ON THE VALIDITY OF A GEOMETRICAL ACOUSTICS MULTIPLE SCATTERING APPROXIMATION FOR THE DESCRIPTION OF THE NEAR AND FAR FIELD PLANE WAVE RESPONSE OF VERY ROUGH HARD SURFACES

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The phenomena of scattering of light and sound from opaque rough (random or otherwise) surfaces have attracted much attention recently due to renewed interest in the enhanced backscattering effect (EBE) observed with random rough surfaces [1,2]. It is now generally recognized that the high-sloped nature of the surfaces employed in the experiments in which EBE was observed signifies that multiple scattering accounts for most, if not the totality of the back-scattered radiation peak. Physical acoustics (also known as the Kirchhoff, tangent plane, Brekhovskikh or Beckmann approximation) cannot be used to explain the EBE since it accounts only for single scattering. Perturbation methods, which seem to account for multiple scattering, can only be used when the characteristic dimensions of each scattering feature are small compared to the wavelength, which is not usually the case of the surfaces employed in the EBE experiments.

We generalize the physical optics method to include multiple scattering. The scattering surface, assumed to be hard and two-dimensional (i.e., it does not depend on one of the cartesian coordinates) is a connected system of inclined or horizontal strips. The basic scattering feature is the groove (of right-triangular shape) and the field within each groove is assumed to be insensitive to the existence or inexistence of neighboring grooves. Within a given groove the field on each strip is calculated by taking into account, by means of a (geometrical acoustics) ray analysis, all the incident (primary wave plus the waves reflected from the adjacent strip) and reflected waves. The surface fields in the grooves are then introduced into a special form (which does not require knowledge of the surface fields on the horizontal strips between the grooves) of the Kirchhoff-Helmholtz integral to compute the field at other-than-surface observation points, notably at infinity (far-field).

The validity of this geometrical acoustics multiple scattering approximation (GA) is established by comparison with (exact) reference solutions obtained by a mode-matching method. Both the near and far-field predictions of the GA are shown to compare well with the corresponding reference solutions for characteristic dimensions of individual scattering features as small as $\lambda/3$. The GA solutions are all the more valid the higher is the frequency.

Examples, obtained with the GA, are given of the far field response of deterministic surfaces and compared to that of random surfaces in connection with the enhanced backscattering effect.

Consider the problem of a compressional plane wave incident on a hard rough (in the mean plane) surface. Assume: 1) the roughness to be two-dimensional and describable by the generating function $\mathcal{F}$: $z=f(x);x\in\mathbb{R}$, with $f'(x)$ piecewise continuous, 2) the wavevector $\mathbf{k}_i$ of the incident wave to lie in the x-z plane, 3) the region $z<f(x)$ to be impenetrable. Let u_i and u be the incident and total pressures in $z\geq f(x)$; both are functions of $\mathbf{x}=(x,z)$ only (assuming an implicit time-dependence $\exp(-i\omega t)$, with ω the angular frequency), i.e., the problem is two-dimensional. Let Ω_0 designate the half-plane $\{z>0;\ x\in\mathbb{R}\}$ and Ω' the remainder of the domain $\{z>f(x);x\in\mathbb{R}\}$. Ω' is composed of N non-connected subdomains Ω_j; $j\in\mathcal{N}$, with $\mathcal{N}=\{1,2,\ldots,N\}$ which are termed "grooves". The number of grooves of the rough surface is N. The junction of Ω_0 with Ω_j is designated by $\mathfrak{I}_j$ and the right- and left-hand x-coordinates of this segment by α_j and β_j respectively. Both Ω_0 and Ω' are filled with an ideal fluid whose equilibrium mass density is ρ_0 and whose adiabatic compressibility is K. The speed of sound and wavenumber in this fluid are $c_0=(K\rho_0)^{-1/2}$ and $k_0=\omega/c_0$ respectively. The pressure $u(\mathbf{x})$ satisfies the Helmholtz equation, the boundary condition of the vanishing of the normal deriviative of u on $\mathcal{F}$ and the Sommerfeld radiation condition.

A special form of the Kirchhoff-Helmholtz integral can be expressed by:

$$u(\mathbf{x}) = u_i(\mathbf{x}) + u_r(\mathbf{x}) + \int_{-\infty}^{\infty} B(k_x)\exp[i(k_x x + k_z z)]\frac{dk_x}{k_z} \quad ; \qquad \mathbf{x}\in\Omega_0 \quad , \tag{1}$$

where

$$u_i(\mathbf{x})=\exp[i(k_{ix}x - k_{iz}z)] \ ; \ k_{ix}=k_0 sin(\theta_i),\ k_{iz}=k_0 cos(\theta_i),\ \theta_i=incident\ angle \tag{2a}$$

$$u_r(\mathbf{x})=\exp[i(k_{ix}x + k_{iz}z)] \ ; \ k_z=(k_0^2-k_x^2)^{1/2},\ \begin{pmatrix}Re\\Im\end{pmatrix}k_z\geq 0,\ f=f(x),\ f'=df(x)/dx, \tag{2a}$$

$$B(k_x) = \sum_{j=1}^{N} B_j(k_x) \ , \quad B_j(k_x)=\left(\frac{-1}{4\pi}\right)\int_{\alpha_j}^{\beta_j} u(x,f)F(k_x;\ x,f)\exp(-ik_x x)dx \quad , \tag{3a}$$

$$F(k_x;x,f) = (f'k_x-k_z)\exp(-ik_z f) + (f'k_x+k_z)\exp(ik_z f) \quad . \tag{3b}$$

u_i, u_r and the integral in Eq.(1) are the incident, reflected and scattered pressures; the third vanishes in the absence of roughness. Conservation of energy is expressed by

$$\int_{-\pi/2}^{\pi/2} |\mathcal{B}(\theta)|^2 d\theta = -2\ Re[\mathcal{B}(\theta_i)] \ , \tag{4}$$

where the left- and right hand sides represent the scattering and extinction cross sections respectively, θ the scattering angle (see figures), $\mathcal{B}(\theta)=B(k_x)=B(k_0 sin\theta)$; $|k_x|\leq k_0$, and $|\mathcal{B}(\theta)|^2$ the bistatic cross section.

Let w_j be the width of $\mathfrak{I}_j$ and 0_j the local origin of Ω_j with coordinates $(d_j,\ 0)$ with respect to the origin 0. d_j (=0 for j=1) is the distance (along the x-axis) of the center line of Ω_j with respect to 0 and $\mathcal{F}_j$: $z_j=f_j(x_j)$ is the form that $\mathcal{F}$ takes in the j-th groove in terms of the local coordinates. Eq.(3b) then becomes:

$$B_j(k_x) = A_j(k_x)\exp(-ik_x d_j) \ , \tag{5a}$$

$$A_j(k_x) = \left(\frac{-1}{4\pi}\right)\int_{-w_j/2}^{w_j/2} u(x_j,f_j)\ F(k_x;x_j,f_j)\ \exp(-ik_x x_j)\ dx_j \quad . \tag{5b}$$

It was shown in Ref.3 that the N (>1)-groove response (i.e., u on the surface of a representative groove) closely follows the one-groove response, especially at high frequencies. It is therefore not unreasonable to assume that $A_j(k_x) \simeq \mathcal{A}_j(k_x)$, with $\mathcal{A}_j(k_x)$ the complex scattering function of the corresponding single-groove configuration. The evaluation of $u(x_j, f_j)$ can be done in an exact manner, as in Ref.3, or an approximation can be made of this function. Since we are assuming the frequency to be rather high, we rely on **geometric acoustics** (GA), and to make things as simple as possible, assume that: a) the (cross section) shapes of the grooves are all that of **right triangles**, b) $|\theta_i|<45°$ so as to obviate shadowing. It is then easy to show that two plane waves (one incident, one reflected) come into play on one portion of one wall of the representative groove surface, and four plane waves (two incident, one of which is the result of reflection from the adjacent wall, and two reflected) come into play on the remaining portion of the wall as well as on the totality of the adjacent wall, so that

$$A_j(k_x) \simeq \mathcal{A}_j(k_x) \sum_{\ell=1}^{3} \sum_{m=1}^{M_\ell} a_{\ell m}(k_x;w_j) \exp[i\varphi_{\ell m}(k_x;w_j)] \ , \tag{6}$$

wherein $M_1 = 2$, $M_2 = M_3 = 4$. The coefficients $a_{\ell m}$ and $\varphi_{\ell m}$ have simple closed-form expressions. The final step is to insert Eq.(6) into Eq(5a) and to insert the latter into Eq.(3a) to obtain the scattering function of the assembly of grooves.

How good is the GA for an isolated groove? Fig.1 provides an eloquent response to this question as concerns the surface field and Fig.2 as concerns the bistatic scattering cross-section. In both of these figures, the so-called exact results have been computed in the manner outlined in Ref.3.

We employed the GA to determine the far-field acoustic response of surfaces with random roughness. Fig. 3 applies to one realization of a rough surface consisting of an assembly of eight contiguous grooves whose widths are randomly distributed. As in Fig. 2, B_1, B_2 and F indicate the directions of expected (from GA) single-bounce backscattering, double-bounce retroreflection and single-bounce forward reflection respectively. The finite width and multiplicity of the peaks are the result of the combined effects of diffraction, interference and the irregular nature of the surface affecting the incident wave. For more irregular surfaces (e.g., grooves not necessarily of the same shape), the angular distribution of scattered energy is much more erratic (speckle) and it is usually impossible to distinguish groups of peaks near B_1, B_2 and F that rise notably above the background. However, averaging over many realizations enables a peak of type B_2 to make its appearance; this is the enhanced backscattering effect. It is expected that this effect will be particularly strong in the present example due to the fact that pronounced backscattering of type B_2 is already observed for a **single** realization.

[1] K.A. O'Donnell and E.R. Mendez, J.Opt.Soc.Am. A 4(19871194.
[2] C. Macaskill and B.J. Kachoyan, J.Acoust.Am. 84(1988)1826.
[3] A. Wirgin and L. Kouoh-Bille, in this issue.

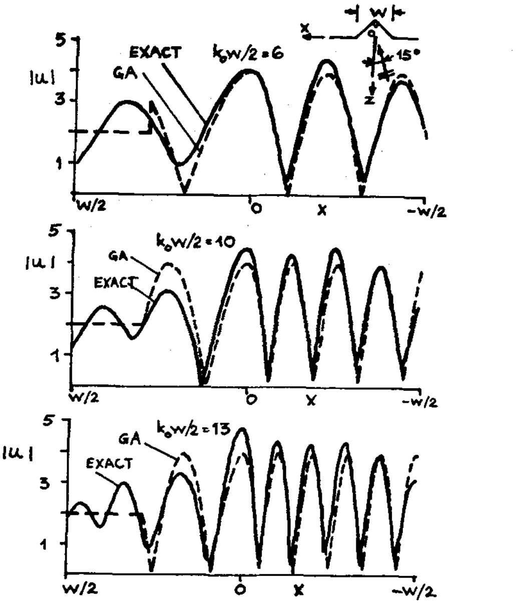

Fig.1 Total surface pressure versus x in a single-groove configuration.

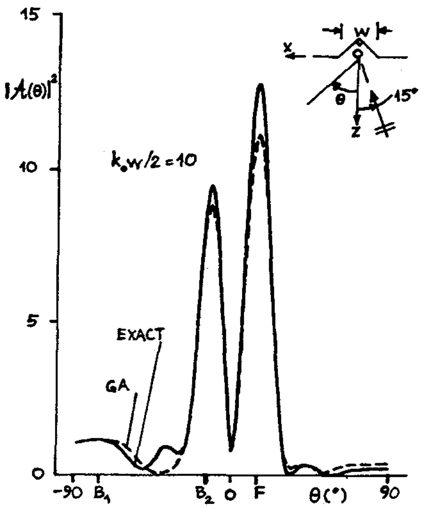

Fig.2 Bistatic cross section for a single-groove configuration.

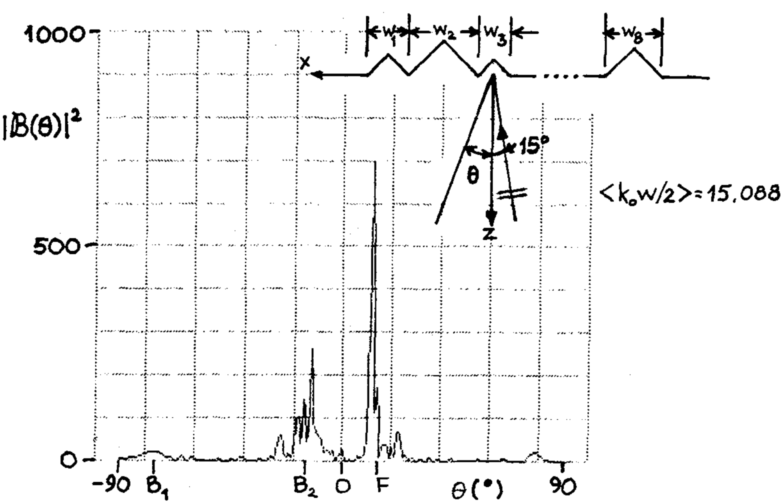

Fig.3 GA bistatic cross section for one realization of a randomly rough surface.