Scattering effects on ocean acoustics coherence for low-frequency long-range propagation

A.G. SAZONTOV and V.A. FARFEL

Institute of Applied Physics RAS, 46 Ulyanov St., 603600 Nizhny Novgorod, Russia

Abstract: The mutual coherence function (MCF) of the received acoustic field is derived using an analysis based upon coupled mode theory. Separations in transverse horizontal position, time and frequency are treated. The mean horizontal array response and the shape of an ensemble-averaged pulse are evaluated from the MCF. The application is illustrated by computation of the considered effects for rough ocean surface scattering in an upper-sound channel.

1. INTRODUCTION

As is well-known (see, e.g. [1]) the problem of finding the correlation characteristics of an acoustic wavefield reduces to evaluating the mutual coherence function (MCF). Previous research examining the behaviour of the MCF in a fluctuating ocean has been carried out in a ray-oriented approach using the path-integral formalism [2].

This report investigates the MCF as a function of separations in time, space and frequency in an irregular oceanic waveguide in the framework of the coupled mode theory which is more suitable for the important case of low-frequency and long-range propagation. We formulated the matrix transport equation for the MCF in terms of the modal structure of a sound pressure field and presented the results ensuring from the approximate analytical solution of this equation for the important correlation properties of an acoustic radiation. The knowledge of the MCF has been then employed to the statistical analysis of the scattering effects on the quality of the space-time matched filtering of a registered signal.

2. FORMULATION OF THE PROPAGATION MODEL

The acoustic pressure field $p(\vec{r}, z, t)$ (where $\vec{r} = (x, y)$ is the horizontal two-dimensional position vector, z is the vertical coordinate and t is the time) in an irregular oceanic waveguide can be formally represented by

$$p(\vec{r}, z, t) = \int_{-\infty}^{\infty} d\omega g(\omega) e^{-i\omega t} \sum_{n=1}^{M(\omega)} p_n(\vec{r}, \omega, t) \varphi_n(z, \omega) \quad (1)$$

Here $g(\omega)$ is the frequency spectrum of a transmitted signal, $\varphi_n(z, \omega)$ denotes the n -th vertical eigenfunction of a regular channel associated with the eigenvalue $\kappa_n^2(\omega)$; M is the number of propagating modes and $p_n(\vec{r}, \omega, t)$ are the random normal mode amplitudes.

We will be interested in the behaviour of the second moment of the sound wave that has passed through a scattering medium with regular refraction:

$$B_p(\vec{r}_1, z_1, t_1; \vec{r}_2, z_2, t_2) = \langle p(\vec{r}_1, z_1, t_1) p^*(\vec{r}_2, z_2, t_2) \rangle. \quad (2)$$

The angular brackets $\langle \rangle$ indicate the ensemble averaging. By substituting Eq.(1) into Eq.(2) we can write:

$$B_p(\vec{r}_1, z_1, t_1; \vec{r}_2, z_2, t_2) = \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\infty} d\omega_2 g(\omega_1) g^*(\omega_2) \Gamma \exp(-i\omega_1 t_1 + i\omega_2 t_2), \quad (3)$$

where $\Gamma(\vec{r}_1, z_1, t_1, \omega_1; \vec{r}_2, z_2, t_2, \omega_2)$ is the MCF defined as

$$\Gamma(\vec{r}_1, z_1, t_1, \omega_1; \vec{r}_2, z_2, t_2, \omega_2) = \sum_{n,m} \Gamma_{nm}(1,2) \varphi_n(z_1, \omega_1) \varphi_m(z_2, \omega_2), \quad (4)$$

$$\Gamma_{nm}(1,2) = \langle p_n(1) p_m^*(2) \rangle.$$

The labels 1 and 2 refer to two different horizontal position points, times and frequencies. Thus, the problem of finding a result for B_p in a waveguide channel now reduces to evaluating the cross-modal mutual coherence functions Γ_{nm} .

In Ref.[3] the basic transport equation for the cross-modal coherence functions Γ_{nm} taken at two horizontal position points $\vec{r}_1 = (x, y_1)$, $\vec{r}_2 = (x, y_2)$ in the same x plane, at two different times and frequencies has been derived with the forward-scattering approximation:

$$\begin{aligned} \left[\frac{\partial}{\partial x} + i(\kappa_n(\omega_1) - \kappa_m(\omega_2)) - i\xi_{nm}^+ \frac{\partial^2}{\partial \rho \partial R} - \frac{i}{2} \xi_{nm}^- \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{4} \frac{\partial^2}{\partial R^2} \right) \right] \Gamma_{nm}(1,2) = \\ = -\frac{1}{2} (\sigma_n(\omega_1) + \sigma_m(\omega_2)) \Gamma_{nm}(1,2) + \sum_{n',m'} A_{nn'}^{mm'}(\rho, \tau; \omega_1, \omega_2) \Gamma_{n'm'}(1,2). \end{aligned} \quad (5)$$

In writing (5) the following notation is used:

$\rho = y_1 - y_2$, $R = \frac{1}{2}(y_1 + y_2)$, $\tau = t_1 - t_2$, $\xi_{nm}^+ = \frac{1}{2}(\kappa_n^{-1}(\omega_1) + \kappa_m^{-1}(\omega_2))$, $\xi_{nm}^- = \kappa_n^{-1}(\omega_1) - \kappa_m^{-1}(\omega_2)$; $\sigma_n(\omega)$ is the scattering coefficient of mode n related to the coupling matrix $A_{nn'}^{mm'}(\rho, \tau; \omega_1, \omega_2)$ by: $\sigma_n(\omega) = \sum_{m=1}^M A_{nn}^{nm}(\rho = 0, \tau = 0, \omega, \omega)$. The concrete form of the quantity $A_{nn'}^{mm'}(\rho, \tau; \omega_1, \omega_2)$ depends on the specific type of the scattering mechanism. Further, for definiteness we restrict ourselves to the consideration of surface scattering phenomena. In this case

$$\begin{aligned} A_{nn'}^{mm'}(\rho, \tau; \omega_1, \omega_2) = \frac{\pi}{2} \frac{[\varphi'_n(0, \omega_1) \varphi'_{n'}(0, \omega_1) \varphi'_m(0, \omega_2) \varphi'_{m'}(0, \omega_2)]}{[\kappa_n(\omega_1) \kappa_{n'}(\omega_1) \kappa_m(\omega_2) \kappa_{m'}(\omega_2)]^{1/2}} * \\ * \int_{-\infty}^{\infty} d\Omega e^{-i\Omega \tau} \int_{-\infty}^{\infty} d\kappa_y e^{i\kappa_y \rho} F_{\zeta}(\kappa_{nm}^+ - \kappa_{n'm'}^+, \kappa_y, \Omega), \end{aligned}$$

where the prime superscript denotes differentiation with respect to the depth z , $F_{\zeta}(\vec{\kappa}, \Omega)$ is the rough surface spectral distribution, and $\kappa_{nm}^+ = \frac{1}{2}(\kappa_n(\omega_1) + \kappa_m(\omega_2))$.

The restrictions to the theory are detailed in Ref.[3].

3. ANALYSIS OF THE COHERENT PROPERTIES OF A RECEIVED SIGNAL

The degree of the received signal coherence is characterized by the correlation coefficient defined as:

$$b_p(1,2) = \Gamma(1,z; 2,z) / [\Gamma(1,z; 1,z) \Gamma(2,z; 2,z)]^{1/2}. \quad (6)$$

The use of Eqs.(4), (5) allows one to estimate from Eq.(6) important statistical properties of the acoustic field. In this section for realistic sound-speed profile from the North-West Pacific (see Fig.1) we present the results of numerical calculations of Eq.(6) employing approximate analytical expressions for Γ_{nm} obtained in Ref.[3]. Figure 2 demonstrates the behaviour of the correlation coefficient as a function of separation in space, time and frequency. The computations have been carried out for source frequency of 250 Hz, source depth of 100 m, receiver depth of 300 m, and propagation distance of 500 km under the assumption that the spectrum of the rough ocean surface is described by the Pierson and Moskowitz distribution. The wind speed V was taken to be 10, 13 and 15 m/s.

Fig.1 Upper part of the sound-speed profile from the winter North-West Pacific.

Fig.2 The signal correlation coefficient as a function of space separation (a), time separation (b), and frequency separation (c).

4. MATCHED SPACE-TIME FILTERING OF A SCATTERED SIGNAL

The space-time filtering is one of the widespread procedures of signal processing.

Assume that a narrow-band transmitted signal is received by a horizontal line array of N uniformly spaced elements and is then subjected to the time filtering. The signal at the processing device output is proportional to:

$$\Psi(\theta, \tau) = \frac{1}{N} \sum_{j=1}^N e^{-ikd(j-1) \sin \theta} \int_{-\infty}^{\infty} p(\vec{r}_j, z, t) S^*(t - \tau) dt, \quad (6)$$

where $p(\vec{r}_j, z, t)$ is the sound pressure field at the j -th hydrophone, $S(t) = \int_{-\infty}^{\infty} d\omega g(\omega) e^{-i\omega t}$ is the envelope of the input pulse, k is the reference wavenumber, d is the interelement space, θ is the steering angle, and τ is the time delay.

We will focus our attention on the mean square power output $\langle \Psi^2(\theta, \tau) \rangle$. It is straightforward to show that $\langle \Psi^2(\theta, \tau) \rangle$ is related to the second moment of the sound wave field (3) as:

$$\langle \Psi^2(\theta, \tau) \rangle = \frac{1}{N^2} \sum_{j=1}^N \sum_{l=1}^N e^{-ikd(j-l) \sin \theta} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 B_p(\vec{r}_l, z, t_1, \vec{r}_j, z, t_2) S^*(t_1 - \tau) S(t_2 - \tau).$$

The knowledge of the MCF allows one to calculate the quantity B_p from Eq.(3) and therefore to analyze the scattering effects on the quality of space-time filtering.

As an example in Fig.3 we plot the normalized mean shape of the spatial filter response as a function of steering angle θ at $N = 256$, $d = \frac{\lambda}{2}$ ($\lambda = \frac{2\pi}{k}$), at the fixed range ($x = 500 \text{ km}$) and the time delay ($\tau = 340 \text{ s}$).

Fig.3 The mean shape of the spatial filter response at arrival angle 30° as a function of the steering angle in the absence of fluctuations (curve 1) and in a random ocean (curves 2,3,4) at different wind speeds: 2 — $V = 10 \text{ m/s}$, 3 — $V = 13 \text{ m/s}$, 4 — $V = 15 \text{ m/s}$.

Figure 4 shows the behaviour of the ensemble averaged pulse amplitude $\langle \Psi^2(\theta = \theta_{\max}, \tau) \rangle^{1/2}$ for the rectangular transmitted signal $S(t) = T^{-1/2} \text{rect}(t/T)$ having the width $T = 50 \text{ ms}$.

Fig.4 Normalized signal amplitude at the matched filter output versus time delay in the presence (a) and absence (b) of sea-surface scattering.

The theoretical curves in Fig.3 and 4 have used calculations of the MCF considered above.

REFERENCES

- [1] Ishimaru A., Wave Propagation and Scattering in Random Media (Academic Press, New-York, 1978)
- [2] Dashen R., Flatte S.M. and Reynolds S.A., *J. Acoust. Soc. Amer.* **77** (1985) 1716-1722.
- [3] Sazontov A.G., *Sov. Phys. Acoust.* **35** (1989) 526-532.