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Bistability and chaos in acoustical resonator filled viscous liquid: experiment and theory

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ABSTRACT

Dispersive acoustic bistability, auto - oscillations and transition to chaotic behavior throw the sequence of doubling - period bifurcations have been realized in the experiment with ultrasonic excitation of resonator filled viscous liquid. Theoretical model is presented, which takes into account thermal mechanism of acoustic nonlinearity. It gives explanation of all the experimental results.

THEORETICAL RESULTS.

This work continues our studies of ultrasonic beam self - action in viscous liquids [1] - [3]. Let's consider a resonator filled with a liquid, limited by the mirrors with intensity reflection coefficients $R_1=R_2=R$, so that one of the mirrors is a transducer of ultrasound of intensity I_i on a frequency ω_0 itself. In assumption that the time t_r of one round of the resonator of length l by sound wave ($l \sim 1\text{cm}$, $t_r = 2l/c_0 \sim 10^{-5}\text{s}$, c_0 - sound velocity) is small in comparison with charact-

eristic times of nonlinear processes, the equation for slow-varying amplitude of the sound pressure p in the liquid is (in the plane - wave approximation) :

$$\partial_{xx}^2 p + (\omega_0/c_0)^2 p = i(\omega_0/c_0)\partial_t p - 2(\omega_0/c_0)^2 \gamma T p, \quad (1)$$

where $\gamma = |\partial \ln c / \partial T|$.

For sound waves, propagating in opposite directions $p = p_f \exp(-ikx) + p_b \exp(ikx)$ and for the temperature lattice $T = T_0 + T_1 \exp(-2ikx) + T_1^* \exp(2ikx)$ the averaging of eq. (1) over the spatial period $2\pi/k = 2\pi c_0/\omega_0$ and the heat conduction equation:

$$\partial_t T - \chi \partial_{xx} T + (\chi/a^2)T = (\delta/2\rho^2 c_p c_0) |p|^2 \quad (2)$$

brings to the shortened equations in assumption that $2ka < 1$, k - wave number:

$$\begin{aligned} \partial_t \theta_0 + \theta_0 &= b_2 (I_f + I_b), \\ \partial_t \theta_1 + b_3 \theta_1 &= b_2 (I_f I_b)^{1/2} + \theta_2 \partial_t \phi, \\ \partial_t \theta_2 + b_3 \theta_2 &= -\theta_1 \partial_t \phi, \\ \partial_x I_{f,b} &= -\theta_2 (I_{b,f})^{1/2} \pm \ln(b_5) I_{f,b}, \\ \partial_x \phi &= \theta_0 + \theta_1 [(I_f/I_b)^{1/2} + (I_b/I_f)^{1/2}]/2, \end{aligned} \quad (3)$$

here are $I_{f,b} = |p|^2/(2\rho c_0 I_i)$, $\phi = \arg p_b - \arg p_f$, $\theta_0 = T_0 2\omega_0 \gamma / c_0$, $\theta_1 = (\omega_0 \gamma / c_0) [T_1 \exp(i\phi) + T_1^* \exp(-i\phi)]$, we also inserted dimensionless variables $t' = t/\tau$ ($\tau = a^2/\chi$), $x' = x/l$ (further all the primes are dropped). Boundary conditions for (3) are:

$$\begin{aligned} I_b(1,t) &= b_1 I_f(1,t), \\ 1 &= I_f(0,t) + b_1 I_b(0,t) - 2[b_1 I_b(0,t) I_f(0,t)]^{1/2} \cos(\phi(1,t) + b_4), \\ \phi(0,t) &= \theta_{0,1,2}(0,t) = 0. \end{aligned}$$

The process is controlled by five parameters:

$$\begin{aligned} b_1 &= R, \\ b_2 &= 2 I_i a^2 \delta / \omega_0 \gamma / \rho c_p c_0 \chi = \tilde{b}_2 I_i, \\ b_3 &= (2ka)^2, \\ b_4 &= 2(kl - \pi n), \quad n - \text{an integer}, \\ b_5 &= \exp(-\delta l). \end{aligned}$$

The relationship between stationary output intensity I_T^s and the input intensity I_i is then ambiguous:

$$I_T^s = b_5 I_i (1 - b_1) / \{1 + (b_1 b_3)^2 - (b_1 b_3) \cos[(2 + b_3^{-1}) \tilde{b}_2 I_T^s / (1 - b_1) + b_4]\} \quad (4)$$

Suppose that all the functions are slowly varying along the axis x ($\theta_2 \ll 1$), and the resonator is durable ($R \cong 1$), we come to the system of equations for $\theta_{1,2}(t) = \theta_{1,2}(1, t)$, $\phi(t) = \theta_0(1, t) + \theta_1(1, t)$ after averaging of eq. (3) over the length of the resonator:

$$\begin{aligned}\dot{\phi} &= [3b_2(1 - \theta_2)J - \phi - (b_3 - 1)\theta_1]/(1 - \theta_2), \\ \dot{\theta}_1 &= -b_3\theta_1 + b_2J(1 - \theta_2) + \phi\theta_2, \\ \dot{\theta}_2 &= -b_3\theta_2 - \phi\theta_1,\end{aligned}\quad (5)$$

where $J = b_5/[1 + (b_1 b_5)^2 - 2b_1 b_5 \cos(\phi + b_4)] = I_T^5/I_1(1 - b_1)$.

The analytical analysis of this system shows, that the input intensity, necessary for observation of auto-oscillatory regime increases with the increase of resonator length (decrease of b_5). The period of oscillations $T \cong 2^{1/2} \pi a^2 b_3^{1/2} / \chi$ depends, in essential, on the parameter b_3 .

The numerical solution of eq.(5) was performed with using the packet AUTO, it allows to test the stationary solutions for stability, to define the positions of the points of bifurcations (BP) and to examine the solutions in their vicinity. The positions of BP depend on the mismatch b_4 weakly. After the transition from the BP one-period dependence I_T experiences the sequence of the doubling of the period, and then stochastic regime is attained in accordance with Feigenbaum universal scenario.

The measurements were made in glycerin under the initial temperature 10C ($\delta = 0,2\text{cm}^{-1}$, $c_0 = 2 \times 10^5 \text{cm/s}$, $\gamma = 10^{-2} \text{K}^{-1}$, $\rho = 1,2 \text{g/cm}^3$, $c_p = 4,1 \text{J/gK}$).

EXPERIMENTAL RESULTS.

In the experimental set-up piezoelectric transducer (the resonance frequency is $\omega_0/2\pi = 2\text{MHz}$) excites an ultrasonic field into the resonator. An oscillator generates electric signal, which is amplified by power amplifier, and then it excites the transducer. The pressure of output wave is measured by a broadband piezoelectric probe connected with a voltmeter that is used as a linear detector for recorder. The transmitted and received electric signals are controlled by an oscilloscope and frequency meter. One of the resonator's mirrors is a transducer (with the aperture $a_0 = 0,75 \text{ cm}$) and other is the metallic plate with reflectivity on inter-

nsity 0,9. The resonator is placed into a temperature-controlled cell filled with glycerin. A few axial modes of the resonator are falling within the width of a transducer resonance line.

After a transient period continued 30 - 200s ($\tau = a_0^2/\chi$) the time dependence of the intensity I_T of output wave are determined by a value of input intensity I_i . If $I_i \leq 1 \text{ Wt/cm}^2$ ($l = 1,2 \text{ cm}$) a stationary regime takes place. For $I_i \leq 0,2 \text{ Wt/cm}^2$ intensity I_T depends linearly on I_i , and in the range $0,2 \leq I_i \leq 1 \text{ Wt/cm}^2$ an acoustical bistability is observed. If input intensity is increased to 1 - $1,5 \text{ Wt/cm}^2$ an oscillatory regime of temporal dependence of I_T is observed. When I_i increases more, initial sinusoidal oscillations turned out to a sum of harmonics. In the range of $I_i = 1,5 - 2 \text{ Wt/cm}^2$ the changes acquire an irregular nature and a rate of signal increases and decreases is determined by the apparatus function of the measuring system. With the increase of cavity length a range of intensity I_i where regular oscillations exist becomes narrower. The changes of an initial tuning of frequency ω_0 from a resonant frequency and the changes of initial temperature don't affect qualitatively the regime of changes I_T .

CONCLUSIONS.

An effective sound-induced retuning of the resonator, that is connected acoustically with the transducer, permits to get a power ultrasonic field on a frequency, that coincides with one of eigen-value frequencies of the system transducer - resonator. This frequency can be changed by self-tuning of the resonator. The approach has got perspectives for realizing a broadband transducer of power ultrasound with smooth sweeping of central frequency.

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