STUDY OF THE BEHAVIOUR OF INHOMOGENEOUS HARMONIC WAVES IN THEIR DIFFRACTION FROM SIMPLE ROUGH SURFACES

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Abstract: A theoretical model is presented to investigate the diffraction of inhomogeneous waves from a traction-free solid-air boundary having simple periodic profile. It is shown how one can calculate the amplitudes, the angular positions and the exponential decays of the various diffracted orders. The numerical results for a brass-air boundary are compared with the experimental results obtained by Jungman et al.[1]. Our model accounts for the first reflection minimum near the fundamental surface resonance frequency, corresponding to Rayleigh wave generation. This minimum is clearly observed experimentally, but fails to show up in the theoretical results of homogeneous plane wave theory.

Résumé: Un modèle théorique est présenté pour investiguer la diffraction d'ondes planes inhomogènes par une surface périodique solide-air. On calcule les amplitudes, les positions angulaires et les décroissances exponentielles des ordres diffractés. Les résultats numériques pour le cas d'une interface cuivre-air sont comparés avec les résultats expérimentaux de Jungman et al.[1]. La théorie des ondes planes homogènes ne décrit pas le minimum du coefficient de réflexion près de la fréquence fondamentale, correspondant à la génération de l'onde de Rayleigh, tandis que notre modèle prévoit un minimum prononcé.

1 The general system of equations

The geometry of the two-dimensional diffraction problem is sketched in Fig.1. The solid is assumed to be isotropic and homogeneous. The solid-air boundary is described by the equation

$$z = f(x) \text{ with } f(x + \Lambda) = f(x). \quad (1)$$

Figure 1: Geometrical configuration for the solid-air problem

A longitudinal inhomogeneous wave with exponential decay β_i , phase velocity v and angle of incidence θ_i^r can be described in the $0'x'z'$ -coordinate system, respectively the $0xz$ -coordinate system, by the velocity potential (the time dependence is suppressed further on)

$$\varphi_i(x', z') = A \exp(\beta_i x') \exp(i \frac{\omega}{v} z'), \quad (2)$$

respectively

$$\varphi_i(x, z) = A \exp[i(k_i x + K_i z)], \quad (3)$$

with

$$k_i = k_d \sin(\theta_i^*) = k_d \sin(\theta_i^r + i\theta_i^c), \quad K_i = k_d \cos(\theta_i^*), \quad k_d = \frac{\omega}{v_d}, \quad (4)$$

ω being the angular frequency of the radiation source and v_d being the longitudinal velocity of the medium. The angle θ_i^c is related to the inhomogeneity β_i and the wavenumber k of the incident wave by

$$\beta_i = -k \sinh \theta_i^c, \quad k = \frac{\omega}{v} = \frac{k_d}{\cosh \theta_i^c}, \quad (5)$$

from which we can deduce the dispersion relation

$$k^2 = k_d^2 + \beta_i^2. \quad (6)$$

The scattered dilatational ($m = d$) and shear ($m = s$) waves are represented by potentials $\varphi_m(x, z)$ which are solutions of the appropriate wave equations [2] and which must satisfy the continuity of stress

$$\sum_{j=1}^3 T_{ij}(\nabla \phi)_j = 0, \quad i = 1, 3 \quad (7)$$

on the boundary $\phi(x, z) = 0$ with $\phi(x, z) = f(x) - z$.

The potentials $\varphi_m(x, z)$ can be described by infinite series of inhomogeneous waves [3]

$$\varphi_m(x, z) = \sum_{l=-\infty}^{\infty} M_l \exp[i(k_l x + K_{ml} z)], \quad (8)$$

$M_l = AD_l$, or $M_l = AS_l$, where D_l and S_l are the unknown reflection coefficients of the l -th order,

$$k_l = \frac{\omega}{v_m} \sin \theta_{ml}^* = \frac{\omega}{v_m} \sin(\theta_{ml}^r + i\theta_{ml}^c), \quad K_{ml} = \frac{\omega}{v_m} \cos \theta_{ml}^*, \quad (9)$$

v_m being the corresponding acoustic velocity of the medium. The same relations as in (5) and (6) can be written down for each scattered D- and S-wave of order l .

The x -components k_l of the wave vectors are calculated from the complex diffraction equation

$$k_l = k_i + l \frac{2\Pi}{\Lambda}, \quad l \in \mathbb{Z}. \quad (10)$$

Equating the real and imaginary part in both members, we obtain a nonlinear system from which the exponential decays β_{ml} and the angular positions θ_{ml}^r of the diffracted D- and S-waves can be calculated

$$\sqrt{k_m^2 + \beta_{ml}^2} \sin \theta_{ml}^r = \sqrt{k_d^2 + \beta_i^2} \sin \theta_i^r + l \frac{2\Pi}{\Lambda}, \quad (11)$$

$$\beta_{ml} \cos \theta_{ml}^r = \beta_i \cos \theta_i^r. \quad (12)$$

Using (2), (9) and (12) in the boundary conditions (7), we find a system of equations which are periodic functions of x so that a sufficient condition for the solution is to require that the Fourier coefficients are equal over $[0, \Lambda]$:

$$\sum_{l=-\infty}^{\infty} [(k_l - k_n + 2\frac{v_s^2}{\omega^2} k_n K_{dl}^2) D_l I_{ln}^d + \sum_{l=-\infty}^{\infty} K_{sl} (2k_l k_n \frac{v_s^2}{\omega^2} - 1) S_l I_{ln}^s] = [(k_n - k_i) - 2\frac{v_s^2}{\omega^2} k_n K_i^2] I_{on} \quad (13)$$

$$\sum_{l=-\infty}^{\infty} K_{dl} (1 - 2k_l k_n \frac{v_s^2}{\omega^2}) D_l I_{ln}^d + \sum_{l=-\infty}^{\infty} [(k_l - k_n) + 2\frac{v_s^2}{\omega^2} k_n K_{sl}^2] S_l I_{ln}^s = K_i (2k_i k_n \frac{v_s^2}{\omega^2} - 1) I_{on} \quad (14)$$

with $n = \dots - 2, -1, 0, +1, +2, \dots$,

$$I_{ln}^m = \int_0^\Lambda \exp[i(k_l - k_n)x] \exp[iK_{ml}f(x)] \frac{1}{K_{ml}} dx$$

and for I_{on} one has to require $k_l = k_i$, $K_{ml} = K_i$.

2 Numerical results and conclusions

We applied our model for normal incidence to a brass sample with sawtooth corrugated boundary. The values of the physical and geometrical parameters are

$$v_d = 4700 \frac{m}{s}, \quad v_s = 2100 \frac{m}{s}, \quad \Lambda = 250 \mu m, \quad h = 66 \mu m.$$

By truncating ($-6 \leq l, n \leq 6$) the infinite system of Eqs.(13)-(14), we calculated to a good accuracy the first 13 diffraction coefficients D_l, S_l .

Fig.2 shows the numerical results for the zero order reflected longitudinal spectrum. We clearly see that plane wave theory ($\beta_i = 0$) doesn't predict the sharp dip around 8 MHz that is observed experimentally [1]. Agreement between theory and experiment is obtained only by considering inhomogeneous waves. In the past, other authors [1,4,5] modified the plane wave theory by introducing an appropriate gating function and finite beam analysis.

The reflection coefficient D_0 becomes zero for $\beta_i = 0.1 \frac{1}{mm}$. By means of Eqs.(11)-(12), we obtain for the longitudinal reflected first order a diffraction angle $\theta_{d1}^r = 89.7^\circ$ and an exponential decay $\beta_{d1} = 22.806 \frac{1}{mm}$, while for the transversal reflected first order we get the values $\theta_{s1}^r = 89.3^\circ$ and $\beta_{s1} = 8.546 \frac{1}{mm}$. Remark that only for normal incident homogeneous plane waves, diffraction parallel with the surface can occur (see Eq.(12)). Using the appropriate dispersion relations (as in Eq.(6)) we find that each of the inhomogeneity parameters mentioned gives rise to a velocity of $1975 \frac{m}{s}$ along the periodic interface. For this reason we interpret those diffracted inhomogeneous waves as components of a first order generated Rayleigh wave ($v_{\text{Rayleigh}} = 1980 \frac{m}{s}$). That both components have a non-negligible amplitude can be seen on Figs. 4 and 5.

Another interesting phenomenon is shown on Fig.3. For normal incident homogeneous plane waves the zero order transversal reflection coefficient S_0 is always zero, while for inhomogeneous plane waves S_0 is not negligible at all and has a sharp extremum at the fundamental resonance frequency.

We also notice that all extrema in the spectra for inhomogeneous plane waves occur at one distinct frequency slightly shifted to the left compared with the frequency position of the extremum given by the homogeneous plane wave theory.

This theory is extended for incident bounded beams described by means of series of inhomogeneous waves as well as for other configurations like liquid-solid and solid-liquid.

FIG. 2

FIG. 3

FIG. 4

FIG. 5

Figures 2 to 5 : zero and first order dilatational (D_0, D_1) and shear (S_0, S_1) reflected spectra for a brass-air interface ($h = 66\mu m, \Lambda = 250\mu m$) and for different values of the inhomogeneity parameter β_i

References

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