Numerical analysis and design of industrial superplastic forming

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Abstract: In the aerospace industry, the production of structural shell components with a prescribed thickness distribution by superplastic forming requires the specification of the initial thickness profile in the undeformed sheet. To this purpose, a finite element simulation methodology developed previously for the analysis of superplastic deformation processes along with the optimum monitoring of the forming pressure is extended to the determination of the initial thickness profile. The utilization of the procedure is shown for the pre-contouring of the blank for the industrial production of a satellite tank. The examination of the produced part confirms that the requested thickness distribution is achieved fairly well and thus proves the appropriateness of the proposed computational approach as a tool for the process design in superplastic net-shape forming.

1. Introduction

Since the early stages of the application of superplastic forming as a production process in aerospace engineering, manufacturers have been able to take advantage of numerical simulation techniques [1]. Initially, numerical analysis was requested for a confirmation of the appropriateness of proposals for the production of complex geometrical shapes. Soon, the requirements extended to tasks relevant to the design of the process such as the optimum monitoring of the forming pressure and the prediction of the initial thickness profile of the blank for the net-shape forming of structural shell components.

A finite element simulation methodology developed at the ICA in compliance with demands from aerospace manufacturers is being applied to the analysis and design of industrial superplastic forming [2]. The method is tailored to the actually nonlinear viscous behaviour of the work-piece material, accounts for grain growth, the development of contact with the die and considers friction phenomena. It proves more efficient than an application of conventional elastoplastic algorithms to the present type of problems. Automated mesh generation and adaptive modification facilitate the handling with the finite element discretization also in conjunction with distributed processing on parallel computer architectures [3].

The material parameters are significant for both the forming process and the numerical procedure. Their effect on the forming [4] is of particular importance for the material selection and when uncertainties necessitate an adjustment of the material model to production data. Optimization of the pressure for a steady development of the superplastic deformation at minimum process time is part of the simulation procedure. It could be demonstrated for particular cases that optimization

substantially reduced the process time required with a pressure history estimated by the industrial manufacturer [5].

The process design for the net-shape forming of satellite tanks with a prescribed thickness profile necessitated an extension of the numerical simulation to the specification of the thickness distribution in the underformed sheet. The procedure conceived for this purpose [4] has been utilized for the determination of the thickness profile in the blank which was used for the production in the factory of the desired hemispherical component [6]. The distribution of thickness as measured in the product compares fairly well with the one requested [7] and thus confirms the reliability of the proposal numerical approach. In view of the significance of the subject for the aerospace industry, the specification of the initial geometry for superplastic net-shape forming is further developed here along with a discussion on the inverse formulation of the forming problem, [8, 9], in association with our particular task.

2. Numerical simulation of superplastic forming

2.1 Material model for superplastic deformation

The following considerations refer mainly, but not exclusively, to manufacturing processes in the aircraft and spacecraft industry where superplastic forming is utilized for the integral production of lightweight structural components. Thereby blanks of Ti-6Al-4V alloy are formed by gas pressure, frequently onto a die, at a constant temperature of 1200 K. At this temperature, the mechanical response of the material may be considered rigid-viscous, the uniaxial characteristic being described by a nonlinear dependence of the flow stress σ on the rate of deformation $\dot{\delta}$. Following Fig. 1, the grain size d enters as a parameter and grows with progressing time in dependence on the local rate of deformation, Fig. 2. As a consequence, the uniaxial material response under superplastic conditions requires the specification of the following two relationships

$$\sigma = f(\delta, d), \quad \dot{d} = g(t, \delta) \quad (1)$$

In the case of the Ti-6Al-4V alloy, we utilized for this purpose in [1] the experimental data reported in [10], which are reproduced in the logarithmic diagrams Fig. 1 and 2.

Superplastic deformation is characterized by the production of high permanent elongations in the material and requires a significant dependence of the flow stress on the rate of deformation which prevents the occurrence of localized necking and failure. The rate sensitivity is defined by the quotient

$$m = d \ln \sigma / d \ln \dot{\delta} \quad (2)$$

the slope of the customary logarithmic representation of the flow stress. For the Ti-6Al-4V alloy, m attains values appropriate to superplasticity at a low $\dot{\delta}$, thus implying slow deformation processes. Expression (2) for the rate sensitivity suggests the following relation for the flow stress at a constant grain size

$$\sigma = k \dot{\delta}^m \quad (3)$$

which may be considered a local linearization of the actual logarithmic diagrams in Fig. 1. The form (3) appears to be a useful approximation for an elementary discussion on the significance of the material parameters k and m for the forming process [4] and for the behaviour of the numerical solution algorithm [1]. In particular $0 < m < 1$ is indicative of the nonlinearity of the viscous response of the solid material. The case $m = 1$ reveals a linear viscous problem, for $m = 0$ the material flows at constant stress as in perfect plasticity. It may be seen that an increasing value of the rate sensitivity m tends to balance inhomogeneities in the deformation and thus impedes the formation of localized necks. Also, a decrease of the factor k in (3) reduces the time scale of the process under otherwise unchanged conditions.

In the multiaxial case, the material response may be described by the relations

$$\boldsymbol{\sigma}_D = 2\mu\boldsymbol{\delta}_D, \quad \sigma_H = 3\bar{\kappa}\delta_V \quad (4)$$

for the deviatoric and hydrostatic/volumetric parts of Cauchy stress and rate of deformation respectively, collected in 6×1 vector arrays. Since superplastic flow is considered isochoric, the factor $\bar{\kappa} \rightarrow \infty$ merely represents a numerical penalty parameter, whereas the material viscosity μ establishes the physical relation between deviatoric quantities and may be deduced from uniaxial test data. To this purpose, the von Mises equivalent stress $\bar{\sigma}$ obtained from the deviatoric relation in (4) is compared to the uniaxial flow characteristic of the material as by

$$\bar{\sigma} = 3\mu\bar{\delta} = f(\bar{\delta}, d) \quad (5)$$

and yields the viscosity coefficient μ for a given equivalent rate of deformation $\bar{\delta}$ and grain size d , thus completely specifying the nonlinear viscous model of (4).

2.2 Finite element simulation procedure

For a numerical simulation of the forming process, both the work-piece material and the die are represented by finite element models. In superplastic forming, the die may be considered rigid and merely defines the geometrical restrictions posed to the motion of the work-piece material. The motion in the discretized system is

$$\mathbf{X} = {}^0\mathbf{X} + \int \mathbf{V} dt \quad (6)$$

where the vector arrays $\mathbf{X}, \mathbf{V}$ collect the coordinates $\mathbf{x}$ and velocities $\mathbf{v}$ respectively of the nodal points of the finite element mesh at time instant t , ${}^0\mathbf{X}$ refers to their original positions. During the course of the quasistatic deformation, the velocity field in the work-piece material is governed by the equilibrium condition. In this connection, the forces at the mesh nodal points resulting from stresses in the work-piece material are collected in the vector array $\mathbf{S}$ and read

$$\mathbf{S}(\boldsymbol{\sigma}, \mathbf{X}) = \mathbf{D}(\mathbf{V}, \mathbf{X})\mathbf{V} \quad (7)$$

Relation (7) may be interpreted as the expression of the viscous material law (4) for the discretized solid, $\mathbf{D}$ being the viscosity matrix of the system. Its dependence on the velocity $\mathbf{V}$ reflects the nonlinearity of the material; the dependence on the evolving microstructure as represented by the grain size d in (5) is not indicated explicitly in (7).

When surface points of the work-piece material are in contact with the die, contact forces $\mathbf{F}_n$ normal to the surface of the die and friction forces $\mathbf{F}_t$ tangential to it are accounted for via the expressions

$$\mathbf{F}_n = -k_n \mathbf{v}_n, \quad \mathbf{F}_t = -k_t \mathbf{v}_t \quad (8)$$

Since the material velocity $\mathbf{v}_n$ normal to the die surface must be suppressed upon contact, $k_n \rightarrow \infty$ is a penalty factor associated with a relaxed contact condition. The factor

$$k_t = |\mathbf{F}_t|/|\mathbf{v}_t| \leq k_{\max} \quad (9)$$

in the above kinematic formulation of a friction force opposed to the tangential velocity $\mathbf{v}_t$, which was introduced in [2], does not pose any restrictions to the friction law for $|\mathbf{F}_t|$ whilst the limitation by $k_{\max}$ corresponds to a penalty approach to the condition of sticking. Also, the velocity components in (8) may be obtained relatively to a moving die as well.

The contact forces act on the work-piece material in addition to the applied loading

$$\mathbf{R}(t, \mathbf{X}) = p(t)\mathbf{R}_p(\mathbf{X}) \quad (10)$$

which we specified here with the intensity of the forming pressure p at instant t and the vector array $\mathbf{R}_p$ associated with the nodal point forces for a unit pressure applied at the deformed geometry $\mathbf{X}$.

The condition of equilibrium may now be stated as

$$\mathbf{S} - \mathbf{F} - \mathbf{R} = [\mathbf{D} + \mathbf{C}]\mathbf{V} - p\mathbf{R}_p = \mathbf{o} \quad (11)$$

and the complete kinematic description of the contact forces as by (8) in conformity with the viscous nature of the deformation problem suggests a modification of the system matrix $\mathbf{D}$ by $\mathbf{C}$. The latter is a diagonal hypermatrix accounting for the joint action of $\mathbf{F}_n$ and $\mathbf{F}_t$ at the points currently in contact with the die. Thereby, the structure of the system matrix is not affected by the variation of the boundary conditions during the course of the forming process. The nonlinear equation (11) for the velocity $\mathbf{V}$ at each instant t , refers to the actual geometry $\mathbf{X}$ of the deforming work-piece material. The latter is determined by an incremental approximation to (6), the integration of the velocity with respect to time. The algorithm for the numerical simulation of the forming process may be summarized as follows

$$\begin{aligned} t &\Leftarrow t + \tau \\ \mathbf{R} &\Leftarrow \mathbf{R}(t, \mathbf{X}) = p(t)\mathbf{R}_p(\mathbf{X}) \\ \mathbf{V}_{i+1} &\Leftarrow \mathbf{V}_i + [\mathbf{D} + \mathbf{C}]_i^{-1}[\mathbf{R} + \mathbf{F} - \mathbf{S}]_i \\ \mathbf{X} &\Leftarrow \mathbf{X} + \int_r \mathbf{V} dt \end{aligned} \quad (12)$$

With the procedure, the time is advanced incrementally by τ up to the completion of the forming process. In each step, the loading is specified and the equilibrium equation is solved for the velocity $\mathbf{V}$. The iteration of the residuum may be performed with any suitable matrix operator including the one based on consistent linearization; use of the regular system matrix as from (11) proves satisfactory. Finally, the geometry is updated either repeatedly as part of the iteration cycle, in conjunction with an implicit scheme for the incremental integration or once after the iterative solution for the velocity at constant $\mathbf{X}$ in connection with an explicit integration. A discussion on the convergence of the iterative solution and the stability of the numerical integration was presented in [1].

2.3 Analysis of process and optimization of forming pressure

The computational methodology presented so far allows a numerical simulation by finite elements of superplastic forming. The algorithm requires the input of the geometry of the blank and of the die, the flow stress as a function of rate of deformation and grain size, and the grain growth kinetics underlying the evolution of the initial grain size. The process of forming may then be followed up for a prescribed history of the applied gas pressure and provides information on the development of the deformation, processing time, distribution of the grain size and thickness in the product. This kind of analysis is relevant to the quality of the product with respect to certain tolerances; the latter may be strict and require a reliable simulation model.

The numerical constituents of the model, i.e. the discretization of the work-piece material and the die by finite elements and the numerical time integration, may be examined to a certain extent without accurate physical data. The physical modelling requires the mechanical properties of the material and the description of friction phenomena upon contact with the die. Due to the scarcity of relevant test data, the simulation of sample processes performed in the industrial laboratory may indicate the necessity for an adjustment of the material model.

In this connection, the discussion on the material properties in Section 2.1 becomes important and is illustrated in Figs. 3 to 5 for the superplastic forming of a pre-contoured blank onto a hemispherical die. The effect of variations of material parameters is demonstrated for frictionless

Fig. 1 Flow stress of Ti-6Al-4V at 1200 K

Fig. 2 Grain growth kinetics of Ti-6Al-4V

Fig. 3 Forming of hemispherical shell ($\sigma = k\delta^m$)

Fig. 4 Effect of rate sensitivity m on final thickness profile

Fig. 5 Effect of k on time scale of forming pressure

contact and the power law (3), considered as a linear approximation to the actually nonlinear viscous response. An initial reference solution was obtained for the parameters $k = 36.3, m = 0.38$. The subsequent simulation with $m \leftarrow 2m, k = \text{const.}$ shows that the increased rate sensitivity tends to reduce differences in the thickness of the product. Also, for $k \leftarrow 0.886k, m = \text{const.}$ the theory [4] predicts a modification of the time scale of the process by 0.5, which is confirmed by the history of forming pressure. Thereby, the thickness profile of the product and the intensity of forming pressure remain practically unchanged.

The variation of the forming pressure in Fig. 5 is associated with a prescribed maximum rate of deformation $\dot{\delta}_o = 2.85 \times 10^{-4} \text{ s}^{-1}$ for the reference process. The adjustment of the forming pressure to an optimum rate of deformation, a standard task in industrial process design, is performed along with the numerical process simulation. Optimized processes are characterised by a minimum process time under observance of the superplastic conditions. The latter ensure that the material will not fail due to localized necking and require that the maximum equivalent rate of deformation in the work-piece material does not exceed the optimum value $\dot{\delta}_o$ associated with the maximum rate sensitivity in of the flow stress. With reference to Fig. 1, both the slope of the flow stress diagrams and the optimum rate of deformation decrease as the grains grow during forming. Also, the rate of grain growth is highest where the maximum rate of deformation appears in the work-piece material, cf. Fig. 2. Accordingly, the optimum value $\dot{\delta}_o$ appertaining to the current grain size at this place should be considered the critical one rather than the higher value associated with the initial uniform grain size in the work-piece material.

The numerical design of superplastic forming for an optimum rate of deformation in the work-piece material, $\dot{\delta}_{\max} = \dot{\delta}_o$, is governed at each instant t by the equilibrium condition in conjunction with the kinematic constraint. The equation system then reads

$$\mathbf{S} - \mathbf{F} = [\mathbf{D} + \mathbf{C}]\mathbf{V} = p\mathbf{R}_p, \quad \bar{\delta}_{\max} = \dot{\delta}_o \quad (13)$$

and the intensity $p(t)$ of the pressure is an unknown quantity. It may be obtained iteratively by

$$\mathbf{V}_{j+1} = [\mathbf{D} + \mathbf{C}]_j^{-1}[p_i\mathbf{R}_{p_j}], \quad p_{i+1}/p_i = f(\dot{\delta}_o)/f(\bar{\delta}_{\max i+1}) \quad (14)$$

The iteration cycle for the pressure starts with an estimate p_i and solves for the velocity $\mathbf{V}_{i+1}$. The associated $\bar{\delta}_{\max i+1}$ and the optimum value $\dot{\delta}_o$ furnish the respective uniaxial flow stress $f(\dot{\delta})$, their quotient determines the new estimate p_{i+1} . Alternatively, the pressure may be adjusted directly within the iteration cycle for the velocity as by

$$\mathbf{V}_{i+1} = p_{i+1}[\mathbf{D} + \mathbf{C}]_i^{-1}\mathbf{R}_{p_i} = p_{i+1}\mathbf{V}_{p_i}, \quad \bar{\delta}_{\max i+1} = p_{i+1}\bar{\delta}_{p_{\max}} = \dot{\delta}_o \quad (15)$$

Each iteration cycle computes first the velocity $\mathbf{V}_{p_i}$ for the unit pressure. The complete solution is established with the determination of p_{i+1} from the kinematic constraint for $\bar{\delta}_{\max i+1}$. For simplicity of the presentation, (14) and (15) do not refer to the solution scheme of (12) for the velocities which, however, was considered in (7).

The significance of a numerical optimization of the forming pressure for the process design was demonstrated in [5]. The superplastic forming of an aircraft component, Figs. 6 to 9, was first simulated for a prescribed pressure history estimated in the factory on the basis of technological experience and engineering intuition. As a result, it could be shown that the process works at the optimum rate of deformation for the 1/3 of the time but then is too slow. The subsequent adjustment of the pressure to $\dot{\delta}_o = 3 \times 10^{-1} \text{ s}^{-1}$ by the simulation algorithm optimizes the process which now requires only 2/3 of the previous time. The computation completely accounts for the properties of the Ti-6Al-4V material as depicted in Figs. 1 and 2. In the symmetric product, which has a diameter of 250 mm, the thickness varies between 0.77 mm and the initial 1.8 mm, the grain size of $8\mu\text{m}$ grows to between $12.9\mu\text{m}$ and $15.1\mu\text{m}$ in the optimized process.

Fig. 6 Discretization of plane sheet

Fig. 7 Discretization of three-dimensional die surface

Fig. 9 Optimised vs. prescribed variation of forming pressure with time

Fig. 8 Development of superplastic deformation

3. Process design for net-shape forming

3.1 Specification of the initial thickness profile in the blank

The present task may be illustrated by Fig. 10. The satellite tank shown in the figure is conceived as a hemispherical shell with a prescribed thickness profile. The distribution of thickness along the meridian is depicted in the lower part of the figure. The shell is to be manufactured by superplastic deformation of a pre-contoured blank. The hemispherical shape may be produced by the geometry of the die whilst the prescribed thickness distribution in the component is to be achieved by the pre-contouring of the blank. This task is left to the numerical simulation.

The methodology proposed in [4] for the specification of the initial thickness distribution in the blank starts with the consideration of a differential element of the sheet with surface area d^oA and thickness os which deforms to dA and s in the final shell product. Since superplastic deformation is considered isochoric, we may write

$$sdA = ^osd^oA, \quad s = (dA/d^oA)^{-1}^os = ^us(\mathbf{x}) \quad (16)$$

which provides the local thickness as a result of the deformation process; this must be equal to us , the thickness ultimately required at position $\mathbf{x}$ on the shell. If the mapping $\mathbf{x}(^o\mathbf{x})$ for material positions does not depend on the initial thickness, os can be obtained immediately from (16). Otherwise, a recurrence scheme may be set up as follows

$$^os_{i+1}(^o\mathbf{x}) = (dA/d^oA)_i ^us(\mathbf{x}_i) = (^us/s_i)^us_i \quad (17)$$

Accordingly, each iteration cycle starts with an estimate os_i for the initial thickness at location $^o\mathbf{x}$ in the blank and determines the mapping $\mathbf{x}(^o\mathbf{x})$ as a result of the forming process. The associated value of the final thickness s and of the prescribed thickness $^us(\mathbf{x})$ then provide a new estimate $^os_{i+1}$ for the initial thickness.

In the finite element discretisation of the problem, the iteration (17) is applied to each representative thickness of the model. The collective notation for the algorithm then reads

$$^os_{i+1} = [^os_i/s_i]^us_i = [^us_i/s_i]^os_i \quad (18)$$

where the vector arrays $\mathbf{s}$ comprise the representative thicknesses. Each iterative cycle for the initial thickness starts with an estimate os_i and implies the simulation of the forming process according to (12). The final thickness $\mathbf{s}_i$ thus computed is used in (18) together with the prescribed values $^us(\mathbf{x}_i)$ for the determination of new estimates $^os_{i+1}$ entering the subsequent iteration. The convergence properties of the iteration scheme (18) are fixed by the dependence of the entities in the diagonal iteration matrix on the initial thickness. On the other hand, the residuum iteration as by

$$^os_{i+1} = ^os_i + \mathbf{h}_i[^os_i - \mathbf{s}_i] \quad (19)$$

allows for a choice of the iteration matrix $\mathbf{h}$ in order to achieve optimum numerical behaviour. For $\mathbf{h} = [^os/s]$, the scheme is equivalent to (18). Convergence of (19), however, requires the condition

$$\|\mathbf{I} + \mathbf{hg}\| < 1, \quad \mathbf{g} = d[^us - \mathbf{s}]/d^os \quad (20)$$

be satisfied.

The above condition refers to the spectral norm of the matrix expression in which the product of the iteration matrix $\mathbf{h}$ and gradient matrix $\mathbf{g}$ is assessed with respect to the unity matrix $\mathbf{I}$. For a substantiation of the system gradient, we notice that the constituent

$$d^us/d^os = [d^us/d\mathbf{x}][d\mathbf{x}/d^os] \quad (21)$$

Fig. 10 Hemispherical satellite tank. Prescribed distribution of thickness in product; thickness profile of blank unknown

Fig. 11 Iterative determination of initial thickness profile

Fig. 12 Specification of blank geometry

Fig. 13 Thickness distribution in the product vs. requested profile

depends on the prescribed function $^us(\mathbf{x})$ and on the sensitivity of the deformation process to the initial thickness.

Concerning the second constituent, one may deduce the form

$$d\mathbf{s}/d^os = \exp[\int_0^t (\partial \dot{\mathbf{s}} / \partial \mathbf{s}) dt] \quad (22)$$

which is based on the time integration of the current sensitivity in the development of the thickness during the course of the process. Its determination implies a number of additional operations within the incremental finite element algorithm.

3.2 Net-shape forming of hemispherical satellite tank

The numerical prediction of the initial thickness profile was motivated by the industrial design of the superplastic forming of the satellite tank in Fig. 10. The originally plane sheet is formed onto a hemispherical die whilst the requested thickness distribution in the component is to be achieved by appropriately pre-contouring the blank. The sheet material employed is of Ti-6Al-4V with an initial grain size of $7\mu\text{m}$. The process of forming takes place at a temperature of 1200 K and the pressure history has to be adjusted to the optimum rate of deformation $\delta_o = 3.0 \times 10^{-4}\text{s}^{-1}$. Both the thickness profile of the blank and the forming pressure are to be specified by the computation procedure.

The discretization of the work-piece material by finite elements is demonstrated in Fig. 3 together with the development of the deformation during the course of the process simulation. Also, preliminary investigations indicated that the work-piece material contacts the die only in the final stage of forming without any tendency to sliding. Accordingly, the effect of friction is not apparent.

The determination of the initial thickness follows the recursive scheme of (19) in conjunction with a relaxation factor in place of the iteration matrix. The procedure is started with a constant thickness $^os_0 = 6\text{ mm}$ of the blank, cf. Fig. 11. The simulation of the forming process refers to the material characteristics of Figs. 1, 2 and is performed by adjusting the pressure to the optimum rate of deformation. It yields the final thickness s_0 as shown in of Fig. 11, which deviates significantly from the requested distribution us . The difference is used as the residuum in (19) and provides a new estimate for the initial thickness profile in the blank. The converged solution os is slightly smoothed on the basis of technological experience, Fig. 12 and is used for the production of the pre-contoured blank in the factory.

The blank, subsequently pre-contoured in accordance with the computational specification of the thickness profile, was subject to superplastic deformation under the determined optimum pressure history and was formed to the hemispherical shell component. The thickness distribution in the shell as measured along two meridians is shown in Fig. 13 and confirms that the produced shell meets satisfactorily with the requirements for the tank component.

3.3 On an inverse formulation of the forming problem

In net-shape forming, the geometry of the final product is specified whilst the initial geometry of the blank has to be determined. Apart from the trial-and-error approach which we proposed and applied so far, we are interested in exploring the possible application of a direct technique avoiding the repeated simulation of the complete forming process. To this end we follow [8, 9] and state the condition of equilibrium for the work-piece material modelled at the end of the process when the component is ultimately shaped. It reads

$$\mathbf{S}(\boldsymbol{\sigma}, \mathbf{X}) - \mathbf{F} - p\mathbf{R}_p(\mathbf{X}) = \mathbf{o} \quad (23)$$

and accounts for contact forces $\mathbf{F}$ from the die as well as for applied pressure. Since the geometry $\mathbf{X}$ of the final component is given, specification of the initial geometry $^o\mathbf{X} = \mathbf{X} - \mathbf{U}$ requires the

displacements $\mathbf{U}$. In the particular application in Section 3.2, points on the external surface of the shell are initially positioned in the corresponding upper plane of the blank. Therefore, their vertical displacements are known. In (23), the direction of the contact forces depends on the velocities which might be approximated for this purpose by the displacements. The still missing knowledge of the intensity of contact may be compensated by the imposition of the aforementioned vertical displacements. Finally, the equilibrium condition becomes an equation for the displacements if the stress resultants are expressed in terms of strains. To this end, we notice that in isochoric inelastic deformation, the deviatoric stress may be related to the rate of deformation as by

$$\boldsymbol{\sigma}_D = \frac{2}{3} \frac{\bar{\sigma}}{\bar{\delta}} \boldsymbol{\delta} \Rightarrow \frac{2}{3} \frac{\bar{\sigma}}{\bar{\gamma}} \boldsymbol{\gamma}, \quad \boldsymbol{\gamma} = \int \boldsymbol{\delta} dt = \boldsymbol{\gamma}(\mathbf{U}) \quad (24)$$

The transition from the original expression to the one in terms of the strain $\boldsymbol{\gamma}$ implies the assumption that the direction of the stress remains constant throughout the deformation process. Nevertheless, the deviatoric stress may then be obtained from strains and is thus related to the displacements; the hydrostatic stress component cannot be physically related to kinematics. Although (24) might introduce the displacements as unknowns in the equilibrium condition, evaluation of the stress for a given strain requires specification of $\bar{\sigma}$ by the uniaxial flow characteristic of the material. In the case of plasticity, the latter is given as $\sigma(\gamma)$; perfectly plastic materials also possess a defined flow stress. In the present case of viscosity, however, the flow stress is a function of the rate of deformation $\sigma(\delta)$ and the evaluation of the stress cannot be based on a given strain. Despite this elementary difference to plastic flow, a one-step computation of the displacements can be considered in superplastic forming also, and may provide an initial estimate for the trial-and-error approach developed in Section 3.1.

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