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A NEW APPROACH TO CHARACTERIZING ACOUSTIC WAVES PROPAGATING UNDER A PERIODICALLY ELECTRODED ARRAY ON PIEZOELECTRIC SUBSTRATE

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Abstract:

The coupling problem between electrodes owing to acoustic wave propagation and electrostatic effect in a periodic metal grating on a piezoelectric substrate is theoretically studied. Harmonic admittance approach is used to obtain the complete acoustic wave spectrum, and mutual admittance is taken to describe wave propagation properties under the grating. Analytical expression for the contribution of surface wave (SAW) is proposed, and its characteristic parameters are numerically determined. A particular feature of the present study consists of using a particular curve-fitting technique to deal with a leaky SAW as well as a conventional Rayleigh SAW. The accurately determined SAW's parameters allow to correctly remove the participation of SAW from the global coupling, so makes it easier to analyze the spurious bulk waves (BAW), especially the surface skimming bulk waves (SSBW) in such devices. The plot as a function of propagation distance, n, of the BAW mutual admittance shows that the BAW's amplitude in the far-field varies according to a power law, i.e., $n^{-p}$, the value of $p$ strongly depends on the substrate configuration.

1. Introduction and Formulation of the problem

Surface acoustic wave (SAW) propagation in a system of metal strips deposited on a piezoelectric substrate is a very important but difficult problem. In spite of the efforts contributed by a number of workers [1-6] with some idealization, a complete description and characterization of all acoustic waves is not available. In this work we present a method for analyzing coupling phenomena between electrodes, taking into account all acoustic waves and electrostatic contributions. Using the concept of "harmonic admittance" and adopting the formulation given by Bløtekjer et al [2] for obtaining harmonic admittance, we put the emphasis of our study on the analytical description and numerical determination of the characteristic parameters of the SAW that can propagate under a grating. In our study, a piezoelectric substrate is assumed to occupy the half-space $x: \leq 0$; an infinitely thin, perfectly conductive grating is deposited from $x_{1}= -\infty$ to $x_{1}= +\infty$ on its plane surface. The width of a strip is $a$ and the period of the grating is $p$. The length $W$ of the strips is assumed to be large enough for edge effects to be neglected.

1.1: Harmonic and Mutual admittance

In the case where the voltages applied on all the strips have the same amplitude $V_0$ but the phase progresses uniformly along the electrodes of the grating at the rate of $kp$ per period, the voltage and current of the $n$th electrode can be written as

$$V_n = V_0 \exp(-jkn) = V_0 \exp(-j2n\pi) \quad I_n = I_0 \exp(-j2n\pi)$$

where $\gamma$ is defined by the fractional part of $(kp/2n)$, and represents the normalized propagation constant. Then, the harmonic admittance, $Y$ is defined as the ratio of the current in the strip, $I_n$ to its voltage, $V_n$, i.e., $Y(\omega, \gamma) = I_0/V_n = I_0/V_0$, so that $Y$ is independent of the strip number, $n$. In these equations both $I_0$ and $V_0$ are functions of $\gamma$ and of the frequency, $\omega$, and they also depend on the substrate configuration (material and orientation). In what follows, the argument $\omega$ will be omitted. The detailed deduction of the expressions of $I_0$, $V_0$ and $Y$ are given in Ref. [2], according to which the harmonic admittance can be written as:

$$Y(\gamma) = [2 \omega e(\infty) W] Y_0(\gamma)$$

where $Y_0(\gamma)$ can be known only in the numerical form via the wavenumber-dependent effective surface permittivity. A more general problem is the case where the electrical potential $V_n$ on the
One of the possible approaches to solving this problem is to use the charge superposition principle. A key step of this approach is to determine the charge distribution in the grating when only one of the strips is excited, all the others being grounded. Without loss of generality, we consider \( I_0 \), the current circulating in electrode number \( n \) due to a potential applied onto electrode number zero, \( V_0 \). The mutual admittance, \( Y_n \), is defined by the relation: \( Y_n(\omega) = I_n/V_0 \). Then it can be shown that the harmonic admittance and mutual admittance is related by the following integral:

\[
Y_n = \int_0^1 Y(\gamma) \exp(-j2\pi n\gamma) \, d\gamma
\]

### 1.2: Identification of diverse participations to \( Y(\gamma) \) and \( Y_n \)

The mutual admittance, \( Y_n \), expressed by eq.(3), upon substitution of \( Y(\gamma) \) from eq.(2), represents the global coupling between any two electrodes separated by \( n \) periods. The coupling occurs from diverse origins: propagation of acoustic waves (namely, SAW and SSBW) and the electrostatic effect (dielectric). To search for analytical development of \( Y_n \) and give an appropriate analytical expression for each physical origin we write:

\[
Y_n = Y_{nd} + Y_{ns} + Y_{nr} ; \quad Y(\gamma) = Y_d(\gamma) + Y_s(\gamma) + Y_r(\gamma)
\]

with "d, s, r" designating respectively "dielectric, surface wave, and residual" part. We propose for the dielectric part \( Y_d(\gamma) \) and the SAW the following analytical expressions:

\[
Y_d(\gamma) = 2j\omega \varepsilon_d \sin(\gamma) ; \quad Y_s(\gamma) = Y_{so} \frac{1 - \cos(2\pi n \gamma)}{\cos(2\pi n \gamma_s) - \cos(2\pi n \gamma)}
\]

where \( \varepsilon_d \) is an equivalent dielectric coefficient, the use of which is aimed at ensuring the continuity of the slope of the curve \( Y_s(\gamma) \) at \( \gamma = 0 \) and 1; and \( \gamma_s \) and \( \gamma_{so} \) are two characteristic parameters of the considered SAW mode, their values will be determined by numerical curve-fitting technique.

The above expression chosen for the SAW contribution, eq.(6) is justified by the facts: firstly, if \( Y_s(\gamma) \) is subtracted from \( Y(\gamma) \), the pole or pseudo-pole of \( Y(\gamma) \) at \( \gamma = \gamma_s \) will be removed; secondly, its inverse transform has the physically correct properties of the electrical potential associated with a SAW. In fact we can demonstrate that \( Y_s(\gamma) \) defined in eq.(5) corresponds in spatial domain to a mutual admittance:

\[
Y_{ns}(\omega) = -\frac{j Y_{so} \sin(\gamma_s) \exp(-j2\pi n \gamma_s)}{1 + j (\sin(\gamma_s) Y_{so})} \quad \text{for} \quad n \neq 0
\]

\[
Y_{ns}(\omega) = -\frac{j Y_{so} \sin(\gamma_s) \exp(-j2\pi n \gamma_s)}{1 + j (\sin(\gamma_s) Y_{so})} \quad \text{for} \quad n = 0
\]

We see from eq.(6) that \( \gamma_s \) represents the propagation constant of the SAW in a shorted grating, the modulus of \( \gamma_{so} \) is directly related to the SAW's amplitude. For an RSAW, \( \gamma_s \) is purely real, \( \gamma_{so} \) purely imaginary; but, both \( \gamma_s \) and \( \gamma_{so} \) have to be complex in the case of an LSAW. The accurate determination of \( \gamma_s \) and \( \gamma_{so} \) is necessary for the behaviour of this kind of SAW to be correctly characterized.

### 2. Numerical results of harmonic admittances and SAW's parameters

The method has been applied to the study of a number of substrate configurations. The harmonic admittance was numerically computed from eq.(2). The integral in eq.(3) was evaluated using a fast fourier transform algorithm. In our numerical computations, 8192 samples were taken for \( \gamma \) in the interval \((0, 1)\). \( Y_n \) obtained in this manner corresponds to a periodic excitation, i.e., every 8192 electrode, instead of "unitary excitation".

On Fig.1(a) and 2(a) is plotted the harmonic admittance, \( Y(\gamma) \) normalized by \((2\omega W \varepsilon(\omega))\). Because of the symmetry, the plot is given only for \( \gamma = 0 \) to 0.5. The RSAW and LSAW modes in a short-circuited grating can be identified on the plots, by respectively a true pole and a pseudo-pole of the function \( Y(\gamma) \); while a true zero and a pseudo-zero of the function \( Y(\gamma) \) are associated with these modes in an open circuited grating, except for \( \gamma = 0 \) and 1. A discontinuity in the slope of the \( Y(\gamma) \) curves indicates the existence of a SSBW mode, whose number varies with crystal orientations, but whose propagation constant (exact position of the discontinuity) does not depend on whether the grating is open or shorted, neither on the ratio \( a/p \). Around a "pole" estimated by observation of the \( Y(\gamma) \) curves, using second equation in (5) for the analytical expression of the SAW harmonic admittance, and the numerical data computed beforehand for the global harmonic admittance \( Y(\gamma) \), we searched for the appropriate values of \( \gamma_s \) and \( \gamma_{so} \) that can best fit the \( Y_s(\gamma) \) defined in eq.(5) into the global \( Y(\gamma) \). In this way we have been able to accurately deduce the two parameters \( \gamma_s \) and \( \gamma_{so} \). Then, the propagation velocity of the SAW, \( V_s \), can be deduced by identification of \( \exp(-j2\pi n x_1/V_s) \) with \( \exp(-j2\pi n x_1/\lambda_s) \) and the attenuation coefficient, \( a \) (in dB/\( \lambda_s \)), is given by \((-20) \times \log_{10}(2n \sin(\gamma_s)^2/\lambda_s) \). These results are given in Table I. The amplitude factor, \( Y_{so} \), was obtained for an array width equal to one wavelength (\( W = \lambda_0 \)).
3. Analysis of the BAW's contribution

Subtracting from the global $Y(y)$ the SAW's contribution and the electrostatic contribution defined in eq.(5), the residue of the harmonic admittance, $Y_r(y)$ results and is plotted on Fig.1(b) and 2(b). This remainder is mainly composed of the SSBW's contributions and they are the main parasitic signals occurring in a SAW device. To improve device performances it will be necessary to take into account these parasitic effects in SAW device design.

Since the residual part of the harmonic admittance is known only in the form of numerical data, its properties in the spatial domain cannot be deduced directly through mathematical analysis as for the SAW or the dielectric contribution. To know the SSBW's behaviour in the physical space, especially the diminution with propagation distance, we have numerically evaluated the integral in eq.(3) with $Y(y)$ replaced by $Y_r(y)$. On Fig.3 is presented, as an example of $Y_{nr}$, the result for 36° YX-LiTaO$_3$. This figure represents Log-Log plots of the module of $Y_{nr}$ as a function of $n$, the number of periods separating the excitation and detection electrodes. This presentation makes it evident that the mutual admittance decreases with $n$ according to a law of the form:

$$Y_{nr} \sim n^{-p}$$  \hspace{1cm} (7)

the slope of the curve's envelope gives directly the value of $p$ to be determined.

Fig.3 shows a very regular envelope up to $n = 10^4$, for $n > 10^4$, the presence of other signals become visible, they give rise to the appearance of small modulation on the envelope, and their influence goes up with $n$ increasing. This phenomenon may be explained by the following: in this substrate, only longitudinal (L) and fast shear (FS) modes of SSBW are strongly coupled, slow shear mode of SSBW being negligibly small, the residual part of the mutual admittance is essentially contributed by these two waves. Since they propagate with different speed, their modulation takes place and forms the envelope of the resultant $Y_{nr}$. The small singularities occurring for large values of $n$ may be attributed, on the one hand, to the residue of pole that was not a hundred per cent extracted by using eq.(5), an error coming from the limited precision implied in numerical iteration process but not from the formula itself; and on the other hand, to the returning signals of L mode and FS mode of SSBW, we recall that the response of $Y_n$ obtained by FFT method corresponds to a periodic excitation, i.e., every 8192 electrodes. The overall envelope of the curve has approximately a slope value of $p = 1.2$. Computations made for other substrates give the value of $p$ is 1.3 for 41° LiNbO$_3$, 0.56 for 42.5° quartz, and $p$ varies with $n$ in YZ LiNbO$_3$.

4. Summary and concluding remarks

We have analyzed the acoustic waves propagating under a periodic metal grating. The use of the harmonic admittance approach allows us to take into account the true and leaky SAW, SSBW, as well as electrostatic effects. Analytical expressions have been proposed for describing the individual participation to electrode coupling due to SAW and electrostatic effect. The two characteristic parameters of SAW have been determined using a curve-fitting technique. Even though all calculations are carried out in the real wavenumber domain, the method is still able to obtain the complex propagation constant of a leaky SAW. The behaviours of SSBW are analyzed after the numerical data resulting from the exact removal of SAW's and dielectric participations. The analysis has been made in both wavenumber domain and space domain. Studies of the mutual admittance associated with SSBW's show that their amplitude diminishes with propagation distance as a power law like $n^{-p}$, with the value of $p$, always positive, varying from one substrate configuration to another. Behaviours of $Y_{nr}$ together with $Y_r(y)$ also show that when only two SSBW are significant, the resultant $Y_{nr}$ has a regular envelope form, but when three or more SSBW are present, the resultant $Y_{nr}$ has a complicated figure, which may appear in the form of local modulation or on the level of overall envelope, this makes it very difficult to deduce the exact properties of SSBW modes in space domain directly from the numerical data that include several SSBW's. An accurate knowledge of the BAW's characteristics requires the identification of the individual contribution due to each SSBW's mode, so that finding an analytical expression appropriate to a SSBW mode will be our next object of studies.

References
Table 1: Substrate configurations, grating and frequency parameters used in the present study and the SAW (Rayleigh and leaky) parameters (propagation velocity, attenuation coefficient and amplitude factor) determined by curve-fitting method using equation (5).

<table>
<thead>
<tr>
<th>Substrates</th>
<th>a/p</th>
<th>f×p (m/s)</th>
<th>V_s (m/s)</th>
<th>α (dB/λ)</th>
<th>-j×tg(ν_y)×Y_{0}</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.5° YX-quartz</td>
<td>0.5</td>
<td>1082.4</td>
<td>5083.3436</td>
<td>4.65524e-1</td>
<td>2.4066e-6, -1.4263e-7</td>
</tr>
<tr>
<td>36° YX-LiTaO_3</td>
<td>0.486</td>
<td>1050</td>
<td>4134.5526</td>
<td>4.32185e-3</td>
<td>4.9049e-4, -1.3595e-7</td>
</tr>
<tr>
<td>41° YX-LiNbO_3</td>
<td>0.5</td>
<td>1200</td>
<td>4444.0654</td>
<td>2.71265e-2</td>
<td>1.5084e-3, -1.1686e-5</td>
</tr>
<tr>
<td>YZ-LiNbO_3</td>
<td>0.486</td>
<td>1400</td>
<td>3429.5085</td>
<td>0.</td>
<td>3.6909e-4, 0.</td>
</tr>
</tbody>
</table>

Fig. 1: Harmonic admittance Y(ν) of 36° YX-LiTaO_3 substrate: (a) before and (b) after removal of SAW's contributions. broken line: negative real part, solid line: imaginary part.

Fig. 2: as figure 1, for YZ-LiNbO_3 substrate.

Fig. 3: Log-log plot of the module of BAW's mutual admittance obtained from figure 1(b), as a function of propagation distance, n. The amplitude of BAW decays as a low of n^{-1.2}.