PHASE CONJUGATION OF THE SOUND WAVE, GENERATED BY THE MOVING SOURCE

V. Strel'Tsov

► To cite this version:

V. Strel'Tsov. PHASE CONJUGATION OF THE SOUND WAVE, GENERATED BY THE MOVING SOURCE. Journal de Physique IV Proceedings, 1992, 02 (C1), pp.C1-899-C1-902. 10.1051/jp4:19921196 . jpa-00251161

HAL Id: jpa-00251161

<https://hal.science/jpa-00251161>

Submitted on 1 Jan 1992

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

PHASE CONJUGATION OF THE SOUND WAVE, GENERATED BY THE MOVING SOURCE

V.N. STREL'TSOV

General Physics Institute of Russian Academy of Science, 38 Vavilov st., 117942 Moscow, USSR

Abstract - We investigate the spatial structure of the contrary sound wave, generated in active parametric layer by the local incident beam, moving along the specimen surface.

One of the most interesting directions in modern physical acoustics is the interaction of sound oscillations with some other collective excitations in solids. Such interaction enables to solve the actual problem of sound phase conjugation (PC) [1-3]. PC can be realized, if phonon-collective interaction (e.c. phonon-plasmon, phonon-magnon etc.) allows the two-phonon decay of the elementary excitation with zero quasi-impulse. Indeed, under this condition, if we excite the collective oscillations with double sound frequency by the external pumping (e.c. electromagnetic field) the travelling incident sound beam in media will induce the generation of the contrary PC wave. Moreover, at high enough pumping level, due to spontaneous two-phonon decay the media becomes unstable. The coupled travelling of incident and contrary waves in unstable regime leads to giant gain in the system. The features of PC process considered essentially depend on system parameters, such as parametric coupling value, dimensions and geometric forms of the active media, beams geometric parameters etc.

In this paper we investigate the spatial distribution of the contrary sound wave, parametrically generated in the layer by the quasi-plane spatially located beam, radiated by the source moving along the specimen surface with steady velocity.

Let the sound source, radiating in self coordinate-system along the Z' -ax monochromatic (with frequency ω) quasi-plane wave with some aperture a , be switched on at the moment $t=0$ and move with steady velocity V along the X -ax parallel the infinite plane (with normal vector $\vec{n}[0,0,1]$) layer of the thickness l . In layer media the effective resonant coupling of the contrary travelling waves exists.

For acoustics field we shall use the hydrodynamics description. For small enough coupling coefficient μ and not very high velocities V , satisfying the condition $V/a \ll \omega$, the density perturbation ρ in the sound field in media may be presented as the sum of direct and contrary waves:

$$\rho = 0.5[\rho_1(x,y,z,t)e^{i(\omega t - kz)} + \rho_2(x,y,z,t)e^{i(\omega t + kz)} + \text{c.c.}]$$

where $\rho_{1,2}$ - slowly varied along z, t amplitudes; $\omega = kc$, c -sound velocity.

Making the transition to the moving coordinate system : $(\xi = x - vt, y' = y, z' = z, t' = t)$ in parabolic approximation for ρ we derive the following coupled system of reduced equations:

$$\begin{aligned} \frac{\partial \rho_2}{\partial z} - \frac{1}{c} \left(\frac{\partial \rho_2}{\partial t} - v \frac{\partial \rho_2}{\partial \xi} \right) - \frac{t}{2k} \Delta_1 \rho_2 &= -\frac{1}{2} \cdot i \mu k \rho_1^* \\ \frac{\partial \rho_1^*}{\partial z} + \frac{1}{c} \left(\frac{\partial \rho_1^*}{\partial t} - v \frac{\partial \rho_1^*}{\partial \xi} \right) - \frac{t}{2k} \Delta_1 \rho_1^* &= -\frac{1}{2} \cdot i \mu k \rho_2 \end{aligned} \quad (1)$$

Neglecting the thermal noise, we have the following initial and boundary conditions :

$$\rho_{1,2}(t < 0) = 0 \quad ; \quad \rho_{1,2}(\vec{r}_1 \rightarrow \infty) = 0 \quad ; \quad \rho_1^*(z=0) = \rho_{inc}^*(\xi, y) \theta(t) \quad ;$$

where ρ_{inc} - the incident beam distribution at the entrance surface,

$\rho_2(z=l)=0$ - the contrary wave at the back surface is absent.

Further, without restriction of generality, we assume that the input signal $\rho_{inc}(\xi, y)$ has the independent distributions along ξ - and y -axes and model ξ -distribution by the function : $\rho_{inc}(\xi, y) = \Phi(y) \exp(-\alpha|\xi|)$ with the characteristic width $a=2/\alpha$. To solve (1) we perform Laplace transform of $\rho_{1,2}$ in time and Fourier transform in space. Then, for images $\tilde{\rho}_{1,2}$ we get :

$$\begin{aligned} \frac{\partial \tilde{\rho}_2}{\partial z} &= \left[\frac{p}{c} - i \frac{v}{c} k_\xi - i \frac{k_\xi^2}{2k} \right] \tilde{\rho}_2 - i \frac{\mu}{2} k \tilde{\rho}_1^* \\ \frac{\partial \tilde{\rho}_1^*}{\partial z} &= \left[-\frac{p}{c} + i \frac{v}{c} k_\xi - i \frac{k_\xi^2}{2k} \right] \tilde{\rho}_1^* - i \frac{\mu}{2} k \tilde{\rho}_2 \end{aligned} \quad (2)$$

with boundary conditions:

$$\tilde{\rho}_1^*(z=0) = \frac{2\alpha}{p(\alpha^2 + k_\xi^2)} \tilde{\Phi}^*(k_y) \quad ; \quad \tilde{\rho}_2(z=0) = 0$$

Solving (2), for the contrary wave output amplitude we obtain:

$$\rho_2(\xi, y, t, z=0) = \frac{i \mu k \Phi^*(y)}{8\pi^2} \int e^{pt + i k_\xi \xi} \frac{2\alpha}{p(\alpha^2 + k_\xi^2) D} (1 - e^{-2l \sqrt{(\frac{p}{c} - i \frac{v}{c} k_\xi)^2 - (\frac{\mu}{2} k)^2}}) dk_\xi dp \quad (3)$$

As one could assume in advance, the output y -profile of ρ_2 reproduces the input y -profile ρ_1 .

The integrand in (3) is regular in complex p -plane, except the pole $p=0$ and poles coincident with roots $D=0$. For moments $t > 2l/c$ the reverse Laplace transform may be carried out with the help of residue theorem : the integration contour is closed in left half of p -plane. Thus, the output signal is presented as a superposition of the modes, corresponding to the above mentioned poles. As it was shown in [4] (see also below), under the condition $\mu k l < \pi$ all the roots $D=0$ have negative real parts. These modes attenuate in time and compose the output signal structure in transient regime. The stationary signal, we are interested in, will be determined by the pole $p=0$. Substitution $p=0$ in (3) immediately yields:

$$\rho_2(\xi, y, t, z=0) = \frac{i \mu k \Phi^*(y)}{2\pi} \int e^{i k_\xi \xi} \frac{(1 - e^{-2l \sqrt{(\frac{p}{c} - i \frac{v}{c} k_\xi)^2 - (\frac{\mu}{2} k)^2}})}{(\alpha^2 + k_\xi^2) D(p=0)} dk_\xi \quad (4)$$

The integrand in (4) as well as in (3) in complex k_ξ -plane has the singularities coincident with roots $D(p=0)=0$. For the transverse coordinate interval $|\xi| > 2\sqrt{l}/c$ we can again use the path integration (note, that in reality the written interval practically completely covers the output and input beams contours). Now, however, the more detailed treatment of the equation $D(k_\xi)=0$ is demanded. This equation was numerically analyzed in [4]. Here we shall treat it in more appropriate for our purpose way.

For the unity of consideration it is convenient to change in (4) the variate k_ξ for $q = -ik_\xi v/c$. Then, making the substitution

$$0.5\mu k l = q - \sqrt{q^2 - (\frac{\mu k}{2})^2} ; \quad q = 0.25\mu k (t + t^{-1})$$

and separating the real and imaginary parts, we can write the equation $D=0$ as a system of equations for the new complex variate t :

$$e^{0.5\mu k l (|t| - |t|^{-1})} \cos \varphi = |t|^{-2} ; \quad 0.5\mu k l (|t| + |t|^{-1}) \sin \varphi = -2\varphi + 2\pi n \quad (5)$$

Note, that (5) is invariant under transformations: $[|t| \rightarrow |t|^{-1}; \varphi \rightarrow \varphi + \pi, |t| \rightarrow |t|; \varphi \rightarrow \varphi; n \rightarrow -n]$.

From the first equation (5) one can easily see, that the solution with positive real part has module $|t|=1$. At the same time for such $|t|$ the second equation will have the solution for φ only under the condition $\mu k l > \pi$. Thus, if $\mu k l < \pi$, all the roots t_0 in (5) have negative real parts. In interval $2 < \mu k l < \pi$ there exists three roots with $|t|=1$: $t_0 = -1$ and two complex conjugated roots, circling (when $\mu k l \rightarrow \pi$) in the left half of q -plane from $t_0 = -1$ to i and $-i$ correspondingly. For the length l far enough from the critical one $\mu k l < 2$ the root with $|t|=1$ is single: $t_0 = -1$. For such small length there exists also two real negative roots, which approach (when $\mu k l \rightarrow 2$) to $t_0 = -1$. Another roots are complex and as can be shown have $|t| \gg 10 < |t| \leq 0.1$.

Returning to (4), we take into account, that for variant q the integration contours rotate by the angle $\pi/2$ about k_ξ -contours.

Then, for $|\xi| > 2\sqrt{l}/c$ the integration path will be closed in right half of q -plane. For the lengths $\mu k l < \pi$, as it follows from the above analyses, the roots $D(q)=0$ lie in left half and the single pole making a contribution in integral is $q=\alpha$. Finally, for $|\xi| > 0$ we obtain

$$\rho_2(z=0) = -0.5\mu k l \Phi(y) e^{-\alpha \xi} \cdot \frac{1 - e^{-2\beta l}}{\frac{v}{c}\alpha + \beta + i - \frac{v}{c}\alpha + \beta + i e^{-2\beta l}} ; \quad \beta = \sqrt{(\frac{v}{c}\alpha)^2 - (\frac{\mu k}{2})^2} ;$$

We see, that the contrary beam, moving together with the incident beam, has the front profile, coincident with the front profile of the last one. We shall look through two limits:

1. "Narrow" beam: $v\alpha/c \gg \mu k/2$.

$$\rho_2(z=0) = -\Phi^*(y) (1 - e^{-2v\alpha l/c}) e^{-\alpha \xi} \cdot \mu k c / 4v\alpha$$

The output amplitude is decreased in compare with input incident beam amplitude.

2. "Wide" beam: $v\alpha/c \ll \mu k/2$.

$$\rho_2(z=0) = -\Phi^*(y) e^{-\alpha \xi} \sin(\mu k l / 2) / [\cos(\mu k l / 2) + \sin(\mu k l / 2) \cdot 2v\alpha / \mu k c]$$

Far from the edge of the stability $\mu k l \ll 1$ the signal is weak. The

growth of the effective layer thickness $\mu k l$ increases the output amplitude and near the critical length ($\mu k l \sim \pi/2$) the high, but finite gain takes place $\rho_2(z=0)/\rho_{inc} = \mu k c / 2v\alpha$.

Now we shall come to the back front consideration: $\xi < -2l/c$. It's convenient to introduce the new variable $\eta = \xi + 2v l/c \leq 0$. Closing the integration contour in the left half of q -plane, we obtain (now all the roots t_o of D make contribution to integral):

$$\rho_2(z=0) = -0.5\mu k \Phi^*(y) \left\{ \frac{(1-e^{-2\beta l})e^{-2\alpha v l/c}}{\frac{v}{c}\alpha + \beta + (\frac{v}{c}\alpha + \beta)e^{-2\beta l}} e^{\alpha \eta} - \sum \frac{(\alpha \frac{c}{v})(t_o - t_o^{-1})^2 e^{-0.5\mu k l(t_o + t_o^{-1})}}{[\alpha^2 - (0.25\mu k \frac{c}{v})^2(t_o + t_o^{-1})^2][2 + 0.5\mu k l(t_o + t_o^{-1})]} e^{\frac{\mu k c}{4v}\eta(t_o + t_o^{-1})} \right\} \quad (6)$$

One can easily verify, using the above analysis, that the complex roots with $|t| \neq 1$ always make the negligible contributions to (6). Up to the length $\mu k l \sim 2$ the real negative roots also can be neglected. Thereby, under the condition $2\alpha v/c \ll 1$ the first term in (6) will be dominant and all the results obtained for front profile are preserved. Thus, for the lengths, far from critical and for not very small transverse dimensions of the incident beam the back front of the contrary wave reproduces the back front of the incident beam, and we may speak about conjugation.

For the lengths $\mu k l \geq 2$ in sum in (6) two roots with $|t|=1$ arises. We shall look again through two limits.

1. "Narrow" beam.

$$\rho_2(z=0) = -\frac{\mu k c}{4v\alpha} \Phi^*(y) \left\{ (1-e^{-2v\alpha l/c})e^{\alpha \eta} + 16\sin^2\varphi \frac{e^{k\mu c \cos\varphi(l-\eta c/v)}}{2+\mu k l \cos\varphi} \right\}$$

The second term monotonously grows, when the length l increases from the value $4\mu k c/v\alpha e$ at the length $\mu k l=2$ up to $2\mu k c/v\alpha$ at $\mu k l=\pi$. Simultaneously its width also rises and near the edge of stability becomes infinite. Thus, beyond the moving contrary beam the sound "tail" of amplitude $\sim 2\mu k c/v\alpha$ exists. The PC no longer takes place.

2. "Wide" beam.

$$\rho_2(z=0) = -\Phi^*(y) \left\{ \frac{\sin(\mu k l/2)e^{-2v\alpha l/c}}{\cos(\mu k l/2) - (2v\alpha/\mu k c)\sin(\mu k l/2)} e^{\alpha \eta} - 4 \frac{v\alpha e^{k\mu c \cos\varphi(l-\eta c/v)} \sin^2\varphi}{c[\alpha^2 - (0.5\mu k \frac{c}{v} \cos\varphi)^2](2+\mu k l \cos\varphi)} \right\}$$

As in previous case, the second term is small at the length $\mu k l \sim 2$. Its amplitude and width grow as length increases and near the edge of stability beyond the moving contrary beam the track of the amplitude $\sim 2\mu k c/2\alpha v$ is formed. Note, that in contrast with narrow beam now the gain takes place: $\rho_2(z=0)/\rho_{inc} > 1$.

REFERENCES

- /1/ Strel'tsov V.N., Kvantovaya Electron. (Moscow) 13, 2144, (1986).
- /2/ Brysev A.P., Strel'tsov V.N. Proc. of 1st France Acoust. Congr. Coll. de Phys., suppl. № 2, v.51, c2-85, (1990).
- /3/ Brysev A.P. et al. Optical & Acoustical Review. 1, 107 (1990).
- /4/ Bobroff D.L., Haus H.A., J. Appl. Phys. 38, 390, (1967).