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► To cite this version:

R. Atanasov, F. Bassani, V. Agranovich. Asymmetric quantum well optics from a mean field polariton theory. Journal de Physique IV Proceedings, 1993, 03 (C5), pp.C5-63-C5-66. 10.1051/jp4:1993511 . jpa-00251597

HAL Id: jpa-00251597

<https://hal.science/jpa-00251597>

Submitted on 1 Jan 1993

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Asymmetric quantum well optics from a mean field polariton theory

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Abstract: We consider the elementary excitations in quantum wells of arbitrary shape, as they result from the coupling between excitons and electromagnetic radiation. The exciton states are obtained from a two-band model [1]. The solutions of Maxwell equations with the appropriate exciton dependent nonlocal susceptibility are obtained by using a mean field approach [2] based on the fact that the radiation wavelength λ is much larger than the well width L . Application to asymmetric wells gives the polariton dispersion modes and the radiative linewidths for all possible well shapes. The splitting of the Z-mode and the values of the radiative lifetimes strongly depend on the well width L and its asymmetry. The reflectivity and absorption are computed and are shown to give strong evidence of polariton modes.

1. NON-LOCAL LINEAR SUSCEPTIBILITY AND DIELECTRIC FUNCTION

We consider an asymmetric quantum well (A.Q.W.) of length L_w . The linear susceptibility is non-local in the $\hat{z}$ direction and can be computed from the results of the microscopic theory on excitation energies and wave functions. Outside the QW regions the susceptibility gives a dielectric constant ϵ_B . Inside the QW regions the susceptibility tensor splits into background term χ_b and two contributions, one due to electronic transitions between subbands and the other to exciton bound states. We have:

$$\chi(E, z, z') = \chi_b \delta(z - z') + \chi^{el}(E, z, z') + \chi^{ex}(E, z, z') ; E = \hbar\omega. \quad (1)$$

Denoting each pair of subbands with indexes i and j , and each polarization with p ($p = x, y, z$),

$$\chi_p^{el}(E, z, z') = \sum_{i,j} \chi_p^{el}(i, j, E) A_{ij}(z) A_{ij}(z') ; A_{ij}(z) = \langle i|z\rangle\langle z|j\rangle \quad (2)$$

where

$$\chi_p^{el}(i, j, E) = \frac{2e^2 P_{pj}^2}{\epsilon_0 V \omega^2 m_0^2} \sum_{k_{\parallel}} \left(\frac{1}{E_{ij}(k_{\parallel}) - E - i\Gamma} + \frac{1}{E_{ij}(k_{\parallel}) + E + i\Gamma} \right). \quad (3)$$

Here, the confinement factor A_{ij} is the product of the electron and hole envelope functions in state i ($i = 1, 2, \dots$) and j ($j = 1\beta, 2\beta, \dots; \beta = hh, lh$) respectively. P_{pj} is the bulk dipolar transition matrix element between states of energy difference $E_{ij}(k_{\parallel}) = E_i - E_j + \hbar^2 k_{\parallel}^2 / 2\mu_{ij}$, where μ_{ij} is their corresponding reduced mass. The contribution of the exciton bound states inside the quantum well is

$$\chi_p^{ex}(E, z, z') = \sum_{i,j} \chi_p^{ex}(i, j, E) A_{ij}(z) A_{ij}(z'), \quad (4)$$

where

$$\chi_p^{ex}(i, j, E) = \frac{2e^2 P_{pj}^2}{\varepsilon_0 L_w \omega^2 m_0^2} \sum_{n=1,2} |F_{ij}^{(n)}(0)|^2 \left(\frac{1}{E_{ij}^{(n)} - E - i\Gamma_{ex}} + \frac{1}{E_{ij}^{(n)} + E + i\Gamma_{ex}} \right), \quad (5)$$

each exciton state (n) (1S, 2S, ..) being characterized by the transition energies $E_{ij}^{(n)}$ and by the amplitude of the exciton envelope function in the quantum well $F_{ij}^{(n)}(\mathbf{r})$.

The constitutive equation of the medium relating the electric induction $\mathbf{D}$ to the electric field $\mathbf{E}$ inside the well are:

$$D_p(z, E) = \varepsilon_b(z) E_p(z) + L_w \sum_{i,j} \chi_p(i, j, E) A_{ij}(z) \langle A_{ij} E_p \rangle \quad ; \quad \langle A_{ij} E_p \rangle = \int_0^{L_w} dz A_{ij}(z) E_p(z), \quad (6)$$

where, $\varepsilon_b(z)$ is the background dielectric constant at point z , and $\chi_p(i, j, E) = \chi_p^{el}(i, j, E) + \chi_p^{ex}(i, j, E)$. Outside the A.Q.W. we have the usual local relation $D_p(z) = \varepsilon_B E_p(z)$.

We next use the approximation that the electric field varies very little on the QW width because $\lambda \gg L_w$, and define an averaged field $\mathbf{F}$ ($\mathbf{F}$ being $\mathbf{E}$ or $\mathbf{D}$) [2]. The averaging is performed inside the well according to the relation $\overline{F_p} = L_w^{-1} \int dz F_p(z) = L_w^{-1} \langle F_p \rangle$. By choosing the polarization parallel to the A.Q.W. plane ($\chi_{||}$), and satisfying the appropriate boundary conditions, we obtain

$$\varepsilon_{||} = \overline{\varepsilon_b} + \sum_{i,j} \chi_{||}(i, j, E) \langle A_{ij} \rangle^2. \quad (7)$$

Analogously, taking into account the discontinuance of E_z trough the barriers, we find the expression for the perpendicular dielectric function $\varepsilon_{\perp}(E)$ near a fixed exciton resonance (n, ij)

$$\varepsilon_{\perp}(E, K_{||}) = \frac{L_w}{\langle \varepsilon_B^{-1} \rangle} + \frac{L_w^2}{\langle \varepsilon_B^{-1} \rangle^2} \sum_{i,j} \frac{\chi_z(E) \langle \varepsilon_B^{-1} A_{ij} \rangle^2}{1 + L_w \chi_z(E) \left(\langle \varepsilon_B^{-1} A_{ij}^2 \rangle - \langle \varepsilon_B^{-1} A_{ij} \rangle^2 / \langle \varepsilon_B^{-1} \rangle \right)}. \quad (8)$$

Fig. 1. Absorption spectra at normal incidence of double A.Q.W. with $\text{Ga}_{0.5}\text{Al}_{0.5}\text{As}$ barriers, 1.4 nm of GaAs, 1 nm of barrier and 1.7 nm of GaAs. The solid line denotes the absorption $\alpha_{||}$, and the long dashed line $\alpha_{\perp}$. In the insert we show the electron and hole confinement potentials with the corresponding envelope functions.

As an application of the above result we find the A.Q.W. absorption coefficient $\alpha_p(E) = (E/n_B \hbar c) \text{Im}(\epsilon_p)$, where $n_B = \sqrt{\epsilon_B}$. We give in Fig. 1 the absorption coefficient of a typical A.Q.W. when the incident electromagnetic wave is polarized in the plane, or perpendicularly to it, as computed from the above expressions with values of the parameters obtained with the method of Ref. [1]. One sees the direct heavy hole ($i=1, j=1hh$) exciton peak E_1HH_1 and the analogous light hole exciton peak E_1LH_1 , as well as weak (2S) exciton peaks on the right of these resonances. In addition, one can observe the crossed exciton state ($i=1, j=2hh$) as a relatively weak peak (E_1HH_2), smaller than those of the direct exciton states.

The light hole resonance in $\alpha_\perp$ is shifted with respect to its position in the spectrum of $\alpha_\parallel$ by approximately 1 meV. Such a splitting has been previously considered on the excitons [4], and has been found to be due to the electron - hole exchange interaction. This in our procedure is due to the fact that the parallel electric field components do not change across the internal surfaces, while the perpendicular ones change according to the boundary conditions.

2. POLARITONS IN A.Q.W.

To find the polariton states in an A.Q.W., we consider the reflection coefficients for an incident wave polarized perpendicularly (R_s) and parallel (R_p) to the plane of incidence. Choosing our frame of reference at the beginning of the quantum well ($z = 0$) and the $\hat{x}$ axis along the exciton wave vector $K_\parallel$, and imposing the boundary conditions on plane wave fields between barriers and wells, we obtain the reflection coefficients

$$R_s = \frac{iQ^2\eta_\parallel}{2\epsilon_B K_z - iQ^2\eta_\parallel} ; R_p = \frac{\epsilon_B^2 K_\parallel^2 \eta_\perp + K_z^2 \eta_\parallel}{2i\epsilon_B K_z - \epsilon_B^2 K_\parallel^2 \eta_\perp + K_z^2 \eta_\parallel}, \quad (9)$$

where $\eta_\parallel = L_w (\epsilon_\parallel - \epsilon_B)$, and $\eta_\perp = L_w (\epsilon_z^{-1} - \epsilon_B^{-1})$, and $K_z = \sqrt{Q^2 - K_\parallel^2}$ is the z - component of the light wave vector in the barrier Q ($Q = \sqrt{\epsilon_B} \omega / c$).

The poles of R_s and R_p are complex quantities in the form $E(K_\parallel) - i\Gamma(K_\parallel)$. According to the general results of the scattering theory [5], these poles correspond to mixed modes of electromagnetic radiation and A.Q.W. excitons (polariton states). The real part gives the polariton dispersion curve, and the imaginary part gives the corresponding radiative line width. We obtain different type of polaritons, according to the values of $K_\parallel$ with respect to the light wave vector Q .

Radiative modes ($K_\parallel < Q$) are resonant states, solutions of the Maxwell equations in the form of plane waves in the barriers and in the wells. Their dispersion laws result to be [6]:

$$E_T \simeq E_L \simeq E_{ij}(K_\parallel) ; E_Z \simeq E_{ij}(K_\parallel) + \frac{1}{\epsilon_B} \chi_\perp(i, j) \langle A_{ij}^2 \rangle ; \chi_\perp(i, j) = \frac{8e^2 P_{\perp j}^2 \hbar^2}{\epsilon_0 \pi (E_{ij}^{(n)})^2 m_0^2 \lambda_{ij}^2}, \quad (10)$$

where $E_{ij}^{LS}(K_\parallel)$ is the corresponding exciton dispersion law. Here, indices T, L and Z refer, respectively, to the transverse ($E \parallel \hat{y}$), longitudinal L and Z - modes ($E \parallel (\hat{x}, \hat{z})$). Numerical results for a typical double A.Q.W. are shown in Fig. 2. The imaginary parts of the reflectivity poles give the polariton radiative line widths, which can be explicitly written as follows:

$$\Gamma_{i,j}^T \simeq \frac{Q^2}{2\epsilon_B K_z} \chi_\parallel(i, j) \langle A_{ij} \rangle^2 ; \Gamma_{i,j}^L \simeq \frac{K_z}{2\epsilon_B} \chi_\parallel(i, j) \langle A_{ij} \rangle^2 ; \Gamma_{i,j}^Z \simeq \frac{K_\parallel^2}{2\epsilon_B K_z} \chi_\perp(i, j) \langle A_{ij} \rangle^2. \quad (11)$$

In the region $K_\parallel > Q$, K_z is an imaginary quantity and we are looking for solutions localized in the $\hat{z}$ direction, and evanescent in the barrier (surface polaritons). In this case the polariton modes have zero radiative linewidths and the following dispersion laws [6]:

$$E_{i,j}^T \simeq E_{ij}(K_\parallel) - \frac{Q^2}{2\epsilon_B K_z} \chi_\parallel(i, j) \langle A_{ij} \rangle^2 ; E_{i,j}^L \simeq E_{ij}(K_\parallel) + \frac{K_z}{2\epsilon_B} \chi_\parallel(i, j) \langle A_{ij} \rangle^2, \quad (12)$$

$$E_{ij}^Z \simeq E_{ij}(K_{\parallel}) + \frac{1}{\varepsilon_B} \chi_z(i, j) \left(\langle A_{ij}^2 \rangle - \frac{K_{\parallel}^2}{2K_z} \langle A_{ij} \rangle^2 \right). \quad (13)$$

The dispersive behavior of the surface polariton states is shown on the right side of the photon line $K_{\parallel} = Q$, and is displayed in Fig. 2.

Fig. 2. Dispersion curves of resonant and surface light hole polaritons in the double A.Q.W. shown in the insert of Fig. 1. The vertical line indicates the photon dispersion $E = \hbar c Q / n_B$.

3. CONCLUSIONS

We have given a procedure to compute the polariton states in semiconductor Q.W. of arbitrary shape and have applied it to the case of a double asymmetric well, thus extending previous work on excitons by Ferreira et. al. [7].

We find in this case a splitting between Z-polarized modes and in-plane modes of about 1 meV. Surface polaritons are also obtained for $K_{\parallel} > Q$.

The asymmetry of the problem gives also crossed exciton transitions HH2-E1, whose resonant life times are three orders of magnitude smaller.

The computed features of A.Q.W.'s are similar to those obtained in the symmetric case, but the presence of crossed transitions and longer lifetimes are expected to be important in nonlinear optics.

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