Comparison of Constitutive Flow Curve Relations in Cold and Hot Forming

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Abstract. Based on experimental results of upsetting tests of an case hardening steel, constitutive stress-strain relations were developed according to various phenomenological and semiempirical models proposed in the literature, covering the strain rate between $\dot{\varphi} = 10^{-2}$ to 10^1 s^{-1} and the temperature range between room temperature and 1200°C including, with special emphasis, the two phase region between A_1 and A_3 .

Résumé. Sur la base des résultats des tests de compression d'un acier de cémentation, des équations constitutives ont été développées correspondant aux modèles d'équations constitutives divers proposés de la littérature. Les études réalisées comprendraient des vitesses de dilatation de 10^{-2} à 10^1 s^{-1} et des températures entre la température ambiante et 1200°C en considération spéciale de la domaine diphasique de A_1 à A_3 .

1 Introduction

Within the last years mathematical simulations in forming became an essential tool to optimise even complicated forming processes. Apart from the correct description of tribological and thermal boundary conditions the experimental determination and the mathematical formulation of stress-strain curves are of decisive importance for the accuracy of such calculations.

Based on the experimental results of compression tests, constitutive stress-strain relations were developed according to various phenomenological and semiempirical models proposed in the literature, covering the whole temperature range investigated including the two phase region between A_1 and A_3 . To provide flow curve models for computer simulations of cold and hot forming processes, these models have to be able to describe the influence of metallurgical processes like strain hardening and dynamical softening on flow stress. The accuracy of the different models has been assessed by statistical methods.

2 Testing techniques

Flow curves of the case hardening steel 20MoCrS4 were measured in uniaxial hot compression tests with an high speed servohydraulic testing machine at constant rates of strain from $0,01 \text{ s}^{-1}$ up to 10 s^{-1} in the temperature range between room temperature and A_3 . Specimens with 10 mm in diameter and 15 mm high in Rastegaev-geometry were used in the tests., lubricants were chosen according the specific forming temperature. Up to a temperature of 500°C , a special high temperature paste Nomynol VI 200-BFTTM and above 500°C , various glass powders were used according to the deformation temperature, in order to avoid bulging and to ensure a quasi-uni-axial stress state.

The specimens were heated inside a special induction coil, which enables also to quench the specimen by an gas shower immediately after testing in order to freeze the structure for subsequent metallographic investigations. All specimens used had a normalised initial structure of ferrite and pearlite with an average grain diameter of ferrite of $8 \mu\text{m}$.

Additional upsetting tests were carried out in the two phase α/γ region between A_1 and A_3 and in the temperature range of stable austenite up to 1200°C at constant rates of strain between $0,01 \text{ s}^{-1}$ and 1 s^{-1} using a deformation dilatometer. Fig. 1 shows the experimental setup. The dilatometry specimens were of cylindrical shape with 5 mm in diameter and 10 mm high.

Figure 1: Testing setup of the deformation dilatometer

3 Constitutive flow curve relations

Table 1 contains an overview of some phenomenological and semiempirical flow-curve models proposed in the literature [1-5]. These models have been evaluated in respect of their suitability and accuracy to describe the stress strain curves in simulations of cold-, warm and hot forming processes. The empirical mathematical

Table 1: Constitutive flow-curve relations considered to evaluate their accuracy

Flow curve equation		Reference
$k_f = A_0 \cdot \varphi^{a_1} \cdot \dot{\varphi}^{a_2} \cdot \exp(-a_3 \cdot T)$	(1)	Hodgson [1] Hajduk [2]
$k_f = A_0 \cdot \varphi^{a_1} \cdot \exp(a_4 \cdot \varphi) \cdot \dot{\varphi}^{a_2} \cdot \exp(-a_3 \cdot T)$	(2)	Spittel, Hensel [3]
$k_f = A_0 \cdot \varphi^{a_1} \cdot \exp\left(\frac{a_4}{\varphi}\right) \cdot \dot{\varphi}^{a_2} \cdot \exp(-a_3 \cdot T)$	(3)	Spittel, Hensel [3]
$k_f = k_{f \max} \cdot \left\{ \frac{\varphi}{\varphi_{\max}} \cdot \exp\left(1 - \frac{\varphi}{\varphi_{\max}}\right) \right\}^c$ $Z = \dot{\varphi} \cdot \exp\left(\frac{Q_d}{R \cdot T}\right) = f(k_{f \max}) = A \cdot \sinh(\beta \cdot k_{f \max})^n$ $\varphi_{\max} = b_0 \cdot d_0^{b_1} \cdot Z^{b_2}$	(4)	Cingara [4]
$k_f = k_{fms} - (k_{fms} - k_{f0}) \cdot \exp(b \cdot \varphi) - (k_{fms} - k_{fs}) \cdot X_{dyn}$ $X_{dyn} = \begin{cases} 0 & ; \varphi < \varphi_c \\ 1 - \exp\left[-2,996 \cdot \left(\frac{\varphi - \varphi_c}{\varphi_s - \varphi_c}\right)^2\right] & ; \varphi \geq \varphi_c \end{cases}$ $k_{f0} = c_0 \cdot d_0^{c_1} \cdot Z^{c_2}$ $k_{fms} = d_0 \cdot \arcsin h\left(\frac{Z^{d_1}}{d_2}\right) ; \quad k_{fs} = e_0 \cdot \arcsin h\left(\frac{Z^{e_1}}{e_2}\right)$ $\varphi_c = f_0 \cdot d_0^{f_1} \cdot Z^{f_2} ; \quad \varphi_s = g_0 \cdot d_0^{g_1} \cdot Z^{g_2}$	(5)	Voce [5]

equations (1) to (3) in table 1 allow to describe the flow stress in dependance of the forming parameters strain, strain rate and temperature. Modifications of eq. (1) have been introduced by Hensel and Spittel [3] in eq. (2) and (3) to take the influence of dynamic softening processes on flow stress into account.

Models (4) and (5) assume that hot forming is a thermally activated process. Under that assumption, the flow stress can be expressed in terms of the Zener-Hollomon Parameter Z , describing the combined influence of strain rate and temperature by means of an Arrhenius function. The influence of the initial grain size as a parameter of structure can be taken into account by empirical equations describing the characteristic strains for the onset of dynamic recrystallisation φ_c , the strain $\varphi_{\max}$ at which the flow stress reaches his maximum value $k_{f\max}$ and the strain φ_s at which the flow stress reaches the steady state value k_g . The occurrence of these characteristic points in flow curves of steels in the austenitic range is attributed to dynamic recrystallisation. In its simple form, equation (4) is only valid up to strains of the order of $\varphi_{\max}$. Equation (5) for the flow stress consists of two terms, whereby the first one describes the strain hardening, first proposed by Voce [5], and the second describes softening by dynamic recrystallisation. This is the only model in table 1 that involves a semi-empirical formulation by means of an Avrami function of the dynamically recrystallized volume fraction X_{dyn} . Therefore model (5) is suitable to take into account either for the strain hardening and for dynamic softening occuring under hot forming conditions.

4 Results

4.1 Temperature range of stable Austenite

Experimentally measured flow curves from hot-compression tests have been used to determine the constants of the flow curve equations (1) to (3) by regression analysis. To obtain the apparent activation energy for hot forming in eq. (4) and (5), regression analysis can be applied by plotting $\ln \sinh(\beta \cdot k_{f\max})$ of the stress-strain curve field versus $\ln \dot{\varphi}$ and $1/T$. Variation can be used to determine the β -value at which the gradient dispersion is minimized. Thus the maximum flow stress $k_{f\max}$ can be expressed in terms of the Zener-Hollomon parameter Z . The strain hardening coefficient c in eq. (4) is obtained for each stress-strain curve from the slope of a plot of $\ln(k_f/k_{f\max})$ versus $(1-\varphi/\varphi_{\max} + \ln(\varphi/\varphi_{\max}))$. The constants for the equation describing the logarithmic strain at which the flow stress maximum occurs in model (4) are to determine empirically from regression analysis of experimental data. The determination of the constants in Flow-curve model (5) are to determine in

Table 2: Material constants and coefficients determined from measured flow curves according to the flow curve equations in table 1 for the temperature range of stable austenite of steel grade 20MoCrS4

Flow curve equation type according to table 1				
(1)	(2)	(3)	(4)	(5)
$A_0 = 20723$	$A_0 = 29495$	$A_0 = 19205$	$c = 0,289$	$b = -14,5$
$a_1 = 0,1332$	$a_1 = 0,2308$	$a_1 = 0,0643$	$Q_d = 332600 \text{ J/mol}$	$c_0 = 6,9578$
$a_2 = 0,1554$	$a_2 = 0,1516$	$a_2 = 0,1532$	$A = 3,8964 \cdot 10^{12}$	$c_1 = -0,254$
$a_3 = 0,003857$	$a_3 = 0,00385$	$a_3 = 0,00385$	$\beta = 0,00868$	$c_2 = 0,09516$
	$a_4 = -0,6275$	$a_4 = -0,0037$	$n = 4,927$	$d_0 = 117,65$
			$b_0 = 0,002182$	$d_1 = 0,1971$
			$b_1 = 0,196$	$d_2 = 310,7$
			$b_2 = 0,1436$	$e_0 = 115,2$
				$e_1 = 0,2068$
				$e_2 = 477,5$
				$f_0 = 0,001746$
				$f_1 = 0,196$
				$f_2 = 0,1436$
				$g_0 = 0,000888$
				$g_1 = 0,285$
				$g_2 = 0,193$

an adequate way.

Table 2 contains all constants of the different flow curve equations obtained from experimental stress-strain curves in this way. An assesment of the accuracy of the mathematical descibtion of experimental stress strain curves can be performed by plotting the cumulative frequency versus the relative error in figure 2. When applying eq.(1) to (3) to calculate stress-strain curves in the temperature range of stable austenite the relative error is about 15 % assuming an error probability of 80%. As can be seen from figure 2, the modifications introduced by [3] to take into account dynamic softening processes cause only little effects on the agreement between measured and calculated stress-strain curves. When introducing the influence of initial structure on the

Figure 2: Cumulative frequency of the relative error when applying eq. (1) to (5) for calculating stress-strain curves of steel grade 20MoCrS4 in the temperature range of stable austenite

Figure 3: Comparision between measured stress-strain curves (solid lines) and calculated stress-strain curves (dotted lines) for different temperatures and a constant strain rate of $0,1 \text{ s}^{-1}$; A) calculated according to eq. (2); B) calculated according to eq. (5)

shape of the stress-strain curves into the mathematical flow curve formulation, as done by eq. (4) and (5), the agreement between measured and calculated flow stress can be improved considerably. The relative error can be decreased to the order of 7%, whereby eq. (4) is only valid up to $\varphi_{\max}$. When applying eq. (5) the best agreement between measured and calculated stress strain curves including the steady state range is obtained. This is due to the implementation of dynamic recrystallisation into the flow curve equation. For example, figure 3 compares measured stress strain curves with calculated curves, according to eq. (2) and (5).

Figure 4: A) Comparison between measured and calculated flow stress maximum $k_{f\max}$ versus temperature; B) comparison between measured and calculated strain at flow stress maximum $\varphi_{\max}$

Figure 5: Ferrite along the former grain boundaries of deformed austenite of steel grade 20MoCrS4, transformed from metastable austenite during deformation at 750°C after austenitization at 1050°C.

Provided that no partial γ/α - transformation takes place before or during the deformation, eq. (4) and (5) can be used to calculate stress strain curves of the metastable austenite in the temperature range between A_1 and A_3 , as has been proved in additional compression tests. This can be seen from the results in figure 4 A) and B). The significant differences between some measured values and the calculated curves in the temperature range between A_1 and A_3 is caused by a partial γ/α - transformation that is enhanced by the plastic deformation. Metallographic observations of these specimens revealed ferrite along the grain boundaries of the deformed former austenite grains (see figure 5).

4.2 Two phase region between A_1 and A_3

In the two phase α/γ -region the maximum flow stress k_{fmax} can not expressed in terms of the Zener-Hollomon parameter in the same good agreement as indicated in figure 4A. Beside that, no significant dependency has been found between characteristic strains like φ_{max} and volume fraction of ferrite, grain size, temperature and strain rate. Thus, a mathematical formulation of stress-strain curves in the two phase α/γ -region according to eq. (4) and (5) can not be applied in that simple form used to describe the behaviour of stable austenite. This is attributed to possible deformation- and/or stress-induced phase transformations and related effects, like transformation plasticity, occuring during deformation. A complete formulation of the stress-strain behaviour in the two-phase region has to include these effects. Under the premise of applying these constitutive flow-curve relations in finite element simulations of forming processes, the calculation effort necessary for an complete description seems not to be justified. From that point of view, the empirical stress-strain relation (2) has been chosen to express the flow stress as an state variable dependent on strain, strain rate and temperature without considering any structural effects, occuring during deformation. Table 3 contains the coefficients, determined by regression analysis according to eq. (2) from measured flow curves. Figure 6 shows calculated stress strain curves in comparision to measured curves in the two-phase α/γ -region. Assuming an error probability of 80%, the application of eq. (2) for calculating stress-strain curves in the temperature

Flow curve equation type (2) for application in the two phase α/γ -region	
A_0	2034
a_1	0,2024
a_2	0,0927
a_3	-0,001644
a_4	-0,4647

Table 3: Coefficients determined from measured flow curves according to flow curve equation (2) in table 1 for the temperature range between A_1 and A_3 of steel grade 20MnCrS4

Figure 6: Measured (solid lines) and calculated stress-strain curves according to table 3 in the two phase α/γ -region

range between A_1 and A_3 causes an relative error of 11%. Eq. (2) with the determined coefficients enables therefore a satisfactoring description of stress strain-curves in the two phase α/γ -region.

4.3 Temperature range below A_1

Deformation induced segregations at temperatures between 100°C and 400°C and the spheroidisation of cementite at temperatures above 600°C, which is enhanced by deformation, exclude the application of the flow curve models (4) and (5) assuming forming as an thermally activated process. As a first approach, again regression analysis of a measured stress-strain curve field in the temperature range up to A_1 has been performed according to eq. (3). Table 4 contains the coefficients determined empirically. In figure 7 can be seen, that the empirical formulation gives a reasonable agreement between measured and calculated flow stress up to 400°C. Significant differences at strains above 0,2 of the stress-strain curve at 600°C are attributed to the spheroidisation of cementite, which is enhanced by deformation. Assuming a probability of error of 80%, the relative error in applying eq. (3) for predicting stress-strain curves up to A_1 is 12%.

The premise that the flow stress is influenced by temperature, strain and strain rate in a mutually exclusive way, on which eq. (1) to (3) is based, is not fulfilled over the whole temperature range due to occuring structural changes like segregation and spheroidisation of cementite. On the other hand, the necessary effort of implementing all these structural changes in constitutive flow-curve equations can not be justified with the aim of applying these relations in finite element modelling of forming processes on an continuum level. For simulation of the microstructural development it is necessary to include occuring structural changes in the formulation.

Flow curve equation type (3) for application in the range between room temperature and A_1	
A_0	1066
a_1	0,102
a_2	0,0007
a_3	-0,0006
a_4	-0,005

Table 4: Coefficients determined from measured flow curves according to flow curve equation (2) in table 1 for the temperature range between 20°C and A_1 of steel grade 20MoCrS4

Figure 7: Measured (solid lines) and calculated stress-strain curves according to table 4 at temperatures below A_1 tested with constant strain rate of 10 s^{-1}

5 Summary

Within the temperature range of stable austenite stress strain curves can be formulated mathematically in good quality using models, which assume hot forming to be a thermally activated process. These models can be

extrapolated in the temperature range below A_3 , provided that no partial γ - α transformation occurs before or during forming.

Due to structural changes during forming in the α/γ -region and at temperatures below A_1 the principle of regarding plastic deformation as an thermally activated process can not be employed in the same quality. An approach on the base of metal physics has to include structural changes occurring during deformation and their influence on flow stress.

The flow stress in the α/γ -region and at temperatures below A_1 can be described in reasonable agreement with measured stress-strain curves by empirical equations.

References

- [1] Hodgson, P. D.; Szalla, J. A.; Campell, P. J., Proc. of 4th Int. Steel Rolling Conf., Deauville 1987, France, Vol. 1, pp. c.8.1 - c.8.13
- [2] Hajduk, M. et al., Hutnicke listy 27(1972), pp. 567 - 571
- [3] Hensel, A.; Spittel, T., Kraft- und Arbeitsbedarf bildsamer Formgebungsverfahren (VEB Deutscher Verlag für Grundstoffindustrie, Leipzig 1978)
- [4] Cingara, A.; McQueen, H.J., J. Mat. Proc. Techn. 36(1992) pp. 31 - 42
- [5] Voce, E., J. Inst. Metals 74(1948) pp. 537 -562