DYNAMIC RUPTURE OF THIN-WALLED CYLINDRICAL SHELLS

A. G. IVANOV

*All-Union Scientific Research Institute of Experimental
Physics, 607200 Arzamas, Nizhni Novgorod region, USSR*

There are some specific features in the rupture behavior of material under dynamic loading. Generally, the short times in which the loads are applied, make it impossible to identify weaknesses and follow the evolution of rupture. The process is still more complicated because of inertial forces involved therein. This rupture may be illustrated, for example, by that of a momentum-accelerated thin-wall ring of plastic material as it expands radially. A lot of papers have been concerned with the solution of this problem aimed to find the rupture strain (or time) and the spectrum (or characteristic size) of fragments. References on this issue are included in papers [1,2]. In recent years, two trends in phenomenological description of this event have distinguished themselves, both based on the energy balance. The papers of the first trend claim, that the work of ring rupture into fragments is done by inertial forces, the alternative trend considering elastic ones.

Some papers by American researchers Grady, Kipp and Hightower [2,3,4] are related with the first trend. Underlying in this set of papers is the one by Grady [5]. Ref. [2] proves the equations of motion and shows the continuity of the solution obtained. Thus, assuming the work of rupture is zero,

the solution found in [2] will transform to the known Mott's solution [6].

The alternative trend has been developed in the papers by the author and his associates [7,8,9] and also by French researchers Olive and Stelly and their colleagues [10,11]. These papers consider elastic forces as dominating.

Let us address ourselves to the analysis of research works on ring ruptures, which have been carried out in terms of inertial force effects.

In ref. [2], the Dugdale-Barenblatt opening crack concept is taken as an approximation mechanism of dynamic rupture of a plastic material. The force acting between the crack edges (T_f) is defined as

$$T_f = T_y (1 - x/x_0) \quad 0 < x \leq x_0, \quad T_f = 0 \quad x > x_0 \quad (1)$$

(T_y is tensile force, x_0 - critical crack opening displacement).

Circumferential extension of a ring when it is moving radially, apparently much resembles that of a bar which ends move in opposite directions at velocity V_0 . Therefore the original equations and final results were obtained for this bar model and extended to the expanding ring case without any foundation. The rupture was assumed to develop in the middle of the bar ($x_0=0$). The problem being symmetric, a half of the bar from $x=0$ to $x=l_0$ was considered. At $x=0$, force $T_y - T_f$ is existing and the boundary velocity $v=0$, and at $x=l_0$ there is tensile force T_y and $v=v_0$. It was claimed, that as the rupture was developing, there formed a region of elastic material in the neighborhood, which was moving as a whole

at the velocity of elastic-plastic boundary particles.

From [6], the equation of motion for this boundary was found from the equality of the bar momentum and the acting force changes. The resulting solution provided formulations for the rupture time (t_0) and strain ($\epsilon_f = \dot{\epsilon} t_0$, where $\dot{\epsilon} = v_0/l_0$) while the double value of the unloaded region by the time of rupture determined the characteristic length of a fragment (x_f).

$$t_0 = 2T_y^{-1} (g \rho \gamma^2 / \dot{\epsilon})^{1/3} \quad (2)$$

$$x_f = [24 \gamma / (\rho \dot{\epsilon}^2)]^{1/3} \quad (3)$$

where ρ , γ and $\dot{\epsilon}$ are weight of the bar unit length, specific work of material tear-off and strain rate, respectively.

It is of interest to ascertain also what source of energy can drive the bar rupture to fragments. Assume, that a bar of l_0 length, with its ends moving at velocities 0 and v_0 , is mentally cut into n equal parts (fig.1). Using the law of conservation of momentum, one would clearly see, that this action makes the bar kinetic energy redistribute. There would be a marked increase in the kinetic energy of the fragments' centers of mass (c.m.) due to sharp fall of that inside the fragments.

Thus, the kinetic energy related to a fragment c.m. is

$$A_n = 2 \int_0^{l_0/2n} 0.5 \rho v^2 dx = M v_0^2 / (24 n^3), \quad (4)$$

$$(v = v_0 x / l_0, M = \rho l_0)$$

while in continuous bar ($n=1$) this is $A_1 = Mv_0^2/24$

The energy which has converted into the c.m. kinetic energy of motion is

$$\Delta A = A_1 - nA_n = (1 - n^{-2})A_1$$

When the stored kinetic energy has been found by formula (4), which may be spent on the rupture, n value can be derived from the equality of E_n and the work of rupture γ . With n known, define x_f value just as l_0/n .

$$x_f = [24\gamma/(\rho\dot{\epsilon}^2)]^{1/3} \quad (5)$$

The x_f value found agrees with formula (4) (This also agrees with both (6) in [4] and (12) in [12]). This is no extraordinary, because x_f value is only determined by the extension of the region where kinetic energy respective to c.m. of fragments has been taken. By formula (5); x_f is independent of both the form and the very quantity of T_f . Neither does it depend on elastic characteristics of material, i.e.

T_y and Young's modulus (E). Therefore, if the bar is of liquid, then x_f value would conserve according to (9).

How suitable is it in considering inertial forces to extend the results obtained from the analysis of unidimensional straight motion, to the motion of cylindrically expanding ring? Clearly, every material point of the ring is involved only in radial motion, its energy being $M_1 v_0^2/2$, where M_1 is ring mass. Is there also any kinetic energy resulting from the velocity gradient along the circumference? By analogy with bar, if A_1 is formally calculated for any arbitrary

point on the ring, then it will be found to be 3.3 times as much as radial motion kinetic energy. Moreover, when the ring is cut into n parts, this would have no effect on its fragment motions, energy value $\Delta A = (1 - n^{-2})A_1$ merely vanishing. There is energy imperfection. The following conclusions can be drawn from the above statements:

a) In considering inertial forces, the results obtained for bar can not be extended to ring.

b) When ring is moving radially, the existing inertial forces are due to only radial motion. Therefore, it would be incorrect to assume that the work of ring fragmentation is done by inertial forces.

Consider the papers of the second trend, which are focused on elastic energy. Thus, in ref. [7], it was assumed without any restrictions posed on ring rupture mechanism, that the work of fracture was due to elastic tension energy. It was supposed, that as the ring starts moving its fracture would begin in some section of this. The required elastic energy for rupture would be taken from the section neighborhood at the unstressing wave velocity. The balance of energy and work of rupture was written as

$$\int_0^V q dV = \lambda S \quad (6)$$

where q and λ are the specific (elastic) energy and work of rupture, and S and V are the ring section and unstressed volume of the section neighborhood. The material was assumed viscoelastoplastic.

Straining equation was taken in the form

$$\sigma = \sigma_0 + \eta \dot{\epsilon} \quad (7)$$

where σ_0 is static flow stress, η - viscosity. Combined solution of (6) and (7) yields a function of the form

$F(\epsilon_f, \dot{\epsilon}, \lambda, \mu)$ which for $\epsilon_f \ll 1$ simplifies to

$$\epsilon_f = \dot{\epsilon} \lambda / (1 + \dot{\epsilon} \mu)^2 \quad (8)$$

$$(\lambda = 4E\lambda / (3c\sigma_0^2), \mu = \eta / \sigma_0)$$

Function F and relation (8) yield maximum value when strength (σ_0) and viscous ($\eta \dot{\epsilon}$) terms are equal in eq. (7).

This approach was successfully extended in refs. [11, 10] to describe the ruptures of radially expanding spheric shells. In ref. [8], the expression for fragments number was found for

$$\epsilon_f \ll 1 \quad n = 3\pi (R_0 \sigma_0 + v_0 \eta)^2 (4R_0 E \lambda)^{-1}$$

A more general equation of straining, than (12), was used in ref. [8]. There the volume swept by unstressing was determined in terms of circumferential material displacement with R increase. Moreover, it was observed, that for liquids $\sigma_0 = 0$ and elastic energy results from only viscous forces ($\eta \dot{\epsilon}$). Therefore, $\epsilon_f(\dot{\epsilon})$ should not be a rising function.

Refer to an experiment, as mentioned above, both approaches under consideration are based on the balance of energy needed for rupture and work of forces (inertial or elastic) capable of this. The results of experimental studies are given in refs. [2-4], [7-11], which are claimed by the authors to support both of the approaches. However, the available experimental data are insufficient to bring the issue to a close, since there are too many simplifying assumptions in the above-menti-

oned papers (such as $T_y = \text{const}$, γ (or λ) = const, $\eta = \text{const}$ etc.). Yet now there are good reasons to favour the approach based on elastic forces. These are as follows

1. For viscoelastoplastic materials, this solution predicts the occurrence of plasticity peak in ring or stripe under biaxial strain [13]. This peak was observed experimentally for soft steels in papers [7,8], its occurrence for steels and aluminum-titanium alloys being pointed out in book [14]. It is noted in [11], that relation $\epsilon_f(\dot{\epsilon})$ having a maximum may account for the falling sections of function $\epsilon_f(\dot{\epsilon})$ for a number of alloys. To illustrate this, figure 2, as taken from [14], shows $\epsilon_f(\dot{\epsilon})$ relations.

2. As shown above, when ring is expanding radially, there are no inertial forces directed circumferentially. These may occur only during the rupture of real ring material into fragments due to their elastic unstressing.

Thus, it is elastic strain energy which is considered a driving factor in the dynamic rupture of material in its deeply-plastic region, as well as in the static fracture.

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n	I_0	$A_{\text{ср}}/A_{\Sigma}$	nA_n/A_{Σ}	A_{Σ}
1	$Mv_0/2$	75%	25%	$Mv_0^2/6$
4	$Mv_0/2$	98,4%	1,6%	$Mv_0^2/6$

Fig.1. Schematic representation of kinetic energy redistribution when bar is cut into n parts ($n=4$).

Fig.2. Unit elongation as a function of strain rate.

1 - steel IX18H9T, 2 - annealed steel 40,
3 - annealed alloy D-16.