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THE MODEL OF VISCOELASTIC COMPOSITE

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résumé : Le développement de modèle composite à deux phases, basé sur une représentation du comportement viscoplastique des matériaux composites, est proposé et réalisé. La méthode de la moyenne phénoménologique des paramètres n'impliquant aucune hypothèse sur la périodicité de la structure, est utilisée. La validité de l'approche proposée est vérifiée par comparaison des résultats des calculs aux résultats expérimentaux.

Abstract - The construction of two-phase composite models based on representation of viscoelastic behaviour of composite materials has been proposed and realized. The method of phenomenological averaging of parameters is used, that doesn't require a hypothesis of composite structure periodicity. The adequacy of the proposed approach has been verified by comparing the experimental and calculation results.

I - INTRODUCTION

Among a lot of experimental studies of properties of composite materials and theoretical constructions of the corresponding models, we can find only some of them that are devoted to investigation of behaviour of composite materials under dynamic loading. The present paper gives the formulation and evaluation of a composite model with viscoelastic components for the problems of intense dynamic deformation. Two-phase composites of two types are under consideration: fibrous unidirectional composites with parallel fibres of one material in a matrix of the other and laminated composites with parallelly alternated layers. Further, the principles of construction can be generalized for more general cases.

In the most of studies known the averaging methods used for finding the efficient characteristics of the state of a composite as continuum are based on the assumption of composite structure periodicity. In this paper the method of the phenomenological averaging is proposed, that allows us to do without a hypothesis about the structure periodicity. In addition to ordinary characteristics, the one (that is tensor characteristic in the general case) of incompatibility of

stresses acting along the fibres (or layers) is introduced. The test calculations have shown the essential effect of this parameter on the $(\sigma-\varepsilon)$ -diagram of deformation. The adequacy of the model constructed was verified by comparison of the calculation and experimental results for the case of a fibrous unidirectional composite.

II - THE MAIN ASSUMPTIONS FOR MODEL FORMULATION

The composite is considered to be continuum which elementary volume contains a lot of fibres or layers. The additional assumptions of mutual arrangement of the reinforcement elements (except the inter-parallelism) are not adopted. The volumetric concentrations and all the required mechanical characteristic of each composite component are assumed to be known.

To describe material behaviour of the composite components, use is made of the viscoelastic Maxwell-like body model [1] applied widely by the authors for homogeneous materials [2,3]. For the composite components, the following relations (where α is the component number) are used, i.e.

$$\rho^{(\alpha)} \frac{\partial u_i^{(\alpha)}}{\partial t} - \frac{\partial \sigma_{ij}^{(\alpha)}}{\partial x_j} = 0, \quad (1)$$

$$\frac{\partial \varepsilon_{ij}^{(\alpha)}}{\partial t} - \frac{1}{2} \left[\frac{\partial u_i^{(\alpha)}}{\partial x_j} + \frac{\partial u_j^{(\alpha)}}{\partial x_i} \right] = - \frac{\sigma_{ij}^{(\alpha)} - \frac{1}{3}(\sigma_{11}^{(\alpha)} + \sigma_{22}^{(\alpha)} + \sigma_{33}^{(\alpha)})\delta_{ij}}{2\mu^{(\alpha)}\tau^{(\alpha)}}.$$

Here $\rho^{(\alpha)}$ - is the density, $u_i^{(\alpha)}$ - is the velocity, $\sigma_{ij}^{(\alpha)}$ - is the stress tensor, $\varepsilon_{ij}^{(\alpha)}$ - is the elastic strain tensor, $\tau^{(\alpha)}$ - is the time of relaxation of shear stresses, $\mu^{(\alpha)}$ - is the shear modulus. The stress-strain relations are in accordance with the Hooke law:

$$\sigma_{ij}^{(\alpha)} = \lambda^{(\alpha)} (\varepsilon_{11}^{(\alpha)} + \varepsilon_{22}^{(\alpha)} + \varepsilon_{33}^{(\alpha)}) \delta_{ij} + 2\mu^{(\alpha)} \varepsilon_{ij}^{(\alpha)}. \quad (2)$$

They can be calculated also with the help of the potentials $E^{(\alpha)}$ and the Murnaghan formulas

$$\sigma_{ij}^{(\alpha)} = \rho^{(\alpha)} \frac{\partial E^{(\alpha)}}{\partial \varepsilon_{ij}^{(\alpha)}}. \quad (3)$$

Now the evolution equations that are analogous to equations (1), should be obtained for the composite as continuum. For this purpose we have to use the averaged efficient characteristics of the state for the elementary volume. It is clear that to describe completely peculiarities of the composite behaviour, additional characteristics for the internal medium state (the microlevel state) are required. Thus, even in the absence of averaged stresses and effect of external forces, the equilibrium microstresses (due to, for example, technology of composite preparation), that don't result in medium displacements, can exist inside the elementary volume. As is shown further, such microstresses can affect significantly the composite behaviour.

In model construction, the velocity field and stresses acting across the fibres inside the element is taken to be uniform, i.e. $u_i = u_i^{(\alpha)}$ and $\sigma_{ij}^{(1)} = \sigma_{ij}^{(2)}$, $(i,j) \neq (1,1)$. To define the characteristics $\langle a \rangle$ averaged over the elementary volume, let us use the rule of mixtures:

$$\langle a \rangle = ca^{(1)} + (1-c)a^{(2)},$$

where c and $(1-c)$ - are the volumetric concentrations of the fibres and matrix, $a^{(\alpha)}$ denote the characteristics of components. Let us also utilize the following notation

$$\tilde{a} = (1-c)a^{(1)} + ca^{(2)} = a^{(1)}a^{(2)}\langle a^{-1} \rangle.$$

III - FIBROUS COMPOSITE MEDIUM

Let the composite consist of parallel fibres of one material that are directed along the x_1 axis and placed in a matrix of the other.

In addition to the mentioned hypotheses

$$u_i^{(\alpha)} = u_i; \quad \sigma_{ij} = \sigma_{ij}^{(1)} = \sigma_{ij}^{(2)}, \quad (i, j) \neq (1, 1); \quad (4)$$

let us assume that

$$\begin{aligned} \sigma_{11} &= \langle \sigma_{11} \rangle; \quad \varepsilon_{ij} = \langle \varepsilon_{ij} - \hat{\varepsilon}_{ij} \rangle, \quad (i, j) \neq (1, 1); \\ \varepsilon_{11} &= \varepsilon_{11}^{(1)} - \hat{\varepsilon}_{11}^{(1)} = \varepsilon_{11}^{(2)} - \hat{\varepsilon}_{11}^{(2)}; \quad \Delta = \varepsilon_{11}^{(1)} - \varepsilon_{11}^{(2)}. \end{aligned} \quad (5)$$

Hypotheses (5) mean that

- the force acting on the composite element along the fibres is equal to a sum of forces acting on each component;
- for $\sigma_{ij} = 0$ there can exist the unavoidable microdeformations $\hat{\varepsilon}_{ij}^{(\alpha)}$ that lead to the non-zero microstresses $\sigma_{ij}^{(1)} \neq \sigma_{ij}^{(2)}$;
- the value of ε_{ij} for $(i, j) \neq (1, 1)$ is the averaged elastic strains of the composite components measured off $\hat{\varepsilon}_{ij}^{(\alpha)}$;
- the elastic strain of each of the components $\varepsilon_{11}^{(\alpha)}$ is a sum of the corresponding value of $\hat{\varepsilon}_{11}^{(\alpha)}$ and common deformation ε_{11} ;
- the parameter Δ characterizes the difference in elastic strains of composite components along the fibre direction, and from $\sigma_{ij} = 0$ it doesn't follow that $\Delta = 0$ and $\sigma_{11}^{(\alpha)} = 0$.

By somewhat awkward, however, rather simple calculations based on hypotheses (4), (5) and the Hooke law (2), the relation of $\hat{\varepsilon}_{ij}^{(\alpha)}$ with Δ and of $\varepsilon_{ij}^{(\alpha)}$ with ε_{ij} , Δ is established. The use of the obtained relations gives the law that connects σ_{ij} with ε_{ij} in the form of the Hooke law for transversally isotropic medium:

$$\begin{aligned} \sigma_{11} &= C_{11}\varepsilon_{11} + C_{12}\varepsilon_{22} + C_{12}\varepsilon_{33}, \quad \sigma_{22} = C_{12}\varepsilon_{11} + C_{22}\varepsilon_{22} + C_{23}\varepsilon_{33}, \\ \sigma_{12} &= C_{66}\varepsilon_{12}, \quad \sigma_{33} = C_{12}\varepsilon_{11} + C_{23}\varepsilon_{22} + C_{22}\varepsilon_{33}, \\ \sigma_{12} &= C_{66}\varepsilon_{12}, \quad \sigma_{23} = \frac{1}{2}(C_{22} - C_{23})\varepsilon_{23}. \end{aligned} \quad (6)$$

where

$$\begin{aligned} C_{11} &= \langle \lambda + 2\mu \rangle - \frac{c(1-c)(\lambda_2 - \lambda_1)}{\tilde{\lambda} + \tilde{\mu}}, \quad C_{12} = \left\langle \frac{\lambda}{\lambda + \mu} \right\rangle \left\langle \frac{1}{\lambda + \mu} \right\rangle^{-1}, \\ C_{22} &= \left\langle \frac{1}{\lambda + \mu} \right\rangle^{-1} + \left\langle \frac{1}{\mu} \right\rangle^{-1}, \quad C_{23} = \left\langle \frac{1}{\lambda + \mu} \right\rangle^{-1} + \left\langle \frac{1}{\mu} \right\rangle^{-1}, \quad C_{66} = \frac{1}{2}(C_{22} - C_{23}) = \left\langle \frac{1}{\mu} \right\rangle^{-1}. \end{aligned} \quad (7)$$

Here λ, μ - are the Lamé constants.

The elastic potential of the composite medium takes the form

$$\rho E = \frac{1}{2} C_{11} \varepsilon_{11}^2 + \frac{1}{2} C_{22} (\varepsilon_{22}^2 + \varepsilon_{33}^2) + C_{12} \varepsilon_{11} (\varepsilon_{22} + \varepsilon_{33}) + C_{12} \varepsilon_{11} \varepsilon_{22} + 2C_{12} (\varepsilon_{11} \varepsilon_{22} + \varepsilon_{11} \varepsilon_{33} + \varepsilon_{22} \varepsilon_{33}) + \frac{1}{2} B \Delta^2, \quad (8)$$

where $\rho = \langle \rho \rangle$, $\rho E = \langle \rho E \rangle$, C_{ij} - are calculated by (7), and

$$B = \frac{c(1-c)E_1 E_2}{\langle E \rangle},$$

where $E_1, E_2, \langle E \rangle$ are the Young moduli. It is easily seen that an analogue of the Murnaghan formulas (3) is fulfilled, i.e.

$$\sigma_{ij} = \frac{\partial \rho E}{\partial \varepsilon_{ij}}.$$

The pulse equations

$$\rho \frac{\partial u_i}{\partial t} - \frac{\partial \sigma_{ij}}{\partial x_j} = 0 \quad (9)$$

are derived from (1) using (4) and the first relation from (5). To derive the evolution equations for elastic strains, use is made of their definition from (5), relations $\varepsilon_{ij}^{(\alpha)}$ with Δ and the corresponding equations (1). The awkward transformations give

$$\frac{\partial \varepsilon_{ij}}{\partial t} - \frac{1}{2} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] = -\phi_{ij}, \quad \frac{\partial \Delta}{\partial t} = -\psi. \quad (10)$$

To show the structure of the right-hand parts of system (10), we write out only two (because of limitations of the paper) relations:

$$\phi_{22} = \alpha_{12} \sigma_{11} + \alpha_{22} \sigma_{22} + \alpha_{23} \sigma_{33} + \beta_2 q, \quad q = \frac{\partial \rho E}{\partial \Delta};$$

$$\alpha_{12} = \frac{1}{12} \langle E \rangle^{-2} \frac{\langle E^2 \rangle}{\langle \mu \tau \rangle} \frac{\langle \lambda \rangle}{\langle \lambda + \mu \rangle} + \frac{1}{12} \frac{\langle \mu \rangle}{\langle (\lambda + \mu)^2 \tau \rangle} + \frac{1}{6} \langle E \rangle^{-1} \frac{\langle E \rangle}{\langle (\lambda + \mu) \tau \rangle} \frac{\langle \lambda \rangle}{\langle \lambda + \mu \rangle} + \frac{1}{4} \frac{\langle 1 \rangle}{\langle \mu \tau \rangle}.$$

Relations (9), (10), along with (8) and (6), (7), form a closed system of equations for the model being formulated. The choice of the right-hand parts as dependences on σ_{ij} and q (as further in the case of a laminated composite) provides the Onzager principle.

Combining (10) with (8) and (6) gives the consequence of the basic equations of the model, i.e. the energy balance equation:

$$\frac{\partial \rho E}{\partial t} - \sigma_{ij} \frac{\partial u_i}{\partial x_j} = -\sigma_{ij} \phi_{ij} - q \psi = -\mathcal{X}.$$

The expressions for ϕ_{ij} , ψ are used to show that $\mathcal{X} \geq 0$, i.e. in the model formulated the energy dissipation condition is satisfied and the model is valid thermodynamically.

IV- MODEL OF LAMINATED COMPOSITE

Consider now a composite medium that consists of plane parallel

layers of two materials alternating in the x_1 -axis direction. As in the previous case, let us formulate the hypotheses, that will be the basis to construct the model, i.e.

$$u_i^{(\alpha)} = u_i ; \sigma_{ij} = \sigma_{ij}^{(1)} = \sigma_{ij}^{(2)} , (i, j) = (1, 1), (1, 2), (1, 3);$$

$$\sigma_{ij} = \langle \sigma_{ij} \rangle ; (i, j) = (2, 2), (2, 3), (3, 3);$$

$$\varepsilon_{ij} = \langle \varepsilon_{ij} - \bar{\varepsilon}_{ij} \rangle , (i, j) = (1, 1), (1, 2), (1, 3);$$

$$\varepsilon_{ij} = \varepsilon_{ij}^{(1)} - \bar{\varepsilon}_{ij}^{(1)} = \varepsilon_{ij}^{(2)} - \bar{\varepsilon}_{ij}^{(2)} ; (i, j) = (2, 2), (2, 3), (3, 3);$$

$$\Delta_{22} = \varepsilon_{22}^{(1)} - \varepsilon_{22}^{(2)} , \Delta_{23} = \varepsilon_{23}^{(1)} - \varepsilon_{23}^{(2)} , \Delta_{33} = \varepsilon_{33}^{(1)} - \varepsilon_{33}^{(2)} .$$

Unlike the previous case, in this one there are three parameters characterizing incompatibility of the elastic deformations of composite materials.

The essence of the assumptions made is the same as that one of the corresponding hypotheses in the case of the fibrous composite. Further the way of deriving the main relations of the model is as that one presented above. It is convenient to introduce new parameters

$$\Delta_1 = (\Delta_{22} + \Delta_{33})/2 , \Delta_2 = (\Delta_{22} - \Delta_{33})/2 .$$

The Hooke law to be obtained will have the same characteristic form (6) with the effective moduli

$$C_{11} = \left\langle \frac{1}{\lambda + 2\mu} \right\rangle^{-1} , C_{12} = \left\langle \frac{\lambda}{\lambda + 2\mu} \right\rangle \left\langle \frac{1}{\lambda + 2\mu} \right\rangle^{-1} , C_{22} = \langle \lambda + 2\mu \rangle - \frac{c(1-c)(\lambda_2 - \lambda_1)^2}{\tilde{\lambda} + 2\tilde{\mu}} ,$$

$$C_{22} = \frac{\lambda_1 \lambda_2 + 2\tilde{\mu} \langle \lambda \rangle}{\tilde{\lambda} + 2\tilde{\mu}} , C_{66} = \left\langle \frac{1}{\mu} \right\rangle^{-1} , C_{22} - C_{23} = 2\langle \mu \rangle .$$

It is in agreement with the analogous relations presented in [4]. The elastic potential takes the form

$$\begin{aligned} \rho E = & \frac{1}{2} C_{11} \varepsilon_{11}^2 + \frac{1}{2} C_{22} (\varepsilon_{22}^2 + \varepsilon_{33}^2) + C_{12} \varepsilon_{11} (\varepsilon_{22} + \varepsilon_{33}) + C_{23} \varepsilon_{22} \varepsilon_{33} + \\ & + 2C_{66} (\varepsilon_{12} \varepsilon_{21} + \varepsilon_{13} \varepsilon_{31}) + 2 \left[(C_{22} - C_{23}) \varepsilon_{23} \varepsilon_{32} + \frac{1}{2} B_1 \Delta_1^2 + \frac{1}{2} B_1 \Delta_1^2 + B_{23} \Delta_{23} \Delta_{32} \right] , \\ B_1 = & \frac{4c(1-c)}{\langle D \rangle} D_1 D_2 , B_2 = 2B_{23} = \frac{4c(1-c)}{\langle \mu \rangle} \mu_1 \mu_2 , D_\alpha = \mu_\alpha \frac{3\lambda}{\lambda_\alpha} + \frac{2\mu}{2\mu_\alpha} . \end{aligned}$$

As previously, we have

$$\rho \frac{\partial u_i}{\partial t} - \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

and

$$\frac{\partial \varepsilon_{ij}}{\partial t} - \frac{1}{2} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right] = -\phi_{ij} , \quad \frac{\partial \Delta_{ij}}{\partial t} = -\psi_{ij} .$$

As a consequence, the energy balance equation has the form

$$\frac{\partial \rho E}{\partial t} - \sigma_{ij} \frac{\partial u_i}{\partial x_j} = -\sigma_{ij} \phi_{ij} - \alpha_{ij} \psi_{ij} = -\mathfrak{A} , \quad \alpha_{ij} = \frac{\partial \rho E}{\partial \Delta_{ij}} .$$

It can be shown that $\mathfrak{A} \geq 0$.

V - TO CONSTRUCTION OF THE MODEL OF AN ARBITRARY STRUCTURE COMPOSITE

For the composites of an arbitrary structure, we can define the tensor Δ_{ij} that characterizes incompatibility of the elastic deformations of the composite materials. In the general case the evolution equations of the tensors ε_{ij} and Δ_{ij} have the form

$$\frac{\partial \varepsilon_{ij}}{\partial t} - \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) = - \alpha_{ijmn} \sigma_{mn} - \beta_{ijk} q_{kr} ,$$

$$\frac{\partial \Delta_{ij}}{\partial t} = - \beta_{ijk} \sigma_{kr} - \gamma_{ijpq} q_{pq} , \quad q_{ij} = \frac{\partial \rho E}{\partial \Delta} .$$

The system is added by the equations

$$\rho \frac{\partial u_i}{\partial t} - \frac{\partial \sigma_{ij}}{\partial x_j} = 0$$

and closed by the constitutive equations

$$\rho E = \frac{1}{2} C_{ijmn} \varepsilon_{ij} \varepsilon_{mn} + \frac{1}{2} B_{ijmn} \Delta_{ij} \Delta_{mn} ,$$

that are used to calculate the stresses

$$\sigma_{ij} = \frac{\partial \rho E}{\partial \varepsilon_{ij}} .$$

To determine the tensors of the coefficients in the above-presented relations C_{ijmn} , B_{ijmn} , α_{ijmn} , β_{ijk} , γ_{ijmn} , use is made of characteristics of the composite component properties and of additional hypotheses that represent peculiarities of its structure. The averaging method formulated above is also applied.

VI - COMPARISON OF TEST CALCULATIONS AND EXPERIMENTAL DATA

To obtain experimental data required for testing the model formulated, use was made of a test fibrous composite of copper wires in an epoxy matrix. The well-known Hopkinson rod method was applied to obtain the $(\sigma-\varepsilon)$ -diagrams for the test composite loaded along the fibres with the strain rates up to 10^4 s^{-1} . Two typical $(\sigma-\varepsilon)$ -diagrams, together with a dependence obtained in the experiments that shows the strain rate change $\dot{\varepsilon}$ during loading, are presented in Figs. 1 and 2 (the experimental points are marked by 1).

For comparison with the experimental data, the solution of the problem of thin rod extension was utilized. The corresponding system of equations is obtained from (9), (10) with the assumptions which functions depend only on time and

$$u_2 = u_3 = 0, \quad \varepsilon_{22} = \varepsilon_{33}, \quad \sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{13} = \sigma_{23} = 0 .$$

Solution of the system obtained for $\dot{\varepsilon}(t)$ which is taken from the experiment gives the $(\sigma-\varepsilon)$ -diagram that can be compared with the experimental one. The calculation dependences are shown in Figs. 1, 2 for different initial values of $\Delta(0) = \Delta_0$ (2- $\Delta_0 = 0$; 3- $\Delta_0 = 0.005$; 4- $\Delta_0 = 0.01$). From comparison it is seen that the model proposed represents quite satisfactorily the experimental $(\sigma-\varepsilon)$ -diagram, especially if the presence of internal microstresses in a composite (deformation incompatibility) is taken into account.

The effect of changes of the reinforcement fibre concentration c and

the level of the initial deformation incompatibility Δ_0 on the form of the $(\sigma-\varepsilon)$ -diagrams of a test composite is illustrated in Figs. 3-5, which present the results of solution of the problem posed for $\dot{\varepsilon}=10^3 \text{ s}^{-1}$ (concentration increases in each subsequent Figure: $c=0.25; 0.5; 0.75$; curves 1-5 in each Figure correspond to $\Delta_0=0.01; 0.005; 0; -0.005, -0.01$). The peculiarity of the $(\sigma-\varepsilon)$ -diagrams is the presence of three typical regions, two of which show the linear elastic deformation of the composite as a solid and the linear elastic deformation of the matrix, the third one represents the plastic deformation of the composite as a solid. These peculiarities are also observed on the experimental $(\sigma-\varepsilon)$ -diagrams. The calculations show also the essential effect of the primary incompatibility of strains on qualitative relation of the $(\sigma-\varepsilon)$ -diagram regions. It is interesting to note that the value of the composite yield point obtained as a stationary point of the system of ordinary differential equations for the thin rod problem is equal to an "average" value of the yield points of composite component materials.

VII - CONCLUSION

The paper presents the formulation of a new approach to construction of models describing behaviour of composites under intense dynamic loading. For description, the viscoelastic Maxwell-like body model is used, that was applied successfully earlier to solve the problems of dynamic deformation of homogeneous media. To calculate the averaged composite characteristics, a new method has been proposed, that doesn't require a hypothesis of composite structure periodicity. The formulations of models of two-phase composites of two types (the fibrous unidirectional composite and the laminated one) have been presented. A comparison of experimental and calculation characteristics of a test composite enables us to apply the models proposed to solution of the dynamic deformation problems and to predict properties of the composite on the basis of information about its components.

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