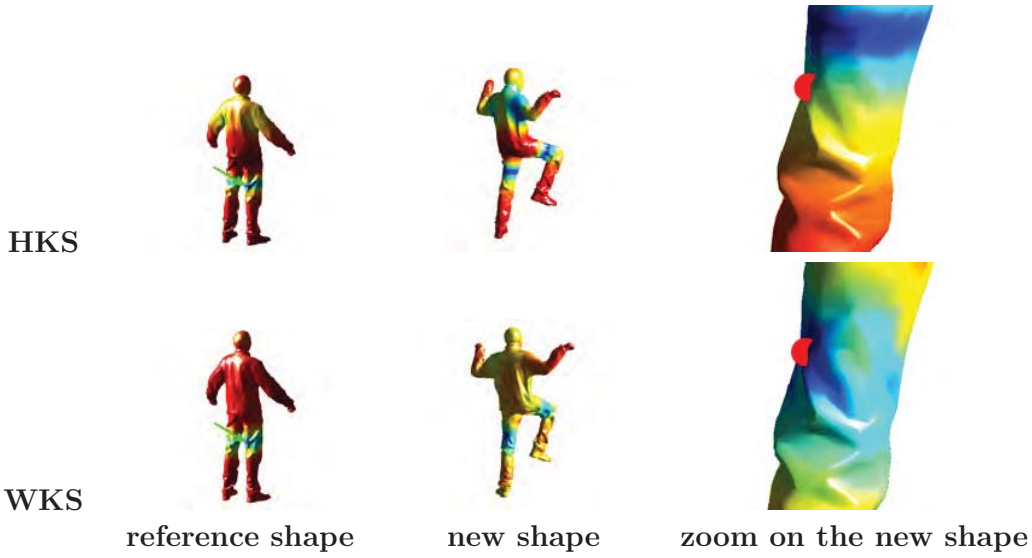


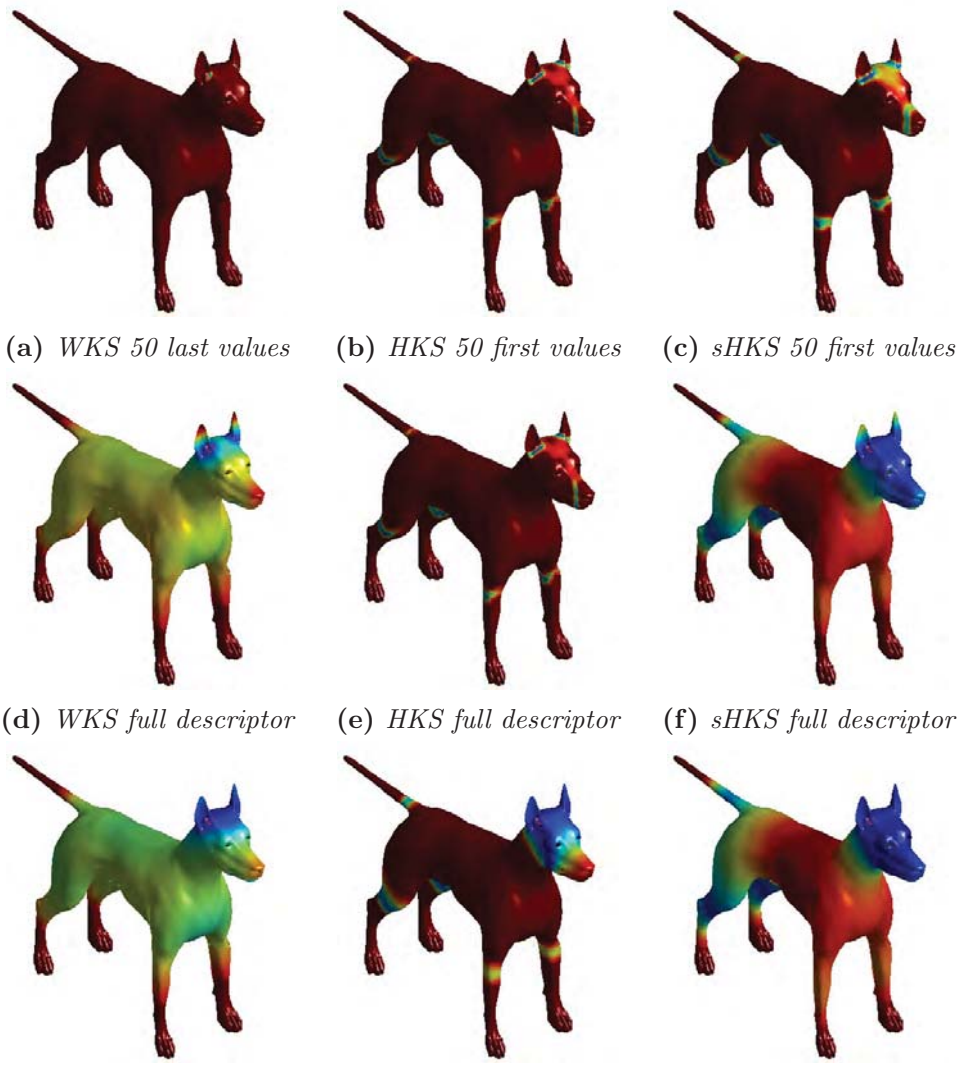
Figure 3.5: *The figures show the logarithmic distance between the descriptor of a point in the reference frame (on the leg) and the descriptor of other points in the same frame (first column) and in another (second and third column). Since it does not overweight the small eigenvalues the Wave Kernel Signature avoids the false matches at the shoulders obtained with the HKS, and provides a highly localized correspondence (shown in the closeup on the right).*

50 first values lead to a coarse localization of the point, while the 50 last values give a very accurate information. On the contrary, for both the HKS and the sHKS points at a completely different location (on the back legs of the dog) have descriptors close to the descriptor of the reference point (on the ear) because the information of the first eigenvectors is repeatedly used.

3.4.1.2 Potential for local signature

Fig. 3.7 shows that the WKS remains discriminative even using only part of the mesh by plotting the descriptor for two different points (on the head and the leg) computed using only the parts of the mesh shown in green in Fig. 3.7a. Indeed, the descriptors are different for the two points, but consistent across deformations.

The main current limitation for a large-scale use of this local descriptor is that it necessitates the computation and diagonalization of a different operator for each point

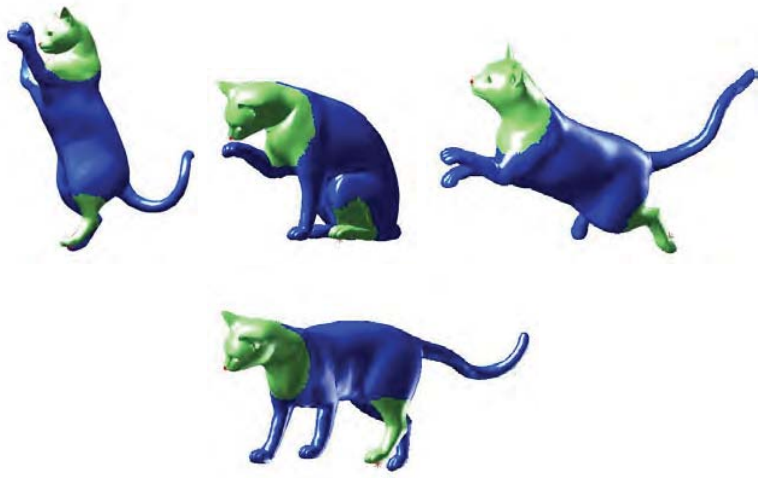


(a) *WKS 50 last values* **(b)** *HKS 50 first values* **(c)** *sHKS 50 first values*

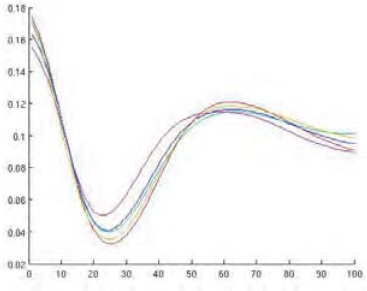
(d) *WKS full descriptor* **(e)** *HKS full descriptor* **(f)** *sHKS full descriptor*

(g) *WKS 50 first values* **(h)** *HKS 50 last values* **(i)** *sHKS 50 last values*

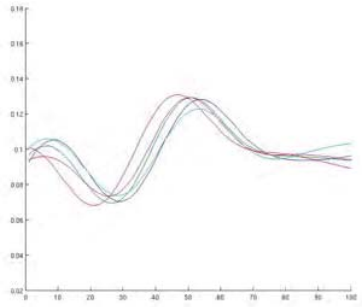
Figure 3.6: *In this figure, each image shows the distance of any point to a reference descriptor (of the point marked with a red circle in the right ear in another shape). The distance is color coded, blue corresponding to close descriptors, red to very different descriptors. The three columns correspond respectively to* WKS*,* HKS *and s* HKS *and the three lines correspond to the scale notion embedded in the descriptor (small scales in the first line, all scales in the middle line, and large scales in the bottom line) In contrast to the usual linear coding between the min and max distance, we used a truncated linear coding, with the maximum (red) corresponding to* $10 \times d(descriptor_1(P), descriptor_2(P))$*, where* P *is the reference point and the descriptors are taken in two different shapes. That allows the contrast to be concentrated in the region of interest and the colors to have an absolute signification. For* HKS *the three figures are similar and show false matches, since* HKS *is dominated by the smallest time values and for all time values* HKS *is influenced mainly by small frequencies.* *s* HKS *gives the same weight to all the descriptor values, but because of the spectral distribution of the* HKS *still overweights large scales. Finally,* WKS *shows qualitatively different results in all cases and no false matches. It is therefor more discriminative than* HKS*, but may not be as good for tasks such as semantic analysis.*



(a) *Part of the mesh used for each point*



(b) *local WKS head point*



(c) *local WKS leg point*

Figure 3.7: *The local WKS descriptor is robust to the shape deformations. This figure shows the descriptors of two points for the cat shape of the TOSCA dataset in 4 different positions. It is computed only using subparts of the mesh colored in green in (a)*

and each scale, leading to prohibitive computational costs. However, we believe that a better understanding of possible approximations can lead to efficient algorithms for its computation on a set of feature points.

3.4.1.3 Robustness to shape perturbation

It was shown in Section 3.3.1 that our descriptor is in theory robust to small changes in the metric. This is also verified in practice for a large class of extreme perturbations. Figure 3.8 shows some examples in the case of noise, mesh sampling and holes in the shape. In the three cases, a point is chosen in the standing shape and its descriptor computed. The colors in both the standing and kneeling shape code the distance from the descriptor of any point to this initial descriptor and the red lines show the top 50 matches. The quality of all the first 50 matches shows the high quality of the descriptors: even if the matches are not the good ones, they are very plausible.

3.4.2 Quantitative evaluation

In this section, we evaluate quantitatively the quality of the WKS for point matching and focus particularly on the analysis of the influence of its variance parameter, associated to its robustness-precision trade-off.

3.4.2.1 Robustness of the descriptor

We begin by evaluating the robustness of the WKS by using the measure introduced in [33] for SHREC 2010. Given two shapes X and Y represented by a triangular mesh, we compute:

$$Q(X,Y) = \frac{1}{|\mathcal{F}(X)|} \sum_{x \in \mathcal{F}(X)} \frac{\| \mathrm{WKS}^Y(y(x)) - WKS^X(x) \|_2}{\| \mathrm{WKS}^Y(y(x)) \|_2 + \| WKS^X(x) \|_2} \tag{3.37}$$



Figure 3.8: *Robustness of the WKS: The red lines connect a reference point on the shape in the background (standing David) with its 50 best matches on the perturbed shape in the foreground (sitting David). The color encodes the feature distance to the reference point, blue indicating proximity and red large distance in the feature space. The experiments visualized here are done with shapes in the strongest perturbation category of the SHREC 2010 [33] feature descriptor dataset.* **Left image:** *the WKS can locate the correspondence of the shoulders despite the strong noise.***Middle image:** *The reference mesh has 52565 vertices, while the perturbed mesh has 2634 vertices.* **Right image:** *The deformed shape has many holes. Note that an isometry invariant feature descriptor cannot distinguish the left and the right of a symetric shape.*

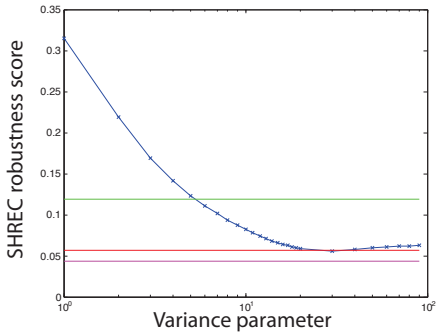


Figure 3.9: *Evaluation of the robustness of the matching using the measure from the SHREC evaluation [33] (equation 3.37, smaller values indicate a more robust descriptor).* **Blue:** *result for the WKS in function of the variance parameter (log-scale).* **Purple:** *results for the HKS.* **Red:** *results for the sHKS.* **Green:** *results for the SI-HKS. As expected, the results for the WKS become better when the variance parameter grows, increasing the robustness. It becomes more robust than the SI-HKS for a variance parameter around 5.*

Table 3.1: *Average area under the cumulative match characteristic curve for the different deformations and descriptors. WKS-5 indicates the performance of the WKS with a variance parameter fixed to 5, and WKS-optimal the performance of the WKS with the optimal variance parameter for each deformed shape. The fact that WKS-5 outperforms the other descriptors in most categories shows that it can be used as an "out of the box" descriptor. The clear improvement provided by WKS-optimal for some deformation categories such as localscale shows when it is worth to optimize the variance parameter.*

	holes	isometry	localscale	microholes	noise	sampling	scale	shotnoise	topology
HKS	0.909	0.953	0.865	0.945	0.910	0.953	0.946	0.943	0.934
sHKS	0.898	0.939	0.869	0.932	**0.925**	0.929	0.931	0.934	0.877
SIHKS	0.942	0.976	0.898	0.982	0.883	0.978	0.982	0.979	0.968
WKS-5	0.952	0.984	0.884	0.992	0.851	0.989	0.992	0.989	0.990
WKS-optimal	**0.962**	**0.985**	**0.909**	**0.993**	0.874	**0.991**	**0.993**	**0.991**	**0.992**

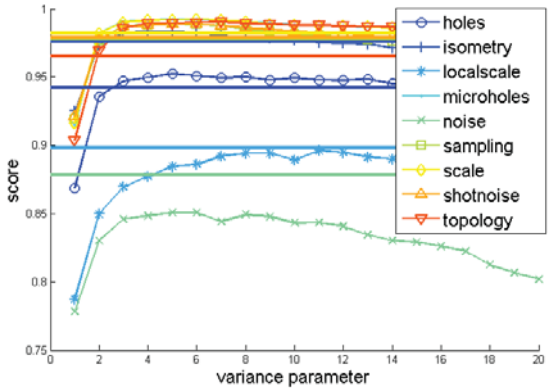
where $\mathcal{F}(X)$ is the set of points in X and $y(x)$ is the point in the shape Y corresponding to the point x in the shape X. Note that this measure evaluates the robustness of the descriptor (i.e. the fact that it does not change too much when the shape is deformed), but not the fact that it is discriminative (i.e. that is different for different points of the shape).

The results of this evaluation are shown in Fig. 3.9 for different variance parameters. As expected, the WKS becomes more robust when the variance parameter grows. It is more robust than the SI-HKS for a variance parameter greater than 6, and as robust as the sHKS for a variance parameter around 20. This evolution corresponds to the intuition developed in section 3.3.1.

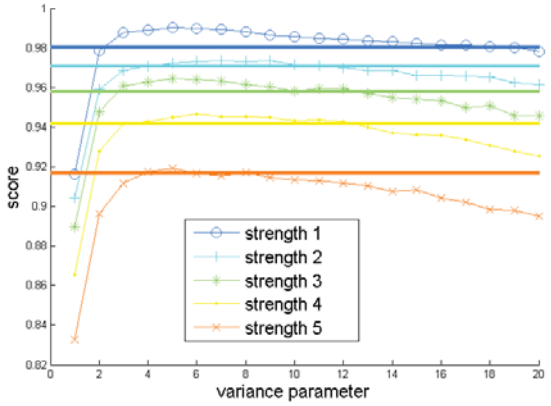
3.4.2.2 Quality of the descriptor for point matching

To evaluate the quality of a descriptor for matching, we repeat the following experiment for randomly selected points:

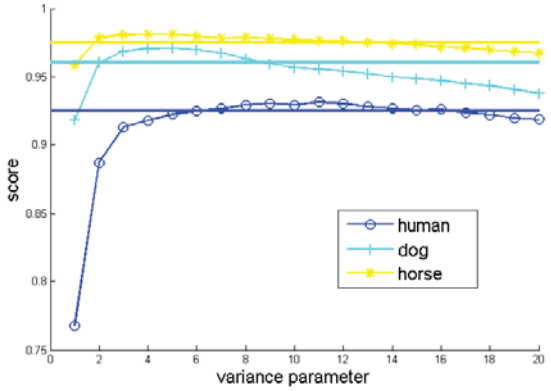
- given a point in a deformed shape, we compute its descriptor
- we compute its distance to all point descriptors in the reference shape
- we sort the points of the reference shape by using the computed distance



(a) *AUC of the CMC for the different deformation categories*



(b) *AUC of the CMC for the different strength of deformation*



(c) *AUC of the CMC for the different shapes*

Figure 3.10: *Area under the cumulative match characteristic curves in function of the variance parameter for different subsets of the SHREC2010 [33] evaluation data. The straight lines represent the baseline given by the SI-HKS.*

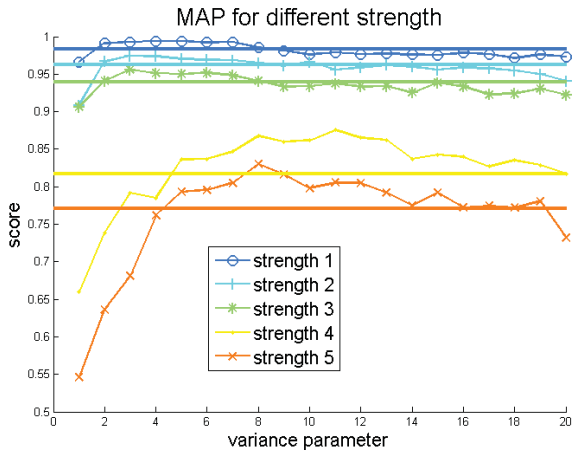


Figure 3.11: *Detailed analysis for the localscale deformation on the dog shape. Area under the cumulative match characteristic curves in function of the variance parameter for different strength of the deformation. The straight lines represent the baseline given by the SI-HKS. It can be seen that for bigger deformations, the optimal variance parameter is higher than for small deformations.*

- we store the rank of the ground truth correspondence

By computing the proportion of good correspondences that are in the first percentage of the matches, we can compute a Cumulative Match Characteristic curve (CMC) that characterize the quality of the descriptor for point matching. To enable easier comparisons, we summarize the quality of the matching by its area under curve, which is reported in table 3.1 and Fig. 3.10 and 3.11. Values close to 1 mean that the descriptor is very good for the considered matching task, while 0.5 is the chance performance.

General results. From the results reported in table 3.1, it can be seen that the WKS used with a variance parameter 5 outperforms the HKS, sHKS and SI-HKS for all of the perturbation categories except *localscale* and *noise*. A reasonable hypothesis is that the WKS does not perform as well for those categories because they would require another variance parameter.

To test this hypothesis, we repeated the same experiment as before but for each shape and transformation we selected the best WKS variance parameter. As expected, it improves all the results. The improvement is especially important for the *holes*, *noise* and *localscale* category. After this parameter selection, the WKS outperforms the other descriptors on all perturbation categories except the *noise* category.

The biggest improvement in using the WKS instead of one of the other descriptors is clearly for the *topology* category, where the results are improved by 3.4% (the second best is 2%). That is easily understood since this perturbation confuses the large scale characteristics of the shape, but not its local ones, which it is the strength of the WKS to encode.

Detailed analysis. Fig. 3.10 provides more details on the influence of the different variations of the dataset and the influence of the variance parameter. For each parameter of the dataset (perturbation category, shape and strength of the perturbation) we compare the performances of the WKS for several variance parameters to the performance of the SI-HKS that usually preforms best among the other spectral descriptors.

Fig. 3.10a shows that the WKS consistently outperforms the SI-HKS for a wide set of variance parameters for all categories of shape perturbations except *localscale* and *noise*.

Looking at the variations of the results in function of the strength of the perturbations in Fig. 3.10b reveals that the strength is indeed correlated to the difficulty of the matching. Interestingly, we can notice that the improvement provided by the WKS is especially important for limited perturbations, showing that it allows to be much more precise than the HKS and its variants for small perturbations. It is not surprising since

the assumption of small perturbations was made in its derivation in 3.3.1.

While the optimal variance parameter for the WKS does not on average depend on the strength of the perturbation, Fig. 3.10c shows that it clearly depends on the shape.

To avoid the dependency with respect to the shape and perturbation category, we performed a more detailed analysis for the *localscale* perturbation for the dog shape in Fig. 3.11. A clear gap in the difficulty can be seen between strength 1, 2 and 3 on one side and strength 4 and 5 on the other side. For the small perturbations, the optimal range of variance parameter is between 3 and 7, and for the bigger perturbations between 7 and 13, showing that increasing the variance increases the robustness at the cost of loosing precision, corresponding to the intuition of 3.3.1.

3.5 Applications

We have shown in the previous section that the WKS could better distinguish points than the competing spectral descriptors in many cases. In this section we show examples of potential other use of the WKS. We intentionally used simple techniques, since our goal is only to prove that those applications are possible and not to design algorithms that makes optimal use of the WKS on each task.

Global shape matching. We used a very simple greedy strategy. We selected a set of feature points in a reference shape, and greedily matched the points in a deformed with the closest feature distance, using a constraint on the geodesic distance for each new match. As shown in figure 3.12 this simple strategy worked well. For a state of the art matching method using the WKS, the reader can refer to [136].

Segmentation. We used a Gaussian Mixture Model (GMM) clustering on the features to segment the shapes. Since we wanted to be able to transfer the segmentation to

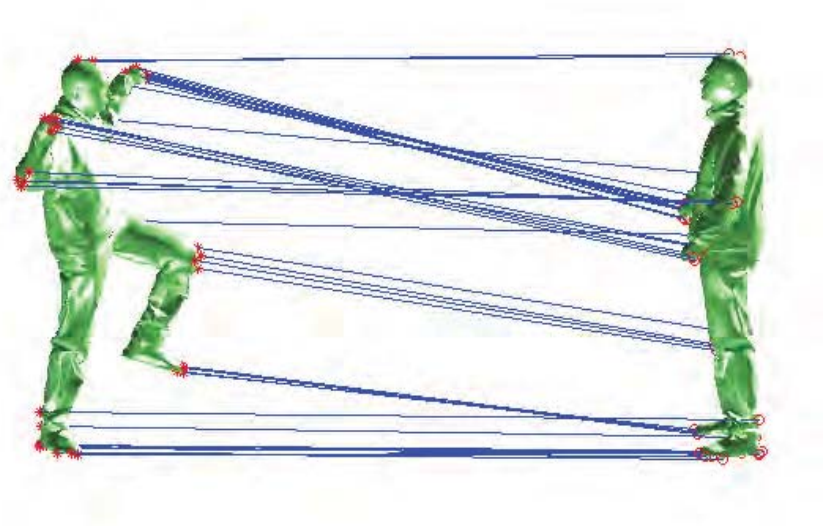


Figure 3.12: *Example of matching using a set of feature points on the left shape.*



(a) *Learning data (2 of 4)* **(b)** *Test data*

Figure 3.13: *Segmentation of a human shape using clustering on the* WKS. *Left: two segmentation (out of four) from the learning set. Right: two test segmentations*



Figure 3.14: *Example of segmentation learned for the different category of the SHREC 2010 dataset [116]. Since our segmentation is robust and relies on a probabilistic model, we can use it for recognition.*



Figure 3.15: *Shape segmentation applied to shape retrieval on the SHREC 2010 dataset [116]. The four columns on the right show representatives of four shape classes. For each shape class a Gaussian mixture distribution was computed as outlined in Section 3.5. The resulting segmentations are color encoded. The log-likelihood of the query shape (leftmost column) with respect to these distributions is displayed below each class.*

new shapes, we used several shapes as training data (without any knowledge about the correspondence between them) to incorporate in the descriptor informations about the possible variations of each part and avoid over-fitting. This method worked well. Especially, it recovers the segments on the different shapes, and it returns spatially coherent parts, without need of any spatial regularization. An example of human shape segmentation in 6 clusters can be seen in figure 3.13.

Recognition. Our segmentation method has the main advantage to rely on a probabilistic model, which can be directly applied for recognition. Given a training set of labelled shapes, we learn a segmentation for each category (see examples in figure 3.14). Thus, for each shape we have a GMM for the WKS distribution of its points.

Presented with a new shape, we compute the WKS of all its points, evaluate its log likelihood with respect to all the learned GMM models, and finally associate it with the label having the biggest log-likelihood, as illustrated figure 3.15.

The SHREC 2010 retrieval dataset [116] contains 10 shape classes with 20 instances of each class. For each class, we used 5 shapes as training, and test on the remaining 150 shapes. We achieved 72% correct assignment, proving that both the WKS and the segmentation are informative.

3.6 Conclusion

We introduced the Wave Kernel Signature in order to characterize points on a 3D shape. It is based on a careful analysis of harmonic functions on shapes and designed to be both discriminative and robust to variations of the metric, with a natural parameter to adjust the trade-off. This Wave Kernel Signature is defined as the time-averaged probability to localize a quantum-mechanical particle of a certain energy distribution at a given point of the shape. We demonstrate that it improves on the state of the art in terms of discrimination and spatial precision and that it is suitable for a large range of applications and in particular for shape alignment.

Chapter 4

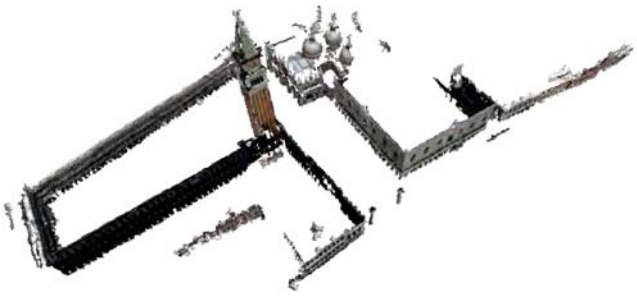
Painting-to-3D Alignment

4.1 Introduction

In the previous chapter, we tackled the problem of describing 3D models for 3D to 3D instance alignment. Here we want to develop a representation of 3D models that enables alignments of 2D depictions to a 3D model. We go beyond what methods reviewed in 2.2.2 and 2.2.3 can do and focus on recovering the viewpoint of historical and non-realistic depictions of an architectural site, such as drawings, paintings and historical photographs, with respect to a 3D model of the site. This is a tremendously difficult task since the appearance and the scene structure in the 2D depictions can be very different from the appearance and the geometry of the 3D model, e.g., due to the specific rendering style, drawing error, age, lighting or change of seasons (see figure 1.4). In addition, we face a hard search problem: the number of possible alignments of a painting to a large 3D model, such as a partial reconstruction of a city, is huge. To address these issues, we develop a new compact representation of complex 3D scenes. The 3D model of the scene is represented by a set of *3D discriminative visual elements* that are automatically learnt from rendered views. Similar to object detection, the set



(a) *Internet paintings and drawings.*



(b) *3D model.*



(c) *Aligned painting to 3D model.*



(d) *Alignment results visualized in Google Earth.*

Figure 4.1: *Our system automatically recovers the viewpoint of paintings, drawings, and historical photographs with respect to a 3D model of an architectural site. Painting in (c) courtesy of Podi Lawrence.*

of visual elements, as well as the weights of individual features for each element, are learnt in a discriminative fashion. We show that the learnt visual elements are reliably matched in 2D depictions of the scene despite large variations in rendering style (e.g. watercolor, sketch, historical photograph) and structural changes (e.g. missing scene parts, large occluders) of the scene. We demonstrate an application of the proposed approach to automatic re-photography to find an approximate viewpoint of historical paintings and photographs with respect to a 3D model of the site. The proposed alignment procedure is validated via a human user study on a new database of paintings and sketches spanning several sites. The results demonstrate that our algorithm produces significantly better alignments than several baseline methods. Example results are shown in figure 4.1.

4.1.1 Motivation

Why is this task important? First, non-photographic depictions are plentiful and comprise a large portion of our visual record. We wish to reason about them, and aligning such depictions to reference imagery (via a 3D model in this case) is an important step towards this goal. Second, such technology would open up a number of exciting computer graphics applications that currently require expensive manual alignment of 3D models to various forms of 2D imagery. Examples include interactive visualization of a 3D site across time and different rendering styles [52, 113], model-based image enhancement [105], annotation transfer for augmented reality [165], inverse procedural 3D modeling [7, 130] or computational re-photography [25, 141]. Finally, reliable automatic image to 3D model matching is important in domains where reference 3D models are often available, but may contain errors or unexpected changes (e.g. something built/destroyed) [32], such as urban planning, civil engineering or archaeology.

4.1.2 From locally invariant to discriminatively trained features

Local feature based methods presented in 2.2.2 represent a powerful tool for matching photographs of the same at least lightly textured scene despite changes in viewpoint, scale, illumination, and partial occlusion. However, appearance changes beyond the modeled invariance, such as significant perspective distortions, non-rigid deformations, non-linear illumination changes (e.g. shadows), weathering, change of seasons, structural variations or a different depiction style (see examples figure 1.4) cause local feature-based methods to fail [84, 147, 160], as illustrated figure 1.5.

Because of these limitations of standard local features methods, we turn towards methods based on discriminative learning similar to those used for object category recognition and presented in 2.3.1.2 and 2.3.1.3. They rely on a weighted spatial distribution of image gradient orientations. The weights are learnt in a discriminative fashion to emphasize object contours and de-emphasize non-object, background contours and clutter. Such a representation can capture complex object boundaries in a soft manner, avoiding hard decisions about the presence and connectivity of imaged object edges. Learnt weights have also been shown to emphasize visually salient image structures matchable across different image domains, such as sketches and photographs [160]. Similar representation has been used to learn architectural elements that summarize a certain geo-spatial area by analyzing (approximately rectified) 2D street-view photographs from multiple cities [55]. Also related is a contemporary work that utilizes similar representation for scene [96] and action [93] classification.

Building on these works we develop a compact representation of 3D scenes suitable for alignment to 2D depictions. In contrast to [55, 160] who analyze 2D images, our method takes advantage of the knowledge and control over the 3D model to learn a representative set of mid-level 3D scene elements robust to a certain amount of viewpoint variation and capable of recovery of the (approximate) camera viewpoint. We show that the learnt mid-level scene elements are reliably detectable in 2D depictions of the scene despite large changes in appearance and rendering style.

4.1.3 Overview

In this chapter, we focus on depictions that are, at least approximately, perspective renderings of the 3D scene and we consider complex textured 3D models obtained by

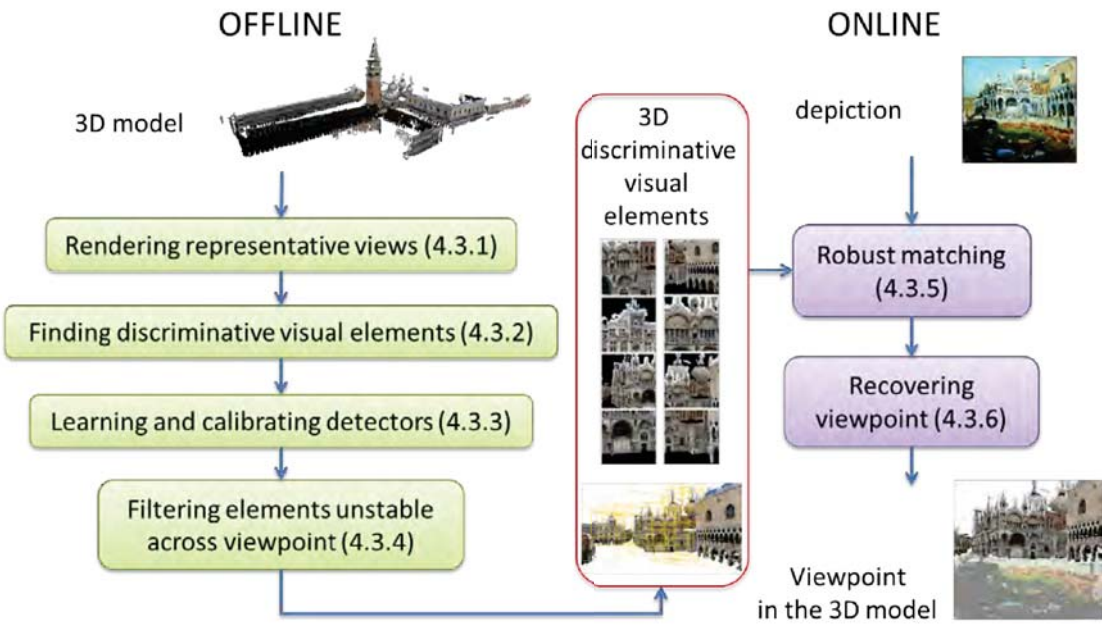


Figure 4.2: ***Approach overview.*** *In the offline stage (left) we summarize a given 3D model as a collection of discriminative visual elements learnt from rendered views of the site (middle). In the online stage (right) we match the learnt visual elements to the input painting and use the obtained correspondences to recover the camera viewpoint with respect to the 3D model.*

recent multi-view stereo reconstruction systems [70] as well as simplified models obtained from 3D modeling tools such as Trimble 3D Warehouse.

In section 4.2 we develop the two key ideas behind our work. First, we introduce our representation of 3D models, 3D discriminative visual elements. Then, we explain how to cast the problem of matching 3D discriminative visual elements to a query image as a classification problem.

The proposed method has two stages: first, in an offline stage we learn a set of 3D discriminative visual elements representing the architectural site; second, in an online stage a given unseen query painting is aligned with the 3D model by matching with the learnt visual elements. The algorithm is detailed in figure 4.2 and in section 4.3.

The input to the offline stage is a 3D model of an architectural site. The output is a set of view-dependent visual element detectors able to identify specific structures of the 3D model in various types of 2D imagery. The approach begins by rendering a set of representative views of the 3D model. Next, a set of visual element detectors is computed from the rendered views by identifying scene parts that are discriminative and can be reliably detected over a range of viewpoints. During the online stage, given an input 2D depiction, we match the learnt visual element detectors and use the top scoring detections to recover a coarse camera viewpoint.

To evaluate our alignment procedure, we introduce a database of paintings and sketches spanning several sites and perform a user study where human subjects are asked to judge the goodness of the output alignments. Our results are presented in section 4.4. We compare with several baseline methods, such as SIFT on rendered views, the coarse viewpoint retrieval step of [147], and Exemplar SVM [160], and show that our algorithm produces more valid alignments than the baselines. Moreover, we evaluate our matching step on the benchmark dataset of [84] and show improvement over local symmetry features [84] and several alternative matching criteria for our system.

4.2 3D discriminative visual elements

In this section, we present the main ideas behind our 3D model representation and its matching to 2D depictions. We first define more precisely 3D discriminative visual elements (section 4.2.1). We then present how we match them to test depictions by seeing the matching problem as a classification task (section 4.2.2).

4.2.1 Learning 3D discriminative visual elements

We define a *discriminative visual elements* of a 3D scene to be a mid-level patch that is rendered with respect to a given viewpoint from a 3D model with the following properties: (i) it is visually discriminative with respect to the rest of the "visual world" represented here by a generic set of randomly sampled patches, (ii) it is distinctive with respect to other patches in nearby views, and (iii) it can be reliably matched across nearby viewpoints. We employ modern representations and recent methods for discriminative learning of visual appearance, which have been successfully used in recent object recognition systems. Our method can be viewed as "multi-view geometry [83] meets part-based object recognition [61]" – here we wish to automatically discover the distinctive object parts for a large 3D site.

We discover discriminative visual elements by first sampling candidate mid-level patches across different rendered views of the 3D model. We cast the image matching problem as a classification task over appearance features with the candidate mid-level patch as a single positive example and a negative set consisting of a large set of "background" patches. Note that a similar idea has been used in learning per-exemplar distances [68] or per-exemplar support vector machine (SVM) classifiers [123] for object recognition and cross-domain image retrieval [160]. Here we apply per-exemplar learning for matching mid-level structures between images. For a candidate mid-level patch to be considered a discriminative visual element, we require that (i) it has a low training error when learning the matching classifier, and (ii) it is reliably detectable in nearby views via cross-validation.

The output for each discriminative visual element is a trained classifier. At run-time,

for an input painting, we run the set of trained classifiers in a sliding-window fashion across different scales. Detections with high responses are considered as putative correspondences with the 3D model, from which camera resectioning is performed.

4.2.2 Matching as classification

We formulate the matching problem as a classification task. Concretely, we want to match a given rectangular image patch q in a rendered view (represented by a descriptor such as HOG [49]) to its corresponding image patch in the painting, as illustrated in figure 4.3. Instead of finding the best match measured by the Euclidean distance between the descriptors, we train a linear classifier with q as a single positive example (with label $y_q = +1$) and a large number of negative examples x_i for i =1 to N (with labels $y_i = -1$). The matching is then performed by finding the patch x^* in the painting with the highest classification score

$$s(x) = w^\top x + b, \tag{4.1}$$

where w and b are the parameters of the linear classifier. Note that w denotes the normal vector to the decision hyper-plane and b is a scalar offset. Compared to the Euclidean distance, the classification score (4.1) measures a form of similarity, i.e. a higher classification score indicates higher similarity between x and q. In addition, the learnt w weights the components of x differently. This is in contrast to the standard Euclidean distance where all components of x have the same weight. Parameters w and b are obtained by minimizing a cost function of the following form

$$E\left(w, b\right) = L\left(1, w^T q + b\right) + \frac{1}{N}\sum_{i=1}^{N} L\left(-1, w^T x_i + b\right), \tag{4.2}$$

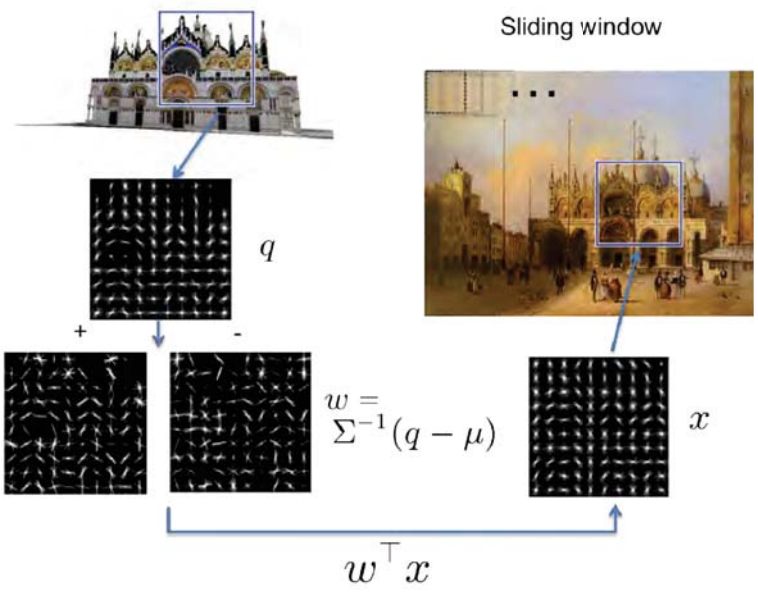


Figure 4.3: Matching as classification. *Given a region and its HOG descriptor q in a rendered view (top left) the aim is to find the corresponding region in a painting (top right). This is achieved by training a linear HOG-based sliding window classifier using q as a single positive example and a large number of negative data. The classifier weight vector w is visualized by separately showing the positive (+) and negative (-) weights at different orientations and spatial locations. The best match x in the painting is found as the maximum of the classification score.*

where the first term measures the loss L on the positive example q (also called "exemplar") and the second term measures the loss on the negative data. Note that for simplicity we ignore in (4.2) the regularization term $||w||^2$, but the regularizer can be easily added in a similar manner to [24, 73]. A particular case of the exemplar based classifier is the exemplar-SVM [123, 160], where the loss $L(y, s(x))$ between the label y and predicted score $s(x)$ is the hinge-loss $L(y, s(x)) = \max\{0, 1 - ys(x)\}$ [30]. For exemplar-SVM cost (4.2) is convex and can be minimized using iterative algorithms [59, 156].



Figure 4.4: Example sampled viewpoints. *Camera positions are sampled on the ground plane on a regular 100×100 grid. 24 camera orientations are used for each viewpoint. Cameras not viewing any portion of the 3D model are discarded. This procedure results in about 45,000 valid views for the depicted 3D model.*

4.3 Discriminative visual elements for painting-to-3D alignment

We seek to identify elements of the 3D model that are reliably detectable in arbitrary 2D depictions. As explained in section 4.2, we build on discriminative learning techniques to identify visually distinctive mid-level scene structures in rendered views of the 3D model . In this section, following the steps described in figure 4.2, we specify the design choices we made to solve our specific problem of matching a painting to the 3D model of an architectural site.

4.3.1 View selection and representation

The aim is to extract from the 3D model a set of view-dependent 2D descriptors suitable for alignment to 2D depictions. This is achieved by sampling representative views of the 3D model and learning visual element detectors from the rendered appearance in the sampled views. We sample possible views of the 3D model in a similar manner to [22, 90, 147]. First, we identify the ground plane and corresponding vertical direc-

tion. The camera positions are then sampled on the ground plane on a regular grid. For each camera position we sample 12 possible horizontal camera rotations assuming no in-plane rotation of the camera. For each horizontal rotation we sample 2 vertical rotations (pitch angles). Views where less than 5% of the pixels are occupied by the 3D model are discarded. This procedure results in 7,000-45,000 views depending on the size of the 3D site. Example sampled camera positions are shown in figure 4.4. Note that the rendered views form only an intermediate representation and can be discarded after visual element detectors are extracted. We render views from the 3D model by adapting the publicly available OpenGL code from [147] to work with our models. The renderer simply ray casts and samples colors from the textured models against a white background, and does not explicitly reason about illumination effects, such as shadows or specularities (although the textured models may implicitly include this information).

Each rendered view is represented by densely sampled patches at multiple scales, with each patch represented by a Histogram of Oriented Gradient (HOG) descriptor [49] (see section 2.3.1.2 for a review of HOG). We use the publicly available implementation of HOG from [61]. We only use the contrast insensitive portion of the HOG descriptor on a 10 x 10 grid of cells with 9 orientations within each cell, which results in an 900 dimensional descriptor. The HOG descriptor is forgiving to small drawing errors thanks to its spatial and orientation binning. In addition, we use a contrast insensitive HOG to enhance the capability of matching across different depiction styles.

4.3.2 Least squares model for visual element selection and matching

Every rendered view has thousands of potential visual elements and the task is to identify those that are distinct and hence likely to be detectable in different depictions. For example, a specific tower on the building may be distinctive for the site, whereas a patch in the middle of a gray wall may not. In the following, we show that using a least squares loss function, the classifier can be computed in closed-form without computationally expensive iterative training. In turn, this enables efficient training of candidate visual element detectors corresponding to image patches that are densely sampled in each rendered view. The quality of the trained detector (measured by the training error) is then used to select only the few candidate visual elements that are the most discriminative in each view (have the lowest training error). Finally, we show how the learnt visual elements are matched to the input painting, and relate the proposed approach to other recent work on closed-form training of HOG-based linear classifiers [73, 81].

4.3.2.1 Selection of discriminative visual elements via least squares regression

In section 4.2.2 we have assumed that the position and scale of the visual element q in the rendered view was given. As storing and matching all possible visual elements from all rendered views would be computationally prohibitive, the aim here is to automatically select a subset of the visual elements that are the most discriminative. First, we note that the optimal value of the cost (4.2) characterizes the separability of a particular candidate visual element q from the (fixed) negative examples $\{x_i\}$ and hence

can be used for measuring the degree of discriminability of q. However, when using a hinge-loss as in exemplar SVM, optimizing (4.2) would be expensive to perform for thousands of candidate elements in each rendered view. Instead, similarly to [24, 73], we take advantage of the fact that in the case of square loss $L(y, s(x)) = (y - s(x))^2$ the w_{LS} and b_{LS} minimizing (4.2) and the optimal cost E^*_{LS} can be obtained in closed form as

$$w_{LS} = \frac{2}{2 + \|\Phi(q)\|^2} \Sigma^{-1}(q - \mu), \tag{4.3}$$

$$b_{LS} = -\frac{1}{2}(q + \mu)^T w_{LS}, \tag{4.4}$$

$$E^*_{LS} = \frac{4}{2 + \|\Phi(q)\|^2}, \tag{4.5}$$

where $\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$ denotes the mean of the negative examples, $\Sigma = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)(x_i - \mu)^\top$ their covariance and

$$\|\Phi(q)\|^2 = (q - \mu)^\top \Sigma^{-1}(q - \mu), \tag{4.6}$$

the squared norm of q after the "whitening" transformation

$$\Phi(q) = \Sigma^{-\frac{1}{2}}(q - \mu). \tag{4.7}$$

We can use the value of the optimal cost (4.5) as a measure of the discriminability of a specific q. If the training cost (error) for a specific candidate visual element q is small the element is discriminative. If the training cost is large the candidate visual element q is not discriminative. This observation can be translated into a simple and efficient

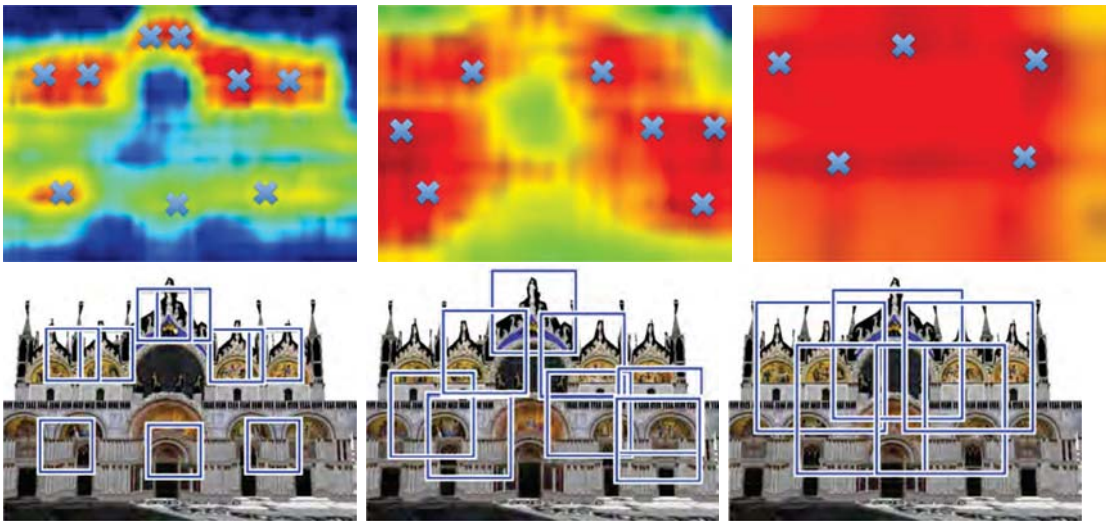


Figure 4.5: Selection of discriminative visual elements. *First row: discriminability scores shown as a heat-map for three different scales. Red indicates high discriminability. Blue indicates low discriminability. The discriminability is inversely proportional to the training cost of a classifier learnt from a patch at the particular image location. Second row: example visual elements at the local maxima of the discriminability scores. The corresponding local maxima are also indicated using "x" in the heat-maps above.*

algorithm for ranking candidate element detectors based on their discriminability. In practice, we evaluate the squared "whitened" norm $\|\Phi(q)\|^2$ of each candidate element q, which is inversely proportional to the training cost. If the whitened norm is high the candidate element is discriminative, if the whitened norm is low the candidate element is not discriminative. Given a rendered view, we consider as candidates visual element detectors of all patches that are local maxima (in scale and space) of the norm of their whitened HOG descriptor, $\|\Phi(q)\|^2$. Non-maximum suppression is performed using a threshold of 0.1 on the standard ratio of area intersection over union between two neighboring patches. Illustration of multi-scale discriminative visual element selection for an example rendered view is shown in figure 4.5.

4.3.2.2 Relation to linear discriminant analysis (LDA)

Recent works [73, 81] have shown that linear HOG-based object detectors computed analytically using linear discriminant analysis (LDA) can reach similar object detection accuracy as detectors learnt by expensive iterative SVM training. The distribution of positive and negative data points is assumed to be Gaussian, with mean vectors μ_p and μ_n, respectively. The covariance matrix $\Sigma_p = \Sigma_n = \Sigma$ is assumed to be the same for both positive and negative data. Under these Gaussian assumptions, the decision hyperplane can be obtained via a ratio test in closed form. Applying this approach to our image matching set-up, we estimate μ_n and Σ from a large set of HOG descriptors extracted from patches that are sampled from a set of ("negative") photographs independent from all sites considered in this work. μ_p is set to be a specific single HOG descriptor q of the particular positive example patch in the given rendered view. Parameters w_{LDA} and b_{LDA} of the linear classifier defining the matching score (4.1)

$$s_{LDA}(x) = w_{LDA}^T x + b_{LDA}, \tag{4.8}$$

can be obtained in closed form as

$$w_{LDA} = \Sigma^{-1}(q - \mu_n), \tag{4.9}$$

and

$$b_{LDA} = \frac{1}{2}\left(\mu^T \Sigma^{-1} \mu - q^T \Sigma^{-1} q\right). \tag{4.10}$$

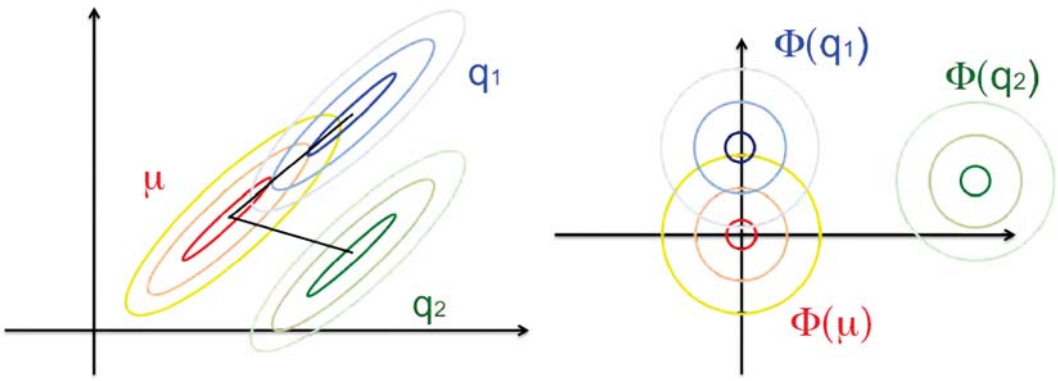


Figure 4.6: Selection of discriminative visual elements - interpretation using linear discriminant analysis. Left: *The negative data distribution (centered at μ) and two example positive data distributions (q_1 and q_2) are modeled as Gaussians with different means but the same covariance.* **Right:** *After "whitening", the negative data is centered at the origin with unit covariance. For fixed negative data, the classifier defined by q_2 is clearly more discriminative than the classifier defined by q_1, as measured by the overlap of the positive and negative data distributions. In the whitened space, this overlap can be measured by the Euclidean distance of the (whitened) mean of the positive data points from the origin. Note that in the original non-whitened space (left) the means of q_1 and q_2 are at the same distance from the mean of the negative data μ.*

Note that the matching score (4.8) can also be expressed using the whitening transformation defined in (4.7) as

$$s_{LDA}(x) \quad = \quad \Phi(q)^T \Phi(x) - \frac{1}{2}\|\Phi(q)\|^2, \tag{4.11}$$

where the first term is a dot-product between whitened q and x, and the second term is an additive normalization factor reducing the matching score for q vectors with large whitened norm. It is interesting to note that under the Gaussian assumptions of LDA, the squared whitened norm $\|\Phi(q)\|^2$ can be interpreted as the Bhattacharyya distance [98] measuring the "overlap" between the Gaussian representing the negative data and the Gaussian representing the positive example q. Discriminative visual elements q with large $\|\Phi(q)\|$ (as described in section 4.3.2.1) correspond to "unusual" examples far from the distribution of the negative data. This intuition is illustrated in figure 4.6.

4.3.2.3 Discussion

Classifiers obtained by minimizing the least squares cost function (4.2) or satisfying the LDA ratio test can be used for matching a candidate visual element q to a painting as described in equation (4.1). Note that the decision hyperplanes obtained from the least squares regression, w_{LS}, and linear discriminant analysis, w_{LDA}, are parallel. As a consequence, for a particular visual element q the ranking of matches according to the matching score (4.1) would be identical for the two methods. In other words, in an object detection set-up [49, 73, 81] the two methods would produce identical precision-recall curves. In our matching set-up, for a given q the best match in a particular painting would be identical for both methods. The actual value of the score, however, becomes important when comparing matching scores across different visual element detectors q. In object detection, the score of the learnt classifiers is typically calibrated on a held-out set of labeled validation examples [123].

4.3.3 Calibrated discriminative matching

We have found that calibration of matching scores across different visual elements is important for the quality of the final matching results. Below we describe a procedure to calibrate matching scores without the need of any labelled data. First, we found (section 4.4.4.3) that the matching score obtained from LDA produces significantly better matching results than matching via least squares regression. Nevertheless, we found that the raw uncalibrated LDA score favors low-contrast image regions, which have an almost zero HOG descriptor. To avoid this problem, we further calibrate the LDA score (4.8) by subtracting a term that measures the score of the visual element q

matched to a low-contrast region, represented by zero (empty) HOG vector

$$s_{calib}(x) = s_{LDA}(x) - s_{LDA}(0) \quad (4.12)$$

$$= (q-\mu)^T \Sigma^{-1} x. \quad (4.13)$$

This calibrated score gives much better results on the dataset of [84] as shown in section 4.4.4.3 and significantly improves matching results on our dataset of historical photographs and non-photographic depictions.

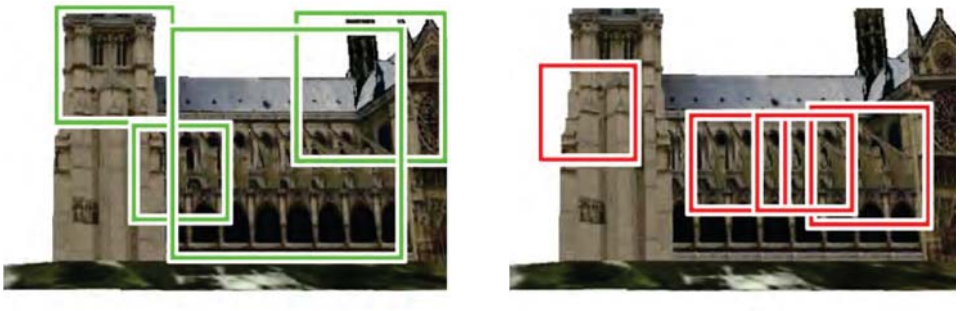
4.3.4 Filtering elements unstable across viewpoint

Here we wish to discard elements that cannot be reliably detected in close-by rendered views. This filtering criterion removes many unstable elements that are, for example, ambiguous because of repeated structures in the rendered view or cover large depth discontinuities and hence significantly change with viewpoint.

We define close-by views based on the visual overlap of imaged 3D structures rather than, for example, the distance between camera centers. In detail, to measure *visual overlap* between views V^1, V^2 we define the following score

$$S(V^1, V^2) = \frac{1}{|\mathcal{V}|} \sum_{\{x_i^1, x_i^2\} \in \mathcal{V}} e^{-\frac{(x_i^1 - x_i^2)^2}{2\sigma_x^2} - \frac{1}{2\sigma_d^2} \frac{(d(x_i^1) - d(x_i^2))^2}{\frac{1}{2}(d(x_i^1) + d(x_i^2))^2}}, \quad (4.14)$$

where $\{x_i^1, x_i^2\} \in \mathcal{V}$ is the set of corresponding points (pixels) in view V^1 and V^2, respectively, x_i^j is the location of pixel i in view j, $d(x_i^j)$ is the depth (distance to the 3D model) at pixel i in view j, and σ_x and σ_d are parameters. The first term in the exponent measures the squared image distance between the corresponding pixels.



(a) *Top 4 stable patches* **(b)** *Top 4 unstable patches*

Figure 4.7: Filtering elements unstable across viewpoint. *Examples of top stable (left) and unstable (right) visual elements detected in one of the rendered views. Unstable elements are typically detected on repeated structures or occlusion boundaries and are removed.*

The second term in the exponent measures the difference between the depths at the corresponding pixel locations normalized by their average depth. The per-pixel scores are then averaged over all corresponding pixels in the two views. The score is one if the two views are identical and zero if the two views have no visual overlap. In our case, two views are deemed "close-by" if their visual overlap score is greater than 0.4. Note that the score depends on camera positions as well as the 3D structure as it measures differences between projected 3D points in the image plane. As a result, the score is, for example, less sensitive to small camera translations if the camera is looking at a far away scene. We found that good values for the parameters are $\sigma_d = 0.3$ and $\sigma_x = (W + H)/10$, where W and H are, respectively, the width and height of the rendered views.

Here we evaluate if each candidate discriminative visual element obtained as described in section 4.3.2 is stable. Equipped with the above definition of nearby views, we test if each candidate visual element can be correctly detected in the set of nearby views using the ground truth locations of the candidate element obtained from the knowledge of the 3D model. In detail, we first select the near-by views in which the visual element is fully visible. Then, we attempt to localize the visual element in each view by applying

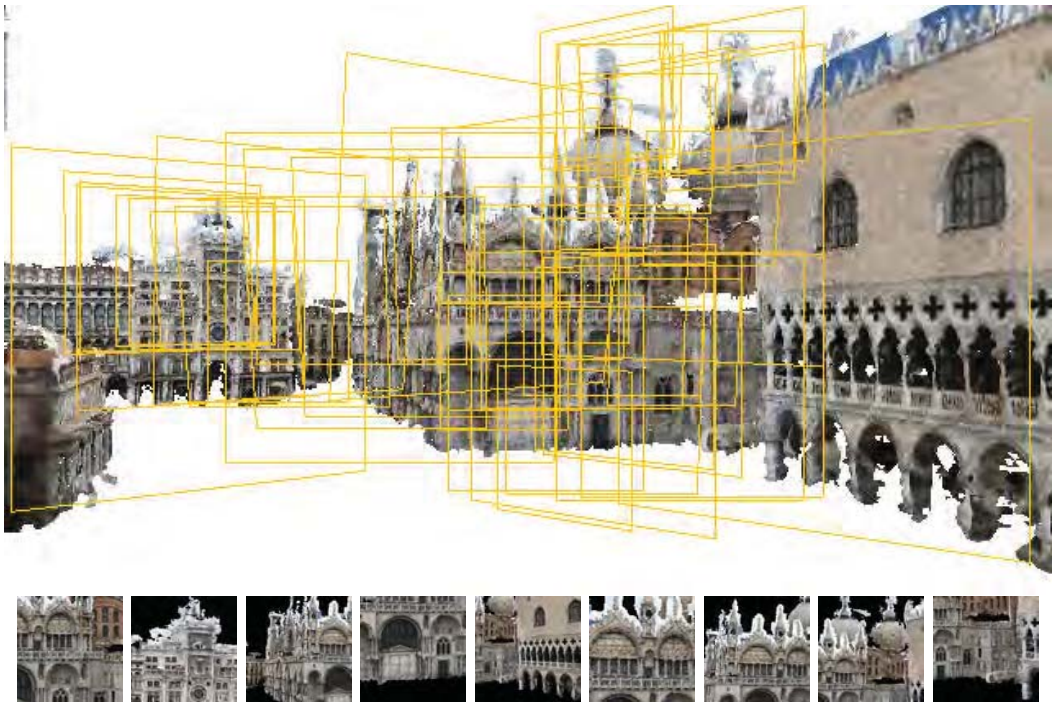


Figure 4.8: Examples of selected visual elements for a 3D site. Top: *Selection of top ranked 50 visual elements visible from this specific view of the site. Each element is depicted as a planar patch with an orientation of the plane parallel to the camera plane of its corresponding source view.* **Bottom:** *Subset of 9 elements shown from their original viewpoints. Note that the proposed algorithm prefers visually salient scene structures such as the two towers in the top-right or the building in the left part of the view. In contrast, some repetitive and non-salient scene structures in the right portion of the picture are ignored.*

the corresponding linear detector given by eq. (4.12) in a sliding window fashion. To suppress potential repeated structures, we require that the ratio between the score of the first and second highest scoring detection in the image is larger than a threshold of 1.04, similar to [121]. We keep visual elements that are successfully detected in more than 80% of the nearby views. Examples of stable and unstable visual elements are shown in figure 4.7.

This procedure typically results in several thousand selected elements for each architectural site. Examples of the final visual elements obtained by the proposed approach are shown in figure 4.8.

4.3.5 Robust matching

Since we wish to obtain matches that are both (i) non-ambiguous and (ii) have a high matching score we perform the following two step procedure to select candidates visual element matches for a given depiction. First, we apply all visual element detectors densely and at all scales on the depiction using the calibrated similarity score (4.12), perform non-max suppression and take the top 200 detections sorted according to the first to second nearest neighbor ratio [121]. This selects the most non-ambiguous matches. Second, we sort the 200 matches directly by score (4.12) and consider the top 25 matches to compute the camera viewpoint as described in section 4.3.6.

4.3.6 Recovering viewpoint

In this section we describe how, given the set of discriminative visual elements gleaned from the 3D model, to recover the viewpoint and intrinsic parameters of an input painting or historical photograph with respect to the 3D model. We assume that the paintings are perspective scene renderings and seek to recover the camera center and rotation via camera resectioning [83].

For detection, each discriminative visual element takes as input a 2D patch from the painting and returns as output a 3D location $\mathbf{X}$ on the 3D model, a plane representing the patch extent on the 3D model centered at $\mathbf{X}$, and a detector response score indicating the quality of the appearance match. Following the matching procedure described in section 4.3.3, we form a set of 25 putative discriminative visual element matches. From each putative visual element match we obtain 5 putative point correspondences by taking the 2D/3D locations of the patch center and its four corners. The patch corners provide information about the patch scale and the planar location

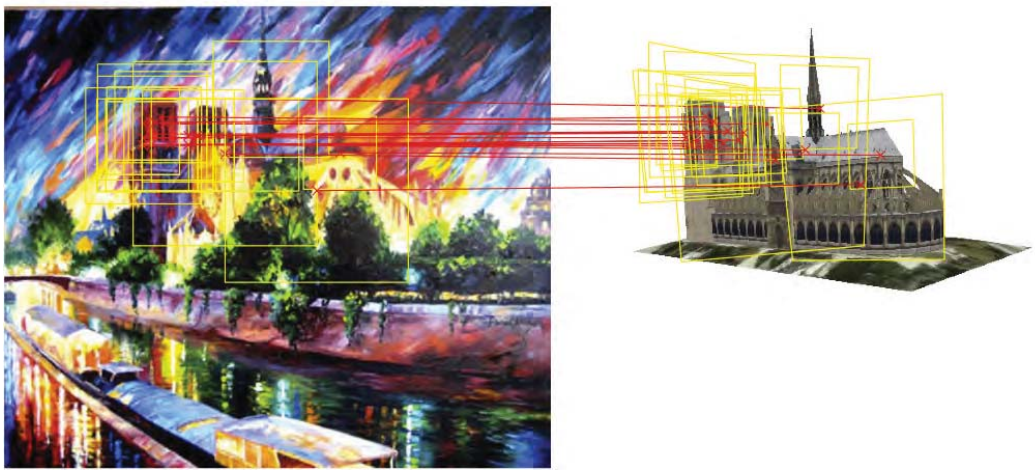


Figure 4.9: Illustration of coarse alignment. *We use the recovered discriminative visual elements to find correspondences between the input scene depiction and 3D model. Shown is the recovered viewpoint and inlier visual elements found via RANSAC. Notice that the visual elements yield inliers across the entire visible part of the site. Painting courtesy of Daniel Wall.*

on the 3D model, and have been shown to work well for structure-from-motion with planar constraints [170]. We use RANSAC [64] to find the set of inlier correspondences to a restricted camera model where the camera intrinsics are fixed to initial values, with the focal length set to the image diagonal length and the principal point set to the center of the image. We use a RANSAC inlier threshold set to 1.5% of the image diagonal length to recover the camera center and rotation. The recovered viewpoint forms a coarse alignment of the input depiction to the 3D model, which is shown in figure 4.9.

4.3.7 Summary

Candidate visual elements $\{q_i\}$ are obtained by finding local maxima of (4.6), which is inversely proportional to the least squares regression training error given by (4.5) as described in section 4.3.2.1. Visual elements that cannot be reliably detected in nearby viewpoint are then filtered out using the procedure described in section 4.3.4. The remaining visual elements are then matched to a painting using the two step matching

procedure described in section 4.3.5 that uses the calibrated LDA score (4.12). The most confident matches are then used to recover the approximate viewpoint of the artist with the algorithm described in section 4.3.6.

4.4 Results and validation

In this section, we first describe our dataset of non-photographic depictions and historical photographs in 4.4.1. We then provide qualitative (4.4.2) and quantitative (4.4.3) results of our full pipeline. Finally, we provide a detailed analysis of our algorithm in 4.4.4.

4.4.1 Dataset for painting-to-3D alignment

We have collected a set of human-generated 3D models from Trimble 3D Warehouse for the following architectural landmarks: Notre Dame of Paris, Trevi Fountain, and San Marco's Basilica. The Trimble 3D Warehouse models for these sites consist of basic primitive shapes and have a composite texture from a set of images. In addition to the Trimble 3D Warehouse models, we also consider a 3D model of San Marco's Square that was reconstructed from a set of photographs using dense multi-view stereo [69]. Note that while the latter 3D model has more accurate geometry than the Trimble 3D Warehouse models, it is also much noisier along the model boundaries.

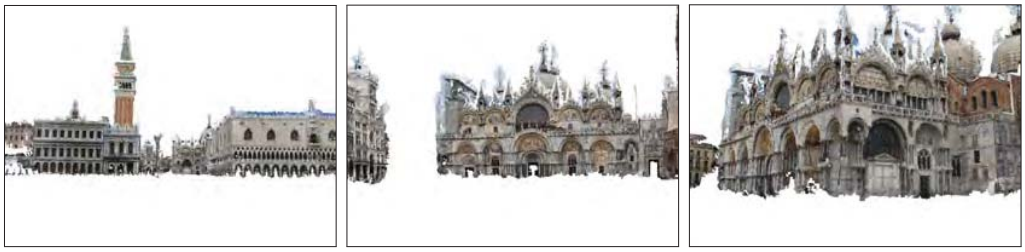
We have also collected from the Internet 85 historical photographs and 252 non-photographic depictions of the sites. We separated the non-photographic depictions into the following categories: 'drawings' (60 images), 'engravings' (45 images) and 'paintings' (147 images). The drawings category includes color renderings and the paintings category includes different rendering styles, such as watercolors, oil paint-

Table 4.1: *Statistics of our collected dataset of historical photographs and non-photographic depictions for the evaluated architectural landmarks. Note that the depictions of San Marco Basilica are also included in the set for San Marco Square, with the total (bottom row) counting the number of unique depictions in our dataset.*

	S. Marco Square	S. Marco Basilica	Trevi Fountain	Notre Dame	**Total**
Hist. photos	44	(30)	0	41	**85**
Paintings	61	(41)	34	52	**147**
Drawings	21	(19)	5	34	**60**
Engravings	15	(9)	10	20	**45**
Total	**141**	**(99)**	**49**	**147**	**337**



(a) *Input historical photographs.*



(b) *Aligned 3D models.*

Figure 4.10: *Alignment of historical photographs of San Marco's Square to their respective 3D models. Photographs courtesy of la Médiathèque de l'architecture et du patrimoine.*

ings, and pastels. Table 4.1 shows the number of images belonging to each category across the different sites.

4.4.2 Qualitative results

Figures 4.10 and 4.11 show example alignments of historical photographs and non-photographic depictions, respectively. Notice that the depictions are reasonably well-aligned, with regions on the 3D model rendered onto the corresponding location for a given depiction. We are able to cope with a variety of viewpoints with respect to the

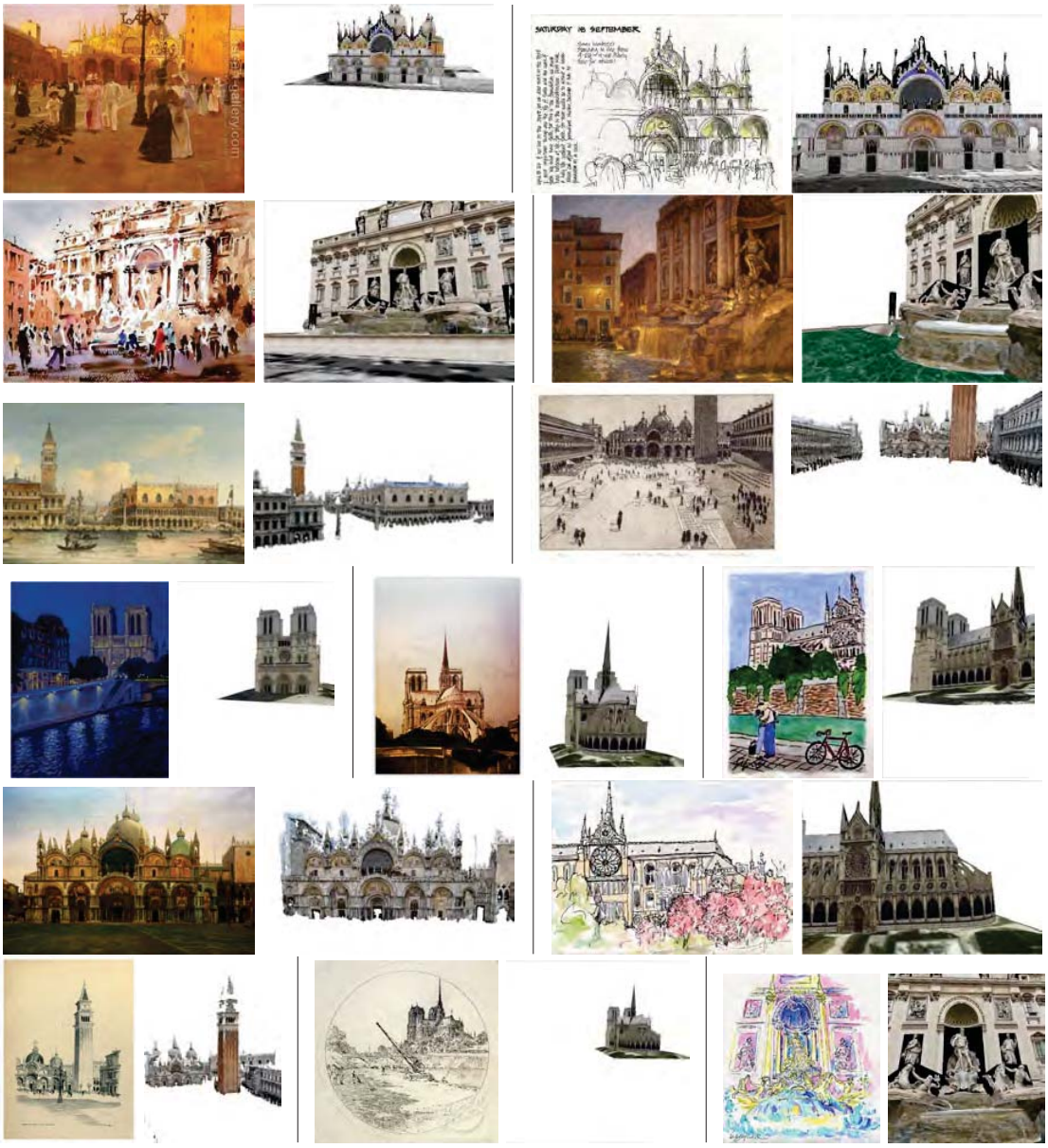
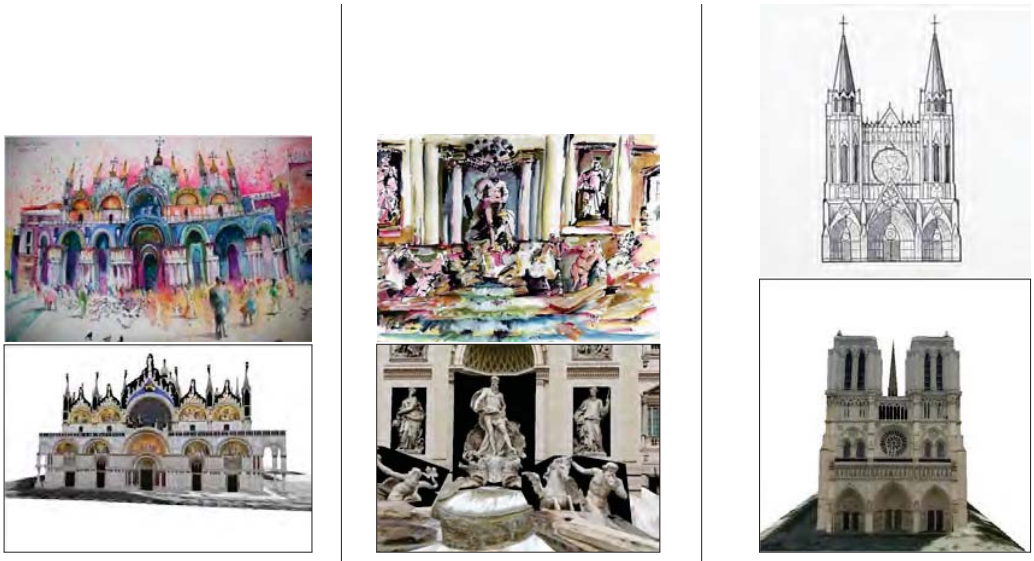


Figure 4.11: *Example alignments of non-photographic depictions to 3D models. Notice that we are able to align depictions rendered in different styles and having a variety of viewpoints with respect to the 3D models. Images courtesy of Liz Steel, Fifi Flowers, Woom Lam Ng, and Noreen Wessling.* ***More results are available at the project website*** *http://www.di.ens.fr/willow/research/painting_to_3d/*



(a) *Scene distortion.* **(b)** *Drawing and 3D errors.* **(c)** *Major structural differences.*

Figure 4.12: *Challenging examples successfully aligned by our method where the assumption of a perspective scene rendering is violated. Note that the drawing in (c) is a completely different cathedral. Image in (a) courtesy of Blythe Scott and in (b) courtesy of Ginette Callaway.*



(a) *Notre Dame of Paris.* **(b)** *Trevi Fountain.*

Figure 4.13: *Trimble 3D Warehouse models and camera frusta depicting the recovered viewpoints of the paintings.*

3D model as well as different depiction styles. Our approach succeeds in recovering the approximate viewpoint in spite of these challenging appearance changes and the varying quality of the 3D models. In figure 4.12 we show alignments to a set of challenging examples where the assumption of a perspective rendering is significantly violated, but the proposed approach was still able to recover a reasonable alignment. Notice the severe non-perspective scene distortions, drawing errors, and major architectural differences (e.g. a part of the landmark may take a completely different shape).

Figure 4.13 shows the camera frusta for the recovered approximate painting viewpoints.

Notice that our system is able to recover viewpoints that are to the rear of the main facade of the Notre Dame cathedral, which has not been possible in prior work [165] due to the lack of reconstructed structure in these areas. Recovering approximate camera viewpoints for paintings and historical photographs opens up the possibility of large-scale automatic computational re-photography for such depictions [25]. The video http://www.di.ens.fr/willow/research/painting_to_3d/data/ND.mp4 shows an example of a virtual tour of an architectural site transitioning between viewpoints of different images in 3D in a similar manner to [165], but here done for the challenging case of historical photographs, non-photographic depictions, and only an approximate 3D model from Trimble 3D Warehouse. Many architectural sites now have 3D models geo-located on a map, which, combined with the proposed approach, would enable geo-locating historical photographs and non-photographic depictions [160] for, e.g., navigation and exploration of non-photorealistic depictions (as shown in our video or, coarsely aligned manually, at http://www.whatwasthere.com) or *in situ* guided tours of historical imagery using mobile or wearable display devices.

A first step into this direction is to use the geo-localization of the reference 3D models to display the depictions on a 3D map, as shown in figure 4.14 where the paintings are visualized in Google Earth.

4.4.3 Quantitative evaluation

In the following we give details of the performed user-study to evaluate the quality of the alignments (section 4.4.3.1)), report the corresponding quantitative results across the 3D sites and depiction styles, and compare performance with several baseline methods (section 4.4.3.2). We also provide a quantitative evaluation of our geo-localization



(a) *Notre Dame*



(b) *San Marco's Basilica.*



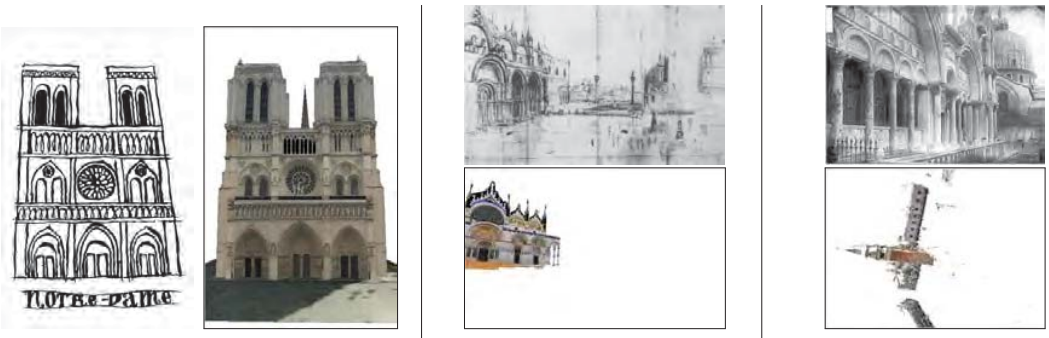
(c) *Trevi Fountain.*

Figure 4.14: *Examples of geo-localized depictions visualized in Google Earth. Note that the proposed method allows us to visualize the specific place across time and through the eyes of different artists.*

application (section 4.4.3.3).

4.4.3.1 Evaluating alignment

To quantitatively evaluate the goodness of our alignments, we have conducted a user study via Amazon Mechanical Turk. The workers were asked to judge the viewpoint similarity of the resulting alignments to their corresponding input depictions by categorizing the viewpoint similarity as either a (a) Good match, (b) Coarse match, or (c) No match, illustrated in figure 4.15. We asked five different workers to rate the viewpoint similarity for each depiction and we report the majority opinion.



(a) *Good match* **(b)** *Coarse match* **(c)** *No mathch*

Figure 4.15: Alignment evaluation criterion. *We asked workers on Amazon Mechanical Turk to judge the viewpoint similarity of the resulting alignment to the input depiction. The workers were asked to categorize the viewpoint similarity into one of three categories: (a) Good match – the two images show a roughly similar view of the building; (b) Coarse match – the view may not be similar, but the building is roughly at the same location in both images, not upside down, and corresponding building parts can be clearly identified; (c) No match – the views are completely different, e.g. upside down, little or no visual overlap. Image in (a) courtesy of Maral Sassouni and in (c) courtesy of la Médiathèque de l'architecture et du patrimoine.*

Table 4.2: *Viewpoint similarity user study of our algorithm across different sites.*

	Good match	Coarse match	No match
S. Marco Square	51%	21%	28%
S. Marco Basilica	45%	39%	15%
Trevi Fountain	55%	20%	24%
Notre Dame	65%	27%	9%
Average	**55%**	**27%**	**18%**

4.4.3.2 Alignment quality and baseline comparisons

Table 4.2 shows the performance of our algorithm for the different 3D sites considered in this work. As expected, the performance varies to some extent across the different models depending on their size, quality and the difficulty of the matching task. However, the failure (no match) rate remains consistently below 30%.

Table 4.3 shows the performance of our algorithm for different depiction styles averaged across the 3D sites. Interestingly, the results are fairly consistent across different depiction styles.

Finally, table 4.4 compares the performance of our algorithm to several baseline meth-

Table 4.3: *Viewpoint similarity user study of our algorithm across different depiction styles.*

	Good match	Coarse match	No match
Historical photographs	59%	20%	21%
Paintings	53%	30%	18%
Drawings	52%	29%	19%
Engravings	57%	26%	17%
Average	**55%**	**27%**	**18%**

Table 4.4: *Viewpoint similarity user study – comparison with baselines on the "San Marco Square" 3D site.*

	Good match	Coarse match	No match
SIFT on rendered views	40%	26%	33%
Viewpoint retrieval [147]	1%	39%	60%
Exemplar SVM [160]	34%	18%	48%
mid-level painting visual elements	33%	29%	38%
3D discrim. visual elements (ours)	**51%**	**21%**	**28%**

ods for the 141 depictions of San Marco Square – the largest 3D model in our dataset with 45K sampled viewpoints. We compare our algorithm against the following four baselines: (i) SIFT on rendered views, (ii) viewpoint retrieval (corresponding to the coarse alignment step of [147]), (iii) exemplar SVM [160], and (iv) mid-level painting visual elements that, similar to [163], learns mid-level visual elements directly from paintings, rather than the 3D model. The implementation details of each baseline are given next.

For the SIFT on rendered views baseline we extract and match SIFT descriptors computed at interest points across scale [121] over each input depiction and all rendered views. We use orientation sensitive descriptors as we found them to be more reliable than orientation invariant descriptors in practice. We perform geometric verification by finding inliers to an affine homography between the input depiction and each rendered viewpoint. Then, we take the rendered viewpoint with the most inliers and perform camera resectioning with RANSAC using the SIFT putative matches for that view. We

return as output a rendering of the final resectioned viewpoint. Note that the matching procedure is not standard since it is extracting descriptors from *rendered views*, which accounts for viewpoint changes. In other words, the SIFT matching step does not need to be viewpoint invariant as we are matching to a similar viewpoint from the rendered set. This baseline is similar in spirit to matching with Viewpoint Invariant Patches (VIP) [178], except no depth or rectification is needed for the paintings. This baseline performs reasonably well, having 40% good alignments compared with 51% for our algorithm. The good performance is largely due to alignments of historical photographs (70% vs. 50% for our method). However, if historical photographs are removed from the dataset, the SIFT on rendered views baseline drops to 27% good alignments, while our algorithm still achieves 52% good alignments.

The viewpoint retrieval baseline consists of matching a global Gist descriptor [133] extracted for each input depiction and all rendered views. The Gist descriptors are compared using L2 distance and the view corresponding to the minimum distance is returned. The Gist descriptor is sensitive to viewpoint, with the matching procedure corresponding to the coarse alignment step of [147]. Our method clearly outperforms the viewpoint retrieval baseline mainly because the sampled rendered views fail to cover the enormous space of all possible viewpoints. Matching the global image-level Gist descriptor would require much denser and wider sampling of views.

To reduce the viewpoint coverage issue, we explore as a baseline the exemplar-SVM approach of [160]. For this a single exemplar SVM detector is trained for each input depiction and is subsequently matched across all scales and 2D locations in sliding window fashion in the rendered views. While the performance improves over Gist matching, nonetheless the results remain limited since the approach cannot handle partial occlusions and significant deformations that are common in non-photorealistic depictions.

Moreover, the procedure is computationally expensive since an SVM detector is trained with hard negative mining for each input painting, with the resulting detector run in a sliding window fashion over all rendered views. In contrast, our approach learns offline a few thousand visual element detectors that compactly summarize an entire architectural site. At run time, only the learnt visual elements are applied to the input depiction.

To overcome the issues with partial occlusion and significant deformations, but keeping the idea of matching the input painting to the rendered views, we extract mid-level visual elements directly from the *input paintings* without any explicit knowledge of the 3D model. In detail, we extract 25 mid-level discriminative visual elements from each input painting using the approach presented in section 4.3.2.1. The painting visual elements are then matched in a sliding window fashion to all rendered views. For each rendered view inlier point correspondences are recovered via camera resectioning with RANSAC over the maximal detector responses. The resectioned view that yields the largest number of inliers is rendered and returned. Note that this baseline is similar in spirit to learning mid-level patches [55, 163] from the input paintings without the explicit knowledge of the 3D model. While this baseline further improves over exemplar-SVM (38% vs. 48% failures), it does not outperform our method mainly because it cannot combine visual element detections from multiple views available to our method via the 3D model. Similar to the exemplar-SVM, an additional drawback of this baseline is the high computational cost as visual elements from each painting must be run densely across all rendered views.

Table 4.5: *The percentage of input depictions that were assigned to the correct architecture site split across different sites (rows) and depiction styles (columns). Note that there are no historical photographs for Trevi Fountain in the database.*

	Paintings	Historical photograph	Engravings	Drawings	**Average**
S. Marco Basilica	83%	87%	89%	94%	**87%**
Trevi Fountain	82%	-	90%	80%	**84%**
Notre Dame	90%	88%	85%	79%	**86%**
Average	**86 %**	**87%**	**87%**	**84%**	**86%**

4.4.3.3 Geo-localization

We explored the potential of our method for geo-localization of historical and non-photographic depictions. We summarized the three Trimble Warehouse models with 15,000 discriminative visual elements each. For each input depiction, we applied all of the 45,000 detectors corresponding to those elements, selected the 25 most confident ones, and performed camera resectioning using RANSAC as described in section 4.3.6, with the constraint that only elements from the same site could be counted as inliers. Thus, our output is both a specific 3D model and a viewpoint.

We report our results on the task of identifying the 3D model of the architectural site. Table 4.5 shows the results separately for the three different sites and across different depiction styles. Despite the difficulty of the task due to the large variety of viewpoints and styles, our method identified correctly the architectural site for 86% of the depictions, which is much larger than the 33% chance performance, showing the potential of our method for geo-localization.



(a) *Original rendering* **(b)** *Drawing style* **(c)** *Painting style*

Figure 4.16: Viewpoint rendering styles. *We explored the possibility of rendering viewpoints from the 3D model in different styles by applying style filters within Photoshop CS4 to the rendered views.*

Table 4.6: *Evaluation of different 3D model rendering styles.*

	Good match	Coarse match	No match
Drawing style	61%	12%	27%
Painting style	54%	18%	28%
Original rendering	**65%**	**27%**	**9 %**

4.4.4 Algorithm analysis

In this section we evaluate variants of the three steps of our algorithm: viewpoint rendering style, visual element selection, and visual element matching. Finally, we show and analyze the main failure modes.

4.4.4.1 Viewpoint rendering style

Since our goal is to align a 3D model to non-photorealistic depictions, we explored the possibility of applying different rendering styles during the viewpoint rendering step of our algorithm. We applied the 'watercolor' and 'accentuated edges' style filters from Photoshop CS4 to our rendered views to generate, respectively, a 'painting like' and a 'drawing like' style. Example filter outputs are shown in figure 4.16. We quantitatively evaluate the output of our full system (using the style filters during rendering) on 147 depictions of the Notre Dame site via a user study on Amazon Mechanical

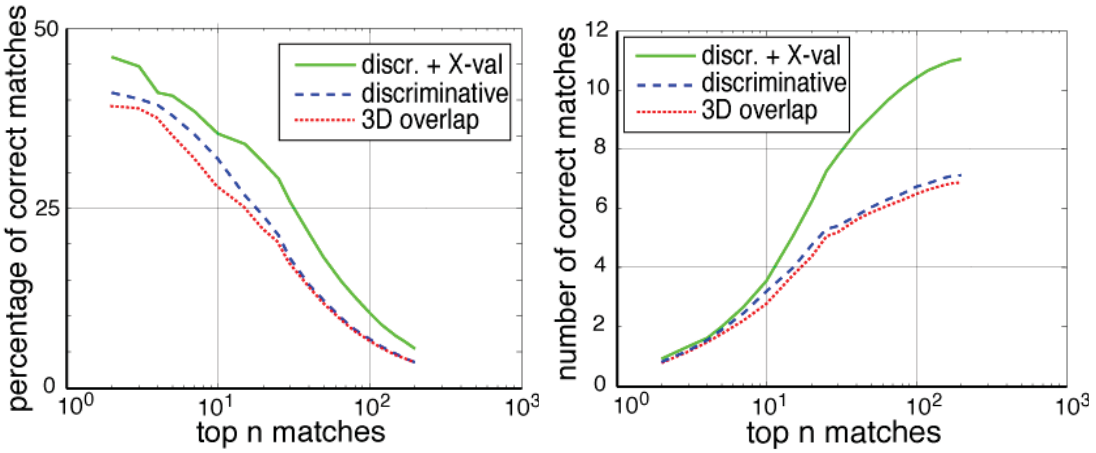


Figure 4.17: Evaluation of visual element selection. *The average percentage (left) and number (right) of correct matches as a function of the top n matches. See text for details.*

Turk. Results are summarized in table 4.6. Both styles result in a decrease of the overall matching performance compared to the original rendering. However, when results are split by depiction (not reported in table 4.6) the drawing style results in a small increase of matching performance on drawings (68% good matches vs. 62% good matches with the original rendering). While this difference amounts to only 3 additional matched depictions, it opens-up the possibility of learning a vocabulary of visual elements specific for each rendering style.

4.4.4.2 Visual element selection

Here we evaluate benefits of the proposed discriminative visual element selection. To measure the improvement in the quality of the selected visual elements we compute the percentage of correct matches (inliers). We consider only the San Marco square 3D model and the ground truth is obtained by visual inspection of the resulting alignments – only correct matches from the good and ok alignments are considered as ground truth inliers. The percentage of inliers gives a finer indication of the quality of visual elements than the overall percentage of correct alignments measured in the

Table 4.7: Evaluation of visual element matching. *We report the mean average precision on the "desceval" task from the benchmark dataset of [84].*

Matching method	mAP ("desceval")
Local symmetry [84]	0.58
Least squares regression (Sec. 4.3.2.1)	0.52
LDA (Sec. 4.3.2.2)	0.60
Ours (Sec. 4.3.3)	**0.77**

previous section as RANSAC will often find the correct alignment even from very few correct candidate correspondences. Results are summarized in figure 4.17. Here 10K discriminative visual elements were learnt from 45K sampled views. We compare three methods for selecting visual elements from the set of rendered views: The "3D overlap" (red) method selects visual elements that significantly overlap the 3D model in rendered views, i.e. where at least 50% of the HOG support is occupied by the 3D model. 10K visual elements are then chosen randomly out of all visual elements that satisfy the 3D model overlap criterion. The "discriminative" (blue) method uses the discriminative selection (section 4.3.2.1), but no cross-validation. The "discr. + X-val" (green) uses the proposed discriminative visual element selection (section 4.3.2.1) with cross-validation (section 4.3.3). For example, inspecting figure 4.17(a) reveals that within the top 10 matches there are 27.9% of correct matches for the 3D overlap method, 31.9% for the discriminative selection, and 35.4% for the discriminative selection with cross-validation. This demonstrates that visual elements selected by the proposed method are more likely to be correctly recovered in the painting.

4.4.4.3 Visual element matching

We evaluate the proposed matching procedure on the 'desceval' task from the benchmark dataset collected in [84]. The benchmark consists of challenging imagery, such as historical photographs and non-photographic depictions of architectural landmarks.

Pairs of images in the dataset depicting a similar viewpoint of the same landmark have been registered by fitting a homography to manual point correspondences. The task is to find corresponding patches in each image pair. Since the ground truth correspondence between points is assumed known via the homography, a precision-recall curve can be computed for each image pair. We report the mean average precision (mAP) measured over all image pairs in the dataset.

Following [84] we perform matching over a grid of points in the two views, with the grid having 25 pixel spacing. In table 4.7 we report the mAP for different visual element matching methods for our system, along with the local symmetry feature baseline of [84]. Our full system using the calibrated matching score (section 4.3.3) achieves a mAP of 0.77, which significantly outperforms both the alternative visual element matching scores obtained by least squares regression (section 4.3.2.1) and linear discriminant analysis (LDA, section 4.3.2.2), as well as the local symmetry feature baseline.

4.4.4.4 Failure modes

We have identified three main failure modes of our algorithm, examples of which are shown in figure 5.14. The first is due to large-scale symmetries, for example when the front and side facade of a building are very similar. This problem is difficult to resolve with only local reasoning. For example, the proposed cross-validation step removes repetitive structures visible in the same view but not at different locations of the site. The second failure mode is due to locally confusing image structures, for example, the vertical support structures on the cathedral in figure 5.14 (middle) are locally similar (by their HOG descriptor) to the vertical pencil strokes on the drawing. The learnt mid-level visual elements have a larger support than typical local invariant features (such

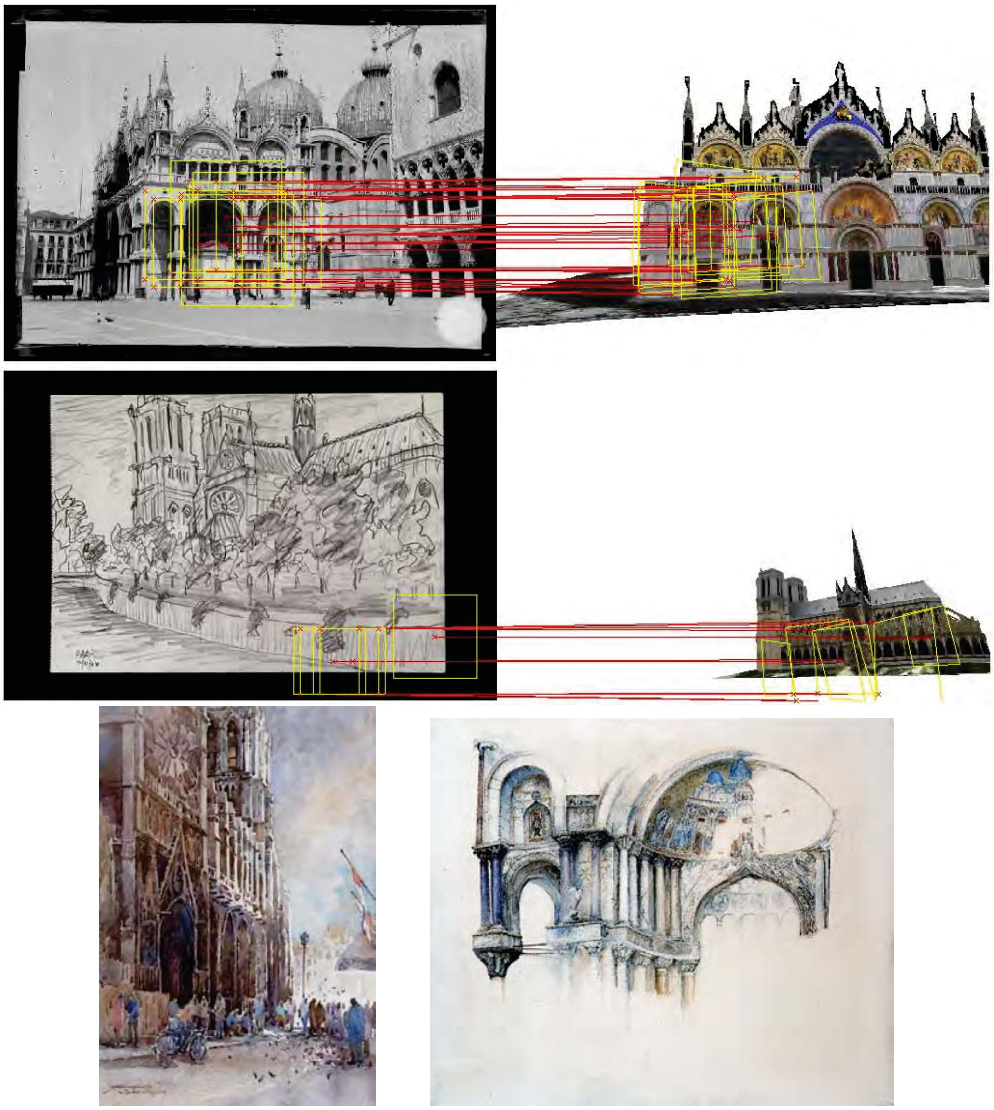


Figure 4.18: Example failure cases. *Top: large scale symmetry. Here arches are incorrectly matched on a building with similar front and the side facades. Middle: locally confusing image structures. Here the vertical support structures on the cathedral (right) are locally similar by their HOG descriptor to the vertical pencil strokes on the drawing (left). Bottom: Two examples of paintings with unusual viewpoints. Image in the top row courtesy of la Médiathèque de l'architecture et du patrimoine, in the middle row courtesy of Norman Ziff and in the bottom row left courtesy of Woom Lam Ng.*

as SIFT) and hence are typically more distinctive. Nevertheless, such mismatches can occur and in some cases are geometrically consistent with a certain view of the 3D model. The third failure mode is when the viewpoint depicted in the painting is not covered in the set of sampled views. This can happen for unusual viewpoints including extreme angles, large close-ups, or cropped views.

4.4.4.5 Computational cost

The most computationally demanding part of our algorithm is the selection of discriminative visual elements, which can be done offline. Our basic rendering engine outputs between 10 to 80 views per minute depending on the 3D model, but modern GPU implementations are capable of much faster rendering speeds. Additionally, it is possible to render the views on-demand only, without ever storing them, which could significantly reduce the storage requirements, specially for large sites. In our Matlab implementation, the visual element selection learning time is dominated by cross-validation. Overall, the algorithm is able to learn about 2,000 elements per hour using 20 cores on a cluster. Note that after the offline learning only the learnt visual elements need to be stored. Each element is represented by an 800-dimensional weight vector, together with the 3D location, scale and orientation of the corresponding planar patch. During the online detection stage, matching 10,000 visual elements to a 450x360 image takes about 22 minutes. The final camera resectioning takes about 25 seconds. Both timings are on a single 4-cores machine with our Matlab implementation.

4.5 Conclusion

We have demonstrated that automatic image to 3D model alignment is possible for a range of non-photographic depictions and historical photographs, which represent extremely challenging cases for current local feature matching methods. To achieve this we have developed an approach to compactly represent a 3D model of an architectural site by a set of visually distinct mid-level scene elements extracted from rendered views, and have shown that they can be reliably matched to a variety of photographic and non-photographic depictions. We have also shown an application of the proposed approach

to computational re-photography to automatically find an approximate viewpoint of historical photographs and paintings. This work is just a step towards computational reasoning about the content of non-photographic depictions. The developed approach for extracting visual elements opens-up the possibility of efficient indexing for visual search of paintings and historical photographs (e.g. via hashing of the HOG features as in [51]), or automatic fitting of complex non-perspective models used in historical imagery [141].

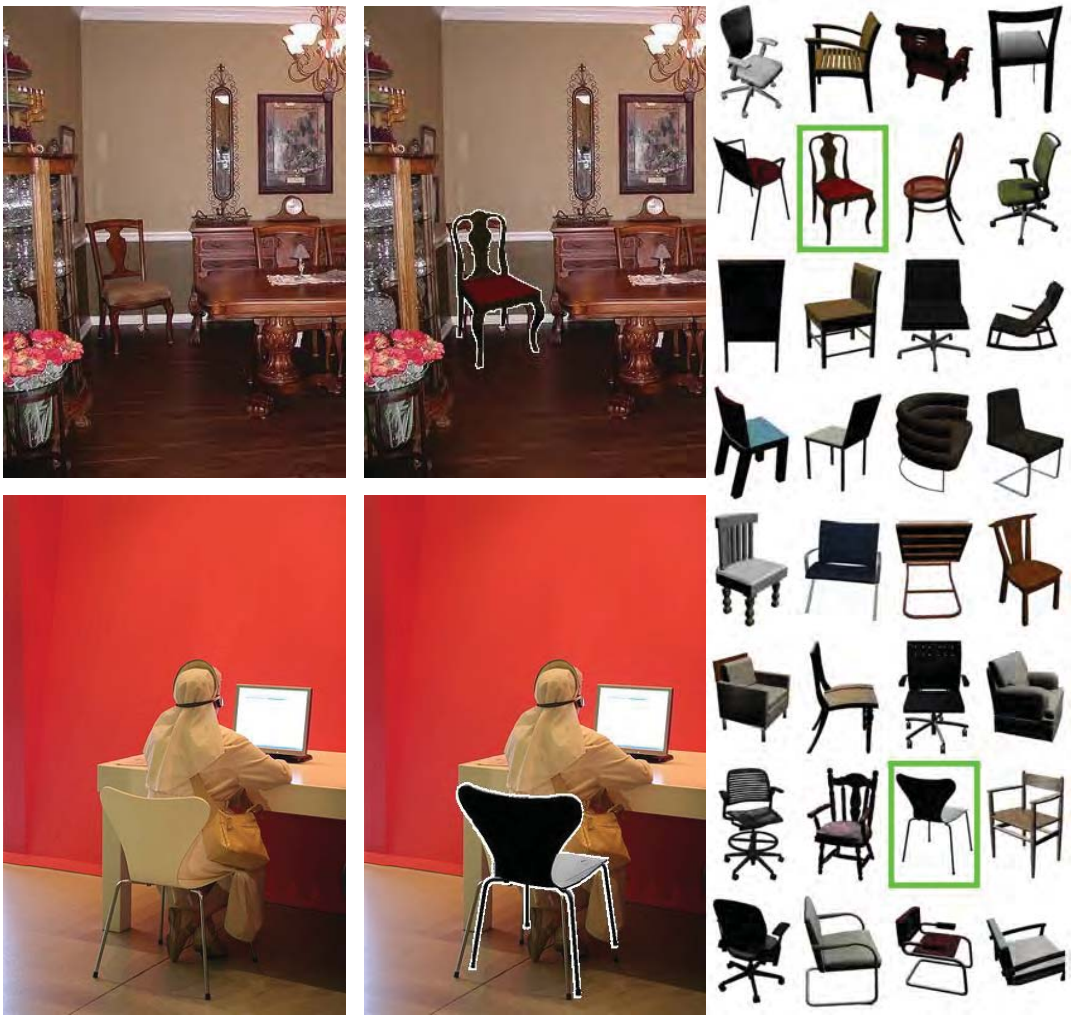
Chapter 5

Seeing 3D Chairs

5.1 Introduction

In chapter 3 we have designed a 3D shape descriptor adapted to 3D instance alignment. In chapter 4, we have introduced a representation of 3D models that enabled matching with approximate 2D perspective rendering of the model. In this chapter we want to go beyond instance-level alignment and perform 2D-to-3D category-level alignment. To achieve this goal, we build on the notion of discriminative visual elements introduced in chapter 4.

We describe an object category by utilizing large quantities of 3D CAD models that have been made publicly available online. Using the "chair" class as a running example, we propose an exemplar-based 3D category representation, which can explicitly model chairs of different styles as well as the large variation in viewpoint. We develop an approach to establish part-based correspondences between 3D CAD models and real photographs. This is achieved by (i) representing each 3D model using a set of view-dependent mid-level visual elements learned from synthesized views in a discriminative fashion, (ii) carefully calibrating the individual element detectors on a common dataset



(a) *Input images* **(b)** *Aligned output* **(c)** *3D chair models*

Figure 5.1: *Given an input image (left), our algorithm searches a database of 1,393 3D chair models to detect all chairs depicted in the image. The algorithm returns a 3D model matching the style of the chair and recovers its viewpoint relative to the camera (outlined in green, right). We overlay a projection of the returned 3D model onto the input image (middle). Notice the agreement of the returned model with the depicted chair style and pose.*

of negative images, and (iii) matching visual elements to the test image allowing for small mutual deformations but preserving the viewpoint and style constraints. We demonstrate the ability of our system to align 3D models with 2D objects in the challenging PASCAL VOC images, which depict a wide variety of chairs in complex scenes. Example results are shown in figure 5.1.

5.1.1 Motivation

From its very beginnings [144] and up until the early nineties [129], object recognition research has been heavily geometry-centric. The central tenet of the time was *alignment*[1], and the act of recognition was posed as correctly aligning a 3D model of an object with its 2D depiction in the test image [89, 122]. The parameters recovered during alignment (object pose, object scale, etc.) served as the output of the recognition process, to be used, for instance, in the perception-manipulation loop in robotics applications. Unfortunately, the success of these 3D model-based methods was largely limited to instance recognition tasks for objects with well-pronounced rectilinear structures (e.g. staplers were a favorite example). As the field moved toward category recognition and objects with more complex appearance, 3D model-based object recognition has been replaced by the new 2D appearance-based methods (e.g. [49, 61, 174]). These methods forgo 3D and operate directly on the 2D image plane (see section 2.3). Thus, instead of a 3D model of an object, they use a large dataset of 2D views of the object class from different viewpoints, as the model. These methods have shown steadily improving performance on a number of challenging tasks, such as the PASCAL VOC dataset [58]. However, their main drawback is that the result of a successful recognition is typically just the name of the object that was found (e.g. "chair") and a bounding box to indicate its rough location within the image. While this type of result is reasonable for tasks such as retrieval (e.g. "find all chairs in this dataset"), it is rather unsatisfying for doing any deeper reasoning about the scene (e.g. "what's the pose of the chair?", "can I sit on it?", "what is this chair occluding?", "how can I fold this chair?", etc). All these questions could be trivially answered, if only we had a 3D

[1]Indeed, one of the oft-told stories is that when a student asked Takeo Kanade what are the three most important problems in computer vision, his reply was: "Alignment, alignment, alignment!".

model of the chair aligned with the image!

The work presented in this chapter aims to combine some of the benefits of the 3D model-based instance alignment methods with the modern, appearance-based object category tools towards getting a best-of-both-worlds object recognition engine. The idea is to use a large library of textured 3D object models that have become publicly available on the Internet to implicitly represent both the 3D shape of the object class, as well as its view-dependent 2D appearance. Our approach can be considered as a marriage between part-based discriminative models [61] and exemplar-based matching [123]. Like part-based models, we represent objects using a set of connected appearance parts. But, like exemplar-based methods, we avoid explicitly training an object model by relying on a large dataset of object instances that serve as their own model, both in 2D as well as 3D.

We picked the "chair" category as the running example in this paper because: 1) it is very hard even for the recent methods [61], achieving only 0.13–0.20 average precision (AP) on PASCAL VOC [1]; 2) it is a category well-represented in the publically-available 3D model collections (e.g. Google/Trimble 3D Warehouse), 3) chairs have huge intra-class variation – whereas there are perhaps only hundreds of types of cars ever made, there are thousands of different types of chairs!

5.1.2 From instance-level to category-level alignment

This work is part of an emerging trend towards reclaiming some of the early successes in 3D recognition, and combining them with modern visual recognition tools.

Most approaches tackled the problem by extending existing approaches in image-based recognition to incorporate explicitly a simplified 3D model of the category (see section 2.3.2)

Here we take another approach and extend the 3D alignment method introduced in the previous chapter to category alignment. We do so using a data driven approach on a much larger scale than previous works that were typically limited to a few dozens of 3D instances. This way we avoid modeling the correspondences between the different instances or specifying a general 3D model for the category.

5.1.3 Approach Overview

Our representation consists of a large set of 3D CAD models, which captures both the large variety of chair styles and their different possible viewpoints. Chair detection in new images is accomplished by finding an alignment between the 2D chair and the most similar 3D chair model rendered at the most appropriate camera viewpoint, as shown in Figure 5.1. Aligning photographed 2D objects with the most similar (but not identical) computer-generated 3D model is a very hard problem. Here we address it by representing the collection of all 3D models by more than 800,000 calibrated view-dependent mid-level visual elements learned in a discriminative fashion from rendered views.

At test time, all the learned visual elements are applied to the test image in parallel. The most spatially and appearance-wise consistent alignment is found, while preserving the style and viewpoint-consistency constraints. The details of the algorithm are described in Section 5.2.

Contributions. Posing object *category* detection in images as a 3D *instance* alignment problem, we: 1) develop an exemplar-based 3D category representation capturing variations across object style and viewpoint; and 2) establish the correspondence between computer-generated 3D models and 2D photographs using a large collection of mid-level visual elements.

5.2 Discriminative visual elements for category-level 3D-2D alignment

Explicitly representing and synthesizing the fine-grained style and viewpoint of the 3D object category significantly simplifies the difficult task of 2D-to-3D alignment. Nevertheless, reliably matching a synthesized view of an object to a real photograph is still challenging due to differences in, e.g., texture, materials, color, illumination or geometry. Furthermore, it is well known that computer generated images have different statistical properties than real photographs. To address these issues, in a similar manner as in chapter 4, we cast the matching problem as a classification task, and represent the collection of 3D models using a large set of *mid-level visual elements* – linear classifiers over HOG features learnt from the rendered views in a discriminative fashion (section 5.2.1). As each of the hundreds of thousands of visual elements is learnt individually, calibrating their matching scores becomes a critical issue. We address this by learning a linear calibrating function for each element on a common dataset of negative images (section 5.2.2). Finally, we wish to be tolerant to small geometric deformations (such as a chair with longer legs or shorter armrests). We develop a matching procedure (section 5.2.3) that allows for small deformations in the spatial configurations of the matched visual elements while preserving consistent viewpoint and style.

5.2.1 Representing a 3D shape collection

Similar to section 4.3.1, we need to specify valid views to represent the 3D models. Here, our input is not a single 3D model but a large collection of 3D models. Each of



Figure 5.2: *We represent the "chair" category by a set of 3D models of chairs of different styles $t \in \{1, ..., T\}$ with $T = 1394$. The figure shows a small set of these 3D models, giving a sense of their variability.*

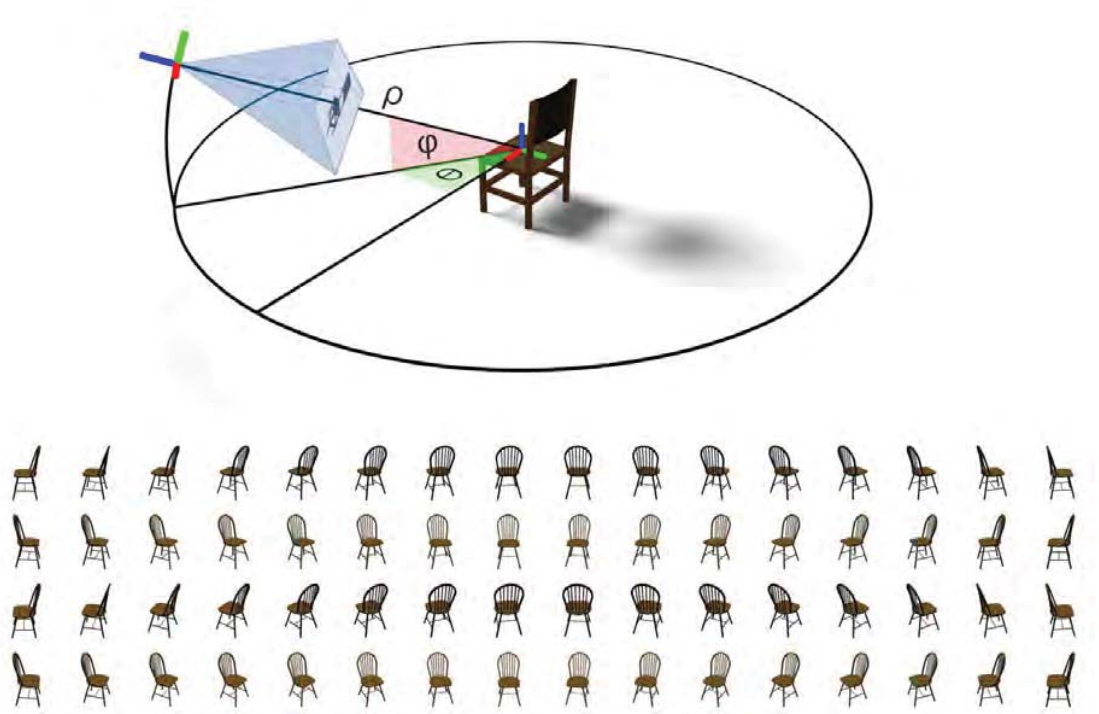


Figure 5.3: *Each 3D model is represented by a set of rendered views. We render the 3D models from 62 viewpoints sampled on a sphere with two different pitch angles ϕ and 31 different azimuth angles θ. This leads to a set of valid views covering most of the usual chair viewpoints. These views are indexed by $o \in \{1, ...O\}$ with $O = 62$.*

these 3D model corresponds to a different style $t \in \{1, ..., T\}$ with $T = 1394$ (see figure 5.2 for a sample of our 3D models and section 5.3.1 for details).

As shown figure 5.3, each 3D chair model was rendered on a white background from

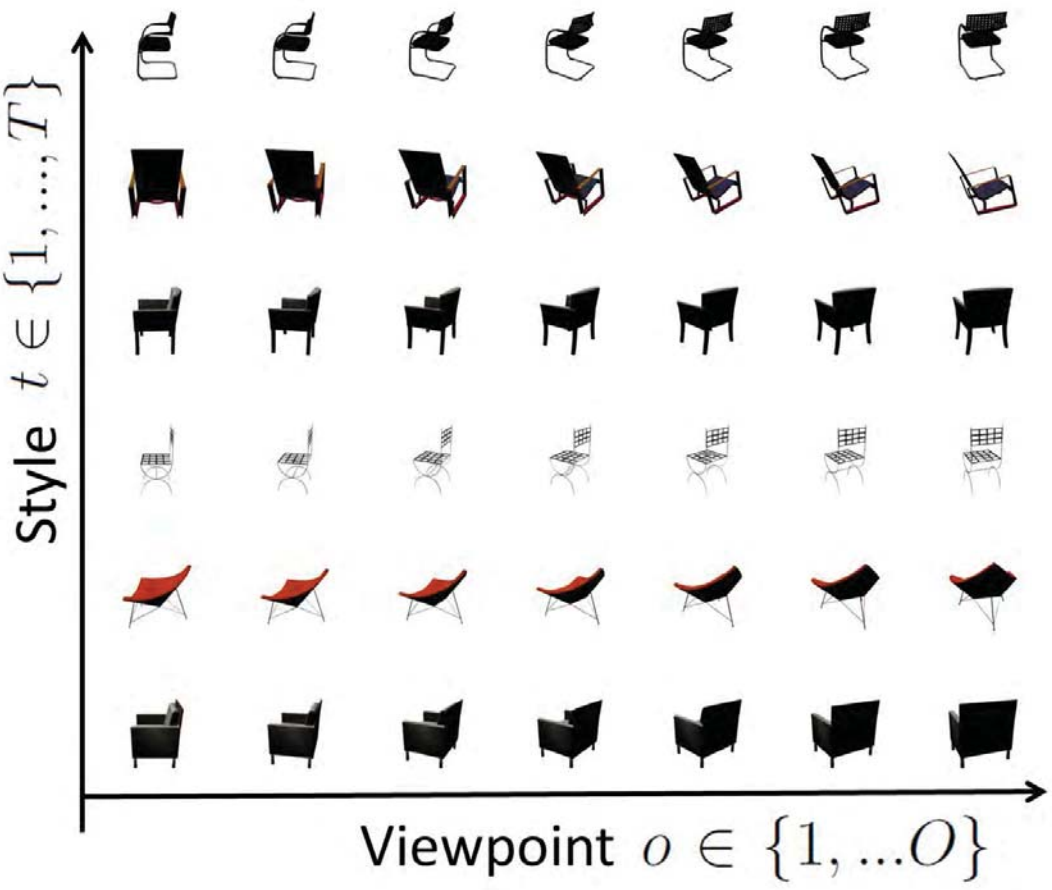


Figure 5.4: *We represent the "chair" category by a set of rendered views covering different styles* $t \in \{1, ..., T\}$ *and viewpoints* $o \in \{1, ...O\}$.

31 different azimuth angles and 2 different pitch angles, leading to a set of 62 views sampled over the upper half of the viewing sphere centered on the chair. Each image is annotated with the chair ID, $t \in \{1, ..., T\}$, indicating the different style, as well as the viewing orientation indexed by $o \in \{1, ...O\}$. This set of rendered views corresponds to a representation of the "chair" category (see figure 5.4).

Following the ideas developed in section 4.2 for instance-level alignment, we represent each view by a set of visual elements. Similar to section 4.3.2, we select for each view the most discriminative patches that minimize the least square classification error. The patches are also represented by a 10x10 contrast invariant HOG descriptor, and the

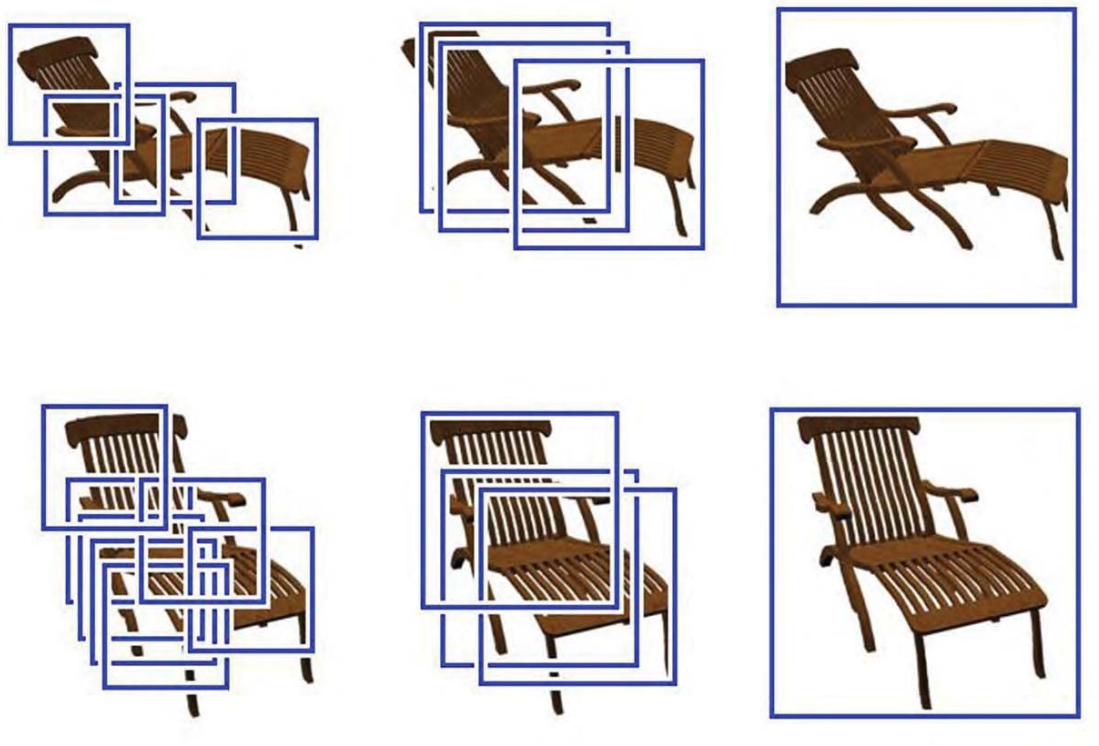


Figure 5.5: *Maxima of the whitened HOG norm at several scales for two viewpoints of the same chair. Those maxima are used to select the set of the $K = 10$ most discriminative visual elements representing each rendered view.*

selection can be performed efficiently by maximizing the whitened HOG norm (see equation 4.6). Example of patches maximizing this norm at several scales are visualized in figure 5.5. As we aim at detecting chairs from each viewpoint, there is no reason to assert the stability of the visual elements across viewpoint, and we simply select the $K = 10$ most discriminative visual elements for each view. The detectors corresponding to each element $k \in \{1, ..., K\}$ are computed using equation 4.8 and calibrated using the method described in section 5.2.2. Thus, the 3D category is finally represented by the discriminative elements of each chair style and each viewpoint.

A chair in a test image is detected by aggregating the calibrated scores of different visual elements detectors. The calibration (section 5.2.2) and detection (section 5.2.3) procedures are described next.

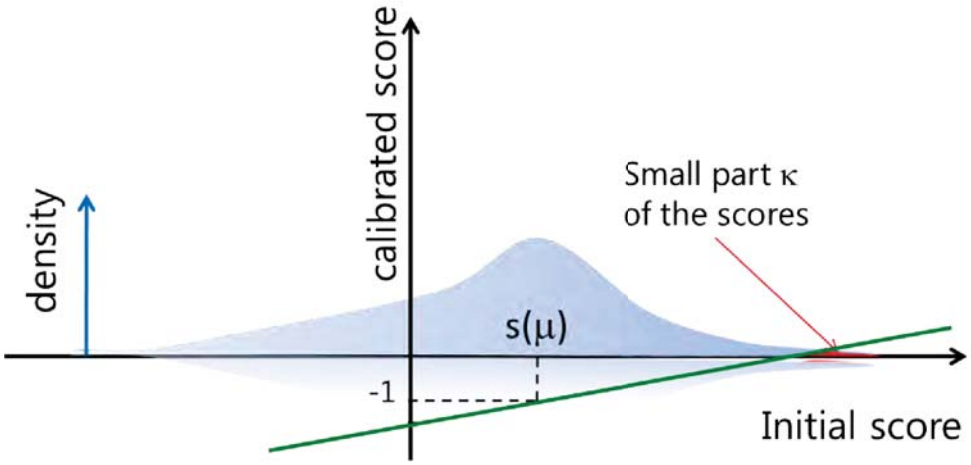


Figure 5.6: *Calibration of the different visual elements detectors. The blue curve represent the distribution of the scores for a given classifier, and the green line the corresponding affine calibration. A small part $\kappa = 0.01\%$ of the scores shown in red is associated to positive calibrated scores.*

5.2.2 Calibrating visual element detectors

As noted in prior work [123] and in section 4.3.3, calibration of matching scores across different visual element detectors is important for the quality of the final detection outputs. As we learn more than 800K element detectors independently, we found careful calibration of their scores to be critical. We address this issue by running all detectors on a common large dataset of 200K negative patches that do not contain the object of interest. Similar to chapter 4, we represent each visual element using a linear classifier, which scores each candidate match x as:

$$S_{t,o,k}(x) = w_{t,o,k}^T x \tag{5.1}$$

where o is the viewpoint, t the model, k the discriminative element ID, and

$$w_{t,o,k} = \Sigma^{-1}(q_{t,o,k} - \mu) \tag{5.2}$$

where $q_{t,o,k}$ is the HOG of the discriminative element, Σ is the covariance of the HOG distribution and μ its mean. This detector is calibrated in an affine way:

$$S^{calib}_{t,o,k}(x) = a_{t,o,k}\, S_{t,o,k}(x) + b_{t,o,k}, \tag{5.3}$$

where for each visual element detector $S_{t,o,k}$ corresponding to the patch k of the viewpoint o of the model t, we seek to find the scalars $a_{t,o,k}$ and $b_{t,o,k}$.

We recover the calibration parameters with respect to two operating points. To select the first operating point, we run the visual element detector on 200K patches that are randomly sampled from the negative image set, which is known to not contain any chairs. We select as operating point the negative patch x_n that yields a false positive rate of κ. In our experiments, we used $\kappa = 0.01\%$, i.e., x_n is the patch having the 99.99 percentile detection score. We choose as the second operating point μ_n the mean HOG feature vector. Given these two points, we set $S^{calib}_{t,o,k}(x_n) = 0$ and $S^{calib}_{t,o,k}(\mu_n) = -1$. This calibration leads to the expected false positive rate of 0.01% when $S^{calib}_{t,o,k}(x) = 0$. We found this to be a good compromise between representing the tail of the score distribution and the amount of time to scan the negative data. The calibration procedure is illustrated figure 5.6 and the influence of the calibration parameter is analyzed in section 5.3.4.2.

5.2.3 Matching spatial configurations of visual elements

For the final object detection, we wish to enforce a consistent spatial layout of the visual element detections corresponding to a rendered 3D view. We assume a star model for the spatial layout, similar in spirit to [61]. The star model provides spatial constraints, while allowing for small spatial deformations. Moreover, the star model

can be run in a sliding-window fashion, enabling detection of multiple object instances in the scene.

More concretely, for all of the visual elements detectors from a single rendered 3D view we compute a dense response map of the calibrated score (5.3) across different position and scales of the 2D test image. For each visual element and each position in the test image, we then replace the response by the maximum response in a 3×3 neighborhood corresponding to the HOG cells at the same scale. This can be seen as a max pooling of the calibrated matching scores which transforms $S^{calib}_{t,o,k}$ into $S^{max}_{t,o,k}$:

$$S^{max}_{t,o,k}(x,y,s) = \max_{i \in \{x-1,x,x+1\}} \max_{j \in \{y-1,y,y+1\}} S^{calib}_{t,o,k}(i,j,s) \tag{5.4}$$

where x, y and s are the coordinate and scale of the potential detection and i and j cover the 3×3 neighborhood of (x,y). The final score $S^{det}(x,y,s)$ for a detection at position (x,y) with scale s is the maximum among all views and chair models of the sum of the positive scores of the different visual element matches:

$$S^{det}(x,y,s) = \max_{t \in \{1,..T\}} \max_{o \in \{1,..,O\}} \sum_{k=1}^{K} \max\left(0, S^{max}_{t,o,k}(x_{t,o,k}, y_{t,o,k}, s_{t,o,k})\right) \tag{5.5}$$

where $(x_{t,o,k}, y_{t,o,k}, s_{t,o,k})$ are the position and scale corresponding to element (t,o,k) for a chair at position (x,y) and scale s and K is the number of discriminative elements per rendered view. Note that we require all visual element detections to come from the same synthesized view, which provides a strong constraint on the viewpoint and style consistency of the detected chair. We found this matching procedure to work well, though the view and style consistency constraints can be potentially relaxed to accumulate matches across multiple close-by views or models with a similar style. This part-based matching between a rendered view and an image is visualized figure 5.7

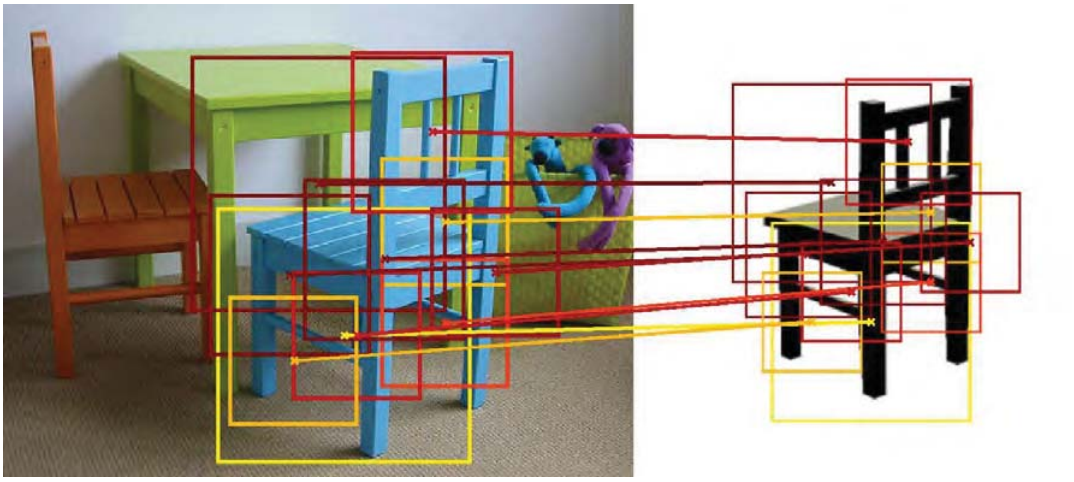


Figure 5.7: *Example of chair detection performed by our method. Each discriminative part of a rendered view (right) is matched to the image and the scores are aggregated. Warmer color represent more confident part correspondences.*

Given a match we deduce a candidate bounding box by adding 10% to the bounding box of the aligned rendered chair. We perform non-max suppression on the resulting bounding boxes in the following manner. Starting from the most confident detection in the image we (i) remove all other detections that overlap this detection with more than 0.5 area ratio overlap but only (ii) downweight (not remove) the remaining detections with non-zero overlap. This procedure is then iterated starting from the next remaining most confident detection. We found this strategy removes well overlapping false positives while preserving highly confident (and correct) close-by detections (e.g. chairs around a table). This motivates a system that would reason about the entire scene jointly.

5.3 Experiments and results

In this section we introduce our new dataset of 3D chairs (section 5.3.1), show qualitative output alignment results of our system (section 5.3.2) and quantitatively evaluate our approach on images from the challenging PASCAL VOC dataset (section 5.3.3).

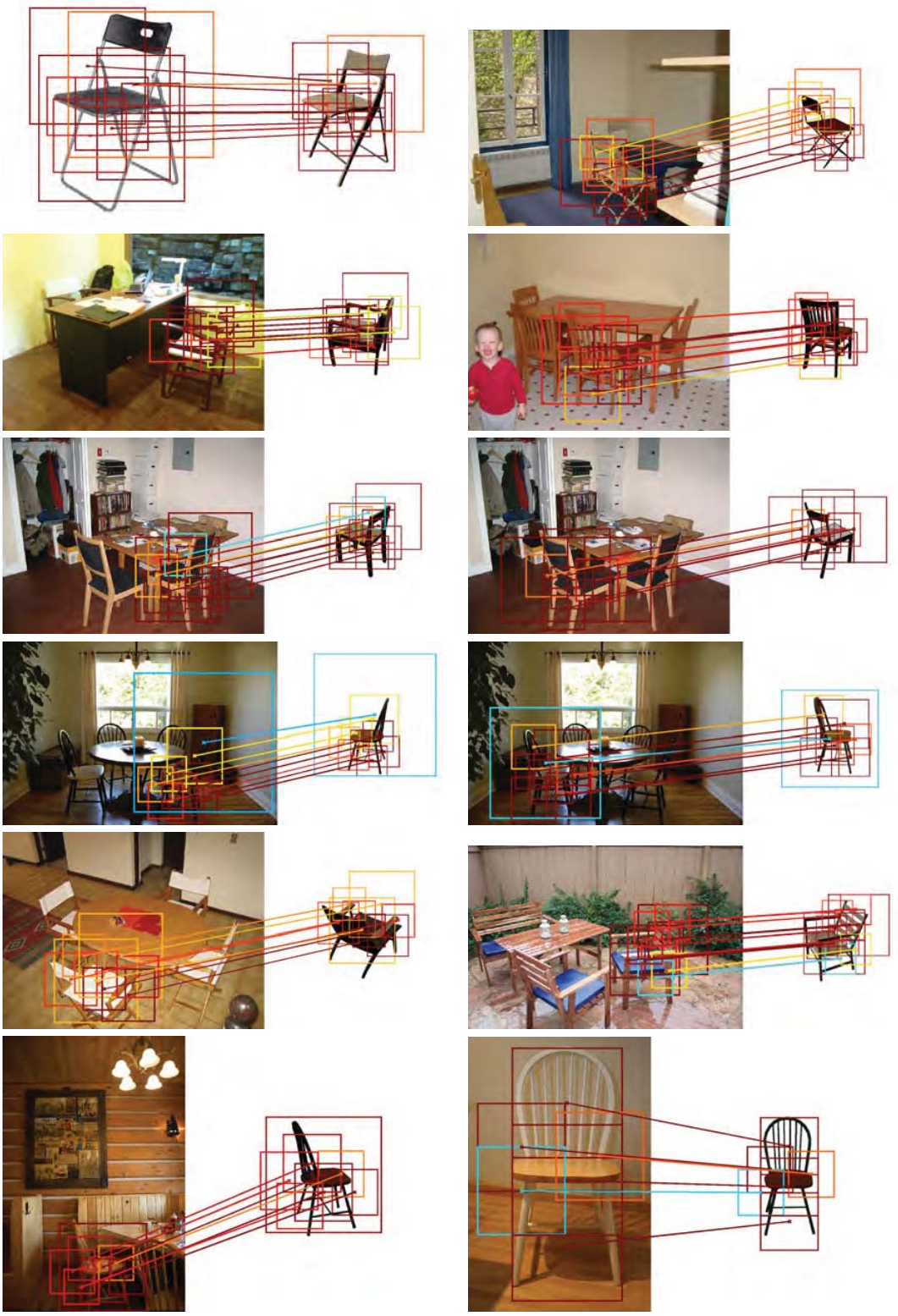


Figure 5.8: *Our output alignments. Parts are colored based on their detection confidence (warmer colors are more confident).* **Please see additional results on the project webpage [4].**

We also evaluate the sensitivity of our system to several key parameters (section 5.3.4).

5.3.1 Large dataset of 3D chairs

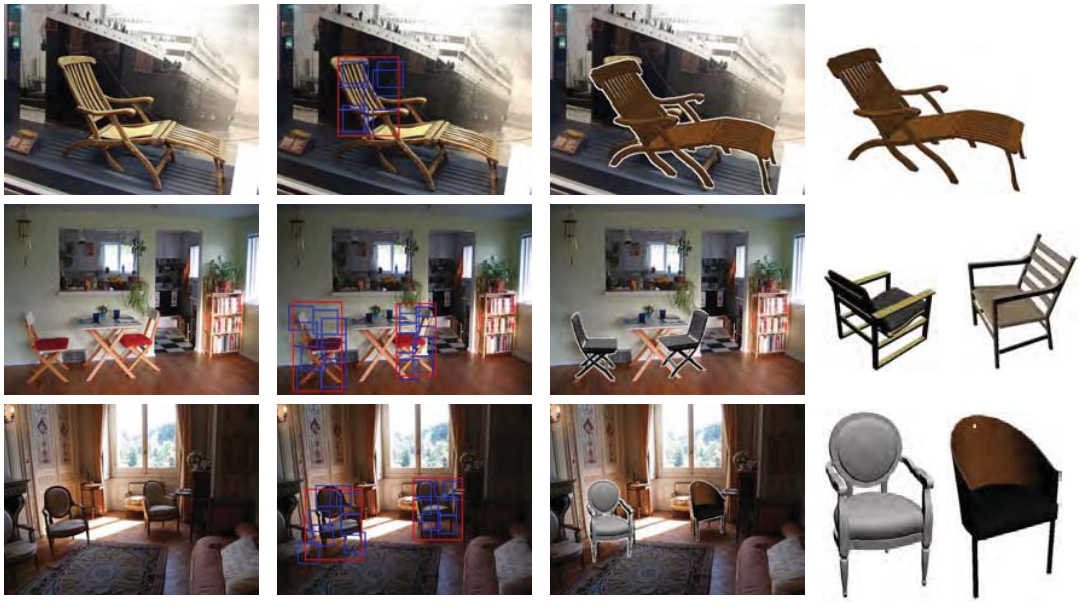
We explicitly represent the shape variation of an object category by a large collection of 3D CAD models. The 3D models used in this work have been downloaded from the Google/Trimble 3D Warehouse, an on-line repository of publicly available, user-contributed 3D graphics content created using Google SketchUp. We initially searched the repository for "chair" and downloaded over two thousand 3D models. However, many of them were not of good visual quality; some weren't even chairs. After manually culling the data, we were left with $1,393$ high-quality 3D chair models, representing a variety of chair styles.

This dataset is our non-parametric representation of the object category explicitly representing the large variety of chair styles. It is publicly available on the project webpage [4].

5.3.2 Qualitative results

In Figure 5.8 we show example output alignments of our algorithm. Notice that our algorithm can detect many different styles of chairs in different poses. For many cases, the predicted chair matches closely the input depicted chair style and pose. In many other cases a similar style is returned, often retrieving an accurate partial match to the depicted chair. Moreover, our approach shows some robustness to background clutter and partial occlusion and cropping.

In Figure 5.9 we compare the output of our algorithm with the Deformable Parts Model (DPM) [61]. While the DPM correctly predicts the 2D location of the depicted chairs, along with the 2D locations of its parts, our algorithm produces a more informative



(a) *Input images* **(b)** *DPM [61] output* **(c)** *Aligned outputs* **(d)** *3D models*

Figure 5.9: *Comparison of our algorithm output with the deformable parts model (DPM) [61]. While the DPM correctly predicts the 2D location of the depicted chairs, along with the 2D location of its parts, our algorithm is able to predict the 3D pose and style of the chair.*



Figure 5.10: *For each image (left), we show the most confident matches (right). Even if we do not have an exact match in our dataset, our algorithm returns reasonable matches.*

result. The aligned 3D chair pose and style allows for true 3D reasoning about the input scene.

For a given chair detection, often there is a set of related, highly confident 3D chair alignments having the same pose and similar style. We visualize these similar chair alignments in Figure 5.10. Notice that the styles are similar, often sharing one or more 3D parts. This suggests that when there is not an exact style match in the database a composite representation could be used to explain the entire input chair by composing well-matching 3D parts from different 3D chairs. Results for the entire dataset are available on the project webpage [4].

5.3.3 Quantitative evaluation

We evaluate the detection accuracy of our algorithm on the PASCAL VOC 2012 dataset [1]. We report detection precision-recall on images marked as non-occluded, non-truncated, and not-difficult in the chairs validation set. While this is an easier set compared to the full validation set, nonetheless it is very challenging due to the large intraclass variation, chair poses, and background clutter. Note that removing these difficult examples nonetheless yields some partially-occluded and truncated chairs, as seen in Figure 5.8. The resulting set contains 179 images with 247 annotated chairs.

In Figure 5.11 we report full precision-recall curves for our algorithm and compare it against two baselines: (i) DPM [61] and (ii) a root template detector using the LDA version of Exemplar-SVM [123]. We train the root template exemplar detector using the whitened HOG formulation described in section 4.3.2.1 assuming a single root template that covers the entire 3D chair for the given viewpoint. We calibrate the template as described in Section 5.2.2. During detection we run the template in a sliding-window fashion across the input image.

Our approach achieves an average precision (AP) of 0.339 on this task. The DPM and root template exemplar detector baselines achieve AP 0.410 and 0.055, respectively.

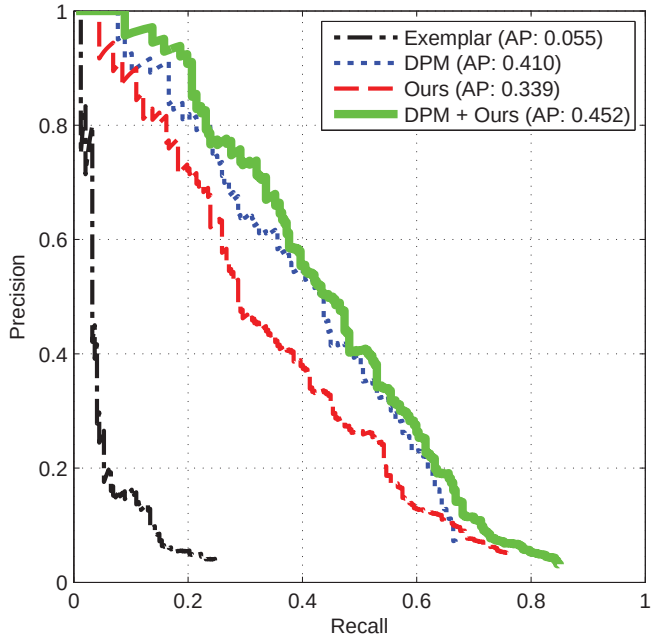


Figure 5.11: *Precision-recall curves on PASCAL VOC 2012.*

	Alignment		Style		
	Good	Bad	Good	Ok	Bad
Exemplar-LDA	52%	48%	3%	31%	66%
Ours	90%	10%	21%	64%	15%

Table 5.1: *Results of user study to evaluate the goodness of the alignment and style recovery. The data used in the user study appears on the project webpage [4].*

Our performance is noteworthy as it does not use any of the PASCAL VOC training images. We investigated combining our algorithm with DPM for the detection task. For this we estimated an affine transformation for the DPM scores to calibrate it in the range of our returned scores. For overlapping detected windows for the two methods, we give one twice the confidence and discard the other. Combining our approach with DPM yields an AP of 0.452, which significantly outperforms the DPM baseline.

We performed a user study to evaluate the quality of the output alignment and returned chair style. For correct detections at 25% recall, users were asked to label the alignment as "Good" (the returned alignment has very similar pose as the depicted chair) or "Bad" (the alignment is incorrect) and to label the returned chair style as "Good" (the returned chair style is an accurate match), "Ok" (part of the returned chair matches

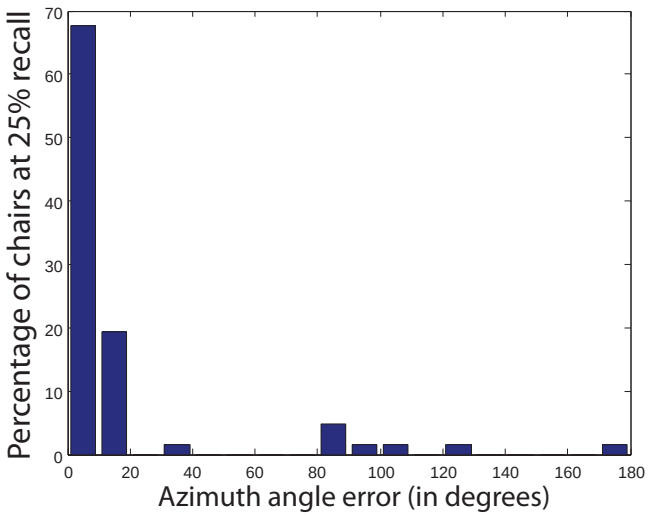


Figure 5.12: *Azimuth angle error.*

the depicted chair style), or "Bad" (no style match).

We report the results of the user study evaluating the quality of the returned alignments and chair styles in Table 5.1. We compare against the root template exemplar baseline and outperform it on both tasks. The fact that the number of exact matches is only 3% for the baseline exemplar detector suggests that there are just too many variations within the chair category. This also motivates using a part-based approach since we are able to obtain high quality partial matches, allowing us to find a close-by chair, whereas the root template detector (Exemplar-LDA baseline) must match the entire chair at once. We found that it was somewhat difficult to judge the returned styles from the exemplar-LDA baseline since the matches were not as good.

Finally, we quantitatively evaluated the accuracy of the estimated chair orientation. For this, we manually annotated the azimuth angle for the same set of detections as used in the user study. Our algorithm returns an azimuth angle within 20° of the ground truth for 87% of the examples. The complete distribution of the orientation errors is shown in figure 5.12.

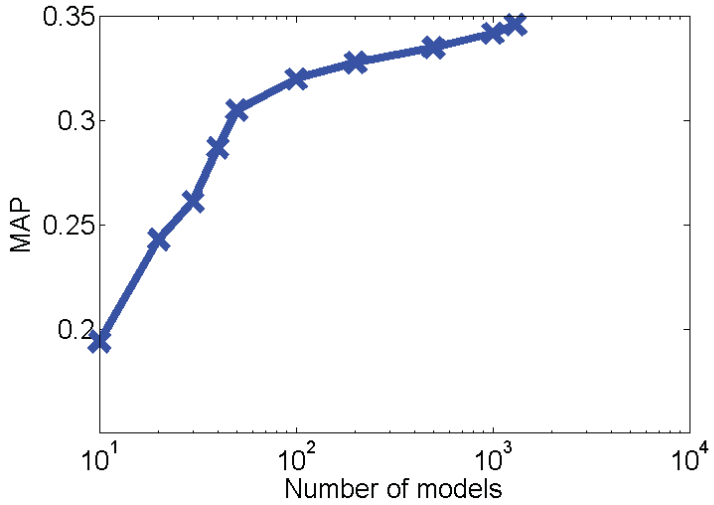


Figure 5.13: *Detection performance of our algorithm as a function of the number of 3D models used to describe the category.*

5.3.4 Algorithm analysis

In this section, we evaluate the influence of several key parameters of our algorithm, namely the number of 3D models (section 5.3.4.1) and the calibration parameter κ (section 5.3.4.2). We also evaluate the closed-form calibration procedure presented in the previous chapter section 4.3.3 (section 5.3.4.2) and the benefit of using discriminative elements instead of elements chosen on a grid (section 5.3.4.3). Finally, we discuss the main failure cases of our method (section 5.3.4.4) and discuss its computational cost (section 5.3.4.5). Except when specified otherwise, all the 1392 3D models are used, the calibration parameter is $\kappa = 10^{-4}$ and the views are used in their original rendering (600×600 pixel images where the size of the chair may vary) and the minimum size of a discriminative element is 100×100 pixels.

5.3.4.1 Number of 3D models

Having a big collection of 3D models is important to have a good representation of an object class. We performed the same detection experiment as the one described in

Table 5.2: *AP evolution with the number of 3D models*

models	10	20	30	40	50	100	200	500	1000	1300	1392
AP (%)	19.4	24.3	26.1	28.7	30.5	32.0	32.8	33.5	34.2	34.6	34.9

Table 5.3: *Dependency on the calibration parameter κ*

calibration parameter κ	0.0001%	0.0005%	0.001%	0.005%	closed form (chapter 4)
AP (%)	35.2	35.2	34.9	32.9	9.3

section 5.3.3 and reported the mean average precision as a function of the number of 3D models (randomly sampled). The results are shown figure 5.13. The corresponding values of the Average Precision are reported table 5.2. The first observation is that using more models consistently improves the result. The curve also seems to indicate that the improvement is still far from convergence and that more 3D models will further improve the performance.

5.3.4.2 Calibration

The calibration procedure we have developed in section 5.2.2 using negative data is different from the one of section 4.3.3, which was defined in the closed form. The one used in this chapter has two main advantages. First, it allows us to discard immediately most of the potential matches, i.e. the 99.99% with scores lower than 0. This makes the alignment procedure more efficient. Second, the calibration of section 4.3.3 makes the 0 score meaningful and thus the fact of considering only positive scores as in equation (5.5) possible.

Because of the inefficiency of the detection method of chapter 4 which requires keeping all candidate matches, we used only 100 chairs models to compare the two methods. The MAP detection score obtained with the discriminative calibration of section 5.2.2 is 0.320, while the score with closed form calibration of chapter 4 is only 0.093, showing a clear advantage for the method developed in this chapter.

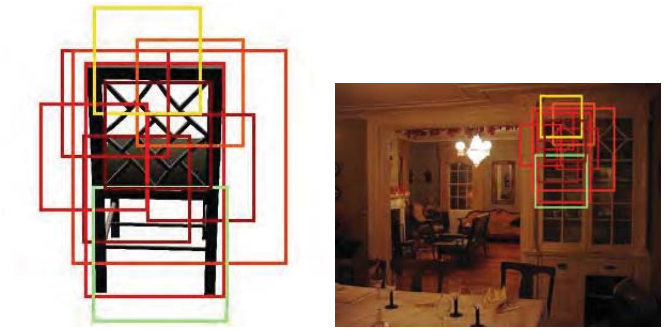
Table 5.4: *Detection AP as a function of the visual elements size (with respect to the larges dimension of the chair)*

relative size	1.8	2.4	2.8
AP (%)	27.2	25.2	17.6

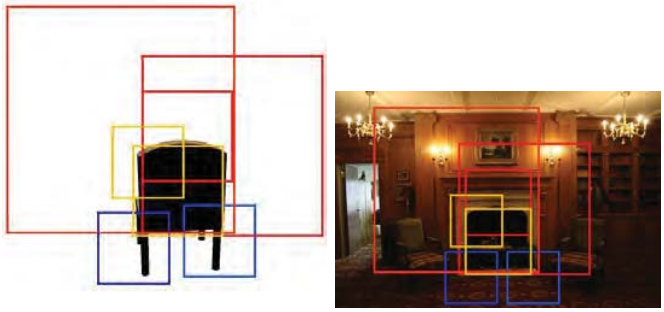
Another interesting question is the choice of the calibration parameter κ. Working with much larger or much smaller value of the calibration parameter κ is problematic for computational reasons. If the parameter is large, we will have to consider many potential matches for the alignment. If it is small, we will have to evaluate the distribution of the scores on a much larger set of negative examples to calibrate the detectors. We experimented with several parameters κ and report the results in table 5.3. The detection score obtained with a calibration parameter $\kappa = 0.001\%$ is 0.352, which is slightly but not significantly better than the 0.349 obtained in exactly the same conditions with $\kappa = 0.01\%$. Qualitatively, the results are also very similar using the two parameters. This shows that the method is robust to the choice of the calibration parameter κ.

5.3.4.3 Deterministic part selection

To demonstrate the benefits of our discriminative part selection, we have defined visual elements manually on a regular grid and tried to match them using the same framework as before. We have overlapped a 3 by 3 grid of elements on top of the rendered views and tested different relative size of the visual elements with respect to the larges dimension of the chair. We have added a global element which overlaps the full chair. This leads to a total of ten elements per rendered view which can be directly compared to the previous results. The results for several relative size of the visual elements are reported table 5.4. It can be seen that that defining the elements in this way strongly harms the performance, leading to a maximum of 27% average precision (to compare with 35.2%



(a) *Confusing discriminative patterns*



(b) *Chair-like structures in the background*



(c) *Errors in pose estimation*

Figure 5.14: *Common failures of our algorithm.*

obtained with our discriminative elements selection method).

5.3.4.4 Failure cases

In Figure 5.14 we show common failure modes of our algorithm. We observed two main causes for false positives. First, some of the database chairs have a particular texture that can produce confident detections on textured regions, such as the rear view of the chair matching the checkered window pattern in Figure 5.14(a). Second, there exist many regions in images that, through the eyes of a HOG descriptor, appear as a chair,

as seen in Figure 5.14(b). We also observed this effect in estimating the pose of the chair, which resulted in confusing the front/back of the chair, as seen in Figure 5.14(c). The chairs our algorithm miss are mainly special chair types, such as sofa-like chairs, for which we do not have training examples, and chairs strongly occluded by other objects and people.

5.3.4.5 Computational cost

Computing the discriminative elements takes 6 seconds per rendered view (1 second for extraction and 5 seconds for calibration) on a single machine, which can be performed offline and parallelized. The main bottleneck is having to match and align an image to the large number of rendered views from the 3D models, which involves detecting over 800K discriminative visual elements. Our Matlab implementation matches and aligns an image in 2 minutes on a 80-core cluster. While this computational complexity is high, there is room for optimism that our approach can be made significantly more efficient. For instance, the recent part-based hashing technique of [51] could be directly applied to our algorithm by applying winner-take-all hashing [181] on the discriminative visual elements. As we have a similar number of part detectors as considered in their work, we believe we can process an image in less than 20 seconds on a single multicore processor.

5.4 Conclusion

We have demonstrated successful detection and alignment of 3D CAD chair models to chairs depicted in 2D imagery. Our approach relied on matching spatial configurations of mid-level discriminative visual elements extracted from a large database of CAD

models having a large number of rendered views. Our algorithm is able to recover the chair pose, in addition to its style. We evaluated our approach on the challenging PASCAL VOC dataset and showed that, when combined with the output of the deformable parts model detector [61], we are able to achieve higher detection accuracy than using either method alone. We also demonstrated that our algorithm is able to reliably recover the chair pose and style, as shown in our user study and orientation error analysis. The output alignments produced by our system open up the possibility of joint 3D reasoning about the depicted objects in a scene toward the larger goal of full 3D scene understanding.

Chapter 6

Discussion

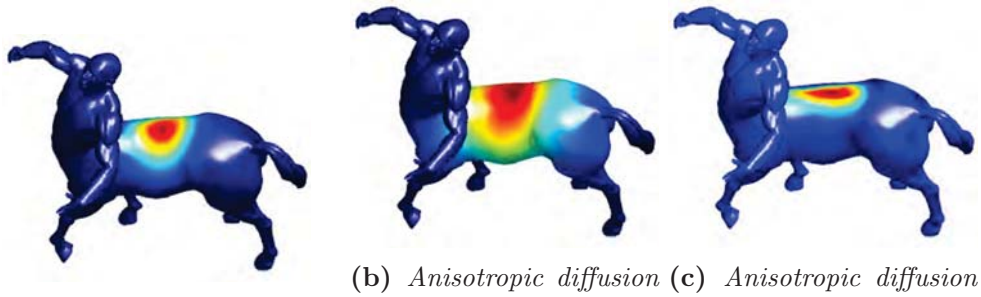
In this chapter, we summarize the contributions of this thesis and present some future extensions.

6.1 Contributions

In this thesis, we presented new representations of 3D models to tackle different alignment problems.

- in chapter 3 we have introduced the Wave Kernel Signature. It improves state of the art results for 3D point descriptors. We have presented a mathematical analysis of the variations of the Laplace-Beltrami eigen-values that indicates the WKS combines the information from different spectral frequencies in an optimal way. Beside being a natural tool for feature-based shape alignment, we have shown that it is also well suited for many shape analysis tasks.

- in chapter 4 we have introduced the concept of 3D discriminative visual elements to represent a 3D model in term of visual appearance. This representation is well adapted for 2D-3D matching, and we demonstrated it can be used to align



(a) *Isotropic diffusion.* **(b)** *Anisotropic diffusion favoring high bending.* **(c)** *Anisotropic diffusion favoring low bending.*

Figure 6.1: *Using anisotropic instead of isotropic diffusion processes can lead to richer and more meaningful analysis of 3D shapes.*

reliably non-realistic depictions to 3D models. Central to the success of our representation is the key idea to view the feature matching problem as a least square classification task.

- in chapter 5 we have extended the notion of discriminative visual elements to represent not only a single 3D model but a full collection of 3D models. We have shown that it allows to perform category-level alignment and recognition. Using a large database of Internet CAD models and a single test image, we can predict the position, pose and an approximate 3D model of an object.

6.2 Future work

6.2.1 Anisotropic Laplace-Beltrami operators

In chapter 3 we have worked to aggregate in an optimal way the informations from the eigen-values and eigen-functions of the Laplace-Beltrami operator. A question that has received little attention is to ask if the Laplace-Beltrami operator itself is an optimal choice. Taking inspirations from works in image processing where anisotropic diffusion is often preferable to isotropic diffusion, we introduced in [10] an anisotropic Laplace-Beltrami operator which corresponds to anisotropic diffusion on shapes (see

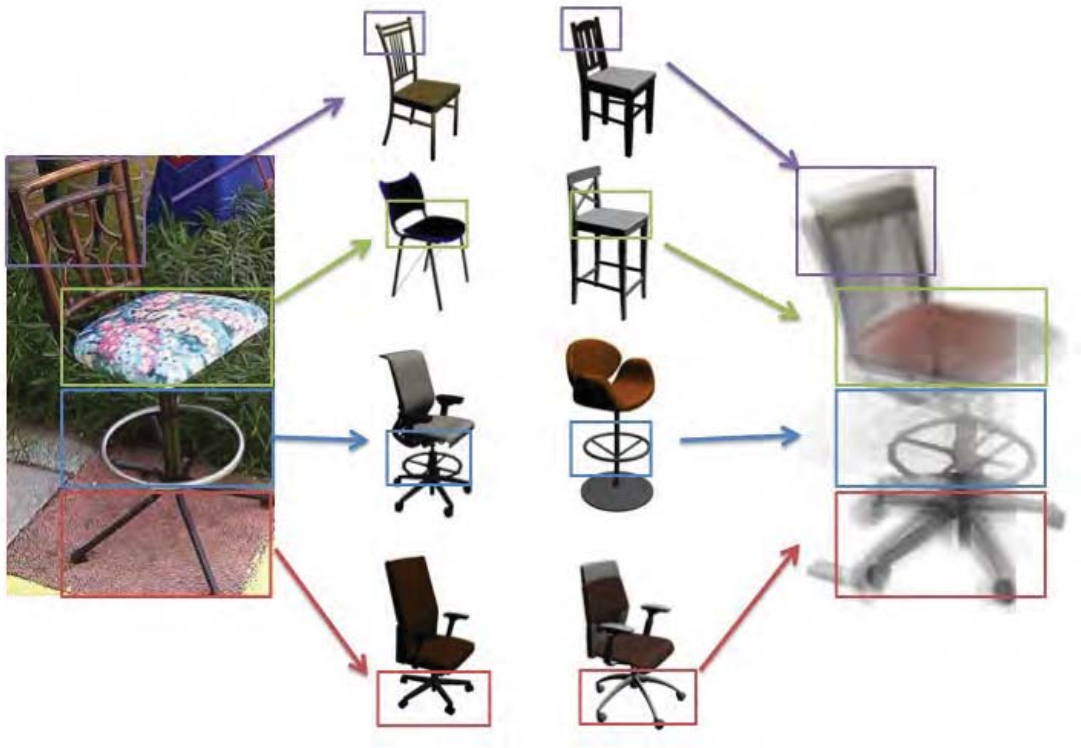


Figure 6.2: *Using parts from different 3D instances, one can explain and analyze an instance that has never been observed.*

figure 6.1). We have shown that it can improve the results of several standard shape analysis algorithms. A more systematic study would be needed to determine how much anisotropy can help different approaches.

6.2.2 Object compositing

In chapter 5 we have presented a method that can recover an approximate 3D model of an object by matching it to many 3D models and selecting the most confident one. However, this method makes little use of the fact that many 3D matches are available. We want to explore the possibility to incorporate informations from several 3D models to explain a given observed instance and thus predict better 3D informations from a single image. This idea is illustrated figure 6.2 for 2D compositing. In the same

direction, [167] recently utilized the analysis of a large shape collection to predict accurately the depth of a segmented 2D test instance.

6.2.3 Use of 3D shape collection analysis

In chapter 5 we have introduced a method to align shape collections with an image. Using 3D tools similar to the one presented in chapter 3, such as the one used in [87], to analyze automatically the 3D collection, we could transfer informations such as parts position to an image, improving the fine grained analysis and understanding of the image content. This exciting perspective could push new research in 3D shape collection analysis.

6.2.4 Synthetic data for deep convolutional network training

As mentioned in section 2.3.1.4, the use of convolutional neural networks has recently improved the performance of many vision algorithms. One of the limitation of these networks is that they need lots of data to be trained efficiently. For example, to train a network predicting the orientation of objects, it would be necessary to annotate a huge database of oriented objects. To avoid this expensive manual annotation, synthetic data could be used. However the possibility to train CNN with synthetic data remains an open research question. In this direction, [80] recently used synthetic depth images to train a CNN for detection from depth data.

6.2.5 Exemplar based approach with CNN features

The statistics and information encoded in CNN features is very different from the one of more standard features. How they can be used to perform exemplar learning, similar to what has been presented in chapter 4 for HOG descriptors is thus an open question.

It is however fascinating because using specifically learned features, very high quality exemplar learning may be possible, bringing computer vision algorithms closer to what a human is capable of doing.

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