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► To cite this version:

D. Leikin. Causality and conservation of energy : their implications for prediction of receiver performance. Journal de Physique IV Proceedings, 1994, 04 (C5), pp.C5-1323-C5-1326. 10.1051/jp4:19945294 . jpa-00253007

HAL Id: jpa-00253007

<https://hal.science/jpa-00253007>

Submitted on 1 Jan 1994

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Causality and conservation of energy: their implications for prediction of receiver performance

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Abstract: the paper is concerned with the analysis of receiver performance in range-dependent waveguides. The problem is shown to be amenable to a simple theoretical treatment. The necessary analytical technique is based on the application of the principle of causality and the law of conservation of energy. The focus of the analysis is on prediction of limiting capabilities of an optimum detector of quasi-deterministic signals in irregular multi-mode channels. Explicit analytical formulae are derived for the probability of detection, the far-field directivity function and the Cramer-Rao bound for mean-root-square errors in estimates of signal parameters. The theory presented reveals clear physical limits on capabilities of signal processing technique in non-uniform oceanic environment.

1. INTRODUCTION

Theoretical prediction of receiver performance in non-uniform channels is a tough problem. Conventional straightforward technique requires accurate transmission models to be specified. Such an approach makes it impossible to examine reception characteristics under realistic conditions since the governing wave equation cannot be solved analytically in range-dependent medium.

In this paper, an alternative approach is proposed. It is demonstrated that application of the first principles allows one to bypass the necessity to solve the wave equation. This enables theoretical analysis to be carried out in ignorance of the actual transfer function of the transmission channel. The reviewed technique provides a vivid insight into the physical essence of the problem.

2. WAVE FRONT REVERSAL

Let us first consider the auxiliary problem of phase-conjugation. Assume that the sound field of a point source in a multi-mode range-dependent channel is reversed by a remote vertical aperture. The complex-valued amplitude distribution of the phase-conjugated wave will be given by the integral

$$A_{sf} \equiv A(\vec{x}_s, \vec{x}_f) = \int G^*(\vec{x}, \vec{x}_s) G(\vec{x}, \vec{x}_f) dz, \quad (1)$$

where $\vec{x} = (x, y, z)$, $G(1,2)$ is the Green function of the governing wave equation, and the subscript s (f) indicates co-ordinates of the source (observation point). At low frequencies sound transmission in the underwater channel is known [1,2] to be characterized by relatively small absorption and scattering losses. This fact enables the Green function $G(1,2)$ to be expressed in terms of the mode-conversion matrix $U(1,2)$ whose elements are given by complex-valued functionals depending on spatial variations of the refractive index of the channel within the given range interval along the propagation route. The principle of causality demands that the conversion matrix should satisfy the relation

$$U(1,2) = U(1,P)U(P,2), \quad (2)$$

where P is an arbitrary intermediate range. There is one more fact that can be learned about $U(1,2)$ from the first principles. Namely, the conversion matrix has to be unitary in order to meet the law of energy conservation:

$$U^+ U = U U^+ = I. \quad (3)$$

The information provided by Eqs. (2,3) is quite sufficient for one to calculate the amplitude distribution (1) near the focal point. In order to cast the final result into the most appropriate form it is convenient to introduce the form factor

$$W_{sf} = A_{sf} / (A_{ss} A_{ff})^{1/2}.$$

In multi-mode channels, the form factor is found to be given by

$$W_{sf} = [2 \sin(g)]^{-1} \int_{-g}^{+g} \exp[ik \vec{q} \cdot \vec{n}(u)] \cos(u) du, \quad (4)$$

where $\vec{n}(u) = [\sin(u), \cos(u)]$ is a unit vector in the range-depth plane, k is the wavenumber,

$$\vec{q} = (\Delta z, \Delta r) \equiv (z_s - z_f, r_s - r_f)$$

denotes spatial separation between the focal point and the observation point, and the limiting angle

$$g = \arcsin(J_s)$$

corresponds to the near-source value of the channel's trapping factor J . (The latter is defined by the ratio of the energy trapped into the channel to the total irradiated energy [2, p.111].) Thus the near-focus amplitude distribution of the wave front reversal is shown to be independent of the channel structure along the propagation route.

3. SIGNAL DETECTION

Suppose that a signal is to be detected in a broad-band Gaussian noise of mean zero. Let the observed sound field $v(x, y, z, t)$ be sampled by a vertical hydrophone array which is able to resolve the propagating modes trapped by the channel. In order to clarify the further analysis assume that: (i) the spectral density of the noise varies insignificantly within the frequency range of the expected signal, (ii) the noise fluctuations do not correlate at different hydrophones. The expected signal is assumed to be produced by a point source emitting into the channel of some non-uniform sound speed distribution $c(x, y, z)$. The emitted signal $s(t)$ is specified, however, the remaining parameters of the source constitute an unknown vector f

$$\vec{f} = (a_s, r_s, z_s),$$

where 'a' is the amplitude of the emitted signal, r is the range measured horizontally from the receiver, and z is the depth measured downward from the surface. The problem of signal detection/estimation can be treated through the use of the likelihood concept [3]. Under the conditions assumed, the optimum receiver should calculate the coherent amplitude of the wave front reversal. If the observation time is sufficiently large, the response of the likelihood detector will be given by

$$l[v(z, t)|\vec{f}] = \int s(\omega) \int G(\vec{x}, \vec{x}_f) v^*(\vec{x}, \omega) dz d\omega, \quad (5)$$

where $s(\omega)$ and $v(\omega)$ denote corresponding signals in frequency domain.

Since statistic l is a Gaussian random variable, the probability of detection for a fixed $\vec{f}$ will be given by the error function integral

$$Q(\vec{f}) = \text{erfc}[D(\vec{f}) - \Lambda],$$

where $D(\vec{f})$ denotes the output signal-to-noise ratio (SNR), and Λ is the pre-assigned decision level. Were the signal parameters known exactly, $D(f)$ would attain its limiting value

$$D_{\text{opt}}^2 = A_{ss} E / N_0 = J_s E / 8\pi^2 N_0 r_s, \quad (6)$$

where E denotes total irradiated energy. It is convenient, therefore, to introduce the loss factor

$$F(\vec{f}) = D(\vec{f}) / D_{\text{opt}}$$

that will indicate the SNR decrease due to mismatch in signal parameters. The loss factor can now be expressed in terms of the "correlation" function of the emitted signal

$$B_s(\tau) = \int s(t)s(t-\tau)dt / \int s^2(t)dt.$$

Using the result (4), one obtains (c_s is the near-source sound velocity)

$$F(\Delta z, \Delta r) = [2\sin(g)]^{-1} \int_{-g}^{+g} B_s[\vec{q}\vec{n}(u)/c_s] \cos(u) du.$$

This general relation determines the limiting far-field directivity function of the optimum detector in non-uniform waveguide channels.

The response of the matched processor (5) will have one principal peak near the true values of signal parameters. The shape of this peak reproduces that of the loss factor,

$$\langle l[v(z, t)|\vec{f}] \rangle \propto F(\Delta z, \Delta r).$$

Since the noise is also present, the principal peak will be surrounded by multiple additional maxima caused by the noise fluctuations. What form will these random peaks have? In order to answer this question one has to calculate the correlation function

$$\Psi(1, 2) = \langle l(1)l(2) \rangle - \langle l(1) \rangle \langle l(2) \rangle$$

of the receiver response $l(\vec{f}) \equiv l[v(z, t)|\vec{f}]$ in the space of the signal parameters. It may seem a surprise, but this problem can also be solved for arbitrary channel, provided that assumption (ii) holds. By using the first principles (2,3) it may be verified that

$$\Psi(1, 2) \approx \Psi(\vec{f}_1 - \vec{f}_2) \propto F(\Delta z, \Delta r).$$

Thus the spurious maxima of the noise clutter will have the same shape as the principal signal peak. Note that the noise clutter is independent of the channel structure along the propagation route.

4. ESTIMATION OF SIGNAL PARAMETERS

The last problem to be discussed concerns the attainable accuracy of the estimates of the signal parameters. Suppose that unbiased estimates $\hat{f}$ of the unknown parameters were obtained. At large SNR the covariance matrix

$$B = \langle \delta \hat{f} \delta \hat{f}^+ \rangle$$

of errors $\delta \hat{f} = \hat{f} - \langle \hat{f} \rangle$ in these estimates can be determined through the Cramer-Rao inequality. The corresponding lower bound is given by the inverse of the information matrix whose elements can be calculated by differentiating the generalized ambiguity function. The latter is found to be related closely to the form factor of the channel:

$$H(1,2) = a_1 a_2 J_s [4\pi(r_1 r_2)^{1/2} N_0]^{-1} \int |s(\omega)|^2 W_{12} d\omega.$$

Calculating the elements of the information matrix (terms of order $1/[\text{range squared}]$ omitted), one obtains

$$B = [c_s / D_{\text{opt}} \Delta\omega]^2 \begin{vmatrix} [a_s \Delta\omega / c_s]^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3/J_s^2 \end{vmatrix},$$

where D_{opt} is the limiting signal-to-noise ratio defined by (6), c_s is the sound velocity near the source, and $\Delta\omega$ is the mean-root-square deviation of the emitted spectrum

$$\Delta\omega^2 = \int |s(\omega)|^2 \omega^2 d\omega / \int |s(\omega)|^2 d\omega.$$

Note that the optimum amplitude-range-depth estimates will always be uncorrelated for arbitrary channel structure along the route.

5. CONCLUSIONS

It has been demonstrated that proper application of the first principles allows limiting capabilities of the matched receiver to be analyzed in ignorance of the transfer function of the channel. Reception characteristics, such as detection probability, the far-field directivity function and the Cramer-Rao bound for errors in amplitude-range-depth estimates, were shown to be determined completely by two physical parameters of the transmission channel, namely, by the near-source values of sound velocity and the trapping factor. The success of the reviewed approach lies in the fact that actual transfer function contains surplus information which is unnecessary for the analysis to be carried out since all important transmission characteristics were already allowed for in the course of waveguide-detector matching.

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