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Nonlinear Structures in Space and Laboratory Plasmas

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Abstract. Coherent nonlinear structures in the form of vortical motions are frequently observed in laboratory devices and in auroral plasmas. Our objective here is to suggest that any realistic description of plasma vortices must involve three-dimensional nonlinear wave models, which account for the nonlinear coupling between finite amplitude electrostatic and/or electromagnetic disturbances. Specifically, we shall discuss properties of various types of vortex structures that are associated with nonlinear convective cells, drift and interchange modes as well as with nonlinear inertial Alfvén waves. We hope that the present investigation shall provide a guideline for understanding the properties of various nonlinear structures in space and laboratory plasmas.

1. INTRODUCTION

In the past, there has been much interest in studying different aspects of vortical motions in magnetized plasmas. Thus, attention has been focused on the generation and the dynamics of various types of vortices involving nonlinear low-frequency electrostatic and electromagnetic disturbances. Recently, observations from both space [1] and laboratory [2] plasmas have conclusively exhibited the signature of coherent nonlinear structures in the form of monopolar, dipolar, as well as chains of vortices. Accordingly, it is of much importance to have appropriate models which can describe the salient features of these nonlinear structures.

In this paper, we shall discuss nonlinear electrostatic and electromagnetic wave models that exhibit various types of stationary vortex solutions which resemble the observed structures. Specifically, we shall focus on the stationary solutions of the nonlinear equations that govern the dynamics of weakly interacting convective cells, drift waves and interchange modes as well as inertial Alfvén waves.

2. DERIVATION OF NONLINEAR EQUATIONS

We consider a nonuniform electron-ion plasma embedded in an external magnetic field $B_0 \hat{\mathbf{z}}$. The equilibrium density gradient $d_x n_0$ is along the x axis, where n_0 is the unperturbed plasma number density. In the presence of a low-frequency (in comparison with the ion gyrofrequency $\omega_{ci} = eB_0/m_i c$, where e is the magnitude of the electron charge, m_i is the ion mass, and c is the speed of light), long-wavelength (in comparison with the ion gyroradius) electrostatic field $\mathbf{E} = -\nabla\phi$ (ϕ is the electrostatic potential), the perpendicular components of the electron and ion fluid velocities are given by, respectively,

$$\mathbf{v}_{e\perp} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla\phi - \frac{c}{eB_0 n_e} \hat{\mathbf{z}} \times \nabla p_e, \quad (1)$$

and

$$\mathbf{v}_{i\perp} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla \phi + \mathbf{v}_g - \frac{c}{B_0 \omega_{ci}} (\partial_t + \mu_i \nabla_\perp^2 + \mathbf{v}_i \cdot \nabla) \nabla_\perp \phi, \quad (2)$$

where $p_e = n_e T_e$ is the electron pressure, $n_e(T_e)$ is the electron number density (electron temperature), $\mathbf{v}_g = -(g/\omega_{ci})\hat{\mathbf{y}} \equiv v_G \hat{\mathbf{y}}$ is the equilibrium ion drift caused by the gravitational field $g\hat{\mathbf{x}}$, $\mu_i = 0.3\nu_i \rho_i^2$ is the ion gyro-viscosity, ν_i is the ion collision frequency, and ρ_i is the ion gyroradius. For convenience, the electron temperature is assumed to be much larger than that of the ions.

First, we consider the dynamics of two-dimensional convective cells in a uniform plasma. Thus, inserting (1) and (2) into the charge conservation equation, and making use of Poisson's equation, we obtain [3]

$$(\partial_t + \eta_i \nabla_\perp^2) \nabla_\perp^2 \phi + \frac{c}{B_0} J(\phi, \nabla_\perp^2 \phi) = 0, \quad (3)$$

where $\eta_i = \mu_i/(1 + \omega_{ci}^2/\omega_{pi}^2)$, $J(a, b) = (\partial a/\partial x)(\partial b/\partial y) - (\partial a/\partial y)(\partial b/\partial x)$ represents the vector nonlinearity, and ω_{pi} is the ion plasma frequency. We note that (3) resembles the Navier-Stokes equation.

Second, when the electron motion is considered to be pseudo-three-dimensional, then for $|\partial_t| \ll v_{te} \partial_z$, where $v_{te} = (T_e/m_e)^{1/2}$, the electrons rapidly thermalize along the $\hat{\mathbf{z}}$ axis so that the corresponding electron number density perturbation is given by

$$n_{e1} = n_0 e \phi / T_e, \quad (4)$$

Substituting for $\mathbf{v}_{i\perp}$ from (2) in the ion continuity equation and imposing the quasi-neutrality condition $n_{i1} = n_{e1}$, we obtain on using (4) the Hasegawa-Mima (HM) equation [4]

$$\partial_t(\phi - \rho_s^2 \nabla_\perp^2 \phi) - \frac{c}{B_0} \rho_s^2 J(\phi, \nabla_\perp^2 \phi) = \mu_i \rho_s^2 \nabla_\perp^4 \phi, \quad (5)$$

where $\rho_s = c_s/\omega_{ci}$ is the ion Larmor radius at the electron temperature, and the ion motion along the $\hat{\mathbf{z}}$ direction has been neglected. In the absence of the nonlinear interaction, (5) gives the viscous damping rate of pseudo-three-dimensional convective cells in a hot magnetoplasma.

Third, we consider nonlinear electrostatic drift wave motions in the presence of the equilibrium density inhomogeneity in a plasma embedded in a homogenous magnetic field. The relevant nonlinear equations for our purposes are then derived by substituting (2) into the ion continuity equation and employing the quasi-neutrality condition with (4). The result is [5, 6]

$$\partial_t(\phi - \rho_s^2 \nabla_\perp^2 \phi) + V_* \partial_y \phi - \frac{c}{B_0} \rho_s^2 J(\phi, \nabla_\perp^2 \phi) + \frac{T_e}{e} \partial_z v_{iz} = \mu_i \rho_s^2 \nabla_\perp^4 \phi, \quad (6)$$

where the parallel (to $\hat{\mathbf{z}}$) component of the ion fluid velocity is given by

$$d_t v_{iz} = -\frac{e}{m_i} \partial_z \phi, \quad (7)$$

where $d_t = \partial_t + (c/B_0)\hat{\mathbf{z}} \times \nabla \phi \cdot \nabla$, and where we have denoted $V_* = \rho_s^2 \omega_{ci}/L_n$ and $L_n = -n_0/d_x n_0$. In the absence of the ion gyroviscosity and the parallel ion motion, the HM equation with the density gradient term $V_* \partial_y \phi$ in (6) resembles the Charney equation [7], which governs the dynamics of geophysical flows.

Fourth, in the presence of a gravitational force, the equations governing the dynamics of three-dimensional electrostatic interchange modes are obtained by substituting (1) and (2) into the electron and ion continuity equations. We obtain [8]

$$d_t(n_0 + n_1) + n_0 \partial_z v_{ez} = 0, \quad (8)$$

and

$$(d_t + v_G \partial_y) \left[(n_0 + n_1) - \frac{n_0 c}{B_0 \omega_{ci}} \nabla_\perp^2 \phi \right] = 0, \quad (9)$$

where $n_1 (<< n_0)$ is the quasi-neutral density perturbation, and the electron and ion motions along the external magnetic field lines have been ignored. The parallel component of the electron fluid velocity is given by

$$d_t v_{ez} = \frac{e}{m_e} \partial_z \phi, \quad (10)$$

where we have assumed that $(c/B_0)\hat{\mathbf{z}} \times \nabla \phi \gg v_{ez}\partial_z$.

Finally, we turn our attention to low-frequency (in comparison with ω_{ci}) electromagnetic disturbances in a uniform magnetoplasma with $8\pi n_0 T_e/B_0^2 \ll m_e/m_i$. Here, the electromagnetic fields are given by $\mathbf{E} = -\nabla \phi - c^{-1} \partial_t A_z$ and $\mathbf{B}_{1\perp} = \nabla A_z \times \hat{\mathbf{z}}$, where A_z is the parallel component of the vector potential. It turns out that for nonlinear electromagnetic perturbations, the perpendicular component of the electron fluid velocity will have an additional term $v_{ez} \mathbf{B}_{1\perp}/B_0$ on the right-hand side of (1), where $v_{ez} \approx (c/4\pi n_0 e) \nabla_{\perp}^2 A_z$. The equation for the conservation of the charge density is then [3]

$$d_t \nabla_{\perp}^2 \phi + \frac{v_A^2}{c} d_z \nabla_{\perp}^2 A_z = 0, \quad (11)$$

where $v_A = B_0/(4\pi n_0 m_i)^{1/2}$ is the Alfvén velocity, and $d_z = \partial_z + (\mathbf{B}_{1\perp} \cdot \nabla)/B_0$.

The parallel component of the electron momentum equation can be written as [3]

$$d_t (A_z - \lambda_e^2 \nabla_{\perp}^2 A_z) + c \partial_z \phi = 0, \quad (12)$$

where $\lambda_e = c/\omega_{pe}$ is the collisionless electron skin depth. Equations (11) and (12) form a pair which governs the nonlinear dynamics of inertial Alfvén waves the phase velocity of which is much larger than the electron thermal velocity.

3. VORTEX SOLUTIONS

The steady-state solution of both the Navier-Stokes and the Charney-HM equation in an inviscid medium is obtained by setting $\partial_t = 0$ in (3) and (5). Thus, the equation governing the steady state nonlinear disturbances can be expressed as

$$J(\phi, \nabla_{\perp}^2 \phi) = 0. \quad (13)$$

Equation (13) is satisfied by

$$\nabla_{\perp}^2 \phi = F(\phi), \quad (14)$$

where $F(\phi)$ is any function of ϕ . On choosing $F(\phi) = -(\phi_0/4)(a^2 - b^2) \sinh(4\phi/\phi_0)$, $b \leq a$, where a, b and ϕ_0 are arbitrary constants, we find that the "sinh-Poisson" equation has an exact solution [9]

$$\phi(x, y) = \phi_0 \operatorname{arctanh} \left[\frac{b \cos(ay)}{a \cosh(bx)} \right], \quad (15)$$

which represents a row of counter-rotating vortices, i.e. a row of vortices of alternating sign. On the other hand, the choice $F(\phi) = (1 - \epsilon^2) \exp(-2\phi)$ gives the Liouville equation

$$\nabla_{\perp}^2 \phi = (1 - \epsilon^2) \exp(-2\phi), \quad (16)$$

where $|\epsilon| < 1$. Equation (16) admits an infinite row of identical Stuart's vortices along the x axis, given by

$$\phi(x, y) = \log(\cosh y - \epsilon \cos x). \quad (17)$$

When $\epsilon = \pm 1$, the potential ϕ represents an infinite row of point vortices and when $\epsilon = 0$ it represents an unperturbed *tanh* mixing layer, viz. $\phi = \log \cosh y$.

Let us now turn our attention to the stationary solutions of the inhomogeneous HM equation including the parallel ion dynamics. We introduce a new variable $\xi = y + \alpha z - ut$, where α and u

are constants, and assume that the wave potential and the parallel ion fluid velocity perturbations are functions of x and ξ only. Thus, in the stationary frame, Eqs. (6) and (7) can be combined in the form

$$D_\xi(\nabla_\perp^2 \phi - \delta \phi) = 0, \quad (18)$$

where $D_\xi = \partial_\xi - (c/uB_0)(\partial_x \phi \partial_\xi - \partial_\xi \phi \partial_x)$ and $\delta = [1 - (V_*/u) - c_s^2 \alpha^2 / u^2] / \rho_s^2$.

For $\delta = 0$, (18) is satisfied by the ansatz

$$\nabla_\perp^2 \phi = \frac{4AK^2}{a^2} \exp \left[-\frac{2}{A} \left(\phi - \frac{uB_0}{c} x \right) \right], \quad (19)$$

where A, K , and a are arbitrary constants. The solution of (19) is given by

$$\phi = \frac{uB_0}{c} x + A \ln [2 \cosh(Kx) + 2(1 - 1/a^2) \cos(Ky)], \quad (20)$$

which exhibits chains of vortices for $a^2 > 1$. For $a = 1$, we have zonal flow.

On the other hand, for $\delta > 0$ and constant value of L_n , (19) admits both dipolar and monopolar vortices. For the latter, we have $\nabla_\perp^2 \phi = \delta \phi$, which exhibits a circular vortex with a monotonic profile for $\delta > 0$. Furthermore, the dipolar vortex solution [11] of (18) is

$$\phi_{out} = \phi_0 K_1(\lambda_1 r) \cos \theta, \quad (21)$$

in the outer region defined by $r = (x^2 + \xi^2)^{1/2} > R$, where R is the vortex radius, ϕ_0 is a constant, K_1 is the modified Bessel function, $\lambda_1 = |\delta|^{1/2}$, and $\cos \theta = x/r$. In the inner region ($r < R$), the solution reads

$$\phi_{in} = [\phi_i J_1(\lambda_2 r) + C_{2i} r / \lambda_2^2] \cos \theta, \quad (22)$$

where ϕ_i is a constant, J_1 is the Bessel function of the first order, $C_{2i} = (\lambda_1^2 + \lambda_2^2) u B_0 / c$, and the constant λ_2 has to be determined by the transcendental equation

$$K_2(\lambda_1 R) / \lambda_1 K_1(\lambda_1 R) = -J_2(\lambda_2 R) / \lambda_2 J_1(\lambda_2 R), \quad (23)$$

where J_2 and K_2 are the Bessel function and the modified Bessel function of the second order, respectively.

It is interesting to note that the inhomogeneous HM equation also admits a global vortex pattern [3, 12] when the density gradient is non-constant in a bounded plasma cylinder. Assuming that the equilibrium plasma number density has a gaussian distribution, namely $n_0(r) = n_0 \exp(-r^2/r_0^2)$, where n_0 and r_0 are constants, the inviscid inhomogeneous HM equation in cylindrical coordinates r, θ, z has the form

$$\partial_t(\Phi - \nabla_\perp^2 \Phi) + (U/r) \partial_\theta \Phi - J_r(\Phi, \nabla_\perp^2 \Phi) = 0, \quad (24)$$

where $\Phi = e\phi/T_e$, $U = 2/r_0^2 = \text{const.}$, and the time and radial variables are in the units of ω_{ci}^{-1} and ρ_s , respectively. The Jacobian in cylindrical co-ordinates is written as $J_r(\Phi, \nabla_\perp^2 \Phi) = (1/r)(\partial_r \Phi \partial_\theta \nabla_\perp^2 \Phi - \partial_\theta \Phi \partial_r \nabla_\perp^2 \Phi)$.

Let us look for a nonlinear structure in the frame $\eta = \theta - \Omega t$. Thus, setting $\partial_t = -\Omega \partial_\theta$ in (24) it can be shown that it can be satisfied by the Ansatz

$$\nabla_\perp^2 \Phi = -\lambda_r^2 \Phi + C_r r^2 + \text{const.}, \quad (25)$$

where $C_r = [(1 + \lambda_r^2)\Omega - U/r]/2$ and λ_r is a constant. A solution of (25) is

$$\Phi(r, \eta) = \Phi_m J_m(\lambda_r r) \cos(m\eta) + C_r(r^2/\lambda_r^2) + C_i, \quad (26)$$

where Φ_m is the amplitude of the vortex, $m = 1, 2, 3, \dots$ is the azimuthal number, and C_i is a constant, to be determined later.

Imposing the boundary conditions ($\Phi = 0$ and $\partial_\eta = 0$) near the center (at $r = 0$) and at the boundary of the plasma column $r = R_0$, we note that we must have $m \geq 1$, $C_i = -C_r(R_0^2/\lambda_r^2)$, and $\lambda_r R_0 = k_n$, where k_n are the zeroes of the Bessel function J_m . We see that the global vortices contain higher harmonics in terms of the azimuthal wavenumbers, and their localization results from the actual boundary condition at the edge of the plasma column. The azimuthal wavenumber determines the number of vortex pairs in the structure. The vortex amplitude Φ_m is determined by the flow velocity at any point within the vortex.

We note that the nonlinear equations (8) to (10) also admit a dipolar vortex and a vortex chain, as possible stationary states of electrostatic interchange mode turbulence.

Finally, we discuss the nonlinear stationary solutions of (11) and (12) in the moving frame, so that we can cast them in the form

$$D_\xi \nabla_\perp^2 \phi = \frac{\alpha v_A^2}{cu} D_{\xi a} \nabla_\perp^2 A_z, \quad (27)$$

and

$$D_\xi \left[(1 - \lambda_e^2 \nabla_\perp^2) A_z - \frac{c\alpha}{u} \phi \right] = 0, \quad (28)$$

where $D_{\xi a} = \partial_\xi - (1/\alpha B_0)(\partial_x A_z \partial_\xi - \partial_\xi A_z \partial_x)$.

Equation (28) can be satisfied by the Ansatz

$$(1 - \lambda_e^2 \nabla_\perp^2) A_z = (c\alpha/u) \phi. \quad (29)$$

Hence, from (27) and (29), we obtain [3]

$$\nabla_\perp^4 \phi + C_1 \nabla_\perp^2 \phi + C_2 \phi - \frac{F_3 u B_0}{\lambda_e^2 c} x = 0, \quad (30)$$

where $C_1 = \beta_1 - F_3 - 1/\lambda_e^2$, $C_2 = F_3/\lambda_e^2$, $\beta_1 = \alpha^2 v_A^2 / u^2 \lambda_e^2$, and F_3 is an arbitrary constant of integration.

The dipolar vortex solution of (30) is [3]

$$\phi = [Q_1 K_1(s_1 r) + Q_2/r] \cos \theta, \quad (31)$$

in the outer region ($r > R$), where $F_3 = 0$, and

$$\phi = \left(Q_3 J_1(s_2 r) + Q_4 I_1(s_3 r) - \frac{F_3 u B_0}{\lambda_e^2 c C_2} r \right) \cos \theta, \quad (32)$$

in the inner region ($r < R$). Here, Q_j are arbitrary constants, $s_1^2 = (\alpha^2 v_A^2 / u^2 - 1) > 0$, $s_{2,3} = [(C_1^2 - 4C_2)^{1/2} \pm C_1]/2$ for $C_2 < 0$, and I_1 is the modified Bessel function.

4. DISCUSSION

In this paper, we have presented nonlinear electrostatic and electromagnetic wave models that are capable of depicting a great variety of stationary nonlinear solutions. The latter are characterized as vortex roles, Stuart's cat's eyes, and dipolar vortices. It is suggested that the present nonlinear solutions are associated with the various types of coherent nonlinear structures that are frequently observed in space and laboratory plasmas.

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