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Numerical study of nonlinear elastic behaviour of structurally inhomogeneous media

A.N. BOGDANOV

N.N. Andreev Acoustic Institute, Russian Academy of Sciences, Shvernika st. 4, 117036 Moscow, Russia

Abstract: Nonlinear elastic properties of structurally inhomogeneous media are investigated in numerical experiment. For these media to be simulated, the lattices of masses (sites), connected by nonlinear springs, are used. For the cases of 2D triangular and 3D fcc lattices, the dependence of linear and nonlinear elastic modules and nonlinearity parameters of second and third orders on the number of defects (absent sites) is obtained. Scaling behaviour of nonlinear elastic modules is studied in the vicinity of percolation threshold (i.e. near the point, where all elastic modules vanish).

1. INTRODUCTION

At present, special attention is assigned to investigations of the media with structural inner inhomogeneities [1-3] (granular, porous, cracked and others), because they can be characterized by great nonlinearity parameter (several orders higher than that of a homogeneous material). In present paper, the results of numerical study of nonlinear elastic properties of structurally inhomogeneous materials are reported and discussed, with the model of the media is taken in realistic form, when certain part of structural elements (grains) is removed at random. This model is treated as an elastic percolation system and, in practice, corresponds to the unconsolidated inhomogeneous rocks and to the forming of branched net of cracks in the initially uniform sample.

2. METHOD AND PROCEDURE

The basic model of structurally inhomogeneous medium is represented by the lattice, which is consisted of masses (sites)

connected with springs. The calculations have been carried out for the 2D triangular lattice and fcc lattice in 3D case. The potential energy of interaction between the elements (energy of spring deformation) was chosen in the form:

$$U = \frac{\alpha}{2} (dr)^2 + \frac{\beta}{n+1} (dr)^{n+1}, \quad (1)$$

where $\alpha, \beta = \text{const}$, dr was deformation of the spring and n was the nonlinearity parameter. The expression (1) corresponds to the nonlinear force of interaction between the sites $F = -\partial U / \partial (dr)$ and can be used as the simple model of contact interaction between the grains in unconsolidated rocks [4]. The parameters of the problem α, β were chosen, so that physical nonlinearity (nonlinearity of Hook's law) was considerably larger than geometrical nonlinearity of the medium.

The simulation of structurally inhomogeneous material was done with the help of well-known Monte-Carlo method [5], in which certain part of sites $q=1-p$ was removed from the lattice at random (p was the probability of a site presence). In given computer experiment the triangular lattice of $30 \times 30 = 900$ sites was modeled in 2D case and the fcc lattice of 1099 sites - in 3D case. When the obtained depleted lattice had been deformed in a specific way (uniform compression, linear compression or pure shear), the usual relaxation method [5] was applied in order to lead the system to new equilibrium position. For the nonlinear elastic modules to be defined the expansion of the stress tensor component σ_{ij} by the degrees of strain tensor u_{ij} was used.

The deformations of compression and shear were investigated separately, which gave us the opportunity to find the dependence of the elastic modules on p . In the present work only quadratic ($n=2$) and cubical ($n=3$) nonlinearities were considered (and, respectively, the modules K_1, K_2, K_3 (uniform compression), C_1, C_2, C_3 (linear compression) and μ_1, μ_2, μ_3 (pure shear), with μ_2 being null for both cases, because of the symmetry of the lattices).

3. RESULTS.

The examples of calculated dependence of the elastic modules on p are shown in the fig. 1 (uniform compression - K_1, K_2, K_3) and fig. 2 (shear - μ_1, μ_3) for fcc net. The vertical axis measures the magnitudes of the elastic modules K_1, K_2, K_3 and μ_1, μ_3 , related to the respective values for the medium without defects $p=1$ $\bar{K}_1(p) = K_1(p)/K_1(1)$ and so on. The value of percolation concentration

Fig.1. Dependence of elastic modules of uniform compression on p .

Fig.2. Dependence of elastic modules of shear on p .

Fig.3. Nonlinearity parameters of 2-nd and 3-rd orders for uniform compression, linear compression and shear.

$p=p^*$, where all the modules vanish, is referred to as percolation threshold [5]. In our calculation, the appropriate thresholds $p^*=0.59\pm 0.01$ and $p^*=0.42\pm 0.01$ have been obtained. In the vicinity of percolation threshold the dependence of all elastic modules on p has scaling character, i.e. their magnitude is proportional to $(p-p^*)^{\nu}$, where ν is the percolation exponent, determined by the dimension of the system d and by the order nonlinearity n (see (1)). Analysis of our results demonstrates the presence of common relation $\nu_1 < \nu_2 < \nu_3$, where ν_1 is the exponent, corresponding to the modules K_1, C_1, μ_1 ; ν_2 to K_2, C_2 ; ν_3 to K_3, C_3, μ_3 . For instance, $\nu_1=2.6\pm 0.5$, $\nu_2=3.3\pm 0.7$, $\nu_3=4.3\pm 1.0$ for triangular lattice and $\nu_1=3.4\pm 0.6$, $\nu_2=4.0\pm 1.0$, $5.5 < \nu_3 < 8.5$ for fcc lattice.

The figure 3 demonstrates the dependence of $\varepsilon_c^{(2)}=\overline{K}_2(p)/\overline{K}_1(p)$, $\varepsilon_c^{(3)}=\overline{K}_3(p)/\overline{K}_1(p)$, $\varepsilon_x^{(2)}=\overline{C}_2(p)/\overline{C}_1(p)$, $\varepsilon_x^{(3)}=\overline{C}_3(p)/\overline{C}_1(p)$, $\varepsilon_\perp^{(3)}=\overline{\mu}_3(p)/\overline{\mu}_1(p)$ on percolation concentration p . These relations have the sense of nonlinearity parameters, which are widely used in acoustics [1]. It is important to notice, that the dependence of nonlinearity parameters $\varepsilon_c^{(2)}$, $\varepsilon_c^{(3)}$ on p has not monotonous character in the wide range of concentration ($0.75\leq p < 1$ in 2D case and $0.6\leq p < 1$ in 3D case), in which the parameters exceed unity. The parameter $\varepsilon_\perp^{(3)}$ always decreases and comes to zero at p^* .

It can be proposed, that the features of nonlinear behaviour of structurally inhomogeneous material, mentioned above, have sufficiently common character and retain (although qualitatively) in the transition to more complex models of the media (anisotropic, media with noncentral forces etc. [4,6,7]).

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