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THE AMPLITUDE-PHASE DISTORTION AT THE PARAMETRIC SOUND WAVE CONJUGATION

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Abstract - We investigate the amplitude distortion of the contrary phase-conjugated wave, generated in the active plane-parallel parametric layer by the incident acoustic beam with Gauss transverse profile. We treat both stable and unstable regimes.

One of the main problems for all mechanisms of the phase conjugation (PC) in acoustics as well as in optics is the quality of the phase reproduction. In acoustics the most promising way of PC realization lies in using of the interaction of the phonon oscillations with some other types of the collective excitations in solids, induced by the external electromagnetic fields [1-4]. If the possibility of two-phonon decay of the quasi particle, corresponding to the certain collective excitation exists, the spatially homogeneous sound velocity modulation at the external field frequency Ω arises. Thus, for zero quasi-particle impulse the travelling of the sound wave with frequency $\omega = \Omega/2$ in such media will be accompanied by the generation of the contrary PC wave.

In this paper we investigate the expansion of the Gauss acoustic beam in the plane-parallel layer, the sound velocity in which is spatially homogeneously modulated in time: $c_0 = c(1 + 2\mu \cos 2\omega t)$, and on this bases determine the amplitude distortion of the output PC beam.

Let the incident quasi-plane acoustic impulse of the duration τ

$$U(z, \vec{r}_1, t) = 0.5U_0(t) \exp(-r_1^2/a^2) \exp[i(\omega t - kz)] + c.c.$$

where U is the displacement in acoustic waves, fall at the moment $t=0$ on the entrance surface $z=0$ of the plane-parallel layer of the thickness l . If the modulation coefficient μ is small enough, the full acoustic field in layer can be presented in the form:

$$U(z, \vec{r}_1, t) = 0.5U^+ \exp[i(\omega t - kz)] + 0.5U^- \exp[i(\omega t + kz)]$$

Then, for slowly varied along the z ax and time t (we assume, that the impulse duration $\tau \gg 1/\omega$) amplitudes $U^\pm$ we possess the following coupled system:

$$\begin{aligned} \left[\frac{1}{2ik} \Delta_1 + \frac{\partial}{\partial z} + \frac{1}{2ik} \frac{\partial^2}{\partial z^2} + \frac{1}{c} \frac{\partial}{\partial t} - \frac{1}{2ikc^2} \frac{\partial^2}{\partial t^2} \right] U^{+*} &= -\mu \left[ik + 2 \frac{\partial}{\partial z} \right] U^- \\ \left[\frac{1}{2ik} \Delta_1 + \frac{\partial}{\partial z} + \frac{1}{2ik} \frac{\partial^2}{\partial z^2} - \frac{1}{c} \frac{\partial}{\partial t} - \frac{1}{2ikc^2} \frac{\partial^2}{\partial t^2} \right] U^- &= -\mu \left[ik + 2 \frac{\partial}{\partial z} \right] U^{+*} \end{aligned} \quad (1)$$

One can easily see, that in parabolic approximation ($\partial^2/\partial t^2 = \partial^2/\partial z^2 = 0$), the output transverse distribution $U(z=0)$ of the contrary wave ideally reproduces the input profile of the incident

beam. Thus, the corrections arise, when we take into account the above mentioned terms. Further, we shall look through the situation, when the pumping intensity corresponds to stationary regime or small exceeds the edge of stability. In this case, as it is shown bellow, we can neglect the terms $\partial^2/\partial t^2$. Further, we shall investigate separately two different regimes.

1. Unstable regime: $\mu k l > \pi/2$.

To solve (1) we perform the Laplas-transform of $U^\pm$ in time and Fourier-transform along the transverse spatial coordinates :

$$\tilde{U}^\pm(\vec{k}_1, p) = \int U^\pm e^{-pt} e^{-i\vec{k}_1 \vec{r}_1} dt d\vec{r}_1$$

Then, for images, we get:

$$\begin{aligned} \left(\frac{1}{2ik} \frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial z} - \frac{k_1^2}{2ik} + \frac{p}{c} \right) U^{+*} &= -\mu \left(ik + 2 \frac{\partial}{\partial z} \right) U^- \\ \left(\frac{1}{2ik} \frac{\partial^2}{\partial z^2} + \frac{\partial}{\partial z} - \frac{k_1^2}{2ik} - \frac{p}{c} \right) U^- &= -\mu \left(ik + 2 \frac{\partial}{\partial z} \right) U^{+*} \end{aligned} \quad (2)$$

We shall look for solution of the system with the boundary conditions:

$$\tilde{U}^+(z=0) = \tilde{U}_0(p) \exp(-k^2 a^2) ; \quad \tilde{U}^-(z=l) = 0 ;$$

the last equality means that the contrary wave at the back plane of the layer is absent. In all previous considerations we assumed that the level of the input signal is much higher, then the level of the thermal noise in media, thus the acoustic radiation of spontaneous sources can be neglected.

The characteristic equation, corresponding to (2) for the dimensionless value $\Lambda (U \exp(\Lambda kz))$ is:

$$\Lambda^2 (1 + \mu\beta) + 2i\mu\beta\Lambda + \mu^2 (1 - \beta^2) = \frac{1}{4}\Lambda^4 + i\Lambda^3 + 4\mu^2\Lambda^2 + 4i\mu^2\Lambda \quad (3)$$

where $\beta = k_1^2 / 2\mu k^2$.

For the wide enough Hauss beams and not very small parametric coefficient in system : $\mu \geq 1/k^2 a^2$, we can build the solution (3) expanding Λ in power series of μ . Then with the accuracy till μ^2 we derive:

$$\Lambda_1 = i\mu(1-\beta) + i\frac{1}{2}\mu^2(3-2\beta-\beta^2) ; \quad \Lambda_2 = i\mu(1+\beta) + i\frac{1}{2}\mu^2(3+2\beta-\beta^2)$$

Finally, for the output Fourier amplitudes of the contrary wave we obtain :

$$\tilde{U}^-(\vec{k}_1, p, z=0) = \pi a^2 \mu \tilde{U}_0^*(p) e^{-\frac{1}{4}k_1^2 a^2} \left(1 - e^{\frac{k(\Lambda_2 - \Lambda_1)l}{\Delta}} \right) \frac{(1+2\Lambda_1)(1+2\Lambda_2)}{\Delta} \quad (4)$$

here $\Delta = \mu - i\frac{p}{k} - 4\mu\beta^2 - \left[-\mu - i\frac{p}{k} + 4\mu\beta^2 \right] e^{-2i\mu k l (1-\mu\beta)}$

Further for definiteness we shall model the input time profile by the rectangular impulse with duration τ and amplitude U_0 . Then

$$\tilde{U}_0^*(p) = U_0^* (1 - e^{-p\tau}) / p.$$

The integrand in (4) is regular in complex plane p , except the simple poles, coincident with the roots $\Lambda=0$. The reverse Laplas-transform for $\tilde{U}^-$ for moments $t > \tau + 2l/c$ can be carried out with the help of the residue theorem. The integral contour is closed in the left half of p -plane. The transcendental equation $\Delta=0$ has the series of roots with negative real parts [5,6] and if, as it was assumed above, the pump small exceeds the edge of stability, the single root with positive real part. Thus, full acoustic field is superposition of time attenuated modes and one exponentially growing component, corresponding to that root. For the long enough time interval the

attenuated modes will make the negligible contribution to the field, thereby only the amplified mode should be taken into consideration.

Near the critical length $l : \mu k l = \varepsilon + \pi/2$, $\varepsilon \ll 1$, the root $\Delta = 0$ we are interested in may be found by the expansion $p = p_0 + \Delta p$, where p_0 is the root, corresponding to $k_1 = 0$. In its turn we can derive p_0 by expanding $\Delta(k_1 = 0)$ in a power row on a small deviation ε . After that, the correction Δp , connected with small transverse terms $\beta \ll 1$, can be obtained by linearizing $\Delta(p) = \Delta(p_0) + \Delta'(p_0)\Delta p = 0$.

Finally, for output Fourier-transform amplitude we have:

$$\tilde{U}^-(\vec{k}_1, p, z=0) = -i\pi\mu a^2 U_0^* e^{-\frac{1}{4}k_1^2 a^2} \left[1 - \left(2 - i\frac{\pi}{4} \right) k_1^2/k^2 \right] \times \\ \times e^{\mu k c (\varepsilon - \frac{\pi}{4} k_1^2/k^2) t} \frac{1 - e^{\mu k c (\varepsilon - \frac{\pi}{4} k_1^2/k^2) \tau}}{\mu (\varepsilon - \frac{\pi}{4} k_1^2/k^2)} \quad (5)$$

It is seen, that gain of the transverse Fourier-component declines as k_1 increases. In reality the input impulse amplification on the spatial interval equaled to the impulse length is small: $\varepsilon k c \tau \ll 1$. Then, expanding the corresponding exponent in (5) in row, after the reversed Fourier-transform we get:

$$U^-(R, t, z=0) = -U_0^* \mu k c \tau e^{\mu c \varepsilon t} e^{-R_1^2/\tilde{a}^2} \left[1 + \frac{2t\pi - 8}{k^2 a^2} \left(1 - \frac{R_1^2}{\tilde{a}^2} \right) \left(\frac{a}{\tilde{a}} \right)^2 \right] \quad (6)$$

here $\tilde{a}^2 = a^2 \left\{ 1 + \pi \frac{\mu k c t}{ka} \right\}$.

As it follows from (6), the PC wave preserves the Gauss profile of the incident beam. The characteristic radius $\tilde{a}$ monotonously increases in time - time dependant defocusing takes place. The speed of $\tilde{a}$ rising depends both on diffraction parameter $1/ka$ and modulation coefficient μ . The level of output signal is proportional to the input impulse duration.

2. Stationary regime : $\mu k l < \pi/2$.

The coupled travelling of incident and contrary waves in stationary regime is described by the systems (1), (2), where the terms with time derivations are to be omitted. The characteristic equation (3) and its roots, as one can verify, remain valid in our case and, since we assume the same boundary conditions, the output Fourier amplitude of the contrary wave is presented:

$$\tilde{U}^-(z=0) = -\frac{i}{4\pi^2} (1 + 3\mu^2) \int \tilde{U}_0 \operatorname{tg}[\mu k l (1 - k_1^2/2k^2)] e^{i\vec{k}_1 \cdot \vec{r}} d\vec{k}_1$$

where $\tilde{U}_0 = U_0 \exp(-0.25k_1^2 a^2)$ - Fourier-image of input signal.

We see, that near the edge of stability ($\mu k l \rightarrow \pi/2$) the tg -multiplier sharply depends on transverse variable k_1 . In its turn, it leads to sharp spectrum distortion of the output amplitude.

The spectrum of input signal cut off the output amplitude spectrum. Then, within characteristic integration interval $k_1 \ll a^{-1}$ we can expand $\operatorname{tg}[\mu k l k_1^2/2k^2]$ in row of small parameter k_1^2/k^2 and hold the main term. It is convenient to present the inverse Fourier-transform in the form:

$$U^-(z=0) = iU_0^* a^2 \pi (1+3\mu^2) b^2 \operatorname{tg}(\mu kl) \left[K_0(br_1) \int_0^r r_1' e^{-r_1'^2/a^2} J_0(k_1 r_1') dr_1' + \right. \\ \left. + J_0(br_1) \int_r^\infty r_1' e^{-r_1'^2/a^2} K_0(br_1') dr_1' \right] \quad (7)$$

where $b^2 = 2k^2/\mu kl \operatorname{tg}(\mu kl)$; J_0 , K_0 - Bessel functions. As it follows from (7), for the length l far from critical $k^2 a^2 \gg \mu kl \operatorname{tg}(\mu kl)$ the quality of amplitude reproduction is high:

$$U^-(z=0) = i(1+3\mu^2)U_0^* e^{-r_1^2/a^2} \left[1 - 2\mu kl (r_1/a)^2 \operatorname{tg}(\mu kl) / k^2 a^2 \right] \operatorname{tg}(\mu kl)$$

At the contrary relation between diffraction parameter and gain in the system for the most interesting transverse coordinate interval $r_1 \leq a$ it's easy to obtain the following approximation, expanding J_0, K_0 in power row of parameter br_1 :

$$U^-(z=0, r_1 \leq a) = iU_0^* \frac{(1+3\mu^2)k^2 a^2}{\mu kl} \left\{ K_0(\varepsilon q) (1 - e^{-q^2}) - \right. \\ \left. - I_0(\varepsilon q) \left[\left(\ln \frac{\varepsilon q}{2} + C \right) e^{-q^2} - \frac{1}{2} E_1(-q^2) \right] \right\}$$

here $\varepsilon = ab \ll 1$; $q = r_1/a$ - dimensionless transverse coordinate; C - Euler constant.

Figure.

In Fig. we show the $U^-(z=0, q)$ curves for various dimensionless $L = \mu kl$ at the typical diffraction parameter value $(ka)^2 = 100$. For comparison the Gauss profiles of the incident beam with the corresponding amplitude transformations are presented - dashed lines. One can see, that the output signal amplitude increases as L increases, but simultaneously the amplitude distortion rises. Thus, the stationary PC at the high gain in the system vanishes.

In conclusion we note that, though the results were obtained for the model distribution of the incident beam, they still have more general significance. The inevitable sharp changes in output spatial spectrum at the arbitrary spectrum of the input signal near the edge of stability will always causes the phase and amplitude distortion of the contrary beam.

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