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Crack stability in rate sensitive ceramics and their rate dependence

T. Nojima

Dept. of Aeronautical Engineering, Kyoto University, Sakyo-ku, Kyoto 606, Japan

Abstract; The crack stability condition in rate sensitive brittle materials is analysed. The analysis shows that a crack begins to extend before a maximum load (P_{max}) appears in fracture toughness (K_{Ic}) tests and the extension depends on the n -value in the $\dot{a} \propto K_I^n$ relation. This suggests that the evaluation of a true K_{Ic} -value is quite difficult. An analytical method for using the K_{Ic} -value (the initial stress intensity evaluated by the P_{max} and initial crack length) is proposed. Experimental data for an alumina ceramic and a glass are analysed by the method. The K_{Ic} - T_f relation of the glass consists of 4 regions over the range $10^{-5} \text{ s} < T_f < 10^7 \text{ s}$ where T_f is the time to failure.

1. INTRODUCTION

It has been experimentally clarified to a great extent that the fracture toughness value K_{Ic} or strength σ_0 of glasses and ceramics shows a considerable rate effect. In some oxide ceramics such as glasses and alumina ceramics [1, 2], because crack extension is affected by the stress corrosion caused by water in the environment, a trimodal curve is frequently obtained for K_{Ic} - $\dot{a}$ relation ($\dot{a}$; the crack velocity); the rate dependence generally consists of 3 regions (Region I~III). In order to interpret these different rate dependences, it is necessary not only to accumulate further precise data but to analyze the data by utilizing fracture mechanics including rate effects.

Nevertheless, the crack stability condition including the rate effect has been left undiscussed and analytical methods for the data on cracked specimens are not available. Moreover, as mentioned above, although it has been well known that the K_{Ic} - $\dot{a}$ relations for oxide ceramics, for example, are not simple, no careful data analysis has been done in determining K_{Ic} - V_0 (or σ_0 - V_0 , V_0 ; the testing speed) relations.

In the first part of the present paper, the crack-stability including the rate effect is discussed and a pseudo-stable phenomenon is explained. A new method to analyze K_{Ic} -data for different initial crack length and specimen geometry is presented. In the second part, some experimental data and analyzed results on a glass and an alumina ceramic are shown.

2. STABILITY CONDITION OF SPECIMENS WITH A CRACK

Consider the fracture toughness tests of brittle materials in a conventional testing machine. By assuming a quasi-static crack extension (kinetic energy=0) during the tests, the cross-head displacement δ -load P relation is expressed by $\delta=P(\lambda+\Lambda^*)$, where Λ^* is the compliance of the testing machine and $\lambda(A)$ is the compliance of the specimen (A ; cracked area). Putting the energy release rate $G=(P^2/2)(d\lambda/dA)$ equal to the crack resistance force R , δ and P can be expressed by

$$\delta=\sqrt{2BR}/[(d\lambda/da)\cdot\{\lambda(A)+\Lambda^*\}], \quad P=\sqrt{2BR}/[(d\lambda/da)], \quad (1)$$

where B is the thickness of the specimen and a is the crack length ($A=Ba$). By taking logarithms of both sides in Eq.(1) and differentiating with respect to the time t , the following Eq.(2) is obtained;

$$2(\dot{\delta}/\delta)=(\gamma^*-\eta^*)\dot{a}+\gamma^{**}\ddot{a}, \quad \gamma^*=(\partial R/\partial a)_\dot{a}/R, \quad \gamma^{**}=(\partial R/\partial \dot{a})_a/R, \quad (2)$$

$$\eta^*(a)=(d^2\lambda/da^2)/(d\lambda/da)-2(d\lambda/da)/(\lambda+\Lambda^*)$$

The parameter γ^* and γ^{**} are related to the material properties of the test samples, while η^* is given by the testing conditions. In crack-rate insensitive materials ($\gamma^{**}=0$), Eq.(2) becomes $2(\dot{\delta}/\delta)=(\gamma^*-\eta^*)\dot{a}$. The following equation, which gives $\dot{a}>0$, when $\dot{\delta}>0$, becomes the stability condition for crack-rate insensitive materials (Gurney and Hunt [3]);

$$\gamma^*-\eta^* > 0. \quad (3)$$

In perfectly brittle materials ($\gamma^*=0$), $\eta^*<0$ gives the condition for a stable crack extension. Fig.1 shows a numerically calculated example for P - δ curves in the materials where $\gamma^*=0$ and $\gamma^{**}=0$, for a single edge cracked 3 point bending specimen (SEC3PB, $x_0=a_0/W$, a_0 ; initial crack length and W ; specimen height, span length $S=4W$), where $\tilde{P}=P/\sqrt{R_0E'B^2W}$, $\tilde{\delta}=\delta/\sqrt{R_0W/E'}$, $E'=E/(1-\nu^2)$ with E and ν the Young's modulus and Poisson's ratio of the specimen, respectively. The points plotted in the figure are for equilibrium for a given $x=a/W$. The top point of each curve is for $x=x_0$. According to the criterion by Gurney and Hunt, the curves which show $d\delta>0$ at $x=x_0$ correspond to stable crack extension (symbol s), while the curves for $d\delta<0$ at $x=x_0$ correspond to unstable crack extension (symbol u). By their criterion, which neglects any rate effect, it has been interpreted that the load shows a sudden drop from its maximum value.

The value of γ^{**} is reported to be positive. Now we consider a test in which cross-head displacement is suddenly stopped during crack extension. Putting $\dot{\delta}=0$ in Eq.(2), Eq.(2) becomes $(\gamma^*-\eta^*)\dot{a}+\gamma^{**}\ddot{a}=0$. From this equation, we easily understand that $\dot{a}$ accelerates with t when $\gamma^*-\eta^*<0$, while $\dot{a}$ gets smaller with t when $\gamma^*-\eta^*>0$. If we define that a crack is unstable when it accelerates itself without any external work ($\dot{\delta}=0$), the stability condition for rate sensitive materials is also expressed by Eq.(3), i.e., $\gamma^*-\eta^*>0$.

In almost all the K_{Ic} tests, the critical K_{Ic} value has been evaluated by using the P_{max} in the tests. Therefore it can be reasonably accepted that the crack is stable until a maximum appears in the P - δ curve. Thus, the P_{max} condition is deduced hereafter. By differentiating P in Eq.(1) with respect to t , the following Eq.(4) is obtained;

$$2(\dot{P}/P)=\{\gamma^*-(d\lambda^2/da^2)/(d\lambda/da)\}\dot{a}+\gamma^{**}\ddot{a}. \quad (4)$$

P_{max} is given by $\dot{P}=0$. In the crack rate insensitive material ($\gamma^{**}=0$), P_{max} will appear when the following Eq.(5) is satisfied;

$$\gamma^*(a)-(d\lambda^2/da^2)/(d\lambda/da)=0 \quad (5)$$

Since $(d\lambda^2/da^2)/(d\lambda/da)>0$ in conventional specimens, P_{max} appears at $a=a_0$ in perfectly brittle materials.

On the other hand, in the materials with positive rate effect ($\gamma^{**}>0$), the following equation holds when P_{max} appears;

$$\gamma^*(a)-(d^2\lambda/da^2)/(d\lambda/da)+\gamma^{**}(\ddot{a}/\dot{a})=0 \quad (6)$$

From Eq.(2), $\ddot{a}$ becomes positive when the instability condition $\gamma^* - \eta^* < 0$ is satisfied, since $\delta/\dot{\delta} > 0$ (from testing condition), $\gamma^{**} > 0$ and $\dot{a} > 0$. By putting $\ddot{a} > 0$ in Eq.(5), the third term becomes positive. The crack length which gives P_{max} in crack-rate insensitive materials is denoted as a_{max}^0 (solution value of Eq.(5)), while the crack length in rate sensitive materials is denoted as a_{max}^* (solution value of Eq.(6)). The comparison between Eqs.(5) and (6) indicates that a_{max}^* is larger than a_{max}^0 . This can be more easily understood by the schematic illustration in Fig.2, where a method to determine these values is shown. Since the value of $\gamma^{**}(\ddot{a}/\dot{a}) = h(a)$ in Eq.(6) is positive, $a_{max}^* > a_{max}^0$ obviously holds as far as $\ddot{a} > 0$. When the specimen geometry, the testing condition and also the rate dependence of R are given, $\dot{a}$ and $\ddot{a}$ in Eq.(6) will be calculated as a function of a and a_{max}^* can be determined (see next section).

Fig.1 P - δ curves for perfectly brittle, crack-rate insensitive materials ($\gamma^*=0$, $\gamma^{**}=0$)

Fig.2 Schematic illustration for determining a_{max}^* and a_{max}^0

3. NUMERICAL CALCULATIONS FOR P - δ RELATIONS AND CRACK EXTENSION Δx BEFORE P_{max}

The $\dot{a}$ - K_I relations in glasses and ceramics are often expressed by a power relation as $\dot{a} \propto (K_I)^n$ (n =constant). By using $G=R=K_I^2/E'$, it follows that $R \propto \dot{a}^{2/n}$. Therefore, in the following calculations, Eq.(7) is adopted as constitutive relations for these materials.

$$R = R_1(a) \cdot (\dot{a}/\dot{a}^*)^{2/n}, \quad (\dot{a}/\dot{a}^*) = (K_I/K_I^*)^n \quad (\dot{a}^* \text{ and } K_I^* \text{ are constants}). \quad (7)$$

In constant velocity tests in a conventional testing machine, i.e., $\delta = V_0 t$, by substituting R in Eq.(7) into Eq.(1), and also putting $\delta_0(a) = \sqrt{2BR(a)/(d\lambda/da)} \cdot \{\lambda(a) + \Lambda^*\}$, (δ_0 =the δ for $\gamma^{**}=0$), we find

$$\delta^2 = \{\delta_0(a)\}^2 (\dot{a}/\dot{a}^*)^{2/n} = (V_0 t)^2. \quad (8)$$

By integrating Eq.(8), $P = \sqrt{2BR_1/(d\lambda/da)} \cdot (\dot{a}/\dot{a}^*)^{1/n}$ becomes

$$P(a) = \sqrt{2BR_1/(d\lambda/da)} [V_0(1+n)/\dot{a}^*]^{1/(1+n)} \left\{ \int_{a_0}^a \delta_0^n da \right\}^{1/(1+n)} / \delta_0. \quad (9)$$

The δ is given by $\delta(a) = P(\lambda + \Lambda^*)$. By taking the logarithm and differentiating Eq.(9), it is found

$$2 \frac{(dP/dt)}{P} = \left[\frac{(dR_1/da)}{R_1} - \frac{(d\lambda/da)}{(d\lambda/da)} \right] + \left[\frac{2}{(1+n)} \frac{\delta_0^n}{\int_{a_0}^a \delta_0^n da} - 2 \frac{(d\delta_0/da)}{\delta_0} \right] \dot{a}. \quad (10)$$

Since $2(d\delta_0/da)/\delta_0 = \gamma^* - \eta^*$, the third term of the right hand side of Eq.(10) becomes positive, when $\gamma^* - \eta^* < 0$, and it corresponds to $\gamma^{**}(\ddot{a}/\dot{a})$ in Eq.(6). By the numerical calculations, it was also clarified that $\ddot{a}$ is positive at least up to P_{max} even when $\gamma^* - \eta^* > 0$.

Fig.3 shows the numerically calculated P - δ curves for SEC3PB ($\gamma^*=0$). Calculated P -values are plotted at an interval of $\Delta x = 0.01$. In the figure, it can be seen that a crack begins to extend before P_{max} . The phenomenon of a load increase when the crack continues to extend is named pseudo-stable phenomenon (PSP). The crack extension (Δx) appears more remarkably as the n -value gets smaller or EBA^* becomes larger. The P_{max} value, which is commonly utilized to evaluate K_{IC} (or K_{ICG}), also depends on the testing condition EBA^* . The dependence comes to appear more remarkably as the n -value becomes smaller. As an important consequence, it is argued that attention should be paid in checking the crack extension before the K_{IC} -value is calculated from the maximum load. In the above discussion on the crack stability in rate sensitive brittle materials, the stable/unstable condition was interpreted by

considering whether a crack is accelerated or not when no external work is supplied. However it will be also natural to consider that a crack is stable up to the maximum load. From this point of view, it can be said that a crack in rate sensitive materials becomes more stable ($\Delta a = a_{max} - a_0$) than that of rate insensitive materials.

By putting $\dot{P}=0$ in Eq.(10), the critical crack length (x_{max}^*) at P_{max} can be calculated numerically. The amount of the crack extension, $\Delta x = \Delta a/W$, is shown in Fig.4 ((A) and (B); for SEC3PB, (C); for single edge cracked tensile specimen, SECT). The figure shows two important facts; (1) the Δx -value is a function of x_0 and it decreases with x_0 and (2) the Δx -value depends on the specimen geometry and the value in SECT is larger than the one in SEC3PB. These facts suggest that in perfectly brittle materials, the values of K_{Ii} (the initial stress-intensity), as conventionally evaluated by utilizing P_{max} and a_0 values, will inevitably increase with x_0 and the K_{Ii} -value in SEC3PB will be larger than the one in SECT. The analyzed predictions have been clarified in a soda-lime glass and an alumina ceramic by using these specimens [4].

Fig.3 Simulated P- δ relations for crack-rate sensitive materials showing pseudo-stable phenomenon (PSP) ($\gamma^*=0$, $\gamma^{**}>0$, SEC3PB)

Fig.4 Non-dimensional crack-extension before P_{max} as a function of EBA^*

4. A NEW ANALYTICAL METHOD FOR EXPERIMENTAL DATA ON RATE SENSITIVE-BRITTLE MATERIALS WITH DIFFERENT INITIAL CRACK LENGTHS

Experimental data for delayed failure in ceramics have been analyzed by a simple formula presented by Evans [2]. The formula can not be applied directly to the data for large initial cracks ($x_0 > 0.1$) [5]. Thus a new method of data analysis for any crack length is presented in this section.

By expressing the non-dimensional compliance of the specimen as $\tilde{\lambda} = \lambda / (2/E'B)$ and substituting this relation into $K_I^2/E' = \{P(t)^2/2B\} \{d\lambda/da\}$, K_I value becomes $K_I = P(t)/(B\sqrt{W}) \cdot \sqrt{d\tilde{\lambda}/dx}$. By substituting the K_I into Eq.(7), the following relation is obtained;

$$(\dot{a}/\dot{a}^*) = \{P(t)/(B\sqrt{WK_I^*})\}^n \{d\tilde{\lambda}/dx\}^{n/2} \quad (11)$$

By putting $\dot{a} = W\dot{x}$ and integrating Eq.(11), Eq.(11) becomes Eq.(12).

$$\int_{x_0}^x (d\tilde{\lambda}/dx)^{-n/2} dx = (\dot{a}^*/W) / (B\sqrt{W} \cdot K_I^*)^n \cdot \int_0^t \{P(t)\}^n dt. \quad (12)$$

By putting $P=\alpha t$ (α =constant) in constant loading rate (CLR) tests (conventional test), the integration of the right hand side in Eq.(12) becomes $(P_{max})^n \cdot T_f^*/(n+1)$ (T_f^* : the time to the maximum load, $P_{max}=\alpha T_f^*$), while in constant load (CL) test, it becomes $P_0^n T_f$. The time to failure in this test is denoted as T_f .

By defining that "a specimen fractures when $x=1$ ", Eq.(12) becomes

$$\int_{x_0}^1 (\tilde{d}\lambda/dx)^{-n/2} dx = (\dot{a}^*/W) (\tilde{P}_{max})^n \{T_f^*/(n+1)\} \quad \text{for CLR tests,} \quad (13)$$

$$\int_{x_0}^1 (\tilde{d}\lambda/dx)^{-n/2} dx = (\dot{a}^*/W) (\tilde{P}_0)^n \cdot T_f \quad \text{for CL tests.}$$

The initial stress intensity K_{Ii} is expressed by $K_{Ii}=P_0/(B\sqrt{W}) \cdot \sqrt{x_0} F(x_0)$, where F is the geometry factor of the specimen, and P_0 is P_{max} or P_0 in the CLR and CL tests, respectively. By introducing this equation into Eq.(13), and also putting $(d\lambda/dx)=x F^2$ (see Appendix), the following equation is obtained;

$$(W/\dot{a}^*) \int_{x_0}^1 \{ \sqrt{x_0} F(x_0) / \sqrt{x} F(x) \}^n dx = (K_{Ii}/K_{If}^*) \hat{T}_f. \quad (14)$$

where $\hat{T}_f$ is $T_f^*/(n+1)$ or T_f in the CLR and CL tests, respectively.

Eq.(14) indicates that the K_{Ii} -values for different initial crack lengths (x_0), the specimen geometry (F) and the size (W) can be inclusively expressed by the following new parameter (non-dimensional normalized time to failure)

$$(\tilde{T}_f)_N = \{ \hat{T}_f / (W/\dot{a}^*) \} I^{-1}, \quad I = \int_{x_0}^1 \left\{ \frac{\sqrt{x_0} \cdot F(x_0)}{\sqrt{x} \cdot F(x)} \right\}^n dx \quad (15)$$

In the above analysis, it was assumed that the K_{Ii} - $\dot{a}$ relation is given by a single power relation. However the K_{Ii} - $\dot{a}$ relations in oxide ceramics often show trimodal curves [1,2] (Fig.5). The figure shows 4 regions including Region 0 for the threshold of $K_{Ii}(K_{If}^*)$. A sharp gap in K_{Ii} appears in Region II (the crack velocity of Region II is about 10^{-3} m/s for an alumina and about $10^{-3.5}$ m/s for a soda-lime glass in an ambient environment).

K_{Ii} - T_f relations were simulated by using the K_{Ii} - $\dot{a}$ relation in Fig.5 ($n_0=\infty$, $n_1=20$, $n_2=0$ and $n_3=20$) [5]. The simulated results shown in Fig.6 clarify that the K_{Ii} - T_f relations depend on the x_0 -value, and the sharp gap appears as a gentle curve in the K_{Ii} - T_f relation. However, if we adopt the normalized time to failure $(\tilde{T}_f)_N$, it is seen in Fig.7 that the K_{Ii} -values can be expressed by a unique function of $(\tilde{T}_f)_N$. When the n -values are different in Region I and III, the experimental data for delayed failure can be analyzed by using their respective values [5].

Fig.5 Schematic illustration of K_{Ii} - $\dot{a}$ relation of oxide ceramics showing 4 regions

Fig.6 $\log K_{Ii}$ - $\log T_f$ relations for various kinds of initial crack length (simulated, SEC 3PB)

Fig.7 $\log K_{Ii}$ - $\log (\tilde{T}_f)_N$ relation ($(\tilde{T}_f)_N$; normalized time to failure, simulated, SEC3PB)

5. EXPERIMENTAL DATA AND THEIR ANALYSIS

Because a crack begins to extend before P_{max} in rate-sensitive materials, it is quite difficult to evaluate true K_{Ii} -values. Thus the experimental data will be analyzed by the method for K_{Ii} in Section 3.

Fracture toughness tests of an alumina ceramic (96% alumina, $B=1\text{mm}$, $W=8\text{mm}$, Kyocera) and a soda-lime glass ($B=1.8\text{mm}$, $W=19\text{mm}$) were carried out on about 250 SEC3PB specimens at $V_0=0.005\text{--}5\text{mm/min.}$, and at two kinds of slower loading rate by using a coil spring installed in loading bar. The obtained K_{Ii} - T_f^* relations are shown in Fig.8; a gentle gap appears at $T_f^* \approx 100\text{s}$. The bending strength σ_0 of the

specimens with a Vickers flaw (alumina; 98N, 15s, glass; 9.8N, 30s) is shown in Fig.9 as a function of T_f^* ($\sigma_0 = 3P_{max}S/(2BW^2)$), alumina; $B=1$, $W=3\text{mm}$, glass; $B=1.8$, $W=3\text{mm}$). The figure shows that the $\sigma_0 - T_f^*$ relations have a gap at the smaller value of T_f^* (about 10s in alumina, and 7s in glass); in the specimens with a small flaw, the $\sigma_0 - T_f^*$ (or $K_{Ii} - T_f^*$) curve shifts to a shorter time.

In Fig.10, the K_{Ii} -value of the glass is shown as a function of $(\tilde{T}_f)_N$, where $\alpha_0 = 30\mu\text{m}$ is taken as an equivalent initial crack length in the specimens with the Vickers flaw, and the experimentally determined n -values, $n_1=16$ and $n_3=40$ are used for the calculation of $(\tilde{T}_f)_N$. Experimental data in constant load tests are also shown in the figure. The figure suggests that the K_{Ii} -values can be expressed by a unique function of $(\tilde{T}_f)_N$ and the $K_{Ii} - (\tilde{T}_f)_N$ relation consists of 4 regions.

6. CONCLUSIONS

By analyzing the stability condition including rate effects, it was clarified that a crack begins to extend before P_{max} appears in conventional fracture toughness tests. The amount of the extension depends on the n -value in the $\dot{a} - (K_I)^n$ relation. A new method to analyze the failure data of the specimens with a crack was presented. Some fracture toughness and bending data in an alumina ceramic and a glass showed that a clear bend appears in the $K_{Ii} - T_f^*$ relation, and the time to failure becomes shorter as the initial flaw (crack) gets smaller. By using a newly presented parameter $(\tilde{T}_f)_N$, the data can be well expressed by a unique function of the parameter.

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Appendix

The compliance of single edge cracked 3 point bending specimen;

$$\lambda = (1/E'B) [E'B\lambda_0 + 2 \int_0^x x F^2(x) dx]$$

$$F(x) = \left(\frac{S}{W} \right) \frac{3[1.99 - x(1-x)(2.15 - 3.93x + 2.7x^2)]}{2(1+2x)(1-x)^{3/2}}$$

$$\lambda_0 = S'/(4EBW^3)$$

Fig.8 Initial stress-intensity $K_{Ii} - T_f^*$ relations in alumina ceramic and soda-lime glass

Fig.9 $\sigma_0 - T_f^*$ relations in the specimens with Vickers flaw

Fig.10 $K_{Ii} - (\tilde{T}_f)_N$ relations (with constant load data in $(\tilde{T}_f)_N > 10^5$)