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Prediction of the Fatigue Crack Propagation Rate in Metals on the Basis of the Local Fracture Energy Criterion

V.T. Troshchenko, P.V. Yasniy* and V.V. Pokrovsky

Institute for Problems of Strength, National Academy of Sciences of Ukraine, 2 Timiryazevskaya Str., Kyev 252014, Ukraine

** Institute for Instrument Engineering, 56 Ruska Str., Ternopol, Ukraine*

Abstract. A model of fatigue crack propagation has been proposed and substantiated which is based on the analysis of the fatigue damage energy accumulation at the crack tip involving the use of the fatigue fracture energy criterion, which makes it possible to separate the dangerous part of the total energy of inelastic deformation. The investigation has been made into the regularities of cyclic inelastic deformation and crack propagation rate in steel 15Kh2MFA of different heat treatment and titanium alloy VT6S. A satisfactory agreement has been found between the theoretical and experimental rates of fatigue crack propagation

I. INTRODUCTION

As is known, the energy-based methods are the most common in the mechanics of a deformable solid. At the same time, their usefulness in the analysis of the fatigue crack propagation is very limited.

This is associated with the complexity of the calculation of the strain energy values at the crack tip, the necessity of taking into account the variation in the material rheological properties under static and cyclic loading, the need for the use of the energy-based fatigue fracture criterion which makes it possible to separate that part of the total strain energy which is related to the fatigue fracture, and the complexity of calculating the total value of the fatigue fracture energy in the local volume taking into account its continuously varying distance from the crack tip during the process of crack propagation.

The paper presents the justification of the model of fatigue crack propagation with the account taken of the above-listed factors, and its experimental verification.

2. A MODEL OF THE MATERIAL FRACTURE AT THE CRACK TIP UNDER CYCLIC LOADING

On the basis of the results obtained in Refs [1,2,3,4], the distribution of stresses and strains perpendicular to the crack plane under plane strain conditions and cyclic loading will take the following form (Fig. 1):

$$\Delta\sigma_{yy} = 2\sigma_y^e \left(\frac{1-\nu^2}{\alpha I} \right)^{1/(n+1)} \left(\frac{\Delta K^2}{\sigma_y^{e^2}} \right)^{1/(n+1)} \frac{1}{r^{1/(n+1)}} \tilde{\sigma}(n), \quad (1)$$

$$\Delta\varepsilon_{yy} = \frac{2\alpha\sigma_y^e}{E} \left(\frac{1-\nu^2}{\alpha I} \right)^{n/(n+1)} \left(\frac{\Delta K^2}{\sigma_y^{e^2}} \right)^{n/(n+1)} \frac{1}{r^{n/(n+1)}} \tilde{\varepsilon}(n). \quad (2)$$

In these formulas r is the distance from the point considered to the crack tip, E is Young's modulus, ν is Poisson's ratio, σ_y^e is the yield stress under cyclic loading, n and α are parameters of the equation $\bar{\Delta\varepsilon}_p = \alpha \bar{\Delta\sigma}^n$; $\bar{\Delta\varepsilon}_p = \Delta\varepsilon_p / 2\varepsilon_y^e$; $\bar{\Delta\sigma} = \Delta\sigma / 2\sigma_y^e$; $\Delta\varepsilon_p$ and $\Delta\sigma$ are plastic strain and stress ranges, respectively; ΔK is the stress intensity factor range, I is the dimensionless function of the strain hardening coefficient n ; $\tilde{\sigma}(n)$ and $\tilde{\varepsilon}(n)$ are normalized functions of n .

When obtaining formulas (1) and (2) it was assumed that the relation between the J -integral and the stress intensity factor ΔK is of the form $\Delta J = \Delta K^2 (1-\nu^2) / E$.

It was suggested that at the fatigue crack tip within the damage zone R_m , a strain-controlled loading is realized, $\varepsilon_a = \text{const}$ (ε_a is the elastic-plastic strain amplitude).

By the damage zone R_m , a zone is meant where stresses exceed the cyclic elasticity limit σ_e^e . In Ref. [5] the following equation was obtained for the plastic zone contour at the crack tip on the basis of the equation of J.Rice and the criterion of R.Mises:

$$R_{n,\theta} = \left[0.239 (K_{\max} / \sigma_y)^2 \left(\cos^2 \theta / 2 + 3/4 \sin^2 \theta / 2 \right) \right]^{(1+n)/(3+n)}. \quad (3)$$

Taking $\theta = 0$, $\sigma_y = \sigma_e^e$, $K = K_{\max}$, we obtain $R_m = \left[0.239 (K_{\max} / \sigma_e^e)^2 \right]^{(1+n)/(3+n)}$; σ_e^e was subsequently determined with a tolerance on residual strain of 2×10^{-5} mm/mm.

It was also suggested that at a distance of x^* from the crack tip the range of cyclic strain and stress was constant. The distribution of stresses and inelastic strains at the crack tip at $R_m \geq r \geq x^*$ is described by equations (1) and (2). An increment in the crack length by a value of x^* occurs when at the distance x^* from the crack tip the accumulated inelastic strain energy attains the value W_f [6].

$$W_f = \sum_1^{N_f} \left[\Delta W_i - \Delta W_R \left(\frac{\Delta W_i}{\Delta W_R} \right)^\beta \right], \quad (4)$$

where N_f is the number of cycles to fracture, ΔW_i is the inelastic strain energy per cycle; ΔW_R is the inelastic strain energy per cycle at stresses equal to the fatigue limit, β is a parameter determined experimentally. The specific inelastic strain energy per cycle was calculated by the formula:

$$\Delta W = \int_0^{\Delta\varepsilon} \Delta\sigma d(\Delta\varepsilon). \quad (5)$$

Using eqns (1), (2) and (5) we get:

$$\Delta W = \frac{4(1-\nu^2)}{IE} \frac{n}{n+1} \frac{\Delta K^2}{r} \tilde{\sigma}(n) \tilde{\varepsilon}(n) \quad (6)$$

The magnitude of x^* is determined from the condition that the specific inelastic strain ΔW^* at the distance x^* from the crack tip at $\Delta K = \Delta K_{th\,eff}$, where $\Delta K_{th\,eff}$ is the effective threshold stress intensity factor is equal to the inelastic strain energy at stresses which are equal to the fatigue limit ΔW_R .

Assuming in formula (6) that $\Delta W = \Delta W_R$, $r = x^*$ and $\Delta K = \Delta K_{th\,eff}$, we get

$$x^* = \frac{4(1-\nu^2)n}{IE\Delta W_R(n+1)} \tilde{\sigma}(n) \tilde{\varepsilon}(n) \Delta K_{th\,eff}^2. \quad (7)$$

It follows from [7] that for the majority of structural steels and alloys the value of $\Delta K_{th\,eff}$ is independent of the stress ratio in a load cycle and is determined by Young's modulus $\Delta K_{th\,eff} = 1.6 \times 10^{-5} E$.

The crack propagation rate is found by the formula

$$V^* = x^* / N_{fi}, \quad (8)$$

where x^* is calculated by eqn (7), N_{fi} is the number of load cycles to fracture at strain ε^* .

Figure 1: A conventional scheme of stress and strain distribution at the crack tip

Figure 2: Variation of the inelastic strain energy per cycle in the zone R_m as the crack propagates.

3. FATIGUE FRACTURE ENERGY

When realizing the above model of the fatigue crack propagation, the determination of fracture energy in a local volume at the crack tip is important. In this case it should be considered, firstly, that not all the cyclic strain energy is related to the fatigue damage, and secondly, that the accumulation of energy in the local volume proceeds under changing conditions which depend upon the distance of this volume from the tip of the propagating crack.

In Ref. [8] the following fatigue failure energy criteria were analyzed along with criterion (4):

$$\sum_1^{N_f} \Delta W_i = W_f \quad (9) \quad \text{and} \quad \sum_1^{N_f} (\Delta W_i - \Delta W_R) = W_f \quad (10)$$

and on the basis of the analysis of a large body of experimental results it was shown that only the energy calculated by criterion (4) is independent of the number of cycles to fracture. Considering this fact, criterion (4) was used in the calculations. The variation of inelastic strain energy per cycle, ΔW , in the zone of fatigue damage, R_m , during the crack propagation into the bulk of the material is shown in Fig. 2.

In accordance with this scheme, as the crack propagates deep in the zone R_m the next successive element at the crack tip will have a higher ΔW value due to an increase in the stress intensity factor. Also some fatigue damage will be accumulated in it by this moment which in total results in a decrease in the number of cycles to fracture of this element.

The condition of fracture of the i -th element of length x^* on the assumption that there is no initial fatigue damage in the zone R_m has the following form:

$$\Delta \overline{W}_{i,1} N_1 + \Delta \overline{W}_{i,2} N_2 + \dots + \Delta \overline{W}_{i,j} N_j + \dots + \Delta \overline{W}_{i,i} N_i = W_f, \quad (11)$$

$$\Delta \overline{W}_{i,j} = \Delta W_{i,j} - \Delta W_R \left(\frac{\Delta W_{i,j}}{\Delta W_R} \right)^\beta,$$

where $N_1, N_2, \dots, N_i$ are the numbers of load cycles for the 1, 2, ..., i -th elements at the maximum plastic strain range, $j=1, 2, \dots, i$ is the index indicative of the crack tip location (at $l = l_0$, $j = 1$).

Expression (11) can also be written in the form:

$$\sum_{j=1}^i \Delta W_{i,j} N_j = W_f. \quad (12)$$

For a more exact calculation of the crack propagation rate it is also necessary to consider that as the crack propagates deep in the zone R_m , the fatigue damage begins to accumulate in the elements relating to the successive dimension R_m . In this case, expression (12) will take the following form:

$$\sum_{j=1}^i (\Delta \overline{W}_{i,j} N_j + W_{s,j}) = W_f, \quad (13)$$

where $W_{s,i}$ is the inelastic strain energy accumulated in the i -th element at $l = l_0 + R_m$.

In Fig. 3 line 1 indicates the variation of the crack propagation rate when the accumulation of the fatigue damage in the zone $R_m \geq r \leq x$ is not taken into account, line 2 corresponds to formula (12), whereas line 3 to formula (13).

Figure 3: Patterns of the crack propagation rate variation at different assumptions.

4. EXPERIMENTAL VERIFICATION OF THE MODEL FOR THE FATIGUE CRACK PROPAGATION

The investigations were carried out on a Schenck servohydraulic testing machine HYDRÖPULSE 40kN equipped with a minicomputer of G A16/240 type [4]. The crack propagation rate was studied on 7 mm and 25 mm thick compact tension specimens at a loading frequency of 25 Hz for steel and 0.5 Hz for alloy VT6S. The fatigue fracture energy W_f and the characteristics of the materials cyclic properties were determined from the test results for cylindrical specimens of 10 mm in diameter under fully-reversed loading at $\epsilon_a = \text{const}$. The moment at which a fatigue crack of 0.5...1.0 mm in length appeared on the specimen surface was taken as the specimen fracture. On completion of the tests, the processing of initial data arrays

Table 1: Mechanical properties of the materials studied

Material	Thermal treatment	Yield stress MPa	Ultimate strength MPa	Elongation %	Reduction of area %
Steel 15Kh2MFA(I) Chemical composition %: 0.18C, 0.27Si, 0.475Mn, 2.58 Cr, 0.16 Ni, 0.62Mo, 0.3V, 0.019S, 0.013P, 0.11T	Oil-quenched for 13 hrs at 1273 K, tempered for 24 hrs at 963 K in vacuum followed by further tempe- ring for 84 hrs in air at 923... 943 K	584	700	21.0	74.6
Steel 15Kh2MFA(II) (The chemical composition is the same)	Oil- quenched for 4 hrs at 1273 K, tempered for 6 hrs at 893 K	1100	1157	16.6	67.2
Titanium alloy VT6S (6.0%W, 4.0%V)	As - received	829	948	10.7	30.3

stored in a magnetic disc was performed with the use of the CYCLE program written in FORTRAN. The materials studied and their mechanical properties are given in Table 1.

Figure 4: A comparison of the experimental and calculated fatigue crack propagation rates. a-steel 15Kh2MFA(I), b-steel 15Kh2MFA(II), c-alloy VT6S. (1-calculations, 2-experiment).

The main characteristics of the properties of the materials studied used to calculate the crack propagation rate are listed in Table 2.

Table 2: The characteristics of the properties of the materials studied

Material	$\sigma_{0.2}$ MPa	σ_e MPa	$E \times 10^{-5}$ M Pa	ν	$\Delta K_{th eff}$ M Pa $\sqrt{m}$	W_f MJ/m ³	α
Steel							
15Kh2MFA(I)	390	200	2.3	0,3	4.5	2090	0.89
Steel							
15Kh2MFA(II)	840	569	2.0	0.3	5.2	1460	0.16
Alloy VT6S	580	425	1.2	0.3	1.95	3372	0.086
Material	n	$\frac{\Delta W_R}{MJ/m^3}$	β	x^* mm	$\tilde{\sigma}(n)$	$\tilde{\varepsilon}(n)$	I
Steel							
15Kh2MFA(I)	6.44	0.072	0.0904	0.066	2.317	0,0176	4.82
Steel							
15Kh2MFA(II)	9.18	0.028	0	0.032	2.463	0.0164	4.60
Alloy VT6S	9.76	0.0172	0	0.051	2.485	0.0157	4.54

Comparison of the fatigue crack propagation rates for steels 15Kh2MFA(I), 15Kh2MFA(II) and alloy VT6S depending on the K_{max} value, which were calculated with the use of eqns (8) and (13) and obtained experimentally (Fig. 4), reveals that the calculated dependences for all the materials studied are close to the experimental ones.

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