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Etude de la propagation acoustique dans des milieux stratifiés inhomogènes, application à la caractérisation des fonds marins

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Abstract : With the aim of identifying marine sediments we study the conditions of validity of two homogenization models giving estimates of the effective velocity in inhomogeneous media. We have considered the Kuster-Toksöz quasi-static model and the Waterman-Truell dynamic model. Here, we compare the two models experimentally by making measurements on inhomogeneous samples made of glass spheres in gel. We then propose a process for identifying the radius of inclusions and their volume concentration from the measured values of the wave velocity versus frequency, well estimated by the Waterman-Truell model.

1. INTRODUCTION.

The sea bed on the continental shelf is composed of marine sediments lying on a rocky elastic base. Recognition by acoustical means of these media is raising problems relating to the heterogeneity of sediments : they are layered media composed of particles with different sizes embedded in a mixture of water and very fine particles (fine sand). The classical methods for calculating the reflection coefficient of a stratified medium are reserved to homogeneous materials [1]. To apply these methods in the case of a sediment, each inhomogeneous layer is considered to be homogeneous. The process of substituting a homogeneous material for a heterogeneous one, but keeping its mechanical behaviour, is called homogenization. Two different cases are treated separately in homogenization theories : when the particles are very small compared to the wavelength of the incident wave (those problems are said quasi-static), and when their size becomes higher or comparable to the wavelengths (these problems are said dynamic). We have studied the Kuster-Toksöz model [2] among the quasi-static ones and the Waterman-Truell model [3] among the dynamic ones. Both were chosen because they were judged as the most representative in their category. We intend to establish the limiting frequencies with regard to the size of inclusions when the quasi-static model becomes inefficient for describing the acoustical propagation in inhomogeneous media and how the dynamic model can replace it. The tests consist of comparing theoretical and experimental values, measured on inhomogeneous samples. We shall then study how we can get the radius of inclusions and their volume concentration from the effective characteristics given by a homogenization model.

2. HOMOGENEIZATION MODELS.

2.1. Kuster-Toksöz quasi-static model

The authors consider a composite medium containing N inclusions of radius a , with bulk modulus K_2 , shear modulus G_2 , density ρ_2 , embedded in an infinite matrix characterized by the constants K_1 , ρ_1 . The inclusions are confined in a finite spherical region V_0 of the infinite matrix, called representative sphere, and a plane compressional wave is incident. Multiple scattering effects are neglected, the

observation point is assumed far from the centre of V_0 and the wavelengths of all waves are assumed far greater than a . Kuster-Toksöz assume the characteristics of the effective medium, K^* and ρ^* , should be well estimated if the displacement field due to scattering by a homogeneous effective sphere occupying the same volume as V_0 should be equal to that due to scattering by the original composite sphere V_0 . Thus they finally obtain the following relationships :

$$\frac{K_1 - K^*}{K^*} = \phi \frac{K_1 - K_2}{K_2}, \quad \frac{\rho_1 - \rho^*}{\rho_1 + 2\rho^*} = \phi \frac{\rho_1 - \rho_2}{\rho_1 + 2\rho_2}. \quad (1a,b)$$

The effective velocity, c_l^* is derived from K^* and ρ^* with the following formula :

$$c_l^* = \sqrt{\frac{K^*}{\rho^*}}. \quad (2)$$

2.2. Waterman-Truell dynamic model

In this formalism the authors consider an array of N identical spherical scatterers randomly embedded in a half space (matrix medium), without any interpenetration nor correlation. The whole is illuminated by a plane wave. Both matrix and inclusions are isotropic. The fundamental assumption leading to the definition of a homogeneous scattering medium assumes the exciting field on each scatterer (incident wave plus scattered waves by all the others obstacles) can be approximated by the total field at this point if the obstacle was not present. This assumption holds when the following criterion is verified :

$$\frac{n_0 Q_s}{k_l} \ll 1, \quad (3)$$

where n_0 is the volume density of the scatterers, Q_s is the scattering cross section of a single sphere, and k_l the wave number in the matrix.

By a process of statistical averaging based on the formalism of configurational averaging of Foldy and Lax [4,5], and including multiple scattering, Waterman and Truell obtain a complex constant propagation accounting for the propagation in a scattering homogeneous medium :

$$\left(\frac{k^*}{k_l} \right)^2 = \left(1 + \frac{2 \pi n_0 f(0)}{k_l^2} \right)^2 - \left(\frac{2 \pi n_0 f(\pi)}{k_l^2} \right)^2, \quad (4)$$

where $f(\theta)$ is the amplitude scattered by a single inclusion in the direction θ with respect to the direction of propagation of the incident wave. The real part of k^* is related to the effective wave speed c^* and its imaginary part to the effective attenuation α^* , due to multiple scattering :

$$k^* = \frac{\omega}{c^*} + i\alpha^*. \quad (5)$$

2.3. Theory-experiment comparisons

For modelling a seabed containing inhomogeneities we consider the case of glass spheres (5mm diameter) embedded in gel. The gel employed as a matrix has the trade name of agar-agar and is usually used for bacterial cultures in biology. Table1 shows the characteristics of these constituents.

Table 1
Characteristics of the constituents

	longitudinal speed m/s	transverse speed m/s	density kg/m ³
gel (matrix)	1480	0	1014
glass (inclusions)	5500	3400	2560

Two comparisons are shown in figure 1 and 2. They lead to the conclusion that the Waterman-Truell dynamic model provides realistic estimates of the effective phase velocity in both long and short wavelengths conditions of propagation (the transition between the two domains occurs at 592 kHz). But others comparisons not shown in this paper have shown that the agreement was limited to volume concentrations less than 30%. On the other hand for ratios close to unity the quasi-static model of Kuster-Toksöz is inefficient for describing an effective propagation in a composite medium.

Figure1. Effectice wave velocity :
Kuster-Toksöz --- , Waterman-Truell ____
and measured values o, at $\lambda/a = 2.4$.

Figure 2. Effectice wave velocity 17% glass/gel
Waterman Truell ____ , measured values —o—

3. PARAMETER IDENTIFICATION

Having now established that the Waterman-Truell dynamic model was describing the effective propagation in a homogeneous medium in a correct way, we shall use this model to get the parameters of the medium. The nature of the matrix and of the inclusions are assumed to be known. We are looking for the radius of the inclusions and their volume concentration from the measured values of the effective wave velocity.

3.1. Radius

We note the presence of a minimum in the variations of the effective velocity with frequency described by the Waterman-Truell model. Once the nature of the inhomogeneous medium is known the frequency of this minimum only slightly varies with the volume concentration. It can then allow to get the radius. Besides it is well determined by measurements of the wave velocity. For volume concentrations of 1% and 40% of glass in gel (corresponding to the range of possible volume concentrations), we calculate the frequency at which a minimum in the effective velocity given by Waterman-Truell occurs. We then obtain two tables each giving the radius associated with a frequency of a minimum of velocity. As the volume concentration is unknown we propose to take the mean value between the two radii, which leads to a maximum error of 1.46% in the determination of the radius. The table 2 below displays the identification of the radius made for various samples.

Table 2
Identification of the radius

Real radius	Radius with 1%	Radius with 40%	Mean radius	Relative error
a = 1.5 mm	1.48 mm	1.52 mm	1.503 mm	0,2 %
a = 2.5 mm	2.42 mm	2.5 mm	2.46 mm	1,6 %
a = 2 mm	2.002 mm	2.06 mm	2.03 mm	1,5 %

3.2. Volume concentration

The radius is now known. We have some values of the measured effective wave velocity, C_i^m , at various frequencies, f_i , ($i=1, \dots, N$). We have assumed that the Waterman-Truell model was realizing a good modelization of the variations of the effective wave velocity with frequency. For the frequencies f_i , the values of Waterman-Truell are denoted C_i^* and they are written in the form :

$$C_i^*(\phi) = C_I / \text{Real} \left(\sqrt{1 + \frac{3}{k_I^3 a^3} f(0) \phi + \frac{9}{4 k_I^6 a^6} (f(0)^2 - f(\pi)^2) \phi^2} \right) \quad (6)$$

where k_I , a , $f(0)$ and $f(\pi)$ are known. We then identify the value of the parameter ϕ which minimizes the function $g(\phi)$:

$$g(\phi) = \sum_{i=1}^N |C_i^*(\phi) - C_i^m|^2 \quad (7)$$

The algorithm for minimizing is based on the Brent method [6]. The Figure 3 illustrates the identification of the volume concentration made for a sample containing 17% of glass in gel. The value identified is 17.75% which realizes a 4.41% relative error.

Figure 3. Waterman - Truelli with the real volume concentration 17% glass in gel —, measured values for identification --o--, Waterman-Truelli with the identified volume concentration 17.75% glass in gel ***.

4. CONCLUSION

In this paper we have presented experimental data on the velocity of ultrasonic waves in inhomogeneous media. They showed that when the conditions of insonification correspond to the transition domain between long and short wavelengths, i.e. for ratios λ/a close to unity, the Kuster-Toksöz estimates were not correct. On the other hand the Waterman-Truelli model estimates are close to the experimental values for long and short wavelengths. This agreement seems to be limited to cases of volume concentrations smaller than 30 % (for glass spheres of 5 mm diameter embedded in gel). We have therefore established that the Waterman-Truelli dynamic model was the most accurate model which accounted for an effective propagation in a composite medium composed of a fluid matrix containing identical spheres. From the variations of the effective velocity with frequency modeled by the Waterman-Truelli model we have proposed a process of separated identification of the radius and the volume concentration very satisfactory.

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