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S. Bielawski and D. Derozier

Laboratoire de Spectroscopie Hertzienne, associé au C.N.R.S.
Université des Sciences et Techniques de Lille Flandres-Artois, F-59655 Villeneuve d'Ascq Cedex, France

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Abstract. — The Nd-doped fiber laser can either operate in the continuous-wave regime or display undamped spiking. In the first case, most of the phenomena may be interpreted in the framework of model of a two-mode laser, in which each mode is associated with one polarization eigenstate. In the situation for which the output becomes spontaneously periodic or chaotic, we present the observed regimes and optical spectra. Then, we propose and check experimentally a feedback technique for suppressing these oscillations.

1. Introduction

In addition to their interest in connection with their application in telecommunication networks as amplifiers, doped fibers are often used as active media in laser systems. Because of the broad gain profile and the laser cavity length, these lasers are strongly multimode. An other particularity of these kinds of laser lies generally in the absence of polarization selective elements in the cavity. Therefore, there exist additional degrees of freedom that can give rise to dynamical effects in which the polarization state of the light changes in addition to its amplitude and its phase. The two polarization eigenstates (states which replicate after a round trip in the cavity) depend on the birefringence of the fiber, but are always linear and orthogonal on the coupling mirrors. They appear to be the relevant basis for describing the laser dynamics. These kinds of laser can operate either in the c.w. [1] or self-pulsing mode [2]. Different hypotheses have been proposed [3] to explain these instabilities like the existence of ion pairs (or clusters) [4] distributed within the fiber, which act as a saturable absorber [5]. However, in spite of their fundamental interest, these effects are generally undesirable and must be suppressed.

The system considered here is the Nd$^{3+}$ doped fiber laser which exhibits the c.w. and self-pulsing behaviors for different adjustments of the cavity. In this paper, we first recall a phenomenological model taking polarization into account [1]. Then, after evaluating its param-

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eters from experiments in the transient regime, we check its validity in two situations. When
the laser operates in the c.w. regime, we compare the experimental and predicted signals ob-
tained with a strong pump modulation. Then, we present the experimental results in a situa-
tion for which the model fails, namely the self-spiking regime. In this latter situation — de-
spite the lack of knowledge of the instability origin — we show that it is possible to suppress
the self-spiking with a simple feedback technique.

2. Experimental Set-Up

The experimental set-up is schematically shown in Figure 1. The active medium of the opti-
cal fiber laser (OFL) is a 5 m long silica fiber doped with 300 ppm Nd$^{3+}$. The core diameter
is 5.8 $\mu$m and the fiber is single mode at the laser operating wavelength ($\lambda = 1.08 \mu$m) but
the two transverse modes LP$_{01}$ and LP$_{11}$ may propagate at the pump wavelength (cut-off
wavelength : 1 $\mu$m). The excitation of the LP$_{11}$ pump mode reduces the pumping efficiency of
the laser because of the poor spatial overlap between this mode and the lasing mode [6].
However, the dynamics of such a transverse single mode laser is not expected to be qualita-
tively affected by the spatial structure of the pump field [7]. The laser cavity is limited by
two dichroic mirrors transparent at the pump wavelength and with reflection coefficients
$R_1 > 99.5\%$, $R_2 = 95\%$ around 1.08 $\mu$m. The former is coupled to the fiber with an index
matching liquid in order to decrease the losses, and the output mirror is simply butt-coupled
to the fiber. The Fabry-Perot effect resulting from the small gap between the fiber and output
mirror allows the tuning of the laser emission around 1.08 $\mu$m. The pump radiation, emitted
at 810 nm by a polarized single mode laser diode (SDL 5400), is focused into the fiber
through two X10 microscope objectives. Between the two objectives, a half wave plate al-
ows us to vary the pump polarization. The two orthogonal polarizations are separated by a
polarizing beam splitter, a half wave plate is used to select the direction of analysis. The laser
intensities are monitored by silicium photodiodes with a bandpass filter ($\Delta \lambda = 50$ nm) to

![Experimental arrangement for a diode pumped monomode fiber laser. Notation used: LD: laser diode, MO: microscope objectives (NG 10), HWP$_1$: half-wave plate (810 nm), CM: coupling mirror ($R > 99.5\%$ @ 1 080 nm, $T_{\text{max}}$ @ 810 nm), DF: Nd-doped fiber, OC: output coupler (95%), HWP$_2$: half-wave plate (1 080 nm), PC: polarizing cube, F: band-pass filter (1 060-1 100 nm), BS: beam splitter, PD: silicon photodiodes.](image)
block the stray pump laser radiation. A beam splitter is used after the polarization selection to send a part of the laser intensity into a monochromator for spectral analysis. The rotation of the half-wave plate permits to calibrate the two detectors and at the same time to analyse the spectrum on the two orthogonal polarizations. When the pump power $P$ is twice the threshold value $P_{\text{th}}$ (i.e. $A = P/P_{\text{th}} = 2$), the laser is strongly multimode with a wide spectrum (70 cm$^{-1}$) which may be tuned around 1.08 $\mu$m by adjusting the mirror positions because of Fabry-Pérot effect in small gaps between the mirrors and the fiber ends.

3. The C.W. behavior of the OFL

a) Modelisation of the OFL. — The strongly multimode nature of the emission together with the large number of transitions that are involved in the laser operation and thus the great number of dynamical variables and laser parameters make the theoretical approach of the OFLs dynamics difficult. Up to now, most of the models have been designed in order to understand the behavior of the OFL in the c.w. regime and to optimize the performances of this kind of laser. These models are based on the rate-equations approximation and allow to study in particular the importance of the spatial effects (mode overlap) [6], the output saturation [8], and the problems due to excited-state absorption [9]. A comparison of model predictions with the observed experimental behavior of an OFL was first performed by Hanna et al. [10]. They showed a good description of the dependence of the relaxation frequency versus the pump power in the framework of the monomode 2-level class-B model. More recently, Le Flohic et al. [11] studied the transient buildup of emission in the Nd$^{3+}$ OFL and compared their results with a 2-level multimode class-B model including amplified spontaneous emission.

Despite the strongly multimode nature of the laser emission, the experimental results reported in the next sections suggest that our OFL behaves essentially as a two-mode laser in which each mode is associated with a polarization eigenstate. Therefore, we will not consider the many longitudinal modes but rather consider the laser as made up of two subsets corresponding to two clusters of longitudinal modes, one in each of the polarization eigenstates. Because of the short relaxation time of the coherences compared to the photon lifetime and the relaxation times of the levels involved in the lasing action, the atomic polarizations may be adiabatically eliminated and we consider rate-equations as in the models of OFLs proposed previously. All the deexcitation times except the lifetime of the upper level of the lasing transition are extremely short compared to the photon lifetime. Therefore, the only populations involved in the laser dynamics are those of the upper level of the laser transition and of the fundamental level. Moreover, the latter can be considered to be almost constant because the available pump power does not produce any bleaching at the pump wavelength. Spontaneous emission should be taken into account in the model because of the wide numerical aperture of the fiber. The photons emitted by this process have a significant probability to be trapped in the LP$_{01}$ mode. Therefore, a non negligible part of these photons are injected in the lasing modes and contributes to the laser dynamics. This effect is also typical of other guided lasers as the semiconductor lasers [12]. The pumping terms may be either equal or different for the two polarizations. To check the pumping anisotropy, we have realized a series of experiments in which the pump polarization is rotated. The polarization eigenstates of the laser remain the same in accordance with our interpretation according to which they are determined by the residual birefringence of the cavity. On the opposite the intensity sharing between the two states of polarization changes. This polarization selective gain occurs because both the interactions between the polarized pump and the ions, and that between the ions and the fields along each direction of polarization depend on the local field experienced by the ions. Therefore, the pumping terms corresponding to the two laser subsystems need to
be taken different in the model. The losses are taken equal for the both linear polarizations. We will consider that all the anisotropies are taken into account by the pumping asymmetry introduced in the model. The two systems are coupled by two kinds of terms, first, a cross gain effect: the intensity of the mode 1 (resp. 2) is amplified by its corresponding population inversion \( d_1 \) (resp. \( d_2 \)) but also to a smaller amount by the population inversion of the other mode \( d_2 \) (resp. \( d_1 \)). Second, the stimulated emission causes a saturation of each population by its corresponding intensity (self saturation), but also by the other one (cross saturation). The two cross-saturation coupling coefficients are taken equal. The equations of such a laser can be written in the following adimensional form [1]:

\[
\begin{align*}
\dot{m}_i &= (d_i + \beta d_j - 1) m_i + a(d_i + \beta d_j) \\
\dot{d}_i &= \gamma[d_j^0(\tau) - (1 + m_i + \beta m_j) d_i]
\end{align*}
\]

(1)

with \( i = 1 \) or \( 2 \) and \( j = 3 - i \)

\( m_i \) and \( d_i \) are the reduced intensities and population inversions of the two laser systems. The dots represent the derivatives with respect to a reduced time \( \tau = t/\tau_c \), and \( \tau_c \) is the photon lifetime in the cavity. \( d_i^0(\tau) \) are the pumping terms of the two laser subsystems \( (d_i^0(\tau) = d_i^0(1 + r \cos \Omega \tau)) \) if the pump amplitude is sinusooidally modulated. The experimental situation corresponds to a fixed value of \( \alpha = d_2^0/d_1^0 \) determining the asymmetry of the system as discussed above. Without loss of generality, we assume that \( \alpha \leq 1 \). \( \gamma = (\tau_c/\tau_i) \) with \( \tau_i \) the population inversion relaxation time. Spontaneous emission is considered through the coefficient \( a \) which includes the spontaneous emission probability and the waveguiding effect. \( \beta \) is the cross-saturation coefficient describing how each laser field is coupled with the population inversion of the other laser system. For sake of simplicity, the cross-coupling is considered to be the same for spontaneous and stimulated emissions.

This phenomenological model shares many common points with those of coupled lasers [13, 14] and simple models of multimode class-B lasers [11, 15, 16]. We must also note that the multimode nature of the laser could be described by a more complete model including spatial hole-burning, similar to the model of Tang, Statz and De Mars [17], but we have preferred to choose a phenomenological model containing a minimum number of dynamical variables (four) and of adjustable parameters of the laser (\( \alpha, \beta, \gamma, a \)).

b) CHARACTERISTICS OF THE FIBER LASER. — We have investigated the evolution of the output power and its state of polarization versus the pump power. Above a first threshold \( P_{\text{th1}} \), and up to a second value \( P_{\text{th2}} \), the laser radiation is linearly polarized. In that range, the laser power linearly increases with the pump power and the emitted spectrum broadens as \( A \) increases (Fig. 2). We may also note that a weak intensity (\(< 5\%\)) is detected along the other polarization, but it is only due to defects in the polarization optics. The direction of polarization of the laser emission for pump powers between \( P_{\text{th1}} \) and \( P_{\text{th2}} \), can be modified by a changing the strain applied to the fiber. However, it appears to be independent of the polarization state of the pump. This indicates that the polarization eigendirections are imposed by the birefringence of the fiber, as expected in the case of small gain anisotropy. When the pump power is increased above \( P_{\text{th2}} \), laser emission also occurs in the polarization direction perpendicular to that observed previously. The spectral range of this emission is very narrow just beyond \( P_{\text{th2}} \) and it broadens as the pump power increases. Simultaneously to the appearance of this second polarization component with intensity \( I_2 \), the output power versus pump power characteristics of the first polarization component \( I_1 \) exhibits a sudden decrease of the slope (Fig. 2) but these characteristics remain linear. Above the second threshold \( P_{\text{th2}} \), the pump power feeds two compet-
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Fig. 2. — C.W. characteristics of the OFL. Between the first and the second thresholds $P_{th1}$ and $P_{th2}$, the laser emits linearly polarized radiation. Above the second threshold, it emits in both polarization directions.

...ing processes, resulting in less efficiency for the first polarization component. The value of the pump parameter $A$ at second threshold $P_{th2}/P_{th1}$ is typically 1-1.2 and may be adjusted by acting, e.g. on the mirror position or on the fiber winding. In particular, $P_{th2}$ can be set equal to $P_{th1}$ and this situation will be exploited later. The existence of two thresholds and the nature of the output versus pump power characteristics are obtained by the model presented in the previous section but the experimental informations are not sufficient to determine all the adjustable parameters of the model. So, we have recourse to the transient response of the OFL to find them.

c) POLARIZATION RESOLVED INVESTIGATION OF THE TRANSIENT RESPONSE OF THE OFL. — The transient following a small perturbation of the pump power provides additional information on the physics of the fiber laser. We consider here a small step modulation of the pump parameter in a situation where the laser remains always above the first threshold $P_{th1}$, i.e. $A > 1$. For $1 \leq A \leq P_{th2}/P_{th1}$, the response to a small perturbation of the pump is a simple damped oscillation with a single frequency and it corresponds to the standard relaxation oscillation of a class B laser. In our OFL, it typically occurs in the 0-50 kHz range. Hereafter, we will refer to these oscillations as relaxation oscillations. Above the second threshold ($A > P_{th2}/P_{th1}$) the laser response is made of the superposition of two damped oscillations with both different oscillation frequencies and damping coefficients. The fast one corresponds to the relaxation oscillations described above and the slower one is associated with a new relaxation mode that
appeared at the second threshold. It will be called the low frequency since its frequency is typically 0-0.4 times the relaxation frequency. The physical variables, i.e. the total intensity and the intensity in each polarization eigenstate are differently affected by the relaxation and the low frequency oscillations. This is shown in Figure 3 which displays the response of the laser in the two orthogonal polarizations corresponding to the laser eigenstates (Fig. 3a and b), together with the total intensity (Fig. 3c). Note that the relaxation oscillations are in phase in Figures 3a and b. A careful examination of the corresponding signals show that their low frequency oscillations are in opposite phase. As their amplitude is almost equal they destructively interfere in the total output intensity which displays only the high frequency relaxation oscillations (Fig. 3c). The apparition of a slow mode which does not affect the total intensity seems to be a general property of two-mode lasers in which the two modes are coupled via cross-saturation terms [14]. These relaxation oscillations provides us with two kinds of quantitative data: the oscillation frequencies and the damping coefficients. Their evolution versus pump

![Fig. 3. — Transient response of the OFL after a small perturbation of the pump intensity (the perturbation is applied between the two dashed lines): (a) and (b) intensities $I_1$ and $I_2$ in the two orthogonal polarization eigenstates and (c) total intensity.](image-url)
power gives complementary informations about the laser system that will give some insight on its dynamics. As mentioned in Section 2, the laser may oscillate in the symmetric configuration in which the thresholds are the same for the two polarizations states. This situation is interesting as it makes easier the comparison with the predictions of the model elaborated for the OFL since analytical calculations are readily available in the symmetric situation [1]. To provide information for the comparison with theory, a series of experiments has been carried out in the symmetric situation. The corresponding measurements of the relaxation frequencies are reported in Figure 4. The square of the frequency of the relaxation (resp. low frequency) oscillations $\omega_r$ (resp. $\omega_{\omega}$) varies linearly with the pump power (Fig. 4b), as predicted by a simple rate equation model of the two-level laser [18]. In the symmetric case studied here they both scale as $(P - P_{th})$ since both thresholds coincide. In the more general case of two different thresholds the low frequency oscillation $\omega_{\omega}$ scales as $(P - P_{th2})^{1/2}$, a general property of

![Graph](image)

**Fig. 4.** — (a) damping of the relaxation oscillations $\Gamma_r$, versus pump power, (b) frequency of the relaxation oscillations $\omega_r$ and of the low frequency oscillations $\omega_{\omega}$ versus pump power. Experiments have been performed in the symmetric case. The full line is the least-square fit of $\Gamma_r$ with the value obtained from the stability analysis of (1). This fit leads to the determination of $a (2.8 \times 10^{-4})$ and $\tau_r (350 \mu s)$. The dashed straight lines are the fits with $\omega_{\omega}$ and the asymptot of $\Gamma_r$. This fit allows to evaluate $\alpha (0.86)$, $\beta (0.43)$ and $\gamma (0.67 \times 10^{-3})$. Above threshold, this latter coincides with the curve $\Gamma_r(A)$ corresponding to $a = 0$. 

two-mode lasers [14]. The damping rate $\Gamma_r$ of the fast relaxation oscillations strongly increases near the first threshold $\Lambda = 1$ (Fig. 4a). This evolution can be explained by the introduction of spontaneous emission in the model. Spontaneous emission is known to give significant contributions to the dynamics of some guided lasers like semiconductor lasers [12] or OFLs [11].

The parameters of the model corresponding to the experimental situation have been determined from the results obtained in these regimes. The variations of the eigenvalues of the model versus the pump power have been fitted (with the classical least squares procedure) with the experimental results in order to evaluate the parameters $\alpha$, $\beta$, and $\gamma$. These fits lead also to estimation of the upper level relaxation time $\tau_r$. We have obtained lifetimes of 350 to 420 $\mu$s close to the actual value (460 $\mu$s in ref. [19]). Therefore we can consider that the model introduced here is able to describe precisely the linear dynamics of the OFL. The case of strong modulation cannot be treated analytically and we have proceeded to numerical simulations to investigate to what extent the model is able to describe the OFL in such conditions.

d) POLARIZATION RESOLVED INVESTIGATION OF THE RESPONSE TO SINUSOIDAL MODULATION. — As shown in the previous section, the transient response of the laser after a small perturbation of the pump exhibits two components with eigenfrequencies $\omega_r$ and $\omega_{tr}$. The former corresponds to the fast relaxation oscillation of the total laser intensity and the latter to the slow motion which is revealed by polarization resolved experiments. These frequencies usually lie in the 0-50 kHz range and modulation of the pump parameter at such frequencies is easily achieved by varying the current injected in the pump laser diode. Under high pump modulation, the laser exhibits several nonlinear effects including hysteresis and a period doubling cascade leading to chaos, crises, generalized bistability [1]. In this section, we investigate the correlation between the radiation emitted in the polarization states defined in the preceding section when the pump power is modulated at a frequency close to that of the fast relaxation $\omega_r$. Figure 5 illustrates the evolution of the response of the laser on the two orthogonal polarization directions, i.e. $I_1$ and $I_2$ versus the frequency of the pump modulation. The modulation amplitude is kept constant and the instantaneous pump power remains always above threshold. Measurements made simultaneously in the two orthogonal polarizations clearly indicate antiphase dynamics in the two polarizations. In the case of a $2T$-periodic response (Fig. 5b), the maximum output intensity in one polarization direction corresponds to a small peak in the other one. The same phenomenon is also observed in the chaotic regime (Fig. 5c): the large peaks in one polarization are associated with small peaks in the other one. The antiphase phenomenon, which is observed between the two total output intensities in the two orthogonal polarization, originates from the strong competition between two coupled laser systems. Similar antiphase motions were observed in other nonlinear systems, including lasers [13, 20]. The calculated variations of the intensities display the same spiking behavior and antiphase phenomena (Fig. 6) as the experimental signals and chaos occurs with modulation amplitudes comparable with the experimental values. To investigate the chaotic regimes, the reconstruction of the attractors and Poincaré sections (or projections of these sections) have been used. Polarization resolved experiments present the advantage of providing measurements of two dynamical variables, namely the intensities $I_1$ and $I_2$ in each polarization eigenstate. A projection of a Poincaré section of the attractor on a two-dimensional plane is readily obtained by combining the sampling technique with polarization resolution. An oscilloscope fed in the $XY$ mode with the sampled values of the intensities $I_1$ and $I_2$ in the two polarization directions displays in real time a projection of a Poincaré section taken in a plane of constant phase of the modulation. The corresponding sections at different points of the inverse cascade C2, and in the fully chaotic regime near the 2C-C transition and just before the boundary crisis are shown in
Fig. 5. — Experimental evidence of antiphase response of the OFL in different dynamical regimes. The two series of curves are related to the intensity in each polarization eigenstate $I_1$ (lower traces) and $I_2$ (upper traces). (a) $T$ response, (b) $2T$ response, (c) chaotic response.

Figures 7a, 7b and 7c respectively. As expected, the Poincaré sections for the C2 regimes appear as 2 clusters of dots periodically visited while they span a wide region of the plane beyond the end of the inverse cascade. These Poincaré sections have special shapes, they are given as a fingerprint of the chaotic attractor to be compared with the corresponding curves given by numerical simulations. The comparison of the Poincaré sections obtained experimentally with those given by numerical simulations shows that even the detailed structure of the reconstructed attractors (Fig. 7) is the same.

e) CONCLUSION. — When the fiber laser operates in the c.w. regime, most of the experimental findings can be reproduced by the model I taking both spontaneous emission and polarization effects into account. In particular the agreement between the reconstructed attractors and their numerical counterparts is excellent. It should be stressed that this was obtained in spite of the drastic approximations which have been introduced in this model. For instance the restriction of a multimode laser to a two-mode system leads to some discrepancies when a quantitative
prediction is aimed but this can be corrected by including suitably corrected parameters. The quality of the agreement on, e.g. the details of the structure of the chaotic attractors indicates that the proposed model catches the key ingredients of the OFL dynamics.

4. The self-pulsing regime

a) DESCRIPTION. — In the absence of any modulation, the Nd-doped fiber laser can either operate in the continuous-wave regime or display undamped spiking, depending on the tilting of the cavity mirrors. We must remark first that these spiking regimes are out of the range of validity of the model 1. Indeed, for fixed parameters, its stable solutions can be only stationary states. Such instabilities are also observed in other systems as the laser with saturable absorber [5], the Tm and Er-doped fiber lasers [21]. Although for the latter, the origin of the instabilities
could be attributed to the existence of ion pairs inside the amplifying medium [4], in the case of the Nd-doped laser, the underlying physical origins remain unclear. The aim of these studies is twofold. The first one is to characterize these unstable regimes, and the second one is to elaborate a general method for stabilizing lasers displaying undamped spiking.

Usually, when the pump power is increased from zero, the instability occurs in the following way. At low pump powers, the laser operates in the continuous-wave regime and one can observe the two thresholds associated with the two polarization eigenstates. When the pump parameter is increased beyond a third threshold $A_{III}$, a Hopf bifurcation occurs: the output powers $I_1$ and $I_2$ oscillate in phase, and just above $A_{III}$, the laser oscillates with a small amplitude at the relaxation frequency $\omega_r$. Increasing further the pump parameter leads to a subharmonic bifurcation, which leads to a 2T-periodic regime with $I_1(t)$ and $I_2(t)$ in antiphase. The signals are very similar to those observed in the modulated laser. Then, we can observe either a transition to chaos through a period-doubling cascade (Fig. 8) or via quasiperiodicity, depending on the twisting of the fiber and the mirror tilting. When these instabilities appear, we can also note a strong modification of the optical spectrum. In Figure 9 are displayed the
Fig. 8. — Bifurcation diagram of the fiber laser operating in the self-pulsing regime. A region of stable continuous-wave regime not represented in this diagram exists for $3.7 \text{ mW} < P < 5.4 \text{ mW}$.

Fig. 9. — Optical spectrum of the laser (a) below and (b) above the Hopf bifurcation. The pump powers are equal to 5.3 and 6.3 mW respectively, and the Hopf bifurcation occurs for $P = P_H = 5.4 \text{ mW}$. 
spectra averaged over many periods (the acquisition time is of the order of 1 ms). Their shapes become more regular and smooth as the pump power increases, unlike in the case of stable emission. This suggests that the observed instability may involve the multimode nature of the laser.

b) STABILIZATION OF THE LASER. — In order to avoid the instabilities, two ways may be used. The more usual one consists in strongly changing the parameters of the system, for instance by decreasing the pump power, or by changing the glass composition in the case of the Er-doped fiber laser [4]. The strategy which we propose here is to use a feedback technique, in order to achieve stabilization by applying only small perturbations. The method relies on the fact that above the Hopf bifurcation, the steady state still exists but is unstable. As a main consequence, any shift from the steady state will lead to a trajectory which diverges and tends to the attractor of the system (e.g. a periodic or chaotic state). However, the existence of such unstable states allows to control the laser, by stabilizing it with a method close to the one of Ott, Grebogi and Yorke for controlling chaos [22].

For a fixed point characterized by a single positive eigenvalue — e.g. in the case of the unstable branch of a bistable system — one can show that a feedback proportional to the difference between the actual state and the fixed point can stabilize the system [23]. However, in the case of the fiber laser, the linear stability analysis shows that this kind of feedback \( A(t) = A_0 + g(I(t) - I_F) \) with \( I_F \) the intensity associated to the unstable state) does not allow stabilization. A more efficient control scheme [1, 24] is achieved by using the derivative of the output intensity: \( A(t) = A_0 + gI(t) \).

Experimentally, for building the feedback signal, we can use either the intensity emitted along one polarization eigenstate or the total intensity. The detected signal is sent to the stabilization system which contains a simple analog derivator, followed by an inverting amplifier with an adjustable gain. The output of the feedback system generates a current which is added to the constant current injected in the pump diode. In experiments, the stabilization of the unstable stationary state is very easily obtained by increasing simply the feedback gain above a threshold which depends on the operating conditions of the laser. When the laser is stabilized, the correction tends to zero value with extremely small fluctuations (\(< 1\%\)) only due to technical noise. Therefore, we can conclude that the stabilized state is identical to the unstable steady state \( X_F \) of the system without feedback, and only the stability of the fixed point is changed. The value of the feedback coefficient can be chosen in a wide range. In fact, increasing the feedback gain \( g \) to the maximum possible value of our system (more than one order of magnitude larger than the threshold value \( g_c \)) does not suppress stabilization. Practically, stabilization is achieved simply by fixing an arbitrary large value for the feedback gain. The transition from a chaotic behavior in the absence of the external feedback, to the stabilized steady state by switching on the control feedback is displayed in Figure 10. The dependence of the output intensity of the stabilized laser on the pump power is reported in Figure 11. It shows that the instabilities that appear above the Hopf bifurcation threshold of the laser \( (P_H = 3.7 \text{ mW}, \text{ in the conditions of Fig. 11}) \) are suppressed by the application of the feedback in the whole range of accessible pump power.

The feedback method can also provide a characterization of the unstable stationary state. Between the laser threshold and the Hopf bifurcation, the stationary state is stable and a small temporal perturbation of the pump power is followed by a damped relaxation oscillation. On the other hand, above the Hopf bifurcation, when the control feedback is desactivated the relaxation oscillations exponentially increase. It has been possible to study the dependence of the oscillation frequency and the divergence rate \( \lambda \) of the relaxation oscillations
Fig. 10. — Typical transient observed after the activation of the feedback, starting from a chaotic regime. (a) Pump power (its value are electronically limited). The maximum amplitude is 10% of the mean value (6 mW). (b) Laser power. After stabilization on the previously unstable state, the correction signal vanishes into the noise. The arrow indicates the time when the correction is switched on.

Fig. 11. — Dependence of the output power of the stabilized laser when the pump power is swept in the whole accessible range. The Hopf bifurcation is observed for $P = 3.7$ mW. The dashed curves represent the maxima and minima of the output power when the feedback is switched off.
on the pump power. As shown in Figure 12, the dependence of $\lambda$ with the pump pumping rate is similar as previously near threshold, but differs significantly at larger pump powers.

The stabilization technique also allows to find precisely the location of the fixed point in the phase space, by providing the measure of the intensities associated to each mode. Experimentally, this is simply achieved by observing the spectrum of the stabilized laser. Significant differences appear between the spectrum of unstable laser (Fig. 13a) which presents a smooth shape, and the spectrum of the stabilized laser (Fig. 13b) which contains isolated peaks. This confirms that the appearance of the instability involves multimode effects, and asks an important question regarding the behavior of the spectrum during the destabilization. Further experiments are in progress in order to know if during this transient all the modes evolve in phase or if a nontrivial spectrototemporal dynamics occurs, e.g. a displacement of the spectrum as in the case of dye lasers [25].

5. Conclusion

The nonlinear dynamics of the Nd-doped fiber laser has been analysed under various conditions, and appears to be dominated by polarization effects. In the c.w. regime, in spite of the multimode nature of the emission, a simple model of two coupled lasers reproduces well the experimental findings. It reveals a collective behavior of the modes associated to the two polarization eigenstates, but the reason for which each cluster of modes acts as one mode is not yet well understood and deserves further studies. The Nd-doped fiber laser can also display undamped spiking which cannot be reproduced by the model presented in Section 3. In this situation which leads to various periodic and chaotic regimes, we have observed a qualitative change in the optical spectrum. It suggests that multimode effects may be involved, however further investigations are needed to identify the origin of the instabilities. The spiking regimes have been suppressed by using a simple feedback technique. In addition to its practical interest, this technique gives access to the properties of unstable states of the system which are usually difficult to obtain. Therefore it offers new comparison points for checking models of unstable fibre lasers. We can also note that similar methods can be used for stabi-
The generality of the used method and the fact that it does not require the knowledge of the system equations should allow one to stabilize other systems such as the Er and Tm-doped fiber lasers [2].

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