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GROUND AMPLIFICATION IN THE NEIGHBORHOOD OF A GROUP OF MUTUALLY PARALLEL TRIANGULAR RIDGE MOUNTAINS SUBJECTED TO SHEAR HORIZONTAL SEISMIC WAVES**A. WIRGIN and L. KOUOH-BILLE***Laboratoire de Modélisation en Mécanique, U.R.A. 229 du CNRS, Université Pierre et Marie Curie, 4 place Jussieu, F-75252 Paris cedex, France*

Experimental (in the laboratory and in the field) and theoretical studies indicate that the presence of topographic irregularities can strongly affect (notably increase) the strength of ground motion arising from seismic disturbances. The causes of these effects, particularly as they concern convex topographic features (i.e., hills or mountains) are not well understood, although it is clear that they depend on: the frequency, the precise location on the mountains or in their vicinity (stronger motion at the crests than in the valleys), the aspect ratio of the mountains (stronger for larger aspect ratios) and the angle of incidence (although it is difficult to establish any systematic trends).

In a recent publication [1] we showed that an isolated ridge of rectangular shape responds to an SH wave in the manner of a flanged Helmholtz cavity, with modes of successively higher-order being excited as the frequency increases and the displacement field being maximally amplified at the resonance frequencies for appropriate incident angles at locations on the ridge that are usually on the crest. Indications were provided that the excitation of free modes is the dominant cause of the amplification of ground motion of mountains of this shape.

As real mountains are more usually of nearly triangular shape, and are never isolated, it is of substantial interest to determine whether the resonance explanation still holds for the more realistic topographies. We examine herein the 2D problem of SH waves striking a group of mutually parallel triangular elastic ridges emerging from flat ground overlying a homogeneous elastic half-space.

Resonances are brought to light in the same manner as in Refs.1, and shown to be excited simultaneously for several orders and over larger bands (than previously), with the result that resonance peaks of displacement are less pronounced and ground amplification is maintained at relatively high levels over wider frequency ranges, in conformity with what is usually observed in terrain measurements. It also turns out that the order of magnitude of the amplification can be predicted by simple geometrical acoustics arguments and that, in many cases, the fields in successive ridges are weakly coupled. The general level of the amplification increases with the aspect ratio of the mountains, but is modest for realistic aspect ratios. The level increases, and approaches that observed in certain experiments if a contrast is introduced in the elastic moduli and densities between the solids occupying the mountains and that of the basement.

The mountains are assumed to be infinitely long, mutually parallel ridges emerging from otherwise flat ground; they are subjected to a seismic disturbance in the form of a plane wave ($\exp(-i\omega t)$ implicit time dependence, ω the angular frequency) with which are associated a propagation vector k_i (pointing upwards) lying in a plane (the sagittal x - z plane S) perpendicular to the ridge generators and a displacement vector u perpendicular to this plane. The resulting motion is shear horizontal (only u_y is non-vanishing) and independent of the coordinate y . In the absence of the mountains, the ground is the flat plane $z=0$; the intersection of this plane with S is the line L . The mountains M_j , $j \in N$ ($N=\{1,2,\dots,N\}$) occupy Ω_j , $j \in N$ in S and lie on (in welded contact with) L . The basement B ($z < 0$) is the half-plane Ω_0 in S . The junctions of Ω_j with B are the segments J_j . Each mountain is an isosceles triangle in S with base J_j and sides S_j . The width, height and included angle at the apex of the mountain occupying Ω_j are w_j , b_j and α_j respectively. B and M_j , $j \in N$ are filled with linear, elastic, homogeneous, isotropic media M_j , $j=0,1,2,\dots,N$ whose mass density and rigidity are ρ_j and μ_j respectively.

Let u_i and u_j , $j \in N$ be the y -components of the (incident and total respectively) displacement vectors u_i and u_j (in Ω_j) respectively. u_j satisfies: 1) the reduced (Helmholtz) wave equation, 2a) the stress-free boundary condition on $L-(J_1+J_2+\dots+J_N)$, 2b) the stress-free boundary condition on S_j , $j \in N$, 3a) the continuity of displacement condition on J_j , $j \in N$, 3b) the continuity of traction condition on J_j , $j \in N$ and 4) the Sommerfeld radiation condition (as concerns u_0).

The displacement field satisfying 1) in $\Omega_{j \geq 1}$ and 2b) on S_j is represented by the (so-called quasi-modal) expansion:

$$u_j(x) = \sum_{n=0}^{\infty} A_n^j \frac{J_{n\pi}}{\alpha_j}(k_j r_j) \cos\left[\frac{n\pi}{\alpha_j} \left(\frac{\pi}{2} + \frac{\alpha_j}{2} - \psi_j\right)\right]; \quad x \in \Omega_{j \geq 1}, \quad (1)$$

with $k_j = \omega/\beta_j$, $\beta_j = (\mu_j/\rho_j)^{1/2}$ the wavenumber and shear wave velocity in M_j , r_j , ψ_j the polar coordinates with respect to the origin O_j located at the apex of M_j , and $J_\nu(\zeta)$ the ν -th order Bessel function of argument ζ . The displacement field satisfying 1) and 4) in Ω_0 is represented by:

$$u_0(x) = u_i(x) + u_r(x) + \int_{-\infty}^{\infty} B(k_x) \exp[i(k_x x + k_z z)] \frac{dk_x}{k_z}; \quad x \in \Omega_0, \quad (2)$$

with (θ_i the angle of incidence with respect to the $+z$ axis):

$$u_i(x) = \exp[i(k_{ix} x - k_{iz} z)] \quad , \quad k_{ix} = k_0 \sin(\theta_i), \quad k_{iz} = k_0 \cos(\theta_i), \quad (3a)$$

$$u_r(x) = \exp[i(k_{ix} x + k_{iz} z)] \quad , \quad x = (x, 0, z), \quad k_z = (k_0^2 - k_x^2)^{1/2}, \quad \begin{pmatrix} \text{Re} \\ \text{Im} \end{pmatrix} (k_z) \geq 0. \quad (3b)$$

The integral vanishes in the absence of $M_{j \geq 1}$, in which case the total field in Ω_0 reduces to the incident (u_i) plus the reflected (u_r) waves and the modulus of the total displacement (at all points) on L to the value 2. The extent to which this conclusion is modified by the presence of the mountains is analyzed hereafter.

We employ Eqs.(1)-(3) in 2a) and 3b) and project on the Fourier base $\{\exp(-ik_x x; k_x \in \mathbb{R})\}$ to obtain:

$$B(k_x) = \mathcal{F}_1(A_n^j; j \in N, n=0,1,2,\dots); \quad k_x \in \mathbb{R}. \quad (4)$$

Employing Eqs.(1)-(3) in 3a) and projecting on the base $\{\cos[\frac{m\pi}{\alpha_j}(\frac{\pi}{2} + \frac{\alpha_j}{2} - \psi_j)]\}$; $m = 0, 1, 2, \dots$ leads to:

$$\mathcal{F}_{2m}[B(k_x) ; k_x \in R] = \mathcal{F}_{3m}(A_n^j ; n=0, 1, 2, \dots) ; j \in N, m=0, 1, 2, \dots \quad (5)$$

The replacement of $B(k_x)$ in Eq.(5) by its expression given in Eq.(4) finally results in a linear system of equations of the form:

$$\sum_{\ell=1}^N \sum_{n=0}^{\infty} E_{mn}^{j\ell} A_n^{\ell} = G_m^j \quad ; j \in N, m=0, 1, 2, \dots \quad (6)$$

the only term of which involving the incident wave parameters is G_m^j . This infinite-order linear system is solved by truncation and the process is repeated as many times (with increasing truncation order) as is necessary to obtain stabilization of the numerical values of the most significant coefficients A_n^{ℓ} . The latter are then introduced into Eq.(5) to obtain the plane-wave amplitudes $B(k_x)$ and into Eqs.(1) and (2) to compute the displacement fields.

Let us restrict the discussion to the surface field on a representative mountain. For the amplification to be a phenomenon of systematic nature, such as is generally observed (this meaning that it can occur practically at any location of the mountain surface and for essentially any incident angle), one or more of the coefficients A_n^{ℓ} must be large; for this to occur independently of the incident angle the determinant of the hypermatrix E in Eq.(6) must be small, at least for certain frequencies. The latter are none other than the eigenfrequencies, because the singularity of E is equivalent to the existence of a natural (eigen) mode, $\det(E)=0$ giving rise to the dispersion relations of the eigenmodes of the structure, and to a resonance which occurs when the driving frequency is equal to the (real part of) one of the eigenfrequencies. The spectral response is lorentzian in the neighborhood of the eigenfrequencies and finite due to radiation damping. From Eq.(6) it is possible to demonstrate that: a) each (n, j) -th quasi-mode becomes large at its own set of frequencies $\omega(j, n, m)$; $m=0, 1, 2, \dots$, with $m=0$ the fundamental and $m \geq 1$ the overtones, b) for a set of identical mountains, the (j, n) eigenfrequencies are degenerate with respect to j and splitting occurs with the appearance of a set of resonant frequencies above and below the resonance frequencies of the isolated mountains.

To illustrate these predictions, consider Fig.1 which applies to a single-mountain configuration. At the apex the field involves only the lowest-order quasi mode, i.e., $|u_1| = |A_0^1|$. The observed peaks are therefore located at the resonance frequencies $\omega(1, 0, 0), \omega(1, 0, 1), \dots$ and these frequencies are close to the real parts of the roots of $\det(E') = 0$, with $E' = \{E_{00}^{11}\}$. Since the successive resonance bands are rather wide and close together, overlapping occurs with the result that the overall level is relatively high, even at the frequencies furthest from the center frequencies of the resonance bands. The figure also reveals that the amplification is rather stable and small for the most realistic mountain shapes ($\alpha = 90, 120^\circ$); however, a slower, less rigid material within the mountain accentuates the resonances (due to better energy trapping) and thereby increases substantially the peak amplifications. The horizontal line at level 2 in Fig.1 represents the geometrical acoustics (GA) prediction of the apex response of the $\alpha=90^\circ$ mountain. It can be observed that this response constitutes a rather good approximation of the over-all response at all except the lowest frequencies. In the case the media filling M_1 and B are identical the GA predicts an apex amplification of $360/\alpha(^{\circ})$, in good agreement with what is observed in the figure.

The effect of neighbors can be ascertained in Fig.2, where the displacement

field is now sampled at a point half-way up the left side of the left-hand mountain of one and two-mountain configurations. Since the plane wave strikes the mountains at normal ($\theta_1=0^\circ$) incidence, only the even-order quasi-modes are excited, but all of the latter contribute to the field on the side of the mountain. Therefore, the response is both the result of order-to-order mode mixing and overtone-to-overtone mixing with the result that the amplifications are smaller than at the apex. Splitting is readily observed, with the largest effects occurring at low frequencies. At higher frequencies, the two-mountain response closely follows the one-mountain response, indicating rather small coupling effects.

- [1] L. Kouoh-Bille, F.J. Sanchez-Sesma and A. Wirgin, C.R.Acad.Sci. II 312(1991)849.

Fig. 1 Amplification $|u_1|/2$ versus frequency at the apex for a single mountain.

Fig. 2 Amplification $|u_1|/2$ versus frequency at mid-height on a mountain of one- ($N=1$) and two- ($N=2$) mountain configurations.