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Solitary wave in a slender rod of constant curvature

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ABSTRACT: An equation is derived to describe the propagation of nonlinear strain waves in a rod of constant curvature. A solution is obtained in the form of a steady-state solitary wave with symmetrical leading and trailing edges.

A number of papers have been devoted to aspects of the propagation of elastic waves in straight rods [1,2]. Samsonov [2] has derived a nonlinear equation for a longitudinal strain wave in a slender rod. The steady-state solution of his equation are the result of balance between a second-order nonlinearity of the hydrodynamic type and dispersion of the form

$$\omega^2 = c_0^2 k^2 - \frac{\nu^2 a^2}{2} (\omega^2 - c_1^2 k^2) k^2, \quad (1)$$

where c_0 and c_1 are the velocities of linear longitudinal and shear waves, $\nu = \lambda(\lambda + 2\mu)^{-1}$, and a is the radius of the rod's cross-section.

If the rod is not straight, but curves into an arc of radius R , another term must be added to the dispersion relation (1) [3]:

$$\omega^2 = c_0^2 k^2 - \frac{\nu^2 a^2}{2} (\omega^2 - c_1^2 k^2) k^2 + c_0^2 R^{-2}.$$

The addition to Eq.(1) because of the curvature of the rod can be decisive in some cases (e.g., when $ak \ll 1$ or $kR \ll 1$). Consequently, it makes sense to restrict the problem to Klein-Gordon dispersion in the

long-wavelength approximation:

$$\omega^2 = c_0^2 k^2 + c_0^2 R^{-2}. \quad (2)$$

Determining the linear part from the dispersion relation (2) and adding a second-order nonlinear term [2] to it, we obtain the nonlinear Klein-Gordon equation

$$u_{tt} - c_0^2 u_{xx} + (c_0/R)^2 u = \beta (u^2)_{xx}, \quad (3)$$

which describes the propagation of finite-amplitude waves in a curved rod. Here β is the nonlinearity parameter of the rod material and u is the x -derivative (the x axis is directed along the rod) of the longitudinal component of the displacement vector, i.e., the longitudinal strain.

Our objective is to find steady-state solutions in the form of solitary waves, i.e., solutions of the form $u = u(\eta)$ ($\eta = x - Vt$, where $V = \text{const}$ is the wave velocity) that are finite for all values of η and vanish together with the derivative v in the limits $\eta \rightarrow \pm\infty$. Integrating once, we really obtain the following expression in the class of steady-state solutions:

$$\left[\frac{dv}{d\xi} \right]^2 = 4 \frac{(v^2 - v^3)}{(3v - 2)^2}, \quad 0 < v < 1. \quad (4)$$

Here we have introduced the new variables v and ξ :

$$u = -\frac{3}{2} \left[1 - \frac{v^2}{c_0^2} \right] \frac{c_0^2}{\beta} v; \quad \xi = \eta R^{-1} \left[1 - \frac{v^2}{c_0^2} \right]^{-1/2}$$

Integrating Eq. (4), we obtain the solution

$$|\xi| = \begin{cases} 3(1-v)^{1/2} + \ln \frac{1 - (1-v)^{1/2}}{1 + (1-v)^{1/2}}; & |\xi| < \xi_0; \quad 2/3 < v < 1 \\ 2\xi_0 - 3(1-v)^{1/2} - \ln \frac{1 - (1-v)^{1/2}}{1 + (1-v)^{1/2}}; & |\xi| > \xi_0; \quad 0 < v < 2/3, \end{cases} \quad (5)$$

where $\xi_0 = 3^{1/2} - \ln(2 + 3^{1/2}) \approx 0.42$. The derivative of function $v(\xi)$ is infinitive at the points $\xi = \pm\xi_0$. Consequently, the wave obtained has two symmetrically located vertical edges. In the vicinity of the edges $v(\xi)$ has a singularity of the form

$$v = \frac{2}{3} \left[1 - 3^{1/4} \operatorname{sgn}(|\xi| - \xi_0) (2||\xi| - \xi_0|)^{1/2} \right],$$

and it is clear just from the procedure used to solve Eq.(3) that this singularity is a consequence of a presence higher derivative of the equation in the nonlinear term.

The tail of the wave, i.e., the parts that decay infinitely in either direction, have the same structure as the tails of a Korteweg-de Vries soliton. As in the case of the Korteweg-de Vries soliton, the amplitude A and the width Δ of the wave (5) depend on its velocity. however, a different relation exists between the amplitude and the width: $\Delta \sim A^{1/2}$.

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