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Bistability and chaos in acoustical resonator filled viscous liquid: experiment and theory

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ABSTRACT
Dispersive acoustic bistability, auto-oscillations and transition to chaotic behavior throw the sequence of doubling-period bifurcations have been realized in the experiment with ultrasonic excitation of resonator filled viscous liquid. Theoretical model is presented, which takes into account thermal mechanism of acoustic nonlinearity. It gives explanation of all the experimental results.

THEORETICAL RESULTS.
This work continue our studies of ultrasonic beam self-acti on in viscous liquids [1] - [3]. Let's consider a resonator filled with a liquid, limited by the mirrors with intensity reflection coefficients \( R_1 = R_2 = R \), so that one of the mirrors is a transducer of ultrasound of intensity \( I_t \) on a frequency \( \omega_0 \) itself. In assumption that the time \( t_r \) of one round of the resonator of length \( l \) by sound wave (\( l \sim 1cm, t_r = 2l/c_o \sim 10^{-5}s, c_o \) - sound velocity) is small in comparison with charact-
eristic times of nonlinear processes, the equation for slow-varying amplitude of the sound pressure $p$ in the liquid is (in the plane-wave approximation):

$$\partial_{xx}^2 p + (\omega_0/\omega)^2 p = \delta p - 2(\omega_0/\omega)^2 \gamma Tp, \quad (1)$$

where $\gamma = |\delta \ln c/\delta T|$.

For sound waves, propagating in opposite directions $p = p_f \exp(-ikx) + p_b \exp(ikx)$ and for the temperature lattice $T = T_0 + T_1 \exp(-2ikx) + T_1 \exp(2ikx)$ the averaging of eq. (1) over the spatial period $2\pi/k = 2\pi c/\omega_0$ and the heat conduction equation:

$$\partial_t T - \chi \partial_{xx} T + (\chi/a^2)T = (\delta/2\rho c^2 c_0) |p|^2 \quad (2)$$

brings to the shortened equations in assumption that $2ka < 1$, $k$ - wave number:

$$\begin{align*}
\partial_t \theta + \theta &= b_1 (\theta + 1) \gamma \\
\partial_t \phi + \phi &= b_2 (\phi + 1) \gamma \\
\partial_t I_{b,b} &= -\frac{1}{2} (\theta_{b,b} \gamma - \ln(b) I_{b,b}) \\
\partial_x \phi &= \Theta_0 + \Theta_0 (I_{b,b}^{1/2} + I_{b,b}^{1/2})/2, \quad \Theta_0 = \Theta_0 (I_{b,b}^{1/2} + I_{b,b}^{1/2})/2 \\
\Theta_0 &= \Theta_0 (I_{b,b}^{1/2} + I_{b,b}^{1/2})/2 \\
\Theta_0 &= \Theta_0 (I_{b,b}^{1/2} + I_{b,b}^{1/2})/2 \\
\Theta_0 &= \Theta_0 (I_{b,b}^{1/2} + I_{b,b}^{1/2})/2
\end{align*} \quad (3)$$

here are $I_{b,b} = |p|^2/(2\rho c_0 I_1)$, $\phi = \arg p_b - \arg p_f$, $\theta_0 = T_0 \omega_0 \gamma I/c_0 \Theta_1 = (\omega_0 \gamma I/c_0) [T_1 \exp(i\phi) + T_1 \exp(-i\phi)]$, we also inserted dimensionless variables $t' = t/\tau$, $T = a^2/\chi$, $x' = x/l$ (further all the primes are dropped). Boundary conditions for (3) are:

$$\begin{align*}
I_b (0,t) &= b I_b (0,t) \\
\phi (0,t) &= 0
\end{align*}$$

The process is controlled by five parameters:

$$\begin{align*}
b_1 &= R_i, \\
b_2 &= 2 I_1 a \omega_0 \gamma / \rho c_0 \chi = b_2 I_1, \\
b_3 &= (2ka)^2, \\
b_4 &= (2kI - mn), n - an integer, \\
b_5 &= \exp(-\delta I). \end{align*}$$

The relationship between stationary output intensity $I^s T$ and the input intensity $I_i$ is then ambiguous:

$$I^s T = b I_i (1-b)/(1+b b_1^2-(b b_1^2) \cos[(2+b_1^2) b_1^2 I_i/(1-b) + b_1^2)] \quad (4)$$
Suppose that all the functions are slowly varying along the axis \( x (\theta_2 < d) \), and the resonator is durable \((R \geq 1)\), we come to the system of equations for \( \theta_{1,2}(t) = \theta_{1,2}(1,t) \), \( \phi(t) = \theta_0(1,t) + \theta_1(1,t) \) after averaging of eq. (3) over the length of the resonator:

\[ \dot{\phi} = [3b_2(1 - \theta_2) + \phi - (b_1 - 1)\theta_2]/(1 - \theta_2), \]

\[ \dot{\theta}_1 = -b_2 \theta_1 + b_2 J(1 - \theta_2) + \phi \theta_2, \]

\[ \dot{\theta}_2 = -b_2 \theta_2 - \phi \theta_2, \]

where \( J = \frac{b_2}{(1 + (b_1 b_2)^2 - 2b_2 b \cos(\phi + b_2)]) = I^0(1 - b). \)

The analytical analysis of this system shows, that in input intensity, necessary for observation of auto-oscillatory regime increases with the increase of resonator length (decrease of \( b_5 \)). The period of oscillations \( T \approx 2^{1/2} \pi a^2 b_3^{1/2}/\chi \) depends, in essential, on the parameter \( b_3 \).

The numerical solution of eq.(5) was performed with using the packet AUTO; it allows to test the stationary solutions for stability, to define the positions of the points of bifurcations (BP) and to examine the solutions in their vicinity. The positions of BP depend on the mismatch \( b_4 \) weakly. After the transition from the BP one-period dependence \( I_T \) experiences the sequence of the doubling of the period, and then stochastic regime is attained in accordance with Feigenbaum universal scenario. The measurements were made in glycerin under the initial temperature \( 10^\circ C \) \((\delta = 0.2 cm^{-1}, \ c_0 = 2 \times 10^5 cm/s, \ \gamma = 10^{-2}K^{-1}, \ \rho = 1.2 g/cm^3, \ \epsilon_p = 4.1 J/gK)\).

**EXPERIMENTAL RESULTS.**

In the experimental set-up piezoelectric transducer (the resonance frequency is \( \omega_0/2\pi = 2\) MHz) excites an ultrasonic field into the resonator. An oscillator generates electric signal, which is amplified by power amplifier, and then it excites the transducer. The pressure of output wave is measured by a broadband piezoelectric probe connected with a voltmeter that is used as a linear detector for recorder. The transmitted and received electric signals are controlled by an oscilloscope and frequency meter. One of the resonator's mirrors is a transducer (with the aperture \( a_0 = 0.75 cm) and other is the metallic plate with reflectivity on inte-
nsity 0.9. The resonator is placed into a temperature-controlled cell filled with glycerin. A few axial modes of the resonator are falling within the width of a transducer resonance line.

After a transient period continued 30 - 200s ($\tau = a_0^2/\chi$) the time dependence of the intensity $I_T$ of output wave are determined by a value of input intensity $I_i$. If $I_i \leq 1\text{Wt/cm}^2$ ($I = 1.2\text{cm}$) a stationary regime takes place. For $I_i \leq 0.2\text{Wt/cm}^2$ intensity $I_T$ depends linearly on $I_i$ and in the range $0.2 \leq I_i \leq 1\text{Wt/cm}^2$ an acoustical bistability is observed. If input intensity is increased to $1 - 1.5\text{Wt/cm}^2$ an oscillatory regime of temporal dependence of $I_T$ is observed. When $I_i$ increases more, initial sinusoidal oscillations turned out to a sum of harmonics. In the range of $I_i = 1.5 - 2\text{Wt/cm}^2$ the changes acquire an irregular nature and a rate of signal increases and decreases is determined by the apparatus function of the measuring system. With the increase of cavity length a range of intensity $I_i$ where regular oscillations exist becomes narrower. The changes of an initial tuning of frequency $\omega_0$ from a resonant frequency and the changes of initial temperature don't affect qualitatively the regime of changes $I_T$.

CONCLUSIONS.

An effective sound-induced retuning of the resonator, that is connected acoustically with the transducer, permits to get a power ultrasonic field on a frequency, that coincides with one of eigen-value frequencies of the system transducer - resonator. This frequency can be changed by self-tuning of the resonator. The approach has get perspectives for realizing a broadband transducer of power ultrasound with smooth sweeping of central frequency.

REFERENCES.