MODAL DECOMPOSITION OF LOW-FREQUENCY SOUND FIELD MEASURED BY VERTICAL ARRAY IN DEEP SEA

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Five methods of modal decomposition of low frequency cw-field measured by vertical array are compared with each other using data numerically simulated and these one obtained in the experiment in Norway Sea.

Cinq méthodes de la decomposition modale du cw-champ de la fréquence basse mesuré par une antenne sont comparées avec chaque autre pour les données reproduites et pour les données mesurées par la antenne verticale a la mer Norvégienne.

Low frequency cw-acoustical field in ocean far enough from the source consists of finite number M of propagating modes and can be completely described by their complex amplitudes a_j . To determine these amplitudes experimentally one can measure acoustical signals p_i at N points with the help of some array. Then for a_j the following linear equations arise:

$$\vec{p} = H \vec{a}, \quad (1)$$

where $H_{ij} = \psi_j(z_i) \exp(i\xi_j x_i)$, and $\xi_j, \psi_j(z)$ -horizontal wavenumber and eigenfunction of j -th mode, z_i, x_i -depth and horizontal coordinate of i -th receiver in the vertical plane of sound propagation. Formal solution of (1) can be represented in the form

$$\vec{a} = (H^+ H)^{-1} H^+ \vec{p}, \quad (2)$$

where $H^+ = (H^*)^T$ -hermitian conjugate. One can check that this solution minimizes mismatch F between measured field and it's approximation with superposition of L modes:

$$F = \|\vec{p} - H\vec{a}\|^2 = \sum_{i=1}^N |p_i - \sum_{j=1}^L H_{ij} a_j|^2 \quad (3)$$

In practice matrix $H^+ H$ has a number of very small eigenvalues and solution (2) appears to be unstable. There are different ways to overcome this difficulty. In this paper some of them are compared with each other and are applied to the experiment with long vertical array in Norway Sea [1,2].

In [3] singular value decomposition (SVD) method was put forward. This method inverts matrix $H^+ H$ taking into account subspace of eigenvectors $\vec{u}_i$ corresponding to large enough eigenvalues λ_i only:

$$(H^+H)^{-1} = \sum_{i=1}^S \frac{1}{\lambda_i} \vec{u}_i \vec{u}_i^T \quad (4)$$

Another way to solve the problem is to minimize mismatch F (see (3)) provided that the total energy of modes should be small enough:

$$F_1 = ||\vec{p} - H\vec{a}||^2 + \beta ||\vec{a}||^2 \quad (5)$$

Here β is the parameter of regularization [4]. The equations $\partial F_1 / \partial a_j = 0$ lead to the solution

$$\vec{a} = (H^+H + \beta I)^{-1} H^+ \vec{p}, \quad (6)$$

here I is the unit matrix. This approach will be referred to as method of regularization (MR).

Point source located at some specific depth $z = z_s$ generates j -th mode with amplitude proportional to $\psi_j(z_s)$. Assuming analogous distribution of amplitudes a_j one can try to suppress modes with relatively small $\psi_j(z_s)$ considering the following function

$$F_2(\vec{a}) = ||\vec{p} - H\vec{a}||^2 + \beta \sum_{i=1}^L |a_i / \psi_i(z_s)|^2 \quad (7)$$

Minimization of F_2 yields the solution

$$\vec{a} = (H^+H + \beta \Psi_s)^{-1} H^+ \vec{p}, \quad (8)$$

where $\Psi_s = \text{diag}(\psi_1(z_s), \psi_2(z_s), \dots, \psi_L(z_s))$. This approach will be referred to as method of unisotropic regularization (MUR).

One can find minimum of function (3) in mode amplitude space step by step. Let us begin iterations from point $a_1 = \dots = a_L = 0$. At each j -th step we fixed all a_i except current a_j which we have to find from equation $\partial F / \partial a_j = 0$ and then move in mode amplitude space by vector $(0, 0, \dots, a_j, \dots, 0)$. After L steps repeat this procedure again from current point and stop it when the function F cease to dwindle. For considered cases the sequence of finding a_j starting from 1-st to L -th mode appeared to be the best. Usually final values a_j practically didn't depend upon L when $L > M$. Such approach will be referred to as method of iterations (MI).

There is another approach to find $\vec{a}$. Let us consider receivers of the array as sources of sound with amplitudes proportional to measured p_i . In this way we can reconstruct primary field at each point of sound route. Mode amplitudes in this case are calculated as follows

$$\vec{a} = H^+ \vec{p} \quad (9)$$

We can also obtain this result for $\vec{a}$ from (2) considering modes to be orthogonal at array, $H^+H = I$. This approach will be referred to as method of algebraic summing (MAS).

All these five methods were applied to the data of the experiment in Norway Sea in 1990 [1,2]. The array with total length 560 meters consisted of 29 equally spaced receivers [5]. The source of 105 Hz was situated at the distance 105 km and at depth 550 m. At first numerical simulation corresponding to the condition of the experiment was made. The amplitudes of modes $\vec{a}_T$ produced by given source were known

and intensity of the source was normalized according to the condition $\vec{a}_T \vec{a}_T^+ = 1$. Then the field at the receivers $\vec{p}_T$ was calculated, and mode amplitudes were extracted from $\vec{p}_T$ with the help of all approaches mentioned above. Matching between mode amplitudes $\vec{a}_E$ deduced from the simulated data $\vec{p}_T$ and primary values $\vec{a}_T$ was gauged as follows:

$$K = |\vec{a}_E \vec{a}_T^+| \quad (10)$$

The amplitudes $\vec{a}_E$ are normalized according to $\vec{a}_E \vec{a}_E^+ = 1$ as well.

Fig.1 demonstrates mode match K and field mismatch F (see(3)) as a function of:

a.) errors δx_i in coordinates of receivers of the array expressed in terms of wavelength (δx assumed to be uncorrelated for different receivers)

b.) errors in given φ - tilt of the array as a whole in space

c.) noise-to-signal ratio n/s (uncorrelated noise at the receivers was assumed)

d.) span between receivers Δz (with fixed total number of them $N=29$).

For all curves "1" corresponds to SVD, "2" - MR, "3" - MUR, "4" - MI, "5" - MAS, solid curves correspond to K and dashed to F divided by $\sum_{i=1}^N |p_i|^2$.

Fig.2 corresponds to actual experiment. The quantity

$$K_0(r_s) = \max_{z_s} |K(r_s, z_s)| = \max_{z_s} |\vec{a}_E \vec{a}_T^+(r_s, z_s)| \quad (11)$$

is plotted here via distance r_s between the array and the source. The propagation condition appeared to be inhomogeneous and $\vec{a}_T$ in (11) was calculated according to adiabatic mode theory. The vector $\vec{a}_E$ was calculated from the experimental data with the help of all methods mentioned above.

All panels demonstrate the main maximum of $K_0(r_s)$ at approximately $r_s = 109$ km, $z_s = 490$ m (instead of $r_s = 105$ km, $z_s = 550$ m) and a number of "sidelobes". The contrast between the main maximum and "sidelobes" appeared to be the best for the method of regularization. The 4 km error in distance and 60 m error in depth is probably due to the insufficient information about the propagation conditions. This mismatch can be removed after fitting the media along the route [6].

- [1] CHEPURIN, YU, A, GONCHAROV, V, V, VORONOVICH, A, G, Proc. of the 19. International Symposium on Acoustic Imaging (in press)
- [2] VORONOVICH, A, G, GONCHAROV, V, V, CHEPURIN, YU, A, Dokl. Akad. Nauk SSSR. Moscow 317(1991)723 (russ.).
- [3] YANG, T, C, J. Acoust. Soc. Am. 82(1987)1736
- [4] TIKHONOV, A, N, ARSEININ, V, D, Methods of solving incorrect tasks. Moscow (russ.)
- [5] DREMUCHEV, S, A, SELIVANOV, V, G, CHEPURIN, YU, A, Oceanology. Moscow 30(1990)866 (russ.)
- [6] GONCHAROV, V, V, VORONOVICH, A, G, (to be published).