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Noise generation by turbulent flow in ducts

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abstract: The aim of this work is to estimate the sound power generated by a turbulent flow in a duct from the knowledge of mean quantities which can be computed with a $K-\epsilon$ turbulence model when only the plane wave can propagate, i.e., when the characteristic frequency of the turbulence is low. It does turn out to be possible for the simplest configuration of a duct free of internal obstruction. The result provides a prediction which gives rise to a U^7 scaling law for the variation of the acoustic power. Nevertheless, there is no experimental data to compare our results with for this idealized configuration. That is why we tackled the problem of a duct obstructed by a diaphragm. Using conformal mapping and Matched Asymptotic Expansions, we have determined a two-dimensional Green's function which allow us to assess numerically the acoustic power. This case is more interesting because existing experimental data confirmed the U^4 scaling law obtained by calculation for the acoustic power variation.

1- Duct free of internal obstruction

In order to estimate the acoustic power generated by a turbulent flow in a duct, we shall solve the Lighthill inhomogeneous aeroacoustic equation governing the sound field, which reads for the pressure

$$\nabla^2 p - \frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = - \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}, \quad (1.1)$$

where the turbulence stress tensor is reduced to $T_{ij} = \rho_0 u_i u_j$ (inviscid flow, adiabatic motion).

1.1- Davies and Ffowcs-Williams results

The first part of this work is based on the paper of Davies and Ffowcs Williams [1] which deals with the problem of estimating the sound field generated by a limited region of turbulence of volume v in an infinitely long, straight, hard walled pipe of square cross-sectional shape and side b .

- figure 1.1 : empty duct -

Equation (1.1) can be solved by means of a Green's function $G(\vec{x}, t|\vec{y}, \tau)$ satisfying the appropriate hard-wall condition on the walls of the duct and the radiation conditions as $x_3 \rightarrow \pm\infty$. This function can be expressed in the Fourier domain as a sum of the normal modes of oscillation, these being cosine terms only in a square hard walled pipe. This simple form of the Green's function allows theoretical developments which led Davies and Ffowcs-Williams [1] to an expression for the acoustic power due to a limited region v of turbulence. They showed that for low Mach numbers, when the characteristic frequency of the turbulence is so low that only the plane wave $m = n = 0$ can propagate, the acoustic power, or rate at which energy is propagating down the pipe, is proportional to

$$P \propto \frac{v}{\rho_0 c_0^3} \int_{-\infty}^{+\infty} \frac{\partial^2}{\partial \tau^2} R_{3333}(\lambda_3, \tau) \Big|_{\tau = \frac{\lambda_3}{c_0}} d\lambda_3 \quad (1.2)$$

where

$$R_{ijkl}(\vec{\lambda}, \tau) = \langle T_{ij}(\vec{y}, t) T_{kl}(\vec{y} + \vec{\lambda}, t + \tau) \rangle$$

and the brackets denote an ensemble averaging.

1.2- Turbulence source modelling

Now the difficulty is to model the Lighthill correlation tensor R_{3333} . We shall neglect the "self-noise" term $u_3'^2$ as small compared with the "shear-noise" term Uu_3' in the Lighthill tensor T_{33} . Assuming that the space-time velocity probability distribution is Gaussian and considering a small convection Mach number $M_c = U/c_0$, it is possible to write the acoustic power per unit volume of turbulence as a function of the second derivative at the origin of the time velocity correlation

$$\frac{\partial P}{\partial v} = - \frac{2\rho_0 \langle u_3'^2 \rangle U^2 L R''(0)}{c_0^3} \quad (1.3)$$

where L is the integral longitudinal turbulence scale. Now in the theory of locally isotropic turbulence, the longitudinal velocity correlation $f(r)$ has, for small $r \ll L$, a parabolic variation with a radius of curvature known as the Taylor microscale λ_T , $1/\lambda_T^2 = -f''(0)/2$. Similarly, for time correlation at a fixed point of space one can introduce a Taylor time microscale t_T through $1/t_T^2 = -R''(0)/2$. We suggest that the microscales in space and time are related by $\lambda_T = u_0 \tau_T$, where u_0 is the r.m.s. velocity fluctuation (in one direction), $u_0 = \sqrt{u'^2}/3 = 2K/3$. That is to say we assume that in the convected reference frame, small-time variations are produced by the "transport" (not uniform convection) of small space scales by the large eddies. λ_T can be written $\lambda_T = \sqrt{20\nu K/\epsilon}$ in terms of energy density K , kinematic viscosity ν and energy dissipation rate ϵ per unit volume, using standard results of homogeneous turbulence theory. Hence t_T^2 , and then $R''(0)$, can be expressed in terms of K and ϵ . Moreover $\langle u_3'^2 \rangle$ can be written as $\langle u_3'^2 \rangle = 2K/3$. Then (1.4) can be written

$$\frac{\partial P}{\partial v} = \frac{4}{45} \frac{\rho_0 K U^2 L \epsilon}{\nu c_0^3} \quad (1.5)$$

It is interesting that all of the quantities occurring on the right side of (1.5) can be computed from the widely-used K - ϵ turbulence model. Codes for models of the K - ϵ kind calculate U at any position and also directly calculate K, ϵ at every point. L can be expressed as an integral of the energy spectrum function $E(k)$, and if $E(k)$ is chosen to have the von Kármán form, the integral can be evaluated in terms of known function whose parameters are fixed once K, ϵ are known.

1.3- Scaling laws and numerical calculations

If it is assumed that K scales with the square of a typical mean velocity U , then it follows from (1.5) that the acoustic power radiated along the duct by a volume $v = D^3$ obeys the scaling relation

$$P \propto \rho_0 U^3 D^2 \left(\frac{UD}{\nu} \right) \left(\frac{U}{c_0} \right)^3, \quad (1.6)$$

in which $\rho_0 U^3 D^2$ is a representation, in scaling terms, of the flow power (the mechanical power required to maintain the given flow), and $(U/c_0)^3$ is the acoustic efficiency factor found for this problem by Davies and Ffowcs-Williams [1]. We predict [2] a U^7 law because of the extra Reynolds number factor UD/ν . This in turn arises from recognition that the source term involves the second derivative of a correlation $R''(0)$ which is inversely proportional to the squared microscale λ_T^2 , the microscale being itself proportional to the square root of the overall Reynolds number.

Computations were carried out using a K - ϵ code in routine use at Electricité de France. The particular case of a duct of 10 cm cross-section side was considered, for mean velocities U_0 (averaged over a section) of 10, 20, 40 and 80 m/s, corresponding to Reynolds numbers, $U_0 D/\nu$, equal in air to 6.6×10^4 , 1.32×10^5 , 2.64×10^5 , 5.28×10^5 . The total acoustic power per unit length, obtained by integrating the results for $\partial P/\partial v$ across the duct cross-section, is plotted against the mean velocity U_0 .

A U_0^7 variation fits the lowest three Reynolds numbers perfectly, nevertheless use of U_0^7 would lead to an overprediction, at the highest Reynolds number, of about 5 dB. This variation arises from weak Reynolds number dependence of the ratio K/U_0^2 , for example, and from different such dependence at different points of the cross-section, giving on integration a result which cannot be traced to one particular effect.

2- Duct obstructed by a diaphragm

Because of the very idealized configuration we studied, there is a lack of experimental data to compare our results with. That is why we looked at the problem of a duct obstructed with a diaphragm. Using conformal mapping and Matched Asymptotic Expansions, we have determined a 2D Green's function which allow us to assess numerically the acoustic power.

2.1- The Green's function

The Green's function in a two-dimensional duct obstructed by a diaphragm reads

$$g(z; z_0|\omega) = -\frac{1}{\pi} \text{Im} \left\{ \sin^{-1} \left(\frac{\sin \frac{z_0 \pi}{2a}}{\sin \frac{b\pi}{2a}} \right) \right\} \times e^{ik_0 y} = -\frac{1}{\pi} v(z) \times e^{ik_0 y}, \quad (2.1)$$

where $z = x + iy$ is the observer coordinate and $z_0 = x_0 + iy_0$ is the source coordinate.

- figure 2.1 -

This Green's function g satisfies the boundary condition $\partial\Phi/\partial n = 0$ on the walls, where $\partial\Phi/\partial n$ denotes the outward normal derivative. Note here that this result is purely two-dimensional.

2.2- An expression for the acoustic power

This complicated expression for the Green's function does not allow to simplify the expression for the acoustic power as much as in the case of an empty duct. We are left with an expression involving two surface integrations and the second derivatives of the Green's function,

$$P \propto \frac{4\rho_0 2a}{\pi^2 c_0} \int \int_{S(\vec{y})} \int \int_{S(\vec{\lambda})} d\vec{y} d\vec{\lambda} \sum U_i(\vec{y}) U_j(\vec{y} + \vec{\lambda}) R_{ij}(\vec{\lambda}) \frac{\partial^2 v(\vec{y})}{\partial y_k \partial y_l} \frac{\partial^2 v(\vec{y} + \vec{\lambda})}{\partial y_k \partial y_l} \quad (2.2)$$

Note here that the mean velocity field (U_1, U_2) is given by the $K-\epsilon$ code. Moreover, the second derivatives of the Green's function are just mathematical expressions easily computable all over the duct. R_{ij} is the usual space velocity correlation and can be expressed in locally isotropic turbulence with help of the longitudinal and lateral correlation functions $f(r)$ and $g(r)$. Those two functions can be estimated in the same manner as the integral scale L in the first part. They can be expressed as integrals of the energy spectrum $E(k)$, and if $E(k)$ is chosen to have the von Kármán form, these integrals are locally computable once K, ϵ are known. Finally, all the terms to be integrated in (2.2) are known from the knowledge of K and ϵ , or analytically computable.

2.3- Numerical results and comparison with experiments

$K-\epsilon$ computations were carried out for two different configurations. First we considered a diaphragm of large aperture $a=4\text{cm}$ and $e = 2b = 55\text{mm}$ (see figure 2.1), for mean flow velocities of 32 and 75 m/s. The second case is a diaphragm of small aperture $a=4\text{cm}$ and $e = 2b = 15\text{mm}$, with mean flow velocities of 14 and 32 m/s.

The total acoustic power, obtained by integrating (2.2) over the duct area, is plotted against the mean velocity U_0 for the two apertures.

Although there is a shift between numerical and experimental results (up to 20 dB), it is interesting that, in both cases, the acoustic power follows the $P \propto U^4$ scaling law experimentally predicted and which is also found by dimensional analysis.

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