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Ratio of Decrement to Modulus Change for Amplitude-Dependent Internal Friction

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Abstract. The ratio r of the amplitude-dependent decrement δ_h to the amplitude-dependent modulus defect $\Delta M/M$ has been analyzed within the framework of theories of dislocation hysteresis based on both the breakaway Granato-Lücke model and the symmetrical pointed loop frictional model. In the particular case of power laws with the same exponent n for both the decrement and modulus defect: $\delta_h = r(\Delta M/M) \propto \sigma_0^n$, analytical expressions for the relationship between r and n have been derived. The theoretical requirements have been compared with the available experimental data. The use of r for identifying the mechanism of dislocation hysteresis is demonstrated.

1. INTRODUCTION

A proportionality between the amplitude-dependent decrement δ_h and the amplitude-dependent modulus defect $\Delta M/M$:

$$\delta_h = r (\Delta M/M) \quad (1)$$

has been first observed by Read [1,2] in zinc single crystals as early as 1940 and then by many other researchers in a large variety of crystalline solids. Read reported that r in zinc varied with orientation from 0.55 to 5.5. Almost all available experimental data for other materials show the value of r within this range (if the amplitude of vibrational strain ϵ_0 is not very large: $\epsilon_0 < 10^{-4}$). Up to now there is no reasonable theoretical explanation on the orientation dependence of r . In the majority of theories main attention has been paid to the decrement, but not to the modulus defect. In order to derive $\Delta M/M$ correctly, Fourier analysis should be applied [3]. Granato and Lücke [4] did it for their model and estimated the value of r of the order of unity, however exact formula was not given. Recently Lebedev [5] analyzed the ratio r within the framework of the Granato-Lücke (GL) model. Two conclusions were made: (i) in the general case, r is a function of the vibrational amplitude σ_0 , but the value of r should not exceed 2; (ii) for the particular case of a power law

$$\delta_h \propto \sigma_0^n, \quad (2)$$

r does not depend on the amplitude and is a function of n only:

$$r = 2n / \left[n + 2 + \frac{2(n+2)}{\pi} B\left(\frac{n+3}{2}, \frac{1}{2}\right) \right], \quad (3)$$

where $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function and $\Gamma(x)$ is the gamma function.

The aim of the present communication is to analyze the case of frictional hysteresis and to compare theoretical $r(n)$ dependence for three types of hysteresis loops, schematically shown in Fig.1, with the available experimental data. Two types of frictional hysteresis have been considered: (i) Davidenkov pointed loop [6] and (ii) pointed loop with no restoring force (NRF). Lazan [7] gave a description of both types with respect to their damping properties, and actually introduced the term "pointed loops" (in contrast to linear elliptical loops).

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2. THEORY

Davidenkov [6] suggested that amplitude-dependent internal friction resulted from nonlinear hysteresis of microplastic strain ϵ_d , and the stress-strain loop had a symmetry axis of the second order (Fig.1b). For a power dependence ϵ_d on the stress σ

$$\epsilon_d = (\sigma + \sigma_0)^m - 2^{m-1} \sigma_0^m, \quad (4)$$

he derived for the decrement

$$\delta_h = 2^{n+2} n M \sigma_0^n / (n+2), \quad (5)$$

with $n=m-1$. Eqn.(4) corresponds to the upper branch of the loop, i.e. when the stress increases (a structural sensitive factor here and in what follows is omitted, it does not influence the ratio r): For more details of the Davidenkov's model see, for example, Lazan [7], Asano[8], and Lebedev and Kustov [9].

The modulus defect can be obtained by substituting $\sigma = \sigma_0 \cos \Theta$ into Eqn.(4) and using Fourier transformation

$$(\Delta M / M) = (2M / \pi) \sigma_0^{m-1} \int_0^\pi (1 + \cos \Theta)^m \cos \Theta d\Theta. \quad (6)$$

Eqns. (5) and (6) give the ratio r as

$$r = 2^{n+1} n \pi / [(n+2) \int_0^\pi (\cos \Theta + 1)^{n+1} \cos \Theta d\Theta]. \quad (7)$$

Fig.1c shows nonlinear symmetrical NRF loop, The strain ϵ_d changes only during increasing the absolute value of σ . If we assume this change to obey a power law, then equation for the upper branch is: $\epsilon_d = \sigma^m - (\sigma_0^m/2)$. Using, e.g., Asano's δ_h - ϵ_d relationship [8] for the decrement and Fourier analysis for the modulus defect, one obtains

$$r = \left[(n+1)^2 \sqrt{\pi} \Gamma\left(\frac{n+1}{2}\right) \right] / \left[(n+2) \Gamma\left(\frac{n+2}{2}\right) \right], \quad (8)$$

where $n=m-1$, as in the previous case. At $n=0$, i.e for linear NRF loop ($m=1$), Eqn.(8) gives $r=\pi/2$. Linear NRF hysteresis leads to amplitude-independent internal friction, as in the case of the LDWRF (limited displacement without restoring force) mechanism suggested by Gremaud [10] to explain peculiarities of internal friction background in solid solution.

Figure 1. Schematic representation of (a) Granato and Lücke hysteresis, (b) Davidenkov hysteresis, and (c) NRF hysteresis.

3. COMPARISON WITH EXPERIMENTAL DATA

Fig.2 shows calculated from Eqns. (3), (7) and (8) curves together with some experimental data [1-3,11-21], which are in agreement with Eqns.(1) and (2). Materials and references are listed in Table 1. In spite of a large scatter of data, some regularities can be observed. Points for pure annealed copper are closer to

Table 1. The ratio r of the amplitude-dependent decrement to modulus defect for various materials in the case of power law:

$$\delta_h = r (\Delta M/M) \propto \sigma_0^n.$$

The symbols have the following meanings: s.-single crystal, p.-polycrystal, eu.-eutectic alloy, Θ - angle between the normal to basal plane and the sample axis, ann.- annealed, ps.- prestrained, q.-quenched.

Row	Material	Impurity %	Frequency kHz	T, K	r	n	Ref.
1	Cu, s., ps.	0.002	33.5	room	4.4	2.1	2
2	Cu, s., ann.	0.01	39-78	room	2.2	2	3
3	Cu, s., ps.			213-306	3.6		
4	Cu, s. ann.	0.01 Si	80	5	1.5	2	11
5		0.01 Si		81	1.9		
6		0.01 Si		294	2.9		
7		0.005 Ge		76-210	2.7		
8	Cu-Al, p., q.	1.38 Al	1	room	3	2.8	12
9	Cu-P, p.	0.002 P	2.7	room	1.8	1.05	12
10		0.006 P			2.3	1.44	
11		0.021 P			2.7	1.52	
12		0.062 P			3.1	1.69	
13		0.207 P			3.3	2.05	
14	Cu-Si-Mn, p.,ps.	1.6 Si, 1.4 Mn	100	7-293	1.4	1.2	13
15	Al, s.	0.66 Si, 0.22 Fe	100	6.5-293	1.4	1.3	14,15
16	Al-Al ₂ Cu, eu.		100	6.5-293	1.3	1.4	16
17	NaF, s., ps.	0.01	100	room	1.8	2	17
18	NaF, s., ps.	0.01	100	4.2-295	1.6	2	18,19
19	MgO, s., ps.	0.5	100	65-295	1.2	1	20
20	Zn, s., $\theta=0^\circ$, ps.	0.003	100	room	1.6	1	21
21	Zn, s.ann., $\theta=88^\circ$	0.001	39	room	0.55	1.7	1
22					1.4	2.9	
23					4.5	3	
24					5.3	4

Figure 2. Comparison of $r(n)$ curves calculated for: a - GL (Eqn.3), b - Davidenkov (Eqn.7), and c - NRF (Eqn.8) loops with experimental data for materials listed in Table 1 (numeration of symbols corresponds to numeration of rows).

the Davidenkov loop, whereas prestrained copper fits the NRF curve. Crystals with the NaCl structure (rows 17-19) are closer to the Davidenkov loop, which is in agreement with direct measurement of stress-strain loops at low (0.5Hz) frequencies [22] and with the loops obtained at higher frequencies (40 kHz [23] and 128 Hz [24]) from the dislocation polarization signal.

Dependence of r on orientation in zinc (rows 21-24 in Table 1) can be interpreted in terms of changing the mechanism of hysteresis from the GL breakaway (for pyramidal dislocations) to the NRF loop (for basal plane). Results of Golyandin and Kustov [25] on direct measurement of σ - ϵ_d loops at 0.5Hz in zinc single crystals, oriented for basal slip, show features of the NRF hysteresis.

The origin of the restoring force in the GL model is the dislocation line tension [4]. In the frictional hysteresis of the Davidenkov loop type, the line tension and/or long-range internal stresses may play an important role as a source of the restoring force [26,27].

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