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Magneto-Optical Faraday Effect of YIG Films with Random Multilayer Structures

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ABSTRACT. Magneto-optical Faraday effect of Bi-substituted yttrium iron garnet (Bi:YIG) films with random multilayer structures was analyzed by using the random matrix approach. It was found that the random multilayer structure is very effective for enhancing the Faraday effect of the Bi:YIG film, which is much superior than that of the periodic multilayer films.

1. INTRODUCTION

Within the last several years, Kohmoto et al. [1] and Iguchi [2,3] showed theoretically that the optical properties of quasi-periodic multilayer films are considerably different from those of the periodic multilayer films due to the localization of light, and suggested that a new class of optical films could be designed by employing the quasi-periodic structures. In their papers, however, magneto-optical (MO) properties of the disorder and/or quasi-periodic films have not been studied, although they are also very sensitive to the geometrical film structures [4,5].

In this article, therefore, the MO Faraday effect of Bi-substituted yttrium iron garnet (Bi:YIG, hereafter) films with random multilayer structures was analyzed theoretically to investigate whether a unique MO properties of the films arises from the random structures. Particular attention has been paid to the Faraday rotation angle and the figure of merit of the random multilayer films, because they are of great importance in opto-electronic communications and high density MO recordings.

2. THEORETICAL APPROACH

As shown in Fig.1, let us consider the magnetic multilayer film with \( N \) layers, which extends boundlessly in the \( X-Y \) plane but has a finite total thickness \( D \). The film is composed of Bi:YIG and SiO\(_2\) layers being stacked with an arbitrary sequence. All magnetic and dielectric layers are, respectively, assumed to have the same thicknesses \( d_m \) and \( d_\sigma \), whose values are given by \( d_m = D P_m/N_M \) and \( d_\sigma = D(1-P_m)/(N_N-N_\sigma) \) with the number of the magnetic layers \( N_M \) and the packing density of the magnetic material \( P_m \). The film structure is designed by a computer-generated random binary number \( b^N \) with \( N \) digit, \( b^N=10010111...01 \) for instance. Each digit of \( b^N \) corresponds to each layer of the film, and "1" and "0" are assigned to the magnetic and dielectric layers, respectively.

The incident light is a linearly-polarized light \((E//X, \text{TM mode})\) having the plane wave form of \( \exp\{ikZ-\omega t\} \) with the wavenumber \( k \) and the angular frequency \( \omega \), which submerges perpendicularly to the film plane at \( Z=0 \). The fundamental equations are given by Maxwell’s equations; \( \nabla \times E = -\mu_0 \partial H/\partial t \) and \( \nabla \times H = \varepsilon_0 c^2 \partial E/\partial t \). For the magnetic layers with the magnetizations parallel to

![Fig. 1 Model used for the analysis.](image1)

![Fig. 2 Changes in the transmission and reflection coefficients (a) and the Faraday rotation angle (b) versus the packing density of the film with a periodic multilayer structure.](image2)
the light propagation axis (Z-axis), the specific dielectric tensor \( \varepsilon \) has five non-zero elements of \( \varepsilon_{m} = \varepsilon_{0}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4}, \varepsilon_{5} \). As for the dielectric layers, \( \varepsilon \) is a scalar. The Maxwell's equations were solved by using the random matrix approach with the state vector notation: The state vector \( \mathbf{r}(Z) = (E_x(Z), E_y(Z), E_z(Z), H_x(Z), H_y(Z)) \) is so defined that it is continuous across all the layer boundaries. Then, from the Maxwell's equations, the state vectors at the film surfaces of \( Z-\Delta_{0} \) and \( Z=\Delta_{0} \) are connected with the following relation:

\[
\mathbf{r}(D) = \mathbf{M} \cdot \mathbf{G} \cdot \mathbf{M} \cdot \mathbf{G} \cdot \mathbf{M} \cdot \mathbf{r}(0),
\]

where \( \mathbf{M} \) and \( \mathbf{G} \) denote, respectively, the transfer state matrices \((4 \times 4)\) of the magnetic and dielectric layers. The multiple sequence of these matrices coincides with the sequence of "1" and "0" in \( b^n \). The state vectors at both film surfaces are obtained independently from the electromagnetic fields of light traveling in the outer space of \( Z \leq 0 \) and \( Z \geq \Delta_{0} \) as

\[
\mathbf{r}(0) = (1 + C_1, C_2, C_3, C_4) \quad \text{and} \quad \mathbf{r}(D) = (C_1, C_4, -C_3, C_2)
\]

with complex constants from \( C_1 \) to \( C_4 \). By combining eqs.\((1)\) and \((2)\), the values of these constants are determined, which correspond respectively to the reflection \((R)\) and transmission \((T)\) coefficients of light with the relations of \( R^{TM} = C_1^2, R^{TE} = |C_1|^2, T^{TM} = C_2^2, T^{TE} = C_4^2 \). The superscripts, TM and TE, denote the modes of light. The Faraday rotation angle is also obtained from the ratio of \( \theta = C_3/C_4 \) as

\[
\theta_F = \left(1/2 N_m d_m \right) \tan^{-1} \left( 2 \text{Re}(\chi) / (1 - |\chi|^2) \right)
\]

3. RESULTS AND DISCUSSION

The numerical calculations were made using the material parameters of Bi:YIG film, which are those of \( \varepsilon_r = 4.75 \) and \( \varepsilon_i = 0.00269 \) (\( \theta_F = 0.2 \text{deg/\mu m} \)) at the wavelength of \( \lambda = 1.15 \mu m \). As for the SiO\(_2\) layers, \( \varepsilon = 2.5 \) was used. The total number of layers is \( N = 100 \) and the total film thickness is \( D = 5 \mu m \).

As a function of the packing density \( P_d \), the transmission, reflection and MO properties of the films are shown in Figs. 2(a) and 2(b) for the case of the periodic multilayer film, and in Figs. 3(a) and 3(b) for the case of the random multilayer film. In both cases, the \( \theta_F \) enhancement takes place when the film structure meets the non-reflection conditions. This is quite similar to the enhancement of the MO Kerr effect. The maximum Faraday rotation angle \( (\theta_F)_{\text{max}} \) in the periodic and random multilayer films are, respectively, 0.29 deg/\mu m at \( P_d = 0.03 \) (total thickness of the magnetic layers is \( D_n = 18 \mu m \)) and \( (\theta_F)_{\text{max}} = 0.49 \text{deg/\mu m} \) at \( P_d = 0.39 \) (\( D_n = 1.95 \mu m \)), suggesting that the maximum Faraday rotation angle is strengthened by employing the random multilayer structure.

This situation is articulated in Fig. 4 showing the MO properties of the thick films with \( P \times n \) and \( R \times n \) \((n = 0-50)\) structures, where \( P \) and \( R \) denote the 5um-thick periodic and random multilayer films with \( P_d = 0.03 \) and 0.18, respectively. In the random multilayer film, \( (\theta_F)_{\text{max}} \) is further enhanced considerably and reaches at 1.6deg/\mu m. This value is 8 times higher than that of the single Bi:YIG film, nevertheless the fraction of the total Bi:YIG thickness in the film is only 18% (approximately 9um). Then, the apparent extinction coefficient \( a \) of the random multilayer film is greatly reduced, which results in a large figure of merit, \( 2\theta_F/a \).

References