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AC response of Abrikosov vortices in layered superconductors

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In a layered high- T_c superconductor an Abrikosov vortex line perpendicular to the layers can be considered as a stack of coupled two-dimensional "pancake" vortices (2DV) with cores residing inside the superconducting layers. When the in-plane penetration length $\lambda_{ab} = \lambda$ is smaller than the Josephson length $\lambda_J = \lambda_c d / \lambda$ (d : layer period) the interaction between 2DV is via a magnetic pair force which is complicated but exactly known. We calculate linear and nonlinear ac responses for this model including randomly distributed pinning centers. In detail, the thermally averaged shift $S_n(t)$ of a 2DV in layer n under the influence of a finite external ac current in the frequency range up to 1 GHz is computed from coupled Fokker-Planck equations for the distribution functions of 2DV positions. In the adiabatic approximation, the Fokker-Planck equations can be cast into a nonlinear differential equation for a continuous shift function $S(z, t)$ using a gradient expansion. The constitutive equation for $S(z, t)$ is solved perturbatively for the linear ac response and for the mixing current at frequency $2\omega_1 - \omega_2$ from two external currents at frequencies ω_1 and $\omega_2 \approx 2\omega_1$ due to a third order process. This pseudo-harmonic mixing probes the nonlinearity of the intervortex force and its interplay with pinning. Explicit results for the film geometry are given for the surface impedance Z_s and the mixing current amplitude $\delta j^{(3)}$ as functions of external frequencies, crossover frequency $\omega_c \propto n_i d \omega_D$ (measuring the pinning), and diffusion frequency ω_D of free 2DV.

Introduction The ac response of type II superconductors in the mixed state has been of interest for decades [0]. The subject is complicated because of the interplay between vortex structure, Ginzburg-Landau-London electrodynamics, and pinning.

Recently, the high- T_c materials which are highly anisotropic and extremely type II have stimulated new research: Anisotropic continuum theories for (dense) flux line lattices have been worked out in great detail [2].

There are new problems in the case of layered high- T_c superconductors if these require the Lawrence-Doniach approach [0]. This is the case for the most anisotropic high- T_c materials, e.g. the Bi and Tl based superconductors. The new concept is based on "pancake" or two-dimensional vortices (2DV) with their cores residing inside the superconducting layers [4]. They form an Abrikosov vortex line by being stacked on top of each other. This leads to an enhancement of the short-range fluctuations of the flux line coordinates.

A study of ac vortex dynamics based on the 2DV concept has been begun in [0] for the simplest case of a single Abrikosov flux line perpendicular to the layers. We have set up Fokker-Planck equations for the distribution functions of the 2DV coordinates and solved them approximately for the thermally averaged shifts $S_n(t)$ (n : layer number). The linear response to an external driving field was calculated taking account of point-like pinning centers arranged randomly along the flux line. The present work continues the program by giving a more detailed theory of the linear response using the full, i.e., untruncated electromagnetic interaction. This becomes possible by solving the Fokker-Planck equations in the adiabatic approximation when scale frequencies are small in comparison to the diffusion frequency of free 2DV. Furthermore, we study pseudoharmonic mixing as an example of a nonlinear ac response.

Basic Equations A pancake vortex in layer n at in-plane position ρ is attracted by another vortex in layer $m \neq n$ and at $\rho = 0$ by the electromagnetic pair force [3]

$$\mathbf{F}_{nm}(\rho) = -ak_B T \frac{d}{\lambda} \left(e^{-|n-m|d/\lambda} - \exp \left(-\sqrt{|n-m|^2 d^2/\lambda^2 + \rho^2/\lambda^2} \right) \right) \mathbf{e}_\rho/\rho, \quad (1)$$

where $a \equiv \Phi_0^2 d / (16\pi^2 \epsilon(\rho) \lambda^2 k_B T)$ is the Kosterlitz-Thouless parameter (Φ_0 : flux quantum, $\epsilon(\rho) \approx 1$ away from the transition). We concentrate on highly anisotropic layered superconductors (or suitable multilayers) when $\lambda \ll \lambda_J = d\lambda_c/\lambda \equiv \Gamma d$ holds. In that case, the Josephson coupling [6,7] can be neglected in comparison with the electromagnetic coupling between pancake vortices.

The electromagnetic force (1) has a range across the layers equal to the penetration length λ which is much larger than the layer thickness d . By using this condition, $\lambda \gg d$, Bulaevskii, Meshkov and Feinberg [0] proposed a mean field approach in which each pancake vortex at position (ρ, n) is exposed to a total force

$$\mathbf{F}_n(\rho) = \sum_{m \neq n} \int d^2 \rho' \mathbf{F}_{nm}(\rho - \rho') \Gamma_m(\rho', t). \quad (2)$$

$\Gamma_m(\rho', t)$ is the probability distribution function for the vortex positions. It obeys the Fokker-Planck equation

$$\frac{\partial}{\partial t} \Gamma_n = -\mu \operatorname{div}_\rho \left\{ [\mathbf{F}_n(\rho) + f_n(t) \mathbf{e}_x + \mathbf{F}_n^{(\text{pin})}(\rho)] \Gamma_n - k_B T \operatorname{grad}_\rho \Gamma_n \right\}. \quad (3)$$

In (3), the force f_n in x -direction due to a transport current j_n in y -direction is

$$f_n(t) = d \frac{\Phi_0}{c} \left[j_y(t) \frac{\cosh nd/\lambda}{\cosh w/\lambda} \right] \Theta(w - |nd|), \quad (4)$$

and a pinning force $\mathbf{F}_n^{(\text{pin})}$ has also been added. Note that $\mathbf{F}_n$ also takes care of the currents created by deformations of the vortex line. We consider a film of thickness $2w \gg \lambda$. The mobility of the pancake vortices is related to the flux flow resistivity ρ_f by $\mu = \rho_f c^2 / (B d \Phi_0)$.

The vortex shifts which are in x -direction follow from

$$\mathbf{S}_n(t) = \int d^2 \rho \rho \Gamma_n(\rho, t) = S_n(t) \mathbf{e}_x \quad (5)$$

and contribute to the electric field in y -direction

$$E_n = \frac{4\pi\lambda^2}{c^2} \frac{\partial j_n}{\partial t} + \frac{1}{c} B \frac{dS_n}{dt}. \quad (6)$$

Our task is to evaluate S_n as function of the driving current $j_y(t)$.

It is very difficult to solve (2) and (3). We therefore present an approximate treatment valid under conditions: i) $\omega \ll \omega_D$ and ii) $\omega_c \ll \omega_D$. The first condition requires the relevant frequencies in $j(t)$ to be much smaller than the diffusion frequency $\omega_D \equiv \mu a k_B T / (2\lambda^2)$. Condition ii) is a restriction on the number of pinning sites n_i per length which defines a scale frequency $\omega_c = O(n_i d \omega_D)$. As shown earlier [5], the distribution function Γ_n is approximated by

$$\Gamma_n(\rho, t) = \Gamma_{en}(\rho - \mathbf{S}_n(t)), \quad (7)$$

where Γ_{en} is the equilibrium distribution function, i.e. the solution of (3) for $f_n \equiv 0$. The corrections to (7) are of the order of $\kappa^2 \lambda^2 = O((\omega_c - i\omega)/\omega_D) \ll 1$, where κ^{-1} is the actual penetration length of vortex perturbations from the surface into the superconducting film (note that $w \gg \lambda$). We also found that Γ_{en} is well approximated by a Gaussian distribution $\Gamma_{en}(\rho) = (\gamma_n^2/\pi) \exp(-\gamma_n^2 \rho^2)$, provided that $\langle \rho^2 \rangle_{\Gamma_{en}} \ll \lambda^2$ holds and $\gamma_n^2 = \langle \rho^2 \rangle_{\Gamma_{en}}^{-1}$ is determined self-consistently. A bracket $\langle \dots \rangle_{\Gamma_e}$ means an average with respect to the equilibrium distribution function Γ_e . The last inequality requires temperatures below the KT -transition such that $a \ln a \gg 1$.

Inserting (7) into (3), using equation (5), and performing $\int d^2 \rho$ on the resulting equation the relation

$$\frac{1}{\mu} \frac{dS_n}{dt} - f_n(t) \mathbf{e}_x - \delta_{np} \langle \mathbf{F}^{(\text{pin})}(\rho + \mathbf{S}_n) \rangle_{\Gamma_{en}} - \left\langle \sum_{m \neq n} \mathbf{F}_{nm}(\rho - \rho' + \mathbf{S}_n - \mathbf{S}_m) \right\rangle_{\Gamma_{en} \Gamma_{em}} = 0 \quad (8)$$

is obtained. It is central to the following calculations. In (8), equivalent pins are assumed and δ_{np} takes account of layers p containing a pin. Eventually, a sum over pins will be performed.

Using (8), we develop two coupled nonlinear equations for free and pinned vortex shifts in a continuous approximation if $S_n \rightarrow S(\zeta = z/\lambda, t)$. Free vortex shifts are denoted as S . Doing a gradient expansion and retaining dominant terms up to the third order, the equation for the slowly varying part in $S(\zeta)$ is obtained:

$$\frac{1}{\mu} \frac{\partial S}{\partial t} + \nu S \left[\frac{AK}{A+K} + \frac{A_2 K^4}{(A+K)^4} S^2 \right] - \frac{\partial^2 S}{\partial \zeta^2} \left[C + C_2 \left(\frac{\partial S}{\partial \zeta} \right)^2 \right] = f(t) \delta(|\zeta| - w/\lambda). \quad (9)$$

According to (4), the quantity $f(t)$ in (9) abbreviates $\Phi_0 j_y(t) d/c$. In view of the condition $\kappa\lambda \ll 1$ and the slow variation of S with coordinate z , the external current profile and consequently that of the driving forces is expressed by δ -functions.

The coefficients A, A_2, C , and C_2 are found to be

$$\begin{aligned} A &= \frac{ak_B T}{2\lambda^2} \ln \left\{ \frac{4\lambda^2}{(\gamma_f^{-2} + \gamma_p^{-2}) e^{\mathcal{E}}} \right\} \equiv CL, \quad C = \frac{ak_B T}{2\lambda^2}, \\ A_2 &= \frac{ak_B T}{16\lambda^4} \ln \left\{ \frac{4\lambda^2}{(\gamma_f^{-2} + \gamma_p^{-2}) e^{(\gamma\mathcal{E} - 9)}} \right\} \equiv C \frac{L_1}{8\lambda^2}, \quad C_2 = -\frac{9}{4\lambda^2} C, \end{aligned} \quad (10)$$

where $\mathcal{E}$ denotes Euler's constant.

The coefficient A measures the force constant, i.e., the linear part in the self-consistent restoring force on a free vortex. As in [0], this leads to the identification $\gamma_f^2 = A/(2k_B T)$. In practical situations, the pins bind strongly with pinning energy $U_p = O(ak_B T)$ [8, 9] and a corresponding spring constant $K \approx ak_B T/\xi_{ab}^2 \gg A$. We neglect thermally activated depinning of pinned pancake vortices and take $\mathbf{F}^{(\text{pin})} = -K\boldsymbol{\rho}$ for all $\boldsymbol{\rho}$. The analogue of γ_f^2 for the pinned vortices is $\gamma_p^2 = (A+K)/(2k_B T) \gg \gamma_f^2$. Since the shifts S_p of pinned vortices are smaller than those of free vortices, S_p has been eliminated perturbatively.

Finally, equation (9) must be supplemented by a boundary condition. By summing (8) over free 2DV and replacing summation by integration, the desired condition for the free shifts $S(z)$ can be cast into the form

$$\lim_{\zeta \rightarrow w/\lambda^-} \left\{ (\partial S / \partial \zeta) \left[C + (1/3) C_2 (\partial S / \partial \zeta)^2 \right] \right\} = f(t). \quad (11)$$

Linear Response The linear response to a driving force $f(t) = f_0 \exp(-i\omega t)$ is easily found by linearization of (9) and fouriertransforming the remaining parts. Solving the equation by using the linearized boundary condition (11), one finds inside the film

$$S(z) = \frac{f_0 \cosh \kappa z}{C \kappa \lambda \sinh \kappa w}, \quad (12)$$

with

$$[\kappa(\omega)]^2 = \frac{1}{\lambda^2} \frac{\omega_c}{\omega_D} \left(1 - \frac{i\omega}{\omega_c} \right). \quad (13)$$

The diffusion frequency is expressed as $\omega_D = \mu C$. Pinning is described by the frequency $\omega_c = n_i d \frac{KL}{A+K} \omega_D \ll \omega_D$. The surface impedance is thus given by

$$Z_S(\omega) = \frac{\rho_f}{1 - \frac{\omega_c}{i\omega}} \frac{\kappa(\omega)}{\tanh \kappa(\omega) w} - i\omega \frac{4\pi\lambda}{c^2}. \quad (14)$$

This generalizes our former result (cf. eq. (32) of [0]) where a simplified interaction model was used.

Pseudoharmonic Mixing Equation (9) will be used to evaluate the nonlinear response of the 2DV system to a transport current

$$j_y(\zeta, t) = \delta(|\zeta| - w/\lambda) \{ j_1 \exp(-i\omega_1 t + i\phi_1) + j_2 \exp(-i\omega_2 t + i\phi_2) + c.c. \} \quad (15)$$

perturbatively.

We start from the linear shifts (12) with $f_0 \rightarrow f_{1,2} = e^{i\phi_{1,2}} d \Phi_0 j_{1,2} / c$ and $\kappa \rightarrow \kappa_{1,2} = \kappa(\omega_{1,2})$, so that $S^{(\text{lin})}(\zeta, \omega_{1,2}) = f_{1,2} \cosh(\kappa_{1,2} \zeta \lambda) / (C \kappa_{1,2} \lambda \sinh \kappa_{1,2} w)$ and we must solve (9) for the following driving force

$$f(t) = \left\{ f_1 e^{-i(\omega_1 t - \phi_1)} + f_2 e^{-i(\omega_2 t - \phi_2)} + c.c. \right\}.$$

In pseudoharmonic mixing, one is interested in the third order electric field $E_\delta^{(3)}$ at the finite frequency $\omega_\delta \equiv 2\omega_1 - \omega_2$ generated by the combination of linear shifts $S^{(\text{lin})}(\zeta, \omega_1)^2 S^{(\text{lin})}(\zeta, -\omega_2)$. $E_\delta^{(3)}$ is associated with a shift δS which is easily shown to satisfy the equation

$$\left(\kappa_\delta^2 - \frac{d^2}{d\zeta^2}\right) \delta S = -3\nu \frac{\bar{A}_2}{C} S_1^2 S_2 + \frac{C_2}{C} \left\{ 2 \frac{\partial^2 S_1}{\partial \zeta^2} \frac{\partial S_1}{\partial \zeta} \frac{\partial S_2}{\partial \zeta} + \frac{\partial^2 S_2}{\partial \zeta^2} \left(\frac{\partial S_1}{\partial \zeta} \right)^2 \right\}, \quad (16)$$

where $\bar{A}_2$ abbreviates $A_2 K^4 / (A + K)^4$ and $S_{1,2}$ denotes the solution (12) of the linearized equation (9) for the frequencies ω_1 and $-\omega_2$, respectively. The renormalized wave number κ_δ is given by (13) with ω specialized to ω_δ .

The solution of (16) is obtained as the sum of a homogenous solution and a particular inhomogenous solution. Eventually, we find the mixing electric fields from (6).

We consider the limit $K \gg A$ and the cases of a thin ($|\kappa(\omega_\nu)\lambda| \ll |\kappa(\omega_\nu)w| \ll 1$ for $\nu = 1, 2, \pm, \delta$) and a thick ($|\kappa(\omega_\nu)w| \gg 1$) film. In a thin film, the electric field $E_\delta^{(3)}$ is homogeneous across the sample and we find for the induced electric field $E_\delta^{(3)}$

$$E_\delta^{(3)} = 3 \frac{i\omega_\delta B}{c} \frac{L_1}{L} \left(\frac{\lambda}{w} \right)^3 \frac{f_1^2 f_2^*}{\lambda^2 (ak_B T)^3 \kappa(\omega_1)^4 \kappa(-\omega_2)^2}. \quad (17)$$

While $E_\delta^{(3)}$ is homogeneous across the (thin) film, the induced current density $j_\delta^{(3)}$ is not. It is confined to surface layers of thickness λ . We thus relate $E_\delta^{(3)}$ to the total current amplitude $I_\delta^{(3)}$ in the film using (14) in the appropriate limit $|\kappa_\delta w| \ll 1$: $E_\delta^{(3)} = -i\omega_\delta \rho_f / (\omega_c - i\omega_\delta) I_\delta^{(3)} / (2w)$. I_δ is related to the current density amplitude on one surface by $I_\delta^{(3)} = 2\lambda j_\delta^{(3)}$. Introducing the scale for the current density $j_0 = \Phi_0 c / (2\lambda(4\pi\lambda)^2)$, which is about the current density generated by a vortex at a distance λ from its core, we finally get the thin film formula

$$j_\delta^{(3)} = -\frac{3}{8} \frac{L_1}{L} \frac{j_1^2 j_2}{j_0^2} \exp[i(2\phi_1 - \phi_2)] \left(\frac{\lambda}{w} \right)^2 \frac{\omega_c \omega_D^2}{(\omega_c - i\omega_1)^2 (\omega_c + i\omega_2)}. \quad (18)$$

The opposite limit of a thick film, $|\kappa_\nu w| \gg 1$, $\nu = 1, 2, \pm, \delta$, can also be considered: The formula corresponding to (18) then reads

$$j_\delta^{(3)} = -\frac{3}{8} \frac{L_1}{L} \frac{j_1^2 j_2}{j_0^2} \exp[i(2\phi_1 - \phi_2)] \frac{\omega_c \omega_D - 6(\omega_c - i\omega_1)\sqrt{\omega_c + i\omega_2}\sqrt{\omega_c - i\omega_\delta} L/L_1}{(\omega_c - i\omega_1)\sqrt{\omega_c + i\omega_2} [2\sqrt{\omega_c - i\omega_1} + \sqrt{\omega_c + i\omega_2} + \sqrt{\omega_c - i\omega_\delta}]}. \quad (19)$$

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