Application of a cavity perturbation method to the measurement of the complex microwave impedance of thin super- or normal conducting films

G. Schaumburg, H. Helberg

To cite this version:

G. Schaumburg, H. Helberg. Application of a cavity perturbation method to the measurement of the complex microwave impedance of thin super- or normal conducting films. Journal de Physique III, EDP Sciences, 1994, 4 (5), pp.917-927. <10.1051/jp3:1994174>. <jpa-00249155>

HAL Id: jpa-00249155
https://hal.archives-ouvertes.fr/jpa-00249155
Submitted on 1 Jan 1994

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.
Application of a cavity perturbation method to the measurement of the complex microwave impedance of thin super- or normal conducting films

G. Schaumburg and H. W. Helberg

III. Physikalisches Institut, Universität Göttingen, Bürgerstraße 42-44, D-37073 Göttingen, Germany

(Received 9 July 1993, revised 3 January 1994, accepted 23 February 1994)

Abstract. — We present the application of a cavity perturbation method on thin super- or normal conducting films. The sample is placed in the center of the cavity in the maximum of the electric field. We have calculated the complex microwave impedance of the films with a new approach from the measured data. The method allows to determine the complex impedance of films with arbitrary thickness. In particular, films with thickness $d$ small compared to the skin depth $\delta$ or the London penetration depth $\lambda$ can be measured. Therefore, the impedance of superconducting films can be measured both in the normal and superconducting state. The method is an important completion to the conventional measurements of the surface resistance which replace one of the cavity walls by the film. This arrangement is problematic for films with $d \approx \lambda$ or $\delta$ because in this case the field transmits through the film out of the cavity.

1. Introduction.

In this paper, we have to distinguish between two impedances: the well-known surface impedance $Z_s$ and the less known film impedance $Z_f$. Both describe a material with equal intrinsic properties but for different sample geometries. $Z_s$ gives the ratio of the electric ($E_{s\parallel}$) and magnetic ($H_{s\parallel}$) field strength on the surface of an infinite half plane [1]

$$Z_s = R_s + iX_s = \frac{E_{s\parallel}}{H_{s\parallel}} = \sqrt{\frac{i\mu_0 \omega}{\sigma' - i\sigma''}}. \quad (1)$$

($\mu_0$ vacuum permeability, $\omega$ angular frequency, $\sigma = \sigma' - i\sigma''$ conductivity). $R_s$ represents the surface resistance and $X_s$ the surface reactance. In this arrangement, the field penetrates with an exponential decrease from the surface into the sample. The depth where the field has dropped to $1/e$ is the skin depth $\delta$ for metals and the London penetration depth $\lambda$ for superconductors.
A conventional method to determine $Z_f$ of a thin film is to replace one of the walls of a microwave cavity by the film e.g. [2-10]. The surface impedance $Z_s$ is calculated from the shift of the center frequency and the shift of the quality factor of the cavity. As the field exponentially decreases, (1) becomes invalid if the film thickness $d < \lambda$ or $\delta$. Moreover, problems arise in this case, because the field transmits through the film out of the cavity [2]. On the other hand, if $d < \lambda$ or $\delta$ the field inside the film is nearly constant. In this situation, the film can be described by the film impedance [11]

$$Z_f = R_f + iX_f = \frac{E_1}{H_{11} - H_{22}} = \frac{1}{(\sigma' - i\sigma'')d}. \quad (2)$$

$R_f$ represents the film resistance and $X_f$ the film reactance. $E_1$ describes the electric field inside the film and $H_{11}$, $H_{22}$ are the magnetic fields at the front and the back side of the film, respectively.

In this paper, we report a method to measure the complex film- and surface impedance of thin films. We use a cavity perturbation method which we have applied for the first time to superconducting thin films and we show that this is a sensitive technique to measure the impedance. Furthermore, a new method is deduced to calculate the impedance from the complex resonant frequency shift for films with arbitrary thickness. All calculations in this and the following chapters are restricted to the so called London limit where $\lambda$ (or $\delta$) is large compared with the coherence length $\xi_0$ (or the electron mean free path $\ell$) and local electrodynamics applies.


We use the conventional cavity perturbation method [12-18]. Figure 1 schematically shows the arrangement of the sample inside the cavity. The experimental arrangement is discussed in detail in [19-21]. At a fixed temperature, the center frequency $f_0$ and quality factor $Q_0$ of the empty cavity are detected. Then, the sample is placed inside the cavity in the maximum of the electric and the antinode of the magnetic field. The center frequency $f_p$ and quality factor $Q_p$ of the loaded cavity are measured. The procedure is repeated for all other temperatures of measurement.

![Fig. 1. — Sketch of the microwave cavity with sample position.](image-url)
A conventional method to determine the quality factor $Q$ and the center frequency $f$ is to sample the cavity power transmission function over the frequency range of the resonance. Then, the theoretical transmission function [20] (Lorentz function) is fitted through the measured points with a non linear square fit algorithm. Finally, $f$ and $Q$ are determined from the fit function. For this purpose, one usually uses a sweep oscillator and a storage oscilloscope or a network analyser. As the power half width $\Delta f$ ($\Delta f/f \approx 10^{-4}$) of the transmission function is quite narrow, the accuracy of the sweep oscillator and the measurement devices both in the frequency and the power domain has to be extremely high. Due to the dynamic sweep of the frequency, the practical implementation of this procedure results in either insufficient accuracy or very complex (and expensive) technical solutions.

To avoid both of these drawbacks, we have developed a new precision technique for the $Q$ and $f$ measurement which we describe in detail in [19, 20]. In contrast to the above described sweep, the power transmission is sampled with only few points at fixed frequencies. Because of the static (at fixed frequency) measurement, high precision devices (synthesizer oscillators in the frequency domain and time integrating power meters in the power domain) can be utilized. Therefore, we are able to measure each single point with extraordinary high precision (see example at the end of this chapter).

The electrical part of the measurement arrangement is simple and sketched in figure 2. It consists of a microwave synthesizer oscillator and two devices (power diodes) to measure the input $P_{in}$ and transmitted $P_{out}$ power of the cavity. The insulators and resistors suppress undesirable resonances in the wave guides. The E-H tuners are used to create a flat power transmission spectrum from the oscillator to the power diodes. The frequency of the oscillator and the power meters are digitally controlled by a computer. At the start of each measurement cycle, the computer runs an algorithm which approximately determines the position and the width of the cavity resonance by systematically sampling the power transmission $P_{out}/P_{in}(f)$. Then, some equidistant points within the range of the double power half width of the resonance are measured. These points are used for the fit to determine $f$ and $Q$ with high precision. It should be noticed that the accuracy of this procedure is independent of the exact position of the measured points on the frequency axis.

![Figure 2](image_url)

Fig. 2. — Sketch of the electrical part of the measurement arrangement.
An example with 11 measured points is plotted in figure 3. For not too low $Q$ values ($Q \geq 1000$) we achieve absolute errors of $10^{-7}$ $f$ for $f$ and $10^{-3}$ $Q$ for $Q$ at frequencies between 4 and 23 GHz. The lowest limit of $Q$ measurement is about 150 (see chapter 6). These values may be enhanced by using measurement devices with higher precision (e.g. oscillator with better frequency stability and lock in techniques for the power measurement).

![Graph](image)

Fig. 3. — Measured power transmission and fitted Lorentz function.

3. Calculation of the film impedance in the quasi static regime.

At first, we define the complex frequency shift

$$\Delta f = \Delta f' + i \Delta f'' = \frac{f_p - f_0}{f_p} + \frac{i}{2} \left( \frac{1}{Q_p} - \frac{1}{Q_0} \right).$$

The perturbation equation [12-18] for a sample placed in the antinode of the magnetic field

$$\Delta f = -(\varepsilon - 1) \frac{\int_{V_p} E_0(x) \cdot E_p(x, \varepsilon) \, dx}{2 \int_{V_c} |E_0(x)|^2 \, dx}$$

connects the measured $\Delta f$ with the complex dielectric constant $\varepsilon' - i \varepsilon''$ of the sample. $E_0(x)$ and $E_p(x)$ describe the electric field strengths inside the empty and the loaded cavity, respectively. $V_p$ and $V_c$ are the volumes of the sample and the cavity, respectively. Equation (3) is problematic because it contains the field in the sample $E_p(x)$ which itself depends on the unknown $\varepsilon$. For this reason, one usually approximates the sample by a threefold ellipsoid. If $d \ll \lambda$ or $\delta$, the field in the film is nearly homogeneous (quasi static approximation; signed by index s) [22]

$$E_{ps} = \frac{1}{1 + N (\varepsilon_s' - 1 - i \varepsilon_s'')} E_0.$$  

(4)
$N$ is the depolarization factor [22-24] of the film. Connecting (3) and (4), we get the well-known e.g. [25-32] equation [33] to calculate $\varepsilon_s$ ($\varepsilon$ in the quasi static approximation) from cavity perturbation measurements

$$
\Delta f = \frac{-(\varepsilon_s - 1)}{1 + N(\varepsilon_s - 1)} \int_{V_c} |E_0(x)|^2 \, dx = \frac{-(\varepsilon_s - 1)}{1 + N(\varepsilon_s - 1)} \alpha .
$$

(5)

The fraction of the integrals reduces to the volume factor $\alpha = \alpha_0 V_p/V_c$. $\alpha_0 (=2)$ is a constant depending on the field geometry in the empty cavity [19, 21]. If we isolate $1/(\varepsilon_s - 1)$ in (5) and use the conductivity,

$$
\sigma = i \omega \varepsilon_0 (\varepsilon - 1) ,
$$

(6)

($\varepsilon_0$ vacuum permittivity) the quantity $1/\sigma_s \sim Z_{fs}$ can be calculated

$$
Z_{fs} = R_{fs} + iX_{fs} = \frac{1}{\sigma_s} = \frac{i \alpha}{\varepsilon_0 \omega d} \left( \frac{N}{\alpha} + \frac{1}{\Delta f} \right)
$$

(7)

or

$$
R_{fs} = \frac{\sigma_s'}{(\sigma_s'^2 + \sigma_s''^2)} d = \frac{\alpha}{\varepsilon_0 \omega d} \frac{\Delta f''}{\Delta f'^2 + \Delta f''^2}
$$

(8)

$$
X_{fs} = \frac{\sigma_s''}{(\sigma_s'^2 + \sigma_s''^2)} d = \frac{\alpha}{\varepsilon_0 \omega d} \left( \frac{\Delta f'}{\Delta f'^2 + \Delta f''^2} + \frac{N}{\alpha} \right).
$$

(9)

Although, (8) and (9) are valid in the whole quasi static range there are two interesting limits.

3.1 LOW DEPOLARIZATION. — In this limit is $N |\varepsilon_s - 1| \ll 1$. Then, the field inside the sample is identical with the field outside the sample, equation (4). From (5) we get

$$
\sigma_s = - \frac{i \varepsilon_0 \omega}{\alpha} \Delta f
$$

or

$$
Z_{fs} = \frac{i \alpha}{\varepsilon_0 \omega d} \frac{1}{\Delta f}
$$

which is independent of the depolarization factor. The complex conductivity is directly proportional to $\Delta f$.

3.2 HIGH DEPOLARIZATION. — In this case is

$$
N |\varepsilon_s - 1| \gg 1 .
$$

(10)

The field inside the sample is much lower than outside and decreases with $iZ_{fs}$

$$
E_{ps} \approx \frac{1}{N (\varepsilon_s - 1)} E_0 = \frac{i \varepsilon_0 \omega d}{N} Z_{fs} E_0 .
$$

(11)

Connecting (10) with (5) finally yields

$$
\Delta f' \approx - \alpha/N \quad \text{and} \quad |\Delta f'| \gg \Delta f''
$$

(12)
The frequency shift $\Delta f'$ largely depends on the sample dimensions. From (7) and (12) follows for the film impedance

$$Z_{fs} = -\frac{N^2}{\alpha \varepsilon_0 \omega d} \left( \Delta f + \frac{\alpha}{N} \right).$$

As two terms of approximately the same magnitude are subtracted, $X_f$ in the limit of high depolarization only can be determined with the exception of the constant $\alpha/N$. Moreover, $N$ limits the sensitivity of the measurement. For high sensitivity, low depolarization factors are required.

It should be noted that the dependence of the film impedance on $\Delta f$ may change between the two limits $Z_{fs} \sim i/\Delta f$ (3.1) and $Z_{fs} \sim -i(\Delta f + \alpha/N)$ (3.2) due to depolarization effects.


If we define the film impedance as $Z_f = 1/\sigma d$, it becomes independent of the actual field distribution in the sample, thus allowing us to calculate $Z_f$ also for films with arbitrary thickness. The problem will be solved if we are able to calculate $\varepsilon' - i\varepsilon''$. Of course, for films with $d > \lambda$ or $\delta$, the decrease of the field inside the sample has to be taken into account. This problem has been solved by others for samples with shape of a rotary ellipsoid and metallic conductivity ($\varepsilon'' \gg |\varepsilon'|$). Some of these solutions only apply in the so called skin effect range, where $r > \delta$ (radius of the ellipsoid) [34-36]. An exact solution for arbitrary conductivity is described in [37] but requires extensive numerical calculations. In order to get a more general solution for both metals and superconductors and for non ellipsoid shaped samples, we use the following approximation. We have applied it to other geometries, too, and described it in detail in [19, 20].

To determine the real $\varepsilon$, we correct $\varepsilon_s$ with the field distribution in the sample neglecting edge effects. We write the perturbation equation (3) both for $\varepsilon_s$ and $\varepsilon$. Connecting the two equations yields the correction equation

$$\varepsilon - 1 = (\varepsilon_s - 1) \xi(\varepsilon)$$

$$\xi(\varepsilon) = \frac{E_{ps} V_p}{\int_{V_p} E_p(x) \, dx^3}$$

$E_{ps}$ represents the homogeneous depolarization field from (4), which corresponds on the sample border to the location dependent real field $E_p$ in the sample. Equation (15) only holds if the field direction in the sample coincides with that of the field outside the sample. As depolarization effects are already considered in $\varepsilon_s$, we are allowed to calculate the field distribution in the presented case for an infinite sheet of thickness $d$. It is given by [38] (see Fig. 4):

$$E_p(x) = E_{ps} \frac{\cosh(ikx)}{\cosh(ikd/2)} \quad k = \frac{\omega}{c} \sqrt{\varepsilon' - i\varepsilon''}$$

($c$ speed of light).

From (15) we get

$$\xi = \frac{E_{ps} d}{\int_{-d/2}^{d/2} E_{ps} \frac{\cosh(ikx)}{\cosh(ikd/2)} \, dx} = \frac{ikd}{2} \cotanh \left( \frac{ikd}{2} \right).$$
Fig. 4. — Field distribution in a thin sheet.

Using the approximation \(|\varepsilon| \gg 1\), we get from (14)

\[ \varepsilon = \varepsilon_s \xi(\varepsilon) \]

or

\[ \left( \frac{ikd}{2} \right) = \left( \frac{ik_s d}{2} \right)^2 \cotanh \left( \frac{ikd}{2} \right) \] (16)

\((k_s = \omega \sqrt{\varepsilon_s / \varepsilon_c})\). Equation (16) is connected by (2) and (6) to the real and static film impedance by

\[ \left( \frac{ik_s d}{2} \right)^2 = i \frac{\omega \mu_0 d}{4 Z_{fs}} \quad \text{and} \quad Z_t = i \frac{\omega \mu_0 d}{4} \left( \frac{ikd}{2} \right)^2 \] (17)

To determine \(Z_t\) from the measured \(\Delta f\), we calculate \(Z_{fs}\) by (7), then \(ik_s d/2\) by (17), then \(ikd/2\) by (16), and at last \(Z_t\) by (17).

There are two limits depending on the film thickness.

4.1 THIN FILMS \((d \ll \lambda \text{ or } \delta)\). — The field in the sample is homogeneous (quasi static limit) and \(|kd/2| \ll 1\). Then \(\cotanh (ikd/2) = (ikd/2)^{-1}\) and

\[ Z_t = Z_{fs}. \]

4.2 THICK FILMS \((d \gg \lambda \text{ or } \delta)\). — The field in the sample rapidly decreases with increasing distance from the border and is far away from the quasi static limit. It is \(\cotanh (ikd/2) = 1\) and \(\xi = ikd/2\). Equation (16) becomes to

\[ Z_t = \frac{(2 Z_{fs})^2}{i \omega \mu_0 d}. \] (18)

5. Calculation of the surface impedance.

If we define the surface impedance in (1) only by the complex conductivity it becomes independent of the field distribution in the sample. Then the surface impedance \(Z_s\) may be calculated from the film impedance \(Z_t\) by connecting (1) and (2)

\[ Z_s = \sqrt{\omega \mu_0 d} Z_t. \] (19)

If this is compared with the film impedance in the limit for thick films ((18) non quasi static...
\[ Z_s = 2 \frac{Z_{fs}}{Z_{fs}}. \]  

(20)

(The index \( s \) in \( Z_s \) represents surface impedance, not the quasi static range.)

For thick films width \( d \gg \lambda \) or \( \delta \), the static film impedance, which is connected to the measured data in simple fashion, becomes equal to the half surface impedance. All equations in chapter 4 remain valid, if \( Z_{fs} \) is replaced by \( Z_s/2 \). In practice, samples which are in the limit (20) are nearly exclusively found in the range of high depolarization. In this case, the field \( E_p \) on the sample surface decreases with \( iZ_s \) (see (11)) and we get the dependence \( Z_s \sim \frac{\Delta f' + \alpha/N}{i} \) (see (13)). A similar equation is developed from other foundations in [3, 4, 39]. In the case of low depolarization the surface impedance is \( Z_s \sim i/\Delta f \).

6. Practical remarks and measurement examples.

For copper cavities, the complex frequency shift \( \Delta f \) is measurable in the following ranges. The lower value both of \( |\Delta f'| \) and \( \Delta f'' \) is \( 10^{-7} \), caused by the final resolution of the quality and frequency measurement. This limits can be lowered, if superconducting cavities are used. The upper value of \( \Delta f'' \) is limited to \( 5 \times 10^{-3} \) because of frequency dependence of the waveguides, resulting in a non Lorentzian shaped resonance function at low quality factors. The upper value of \( |\Delta f'| \) should not become much larger then \( 10^{-2} \). Otherwise, the cavity perturbation equation (3) may become invalid [17].

The limits for \( Z_{fs} \) depend on the electrical state of the film.

6.1 Low depolarization.

\[ \Delta f \sim V_p (\varepsilon_s - 1) \sim V_p (Z_{fs} d). \]

The sensitivity of \( \Delta f \) increases with the volume \( V_p \) of the film. For a 50 nm thick film with an area of \( 4 \times 0.2 \) mm\(^2\) at 10 GHz a typical measurement range for \( |Z_{fs}| \) is \( 10^2 \ldots 10^7 \) \( \Omega \). These values may be shifted by a factor 10, if special cavity arrangements are used. Mostly, thin films are grown on a substrate which gives a significant contribution to \( \Delta f \). If the substrate is in the range of low depolarization too, the fields in the film and the substrate are equal to the outer field in the cavity and the contribution of the substrate can be subtracted from the total \( \Delta f \) (see (3)).

6.2 High depolarization.

\[ \Delta f \sim \frac{V_p d}{N^2} \frac{1}{Z_{fs}} \sim \frac{V_p}{N^2} \frac{1}{(\varepsilon_s - 1)} \]

The sensitivity of \( \Delta f \) largely depends on film geometry. For high sensitivity, low depolarization factors \( N \) are required. Therefore the geometry of the film should be as long and slim as possible. As \( N \sim d \), the sensitivity of \( \Delta f \) increases linear with decreasing film thickness \( d \). Therefore, especially extraordinary thin films with thicknesses down to 10 nm can be measured with high precision. A typical range of measurement for the above mentioned film is \( |Z_{fs}| \approx 10^{-3} \ldots 10^2 \) \( \Omega \). The contribution of a substrate in the state of low depolarization is smaller as in 6.1, because the field in the substrate is weakened by the depolarization field of the film approximately by a factor \( |N \varepsilon| \) (see (11)). Therefore, substrates with low loss and low dielectric constant can be neglected [40]. Nevertheless, the volume of the substrate should be kept as small as possible.
A measurement example at 10.2 GHz of a 44 nm thick YBa$_2$Cu$_3$O$_{7-\delta}$ superconducting film on a MgO substrate is shown in figure 5. The superconducting transition is observed at 86 K both in $\Delta f'$ and $\Delta f''$. The results are discussed in detail in [40].

![Figure 5](image_url)

Fig. 5. — Measured quality factors and center frequencies of an YBa$_2$Cu$_3$O$_{7-\delta}$ thin film with 44 nm thickness on a MgO substrate.

Other examples are the measurement of the conductivity in thin crystal needles of organic conductors, e.g. [41], or in thin films of reticulate doped polymers [42, 43]. Due to (12), the linear thermal expansion of needlelike samples can be measured [44]. Also the filling volume of sintered materials can be determined [45].

7. Conclusion.

We have shown that the cavity perturbation method is a sensitive technique to measure the impedance of thin films with arbitrary thickness and deduced a new method to calculate the impedance from the measured data. If $d < \lambda$ or $\delta$, the films can be described by the static film impedance $Z_f$. In the limit $d \gg \lambda$ or $\delta$, we have shown that $Z_f$ becomes equal to the half surface impedance $Z_s$. Moreover we have presented a new precision technique for the measurement of the center frequency and the quality factor of microwave cavities.

References


