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The Frequency Drift of the Envelope Solitary Waves Near ω_{cH^+} in Observations of Freja

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Abstract. In this paper, the ion cyclotron envelope solitary waves with a "cigar" shape observed by Freja satellite are reported and studied, especially, the drift of the frequency of the waves is contributed to the non-linear effects on the resonance frequency, which predicts that the frequency drift is directly proportional to the square of the amplitude of the oscillations. This result is comparable with the observations of the Freja satellite.

1. Introduction

There are plenty of different solitary structures in the lower auroral acceleration regions observed by Freja satellite. The authors discussed several cases of these solitary structures, such as Alfvén solitary waves associated with the plasma density solitons (rarefactive or compressive) and lower-hybrid envelope solitons (or called spikelets) accompanied with the plasma density cavitons [Huang *et al.*, 1995a,b]. The main purpose of the authors in these papers is to determine the eigenmodes of these solitary waves by the dependence of their criterions on the linear dispersion relation, then, to predict their frequency range, phase velocity, polarization sense, propagation direction and etc.,

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which are compared with the observations of Freja.

It is noted that the envelope solitons are observed not only close to the lower-hybrid frequency, but also sometimes in the range of the ion cyclotron waves [Huang *et al.*, 1995b], in which a new interesting phenomenon will be reported and studied in this paper. The duration of these special ion cyclotron envelope solitary waves is much longer than that of the lower-hybrid case, the rising time of these waves is very short, but the amplitude of them decays with time very slowly, so, the envelope looks like a "cigar" shape.

A recent paper of Shapiro *et al.* [1993] for the lower-hybrid spikelets also predicts the envelope with a "cigar" shape and the collapse of these spikelets in the vicinity of the low hybrid resonance from low frequency to high frequency, which is resulted from the the non-linear coupling of the lower-hybrid waves with the density perturbation at much lower frequency of the ion acoustic waves. The collapse of the lower-hybrid spikelets is also predicted by another paper [Seyler, *et al.*, 1994].

In this paper, the authors will try to explain the frequency drift of the ion cyclotron envelope solitary waves with a "cigar" shape by a new idea.

2. Observations

It is possible to measure several wave fields simultaneously by a snapshot technique in the F4 experiment of Freja on plasma waves. In the case of low frequency (LF), there are four channels of data sampled at $4ksamples/sec$ and the snapshot is $0.75sec$ long for every $4sec$. The instruments of F4 experiments are described in detail in the paper of Holback *et al.* [1994].

A typical example of these waves observed by Freja on July 6, 1993 between 00:38:43.9 and 00:38:44.5 is shown in Fig.1. The basic plasma parameters of this event are listed in Table 1. The altitude of this event is about $1760km$. The ratio between the density of the oxygen ions and the density of ambient plasma is 50% in average, in which the pro-

portion of helium ions is neglected. The magnetic local time is about 16.0, the magnetic latitude is about 70.0.

Table 1: Plasma Parameters

α	n_α	$f_{p\alpha}$	B_0	$f_{c\alpha}$
e	$4600cm^{-3}$	$600kHz$	$25000nT$	$700kHz$
H^+	$2300cm^{-3}$	$10kHz$	$25000nT$	$380Hz$
O^+	$2300cm^{-3}$	$2.5kHz$	$25000nT$	$20Hz$

Here, $f_{p\alpha} = \frac{1}{2\pi}(\frac{4\pi n_\alpha q_\alpha^2}{m_\alpha})^{1/2}$, $f_{c\alpha} = -\frac{1}{2\pi} \frac{q_\alpha B_0}{m_\alpha c}$. n_α , q_α and m_α refer to the density, the charge and the mass with the species α of plasma particles, respectively. e , H^+ , O^+ represent electron, hydrogen ion and oxygen ion, respectively. B_0 is the ambient magnetic field strength, c is the velocity of light. Note that the plasma cyclotron frequency defined by $f_{c\alpha}$ is positive for electrons but negative for ions.

The general analysis for the eigenmode of the envelope solitons in *Huang et al.* [1995b] is still available for these ion cyclotron envelope solitary waves with a "cigar" shape, which is also corresponding to the plasma density cavitons (Fig.1). However, besides their "cigar" shape, there is an important difference of these waves from the examples discussed in *Huang et al.* [1995b], it is found in Fig.1 that the characteristic frequency of these waves drifts gradually from high to low values. The power spectrum of Figure 1 is shown, in which the frequency drifts from about 800 Hz to 200 Hz.

3. The Frequency Drift in Non-linear Resonance

It is shown that the non-linear effects will produce a drift of the resonance frequency, which depends on the amplitude of the oscillation and is very important difference between the non-linear and linear resonance [Pippard, 1978; Putterman, 1989]. It is also predicted that this frequency drift is very evident at low frequencies [*Huang et al.*, 1995c], which may explain the frequency drift of the ion cyclotron envelope solitary waves with

a "cigar" shape.

The typical equation for the non-linear resonance is written as:

$$\ddot{x} + \omega_0^2 x + ax^2 + bx^3 = 0, \quad (1)$$

Here, a single harmonic vibration is only considered, the coefficients of the non-linear terms a and b are derived in *Huang et al.* [1995b]. If the perturbation theory is used:

$$x = x_1 + x_2 + x_3, \quad (2)$$

and the first order of the non-linear solution is:

$$x_1 = B \cos \omega_0 t. \quad (3)$$

Then, the second and third orders of the solution can be derived:

$$x_2 = -\frac{aB^2}{2\omega_0^2} + \frac{aB^2}{6\omega_0^2} \cos 2\omega_0 t, \quad (4)$$

$$\ddot{x}_3 + \omega_0^2 x_3 = B^3 \left(\frac{b}{4} - \frac{a^2}{6\omega_0^2} \right) \cos 3\omega_0 t + B^3 \left(\frac{5a^2}{6\omega_0^2} - \frac{3b}{4} \right) \cos \omega_0 t. \quad (5)$$

It is evident that the the second term at the right side of Eq.(5) will make the solution (x_3) diverge with time, which means that the perturbation theory may not be used for long time evolution in this case. To overcome this problem, the resonance frequency is supposed to drift from original value:

$$x_1 = B \cos \omega t. \quad (6)$$

$$\omega = \omega_0 + \omega_1 + \omega_2. \quad (7)$$

Hence, the equation for the second order of the solution is written as:

$$\ddot{x}_2 + \omega_0^2 x_2 = -ax_1^2 - 2\frac{\omega_1}{\omega_0}\ddot{x}_1. \quad (8)$$

Because the second term at the right side also makes the solution (x_2) diverge with time, it is necessary to choose:

$$\omega_1 \equiv 0. \quad (9)$$

Then, the equation for the third order of the solution is written as:

$$\ddot{x}_3 + \omega_0^2 x_3 = B^3\left(\frac{b}{4} - \frac{a^2}{6\omega_0^2}\right)\cos 3\omega_0 t + B\left(2\omega_0\omega_2 + \frac{5a^2B^2}{6\omega_0^2} - \frac{3bB^2}{4}\right)\cos \omega_0 t. \quad (10)$$

The drift of the resonance frequency (ω_2) is chosen as following:

$$\omega_2 = \left(\frac{3b}{8\omega_0} - \frac{5a^2}{12\omega_0^3}\right)B^2, \quad (11)$$

to make the second term at the right side of Eq.(10) equal zero.

4. Discussions

(1) As pointed by the authors, the drift of the resonance frequency in Eq.(11) depends on the amplitude of the oscillation (directly proportional to B^2) and is very evident at low frequencies (inversely proportional to the resonance frequency). Because the frequency drift of the ion cyclotron envelope solitary waves with a "cigar" shape is

observed from high to low values with the decay process of their amplitude (Fig.1-2), it is possible to be interpreted by this model.

(2) To estimate the value of the frequency drift, the third order of the non-linear effect is considered at first, because the envelope soliton is predicted by the cubic non-linear Schrödinger equation [Chang, 1993]. The coefficients of the third order of the non-linear current is derived in Huang *et al.* [1995b]:

$$b = \frac{V_0^2}{c^2} \sum_{\alpha} \frac{\omega_{p\alpha}^2}{(\omega^2 - \omega_{c\alpha}^2)^2} \frac{q_{\alpha}^2}{m_{\alpha}^2}. \quad (12)$$

Here, V_0 and c is the phase velocity of the waves and the light, respectively. α represents the species of the particles in plasma, $\omega_{p\alpha}$, $\omega_{c\alpha}$, q_{α} and m_{α} are the plasma and the cyclotron frequencies, the charge and mass for the species α , respectively. The relative frequency drift can be estimated from Eq.(5) and (6) (the second order of non-linear current is neglected):

$$\frac{\omega_2}{\omega_0} \approx \frac{3bB^2}{8\omega_0^2}. \quad (13)$$

From Eq.(12)-(13) and the plasma parameters in Table 1 ($B \sim 10^{-3}V/m$, $V_0 \sim V_A \sim 0.01c$), the order of magnitude of the relative frequency drift is $\frac{\Delta\omega}{\omega_0} \sim 10^0$, so the value of the frequency drift (ω_2) is comparable with the resonance frequency (ω_0) itself. In the typical example of the observations of Freja satellite (Fig.1), the frequency drifts is about $600Hz$ (from $800Hz$ to $200Hz$), in the meantime, the amplitude of the oscillation decreases from $1.5mV/m$ to $1.1mV/m$, which means that the relative frequency drift is approximately proportional to the square of the amplitude. There are also several other similar examples in the observations of Freja, which can support this theoretical result.

(3) Another interesting property found in Fig.1 is the frequency of the waves drifts at last close to a very special value: $\frac{1}{2}\omega_{cH+}$, which is characteristic frequency of the electromagnetic ion cyclotron waves observed by Freja and other satellites and may be

resulted from the non-linear parametric resonance [Huang *et al.*, 1995c].

(4) Therefore, the non-linear effects may produce a drift from the resonance frequency, which is shown in both of theory and observations of Freja. Another important effect is that the non-linear terms is the necessary condition for the saturation of non-linear resonance [Huang *et al.*, 1995c], so and for the other solitary waves, such as Alfvén solitary waves (Huang *et al.*, 1995b), and low-hybrid envelope solitons [Huang *et al.*, 1995a]. This result means that these solitary waves may be relative to the non-linear resonance.

(5) The general mechanism and the eigenmode of the envelope solitons is discussed in Huang *et al.* [1995b], which predicts that the width of the solitons is very sensitive to the value of $\frac{d^2\omega}{dk^2}$ (ω and k are the frequency and wave vector of the waves) as well as the phase velocity of the waves, hence, the "cigar" shape may be resulted from the effects of the non-uniformity of the plasma parameters on the property of the dispersion curve, which will be studied further.

5. Summary

In this paper, the ion cyclotron envelope solitary waves with a "cigar" shape observed by Freja satellite are reported and studied, especially, the drift of the frequency of the waves is contributed to the non-linear effects on the resonance frequency, which predicts that the frequency drift is directly proportional to the square of the amplitude of the oscillations. This result is comparable with the observations of the Freja satellite.

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Fig.1: The typical example of the ion cyclotron envelope solitary waves with a "cigar" shape observed by Freja on July 6, 1993 between 00:38:43.9 and 00:38:44.5.