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Flutter control of incompressible flow turbomachine blade rows by splitter blades

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Abstract. — Splitter blades as a passive flutter control technique are investigated by developing a mathematical model to predict the stability of an aerodynamically loaded splitted-rotor operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The two-dimensional oscillating cascade unsteady aerodynamics, including steady loading effects, are determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode flutter of both uniformly spaced tuned rotors and detuned rotors are predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model. This model is then utilized to demonstrate that incorporating splitters into unstable rotor configurations results in stable splitted-rotor configurations.

Nomenclature.

- $b$ airfoil semichord, $C/2$
- $\bar{C}_p$ steady surface static pressure, $P/\frac{1}{2} \rho U_\infty^2$
- $C_p$ unsteady pressure distribution, $P'/\rho U_\infty^2 \bar{\alpha}$
- $[CM]^n$ motion-induced influence coefficient of airfoil $n$
- $\Delta C_p$ unsteady pressure difference, $\Delta P'/\rho U_\infty^2 \bar{\alpha}$
- $I$ mass moment of inertia
- $K$ linear spring constant
- $k$ reduced frequency, $\omega b/U_\infty$
- $P'$ unsteady pressure
- $Q$ complete flow field
- $Q_0$ steady mean flow
- $Q'$ unsteady flow
- $r_s$ dimensionless radius of gyration
- $S$ airfoil spacing
Flutter is well-known to have a negative impact on the development of advanced design axial flow rotors. As a result, two research techniques have been proposed to eliminate flutter from the operating envelope-structural detuning and aerodynamic detuning.

Structural detuning, defined as designed blade-to-blade differences in the natural frequencies of a blade row, has been considered for passive aeroelastic control. Several studies of the effect of structural detuning on rotors operating in supersonic flow fields with subsonic axial components have shown that structural detuning has beneficial effects on flutter. Crawley and Hall [1] have shown that alternate frequency structural detuning is nearly optimal for stabilizing torsional flutter. However, structural detuning is not a universally accepted passive stability control mechanism because of the associated manufacturing, material, inventory, engine maintenance, control and cost problems.

Aerodynamic detuning is a relatively new concept for passive aeroelastic control. It is defined as designed blade-to-blade differences in the unsteady aerodynamic flow field of a blade row. Thus, aerodynamic detuning creates blade-to-blade differences in the unsteady aerodynamic forces and moments, thereby affecting the fundamental driving force. This results in the blades not responding in a classical traveling wave mode typical of a conventional uniformly spaced aerodynamically tuned rotor. Studies of rotors operating in both incompressible flow fields [2] and supersonic flow fields with subsonic axial components [3, 4] have shown that aerodynamic detuning is beneficial to flutter stability.

Splitter blades introduce combined aerodynamic-structural detuning into a rotor. The aerodynamic detuning results from the differences in the principal blade passage unsteady aerodynamics due to the splitters, with the structural detuning achieved through the higher natural frequencies of the splitter blades as compared to the full chord principal rotor blades. Hence, the incorporation of splitters into a rotor may not only result in improved aerodynamic
performance, but also in increased flutter stability, i.e., the combined aerodynamic-structural detuning resulting from the splitter blades may take advantage of the enhanced flutter stability due to both aerodynamic and structural detuning while eliminating the difficulties associated with structural detuning.

Splittered rotors may thus be able to be designed for safe operation in regions of the performance map wherein an unsplitted rotor would encounter flutter. In this regard, it should be noted that splitter vanes are routinely used for improved performance in high pressure ratio centrifugal impellers, with their application to axial rotors in the research stage [5-9].

For a supersonic axial flow with a subsonic axial component, splitters have been shown to be a viable passive control technique for flutter and forced response [10, 11]. However, these studies are based on classical linear theory, with the steady flow assumed to be uniform and the principal blades and splitters modeled as flat plates. Thus, the effects on the steady flow field of incorporating the splitters into the rotor as well as the effect of the resulting steady flow field on stability were not considered.

This paper is directed at investigating the effect on stability of incorporating splitter blades into a rotor. This involves the investigation of the viability of splitters as a passive torsion mode flutter control technique of an aerodynamically loaded rotor operating in an incompressible flow field. Thus, this research significantly extends the previous modeling and understanding of the fluid dynamics and aeroelasticity of splitted rotors. This is accomplished by developing a mathematical model and utilizing it to demonstrate the enhanced torsion mode stability associated with incorporating splitters into a loaded rotor.

A complete first order unsteady aerodynamic model is developed, i.e., the thin airfoil approximation is not utilized, to analyze the oscillating airfoil motion induced aerodynamics of both conventional uniformly spaced rotors without splitters and rotors incorporating variably spaced splitters between principal blades, including the effects of steady aerodynamic loading. An unsteady aerodynamic influence coefficient technique is then utilized, thereby enabling the stability of both conventional rotors and Splittered rotors to be determined.

The analysis of the torsional mode stability characteristics of a rotor requires the prediction of the unsteady pressure resulting from the harmonic torsional motions of the cascade. The steady and unsteady aerodynamics acting on the typical two-dimensional airfoil sections of a blade row are determined by considering: (1) a single principal blade passage with periodic boundary conditions for the conventional tuned uniformly spaced rotor; and (2) two principal blade passages with two passage periodic boundary conditions for the aerodynamically detuned rotor. The flow field is assumed to be linearly comprised of a steady potential mean flow and a harmonic unsteady flow field. The steady and unsteady potential flow fields are individually described by Laplace equations, with both the steady and unsteady potentials further decomposed into circulatory and noncirculatory components. The steady flow field is independent of the unsteady flow. However, the unsteady flow is coupled to the steady flow field through the unsteady boundary conditions on the airfoil surfaces.

A locally analytical solution is developed in which the discrete algebraic equations which represent the flow field equations are obtained from analytical solution in individual grid elements. A body fitted computational grist is utilized. General analytical solutions to the transformed Laplace equations are developed by applying these solutions to individual grid elements, with the complete flow field then obtained by assembling these locally analytical solutions.

The locally analytic method for steady two-dimensional fluid flow and heat transfer problems was initially developed by Chen et al. [12, 13]. They have shown that the locally analytical method has several advantages over the finite-difference and finite-element
methods. For example, it is less dependent on grid size and the system of algebraic equations is relatively stable. Also, since the solution is analytical, it is differentiable and is a continuous function in the solution domain. One disadvantage is that a great deal of mathematical analysis is required before programming.

Mathematical model. Tuned cascade.

Figure 1 presents a schematic representation of a thick, cambered airfoil cascade at finite mean incidence $\alpha_0$ to the farfield uniform mean flow, executing torsion mode oscillations. The complete flow field $Q(x, y, t)$, is assumed to be comprised of a steady mean flow and an harmonic unsteady flow field, equation (1). The unsteady flow field $Q$ corresponds to the motion-induced unsteady flow field.

$$Q(x, y, t) = Q_0(x, y) + Q'(x, y) \exp[ik_1 t].$$  \hspace{1cm} (1)

![Fig. 1. — Cascade and flow geometry.](image)

STEADY FLOW FIELD. — For the steady flow of an incompressible, inviscid fluid, a velocity potential function can be defined. The complete flow field is then described by the Laplace equation.

$$\nabla^2 \Phi_0(x, y) = 0$$  \hspace{1cm} (2)

where $Q_0(x, y) = \nabla \Phi_0(x, y)$.

Since the Laplace equation is linear, the velocity potential can be decomposed into components by the superposition principle. In particular, the steady potential is decomposed into noncirculatory and circulatory components, $\Phi_{NC}(x, y)$ and $\Phi_C(x, y)$.

$$\Phi_0(x, y) = \Phi_{NC}(x, y) + \Gamma \Phi_C(x, y)$$ \hspace{1cm} (3)

where $\nabla^2 \Phi_{NC} = 0$; $\nabla^2 \Phi_C = 0$; and $\Gamma$ is the unknown steady flow circulation constant.

To complete the steady flow mathematical model, farfield inlet, farfield exit, airfoil surface, wake dividing streamline, and cascade periodic boundary conditions must be specified.
The steady farfield inlet flow is uniform (Eq. (4)), with the mass flow rate specified by the farfield exit boundary conditions (Eq. (5)). Also, a zero normal velocity is specified on the airfoil surfaces.

\[
\begin{align*}
\Phi_{\text{NC}} \mid \text{farfield inlet} & = U_\infty x \\
\Phi_C \mid \text{farfield inlet} & = 0 \\
\frac{\partial \Phi_{\text{NC}}}{\partial n} \mid \text{farfield exit} & = U_\infty \cos (\alpha_0 + \theta) \\
\frac{\partial \Phi_C}{\partial n} \mid \text{farfield exit} & = 0
\end{align*}
\] (4a), (4b), (5a), (5b)

where \( n \) is the surface unit normal.

The steady velocity potential is discontinuous along the airfoil wake dividing streamline. This discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuity in the circulatory velocity potential equal to the steady circulation \( \Gamma \). The Kutta condition is also applied, thereby enabling the steady circulation constant to be determined. It is satisfied by requiring the chordwise velocity components on the upper and lower airfoil surfaces to be equal in magnitude at the airfoil trailing edge. In addition, the cascade periodic steady velocity potential boundary conditions are satisfied by requiring both the normal and chordwise velocity components to be continuous between the upper and lower periodic boundaries.

**Unsteady Flow Field. —** The unsteady potential component \( \Phi' \) is also described by a Laplace equation. The solution is determined by decomposing this unsteady potential component into circulatory and noncirculatory components, \( \Phi_C'(x, y) \) and \( \Phi_{\text{NC}}'(x, y) \), each of which is individually described by a Laplace equation

\[
\begin{align*}
\Phi' & = \Phi_{\text{NC}}' + \Gamma' \Phi_C' \\
\nabla^2 \Phi_C' & = 0 ; \quad \nabla^2 \Phi_{\text{NC}}' = 0
\end{align*}
\] (6a), (6b)

where \( \Gamma' \) is the unsteady flow circulation constant.

The farfield inlet unsteady velocity potential boundary conditions (Eq. (7)), are obtained by using a Fourier series to satisfy the periodicity condition at the far upstream [14]. The farfield exit unsteady noncirculatory potential boundary condition (Eq. (8a)), is obtained in an analogous manner. Since the wake does not attenuate in the farfield, the unsteady circulatory potential farfield exit boundary condition (Eq. (8b)), is obtained by solving the Laplace equation at the farfield exit and satisfying the blade-to-blade periodicity condition [14]. Also, the airfoil surface boundary conditions specify that the normal velocity of the flow field must be equal to that of the airfoil (Eq. (9)).

\[
\begin{align*}
\Phi_{\text{NC}}' \mid \text{farfield inlet} & = - \left| \frac{S}{\beta_0} \left| \frac{\partial \Phi_{\text{NC}}'}{\partial n} \right| \text{farfield inlet} \\
\Phi_C' \mid \text{farfield inlet} & = - \left| \frac{S}{\beta_0} \left| \frac{\partial \Phi_C'}{\partial n} \right| \text{farfield inlet} \\
\Phi_{\text{NC}}' \mid \text{farfield exit} & = - \left| \frac{S}{\beta_0} \left| \frac{\partial \Phi_{\text{NC}}'}{\partial n} \right| \text{farfield exit}
\end{align*}
\] (7a), (7b), (8a)

where \( \beta_0 \) is the interblade phase angle.
\[ \Phi_C \big|_{\text{farfield exit}} = -\Delta \Phi' e^{-i \lambda} \left( \frac{e^{ikv}}{1 - e^{(k - i \nu) S \cos \delta}} + \frac{e^{-ikv}}{1 - e^{-(k + i \nu) S \cos \delta}} \right) \]  

(8b)

where \( \nu = \frac{\beta_0 + k \sin \delta / S}{S \cos \delta} \) and \( \Delta \Phi' \) is the unsteady velocity potential discontinuity at the farfield exit.

\[ \frac{\partial \Phi'_c}{\partial n} \bigg|_{\text{airfoil}} = 0 \]  

(9a)

\[ \frac{\partial \Phi'_c}{\partial n} \bigg|_{\text{airfoil}} = W'(x, y). \]  

(9b)

The normal velocity boundary condition on the oscillating airfoils \( W'(x, y) \) is a function of both the position of the airfoil and the steady flow field. Thus, it is the boundary condition which couples the unsteady flow field to the steady aerodynamics. For an airfoil cascade executing harmonic torsion mode oscillations about an elastic axis location at \( x_0 \) as measured from the leading edge, the normal velocity on the surfaces of the airfoil is defined in equation (10).

\[ W'(x, y) = \hat{\alpha} \left[ \frac{ik \left[ (x - x_0) + y \frac{\partial f}{\partial x} \right] + U_0 + V_0 \frac{\partial f}{\partial x}}{1 + \left( \frac{\partial f}{\partial x} \right)^2 \right]^{1/2} \right. \]

\[ + \left. \frac{\partial U_0}{\partial y} \left[ (x - x_0) \frac{\partial f}{\partial x} + y \right] - \frac{\partial V_0}{\partial y} \left[ (x - x_0) - y \frac{\partial f}{\partial x} \right] }{1 + \left( \frac{\partial f}{\partial x} \right)^2 \right]^{1/2} \]  

(10)

where \( U_0 = U_0(x, y) \) and \( V_0 = (V_0(x, y)) \) are the steady airfoil surface velocity components, \( f(x) \) denotes the airfoil profile, and \( \hat{\alpha} \) is the amplitude of the torsional oscillations.

The unsteady velocity potential is discontinuous along the airfoil wake dividing streamline. This discontinuity is satisfied with a continuous noncircular and a discontinuous circulatory velocity potential. The unsteady circulatory velocity potential discontinuity is specified by requiring the pressure to be continuous across the wake and then utilizing the unsteady Bernoulli equation to relate the unsteady velocity potential and the pressure. The Kutta condition is also applied to the unsteady flow field, enabling the unsteady circulation constant \( \Gamma' \) to be determined. It is satisfied by requiring no unsteady pressure difference across the airfoil chordline at the trailing edge. In addition, the cascade periodic unsteady potential boundary conditions are satisfied by requiring the normal and chordwise velocity components to be continuous in magnitude and to be discontinuous in phase, with the discontinuity equal to the interblade phase angle \( \beta_0 \) between the upper and lower periodic boundaries.

The unsteady dependent variable of primary interest is the unsteady pressure \( P' \) from which the unsteady aerodynamic moment on the airfoil is calculated. It is determined from the solution for the steady flow field, the unsteady velocity potential, and the unsteady Bernoulli equation. Also, the unsteady airfoil surface boundary conditions were applied on the mean position of the airfoil (Eq. (10)). After transfer to the instantaneou airfoil position [15]:

\[ p' = -\nabla \Phi_0 \cdot \nabla \Phi' - ik \Phi' + \partial U_0/\partial y (-U_0 x + V_0 y) - \partial V_0/\partial y (V_0 x + U_0 y) \]  

(11)

where the last two terms account for the transfer of the pressure value from the mean position of the airfoil to its instantaneous position.

The unsteady aerodynamic moment on the reference airfoil is calculated by integrating the unsteady surface pressure difference across the chordline.
\[ M_R = \int [P'(x - x_0) \, dx + P' \, y \, dy] = \bar{\alpha}_R \, C_{\alpha\alpha} \]  

(12)

where \( \bar{\alpha}_R \) is the amplitude of oscillation of the reference airfoil and \( C_{\alpha\alpha} \) is the conventional torsion mode unsteady aerodynamic moment coefficient.

**Locally Analytical Solution.** — A boundary fitted computation grid generation technique is utilized for the numerical solution [16]. A Poisson type grid solver is used to fit a C-type grid around a reference airfoil in the cascade. This method permits grid points to be specified along the entire boundary of the computational plane.

Laplace equations describe the complete flow field including the unknown velocity potentials \( \Phi_{NC}, \Phi_C, \Phi_{NC}' \), and \( \Phi_C' \), equations (3) and (6). In the transformed \((\xi, \eta)\) coordinate system, the Laplace equation takes on the following nonhomogeneous form:

\[ \frac{\partial^2 \Phi}{\partial \xi^2} + \alpha \frac{\partial^2 \Phi}{\partial \eta^2} - 2 \alpha \beta \frac{\partial \Phi}{\partial \eta} - 2 \gamma \frac{\partial \Phi}{\partial \xi} = F(\xi, \eta) \]  

(13)

where \( \Phi \) is a shorthand method of writing these four velocity potentials in the transformed plane, i.e., \( \Phi \) denotes \( \Phi_{NC}(\xi, \eta), \Phi_C(\xi, \eta), \Phi_{NC}'(\xi, \eta) \) or \( \Phi_C'(\xi, \eta) \); \( F(\xi, \eta) \) contains the cross derivative term \( \partial^2 \Phi / \partial \xi \partial \eta \) and the coefficients \( \alpha, \beta, \) and \( \gamma \), are functions of the transformed coordinates \( \xi \) and \( \eta \) which are treated as constants in each individual grid element.

To obtain the analytical solution to the transformed Laplace equation, it is first rewritten as a homogeneous equation by defining a new dependent variable \( \phi(\xi, \eta) \).

\[ \frac{\partial^2 \phi}{\partial \xi^2} + \alpha \frac{\partial^2 \phi}{\partial \eta^2} - (\gamma^2 + \alpha \beta^2) \phi = 0 \]  

(14)

where

\[ \phi = \phi \exp\{\gamma \xi + \beta \eta\} - \frac{F(\gamma \xi + \beta \eta)}{2(\gamma^2 + \alpha \beta^2)}. \]

The following general solution for \( \phi \) is determined by separation of variables.

\[ \phi(\xi, \eta) = [A_1 \cos(\lambda \xi) + A_2 \sin(\lambda \xi)][B_1 \cos(\mu \eta) + B_2 \sin(\mu \eta)] \]  

(15)

where \( \mu = [(\gamma^2 + \alpha \beta^2 + \lambda^2) / \lambda^2]^{1/2} \) and \( \lambda_1, A_1, A_2, B_1, \) and \( B_2 \) are constants to be determined from the boundary conditions.

Analytical solutions in individual computation grid elements are determined by applying proper boundary conditions to evaluate the unknown constants in the general velocity potential solution specified in equation (15). The solution to the global problem is then determined through the application of the global boundary conditions and the assembly of the locally analytical solutions.

**Aerodynamic Model-Detuned Cascade.** — For the aerodynamically detuned rotor configuration of interest herein, i.e., variable circumferential spacing and chord length, an analogous cascade unsteady aerodynamic model is developed by considering two passages with two passage periodic boundary conditions.

In this model, the rotor will incorporate splitters (short chord airfoils) between each pair of full chord airfoils. As schematically depicted in figure 2, the splitters are not required to have
the same airfoil shape as the full chord airfoils, nor are they restricted to particular circumferential or axial positions between each pair of the full chord airfoils.

There are two distinct passages: (1) a reduced spacing, or increased solidity, passage; and (2) an increased spacing, or reduced solidity, passage. There are also two distinct sets of airfoils, with the two reference airfoils denoted by \( R_0 \) and \( R_1 \). These individual sets of airfoils can be considered as cascades of uniformly spaced airfoils each with twice the spacing of the associated baseline aerodynamically tuned uniformly spaced cascade. The circumferential spacing between these two sets of airfoils, \( S_1 \) and \( S_2 \), is determined by specifying the level of aerodynamic detuning, \( \varepsilon \).

\[
S_{2,1} = (1 \pm \varepsilon) S
\]

where \( S \) is the spacing of the baseline uniformly spaced cascade, and \( S_1 \) and \( S_2 \) denote the spacings of the detuned cascade.

An interblade phase angle for this aerodynamically detuned cascade configuration can be defined. In particular, each set of airfoils is individually assumed to be executing harmonic torsional oscillations with a constant aerodynamically detuned interblade phase angle \( \beta_d \) between adjacent airfoils of each set (Fig. 3). Thus, this detuned cascade interblade phase angle is two times that for the corresponding baseline tuned cascade.

\[
\beta_d = 2 \beta_0
\]
where $\beta_0$ is the tuned baseline cascade interblade phase angle, defined between adjacent airfoils on the rotor.

For a rotor with $N$ uniformly spaced blades, Lane [17] showed that the values of $\beta_0$ must satisfy the following condition.

$$\beta_0 = \frac{2 \pi r}{N}, \quad r = 0, \pm 1, \pm 2, \ldots, \pm N - 1$$

where $\pm$ refers to forward and backward traveling waves, respectively.

**Steady Aerodynamics.** — The steady aerodynamic model developed for the baseline uniformly spaced cascade can be applied directly to the two reference passage model of the detuned cascade with the addition of the steady potential for the detuned cascade, $\Phi_{dNC}(x, y)$, and two circulatory components, one associated with each of the two reference airfoils, $\Phi_{CR_0}(x, y)$ and $\Phi_{CR_1}(x, y)$.

$$\Phi_{d_0}(x, y) = \Phi_{dNC}(x, y) + \Gamma_{R_0} \Phi_{CR_0}(x, y) + \Gamma_{R_1} \Phi_{CR_1}(x, y)$$

where $\nabla^2 \Phi_{dNC} = 0$; $\nabla^2 \Phi_{CR_0} = 0$; $\nabla^2 \Phi_{CR_1} = 0$; and $\Gamma_{R_0}$ and $\Gamma_{R_1}$ are the unknown steady flow circulation constants for the reference airfoils $R_0$ and $R_1$, respectively.

The circulatory component $\Phi_{CR_0}(x, y)$ is discontinuous along the wake dividing streamline of the reference airfoil $R_0$ and is discontinuous along the wake dividing streamline of the reference airfoil $R_1$, while $\Phi_{CR_1}(x, y)$ is continuous along the wake of $R_1$ and continuous along the wake of $R_0$. The steady potential discontinuity is satisfied by a continuous noncirculatory velocity potential, with the discontinuities in the circulatory components $\Phi_{CR_0}$ and $\Phi_{CR_1}$ equal to the steady circulation, $\Gamma_{R_0}$ and $\Gamma_{R_1}$, respectively. Also, the steady circulation constants $\Phi_{R_0}$ and $\Gamma_{R_1}$ are determined by simultaneously applying the Kutta condition to the two reference airfoils.

**Unsteady Aerodynamics.** — The unsteady potential for the detuned cascade $\Phi_{d}^*$ is also decomposed into one noncirculatory $\Phi_{dNC}^*(x, y)$, and two circulatory components, one associated with each of the two reference airfoils, $\Phi_{CR_0}^*(x, y)$ and $\Phi_{CR_1}^*(x, y)$.

$$\Phi_{d}^*(x, y) = \Phi_{dNC}^*(x, y) + \Gamma_{R_0}^* \Phi_{CR_0}^*(x, y) + \Gamma_{R_1}^* \Phi_{CR_1}^*(x, y)$$

where $\Phi_{CR_0}^*(x, y)$, $\Phi_{CR_1}^*(x, y)$, $\Gamma_{R_0}^*$, and $\Gamma_{R_1}^*$ are defined analogous to the corresponding detuned cascade steady aerodynamic quantities.

**Influence Coefficient Technique.** — The unsteady airfoil surface boundary conditions specified in equation (10) require that the cascaded airfoils oscillate with equal amplitudes. Also, the interblade phase angle between adjacent nonuniformly spaced airfoils must be specified. Neither of these requirements is appropriate for the aerodynamically detuned cascade. To overcome these limitations, an unsteady aerodynamic influence coefficient technique is utilized.

The unsteady aerodynamic moment acting on the two reference airfoils is expressed in terms of influence coefficients.

$$M_{R_0, R_1} = (\hat{\alpha}_{R_0} [CM]_{R_0, R_1}^0 + \hat{\alpha}_{R_1} [CM]_{R_0, R_1}^1) e^{i \omega t}$$

where $\hat{\alpha}_{R_0}$ and $\hat{\alpha}_{R_1}$ are the unknown complex oscillatory displacements for the reference airfoils $R_0$ and $R_1$, respectively.
The unsteady aerodynamic moment influence coefficients \([CM]_{R_0, R_1}^0\) and \([CM]_{R_0, R_1}^1\) are determined by analyzing the two reference passages with the unsteady detuned cascade model developed above. \([CM]_{R_0, R_1}^0\) are determined by considering a unit amplitude motion of only the reference airfoil \(R_0\), with the interblade phase angle \(\beta_0\), as shown in figure 3. The influence coefficients \([CM]_{R_0, R_1}^1\) are obtained in an analogous manner but with a unit amplitude motion of only the reference airfoil \(R_1\).

**Aeroelastic model.**

The equations describing the single-degree-of-freedom torsional motion of the two reference airfoils of the aerodynamically detuned cascade are developed by considering the typical airfoils depicted in figure 4. The elastic restoring forces are modeled by linear torsional springs at the elastic axis location, with the inertial properties of the airfoils represented by their mass moments of inertia about the elastic axis. The equations of motion, determined by Lagrange's technique, are

\[
I_{\alpha R_0} \ddot{\alpha}_{R_0} + (1 + 2i g_{\alpha R_0}) I_{\alpha R_0} \omega_{\alpha R_0}^2 \alpha_{R_0} = M_{R_0}
\]

\[
I_{\alpha R_1} \ddot{\alpha}_{R_1} + (1 + 2i g_{\alpha R_1}) I_{\alpha R_1} \omega_{\alpha R_1}^2 \alpha_{R_1} = M_{R_1}
\]

where \(I_{\alpha R_p, R_i}\) are the mass moments of inertia about the elastic axis, \(g_{\alpha R_p}\) and \(g_{\alpha R_i}\) denote the structural damping coefficients for the reference airfoils, and the undamped natural frequencies are \(\omega_{\alpha R_p}^2 = k_{\alpha R_p}/I_{\alpha R_p}\) and \(\omega_{\alpha R_i}^2 = k_{\alpha R_i}/I_{\alpha R_i}\).

![Fig. 4. — Single degree-of-freedom detuned cascade model.](image)

Considering harmonic time dependence of the reference airfoils and utilizing the total unsteady aerodynamic moments defined in equation (22), the equations of motion are written in matrix form.

\[
\begin{bmatrix}
\mu_0 & [CM]_{R_0, R_1}^0 \\
[CM]_{R_0, R_1}^0 & \mu_1
\end{bmatrix}
\begin{bmatrix}
\ddot{\alpha}_{R_0} \\
\ddot{\alpha}_{R_1}
\end{bmatrix} = 0
\]

(23)

where

\[
\mu_0 = \mu_{R_0} r_{\alpha R_0}^2 + [CM]_{R_0, R_1}^0 - (1 + 2i g_{\alpha R_0}) \mu_{R_0} r_{\alpha R_0}^2 \gamma_{\alpha R_0} \gamma
\]

\[
\mu_1 = \mu_{R_1} r_{\alpha R_1}^2 + [CM]_{R_0, R_1}^1 - (1 + 2i g_{\alpha R_1}) \mu_{R_1} r_{\alpha R_1}^2 \gamma_{\alpha R_1} \gamma
\]
\[
\mu \frac{m_{R_0 R_1}}{\pi \rho b^2} = \frac{l_{\alpha R_0 R_1}}{m_{R_0 R_1} b^2} ; \quad r_{\alpha R_0 R_1} = \frac{l_{\alpha R_0 R_1}}{m_{R_0 R_1} b^2} ; \quad \gamma_{\alpha R_0 R_1} = \frac{\alpha_{R_0 R_1}}{\omega_0} ; \quad \gamma = \frac{\omega_0}{\omega^2}
\]

and \(\omega_0 = \) reference frequency.

Equation (23) represents a complex eigenvalue problem, with the eigenvalues being \(\gamma\), the ratio of the reference frequency to the flutter frequency. The flutter stability of both conventional uniformly spaced full chord airfoil cascades and cascades incorporating splitters are specified by the real part of the eigenvalue \(\mu\). When \(\mu\) is negative, the amplitude of the harmonic motion of the airfoils will decay and the cascade is stable.

\[
\frac{i \omega}{\omega_0} = \frac{1}{\sqrt{\gamma}} = \mu + i \nu = \text{Eigenvalue}.
\]  

(24)

The required motion dependent unsteady aerodynamic loading is obtained by specifying the detuned interblade phase angle \(\beta_d\) with each permissible interblade phase angle utilized to find the most unstable mode. The permissible values for \(\beta_d\) are obtained from equations (17) and (18) by setting \(N\) equal to the number of blades on the baseline uniformly spaced tuned rotor being analyzed. Since equation (23) represents two equations, there will be two eigenvalues obtained for each value of \(\beta_d\). These complex eigenvalues are related to the exponent \((i \omega)\) which specifies the harmonic motion of the airfoils.

The effects of structural detuning are included through the frequency terms \(\gamma_{\alpha R_0 R_1}\). These terms represent the ratios of the natural frequency to a specified reference frequency. Defining the reference frequency \(\omega_0\) as the torsion mode natural frequency, a structurally tuned cascade would have \(\gamma_{\alpha R_0 R_1}\) equal to unity. For the case when the cascade is structurally detuned, the values of \(\gamma_{\alpha R_0 R_1}\) are altered.

In this regard, splitters enable combined aerodynamic and structural detuning to be introduced into a rotor. In particular, the natural torsion mode frequency of an airfoil is a function of its chord length, thickness, and span as well as its material properties. As a result, the splitters may have a higher natural frequency than the full chord airfoils.

The vibrational characteristics of the full chord and splitter airfoils are determined by modeling each airfoil as a thin rectangular cross-section cantilevered slender beam. The torsion mode natural frequency, \(\omega_{\alpha}\), is then specified in equation (25).

\[
\omega_{\alpha} = \sqrt{\frac{G}{2 \pi^2 \rho_m \frac{T}{C L}}}
\]  

(25)

where \(G\) is the material modulus of rigidity, \(J\) is the 2nd moment of inertia about the rotation axis, \(L\) is the airfoil span, \(T\) is the airfoil thickness, \(C\) is the chord length, and \(\rho_m\) is the material density.

Two types of splitters are of interest: (1) splitters with the same thickness-to-chord ratio as the full chord airfoils and (2) splitters with the same thickness as the full chord airfoils.

For the first type, the splitters and full chord airfoils have the same natural frequency when their material properties and spans are the same, with the splitters having a higher natural frequency when the material properties are different or the splitter span is smaller than that of the full chord airfoils. Hence, these splitters incorporate either aerodynamic detuning or combined aerodynamic-structural detuning into a rotor design.
For the second case, the splitters have a higher natural frequency than the full chord airfoils as long as the two sets of airfoils have the same material properties and spans, with the possibility of the splitters and full chord airfoils having the same natural frequency if different materials are utilized or the splitters have increased span. Thus, these splitters can also introduce either aerodynamic detuning or combined aerodynamic-structural detuning into the rotor.

Model verification.

To demonstrate the steady flow predictive capability of this model and locally analytical solution, the theoretical cascade initially considered by Gostelow [18] is analyzed. This cascade is characterized by a stagger angle of 37.5°, a solidity of 1.01, and a mean flow incidence angle of 1.0°. The correlation of the predicted chordwise distribution of the airfoil surface static pressure coefficient obtained with the model developed herein with that of Gostelow is shown in figure 5. There is very good correlation between the two analyses.

![Comparison of cascade steady prediction and Gostelow solution.](image)

To verify the unsteady cascade modeling and solution, unsteady predictions are correlated with those from the classical model of Whitehead [19]. In particular, a flat plate airfoil cascade with a solidity of 1.01, and a stagger angle of 37.5° executing harmonic torsion mode oscillations at a reduced frequency of 0.8 with a 180° interblade phase angle is considered. The resulting predictions from the two models for the chordwise distribution of the unsteady surface pressure difference are presented in figure 6. The excellent agreement between the two analyses is clearly seen.

Results.

To take advantage of the enhanced flutter stability associated with combined aerodynamic-structural detuning and aerodynamic detuning alone while eliminating the difficulties
associated with structural detuning, splitter blades are introduced into an unstable rotor design. Two detuned rotor-splitter configurations are considered: (1) alternate blades in a baseline unstable rotor are replaced by splitters, resulting in a twelve bladed rotor with six splitters and six full chord airfoils; (2) splitters are introduced into alternate passages of an unstable baseline rotor, resulting in a twenty-four bladed rotor with twelve splitters and twelve full chord airfoils.

**BASELINE ROTOR CONFIGURATIONS.** — The baseline rotors are uniformly spaced, characterized by a Gostelow cascade geometry [18] with a stagger angle of 40°, a mean flow incidence angle of 24°, and an airfoil mass ratio, μ, and radius of gyration, \( r_g \), of 193.776 and 0.3957, respectively, values typical of the tip region of modern fan blades. The solidity of the baseline twelve bladed rotor is 1.67, with the baseline twenty-four bladed splitted-rotor solidity being 0.83.

**SPLITTERED-ROTOR CONFIGURATIONS.** — Aerodynamic detuning is demonstrated by introducing splitters with the same thickness-to-chord ratio as the full chord airfoils, and thus, the same natural frequencies. The aerodynamic detuning results from both the splitted decreased chord and their circumferential position between adjacent full chord airfoils. Three circumferential splitter locations are considered: 40 %, 50 % and 60 %. Figure 7 shows examples of these splitted-rotor flow geometries and the associated computational grids.

Combined aerodynamic-structural detuning is accomplished with splitters having the same thickness but a different chord length than the full chord airfoils positioned such that the splitter trailing edge is aligned with that of the full chord airfoils.
Aerodynamically detuned 12 bladed splitted-rotor flow geometries and computational grids.

Steady Aerodynamic Performance. — Steady rotor performance is defined by the chordwise distributions of the airfoil surface steady static pressure coefficient. For the baseline and splitted-rotors, these are presented in figures 8 and 9. As seen, the introduction of the splitters into the rotor has a noticeable effect on the steady loading distribution of the full chord airfoils, with this effect being a function of the splitter circumferential position.

Baseline Rotor Stability. — For a midchord elastic axis location and a reduced frequency of 0.8, the torsion stability of the 12 and 24 bladed baseline rotors are shown in figures 10 and 11. The 12 bladed baseline rotor is unstable in traveling wave modes characterized by an interblade phase angle of $-30^\circ$, with the 24 bladed rotor unstable for interblade phase angles of $0^\circ$ and $30^\circ$. Also, a comparison of these baseline stability results shows that the 24 bladed baseline rotor is more unstable than the 12 bladed one.

Splittered-Rotor Stability. — Aerodynamically detuning the unstable baseline rotor by the splitters affects stability, with the splitted-rotor stability a function of the splitter circumferential position, i.e., the aerodynamic detuning is a strong function of the splitter circumferential position, figures 12 through 14 for the 12 bladed rotors and figures 15 through 17 for the 24 bladed rotors. Positioning the splitters at 40% of the full chord airfoil passage spacing results in a stable splitted-rotor configuration. The splitted-rotor is neutrally stable when the splitters are at midpassage, becoming even more unstable than the baseline rotor with the splitters at 60% passage spacing. Also, the aerodynamic detuning results in a separation of the splitter and full chord airfoils, with the splitters always more stable.

Combined aerodynamic-structural detuning due to the splitters has a significant effect on rotor stability. In particular, the higher natural frequency of the splitters results in increased torsion mode stability for all of the detuned splitted-rotors as well as increased splitter and...
Fig. 8. — Steady aerodynamic performance of 12 bladed rotors.
Fig. 9. — Steady aerodynamic performance of 24 bladed rotors.
full chord airfoil separation, with the splitter circumferential position only affecting the stability margin. This stability enhancement is shown in figures 18 and 19 for a midpassage circumferential splitter location.

**Effect of Splitters on $\Delta C_p$ Phase.** — The aerodynamic detuning creates blade-to-blade differences in the unsteady aerodynamics, thereby affecting the fundamental driving force — the unsteady pressure difference across the airfoil chords. This is demonstrated by considering the effect of the aerodynamic and combined aerodynamic-structural detuning introduced into
Fig. 13. — Stability of aerodynamically detuned 12 bladed splitted-rotor, splitters at 50% passage.

Fig. 14. — Stability of aerodynamically detuned 12 bladed splitted-rotor, splitters at 60% passage.

Fig. 15. — Stability of aerodynamically detuned 24 bladed splitted-rotor, splitters at 40% passage.

the unstable 12 bladed baseline rotor by the splitters on the complex unsteady pressure difference across the airfoil chordline, $\Delta C_p$.

The unstable baseline traveling wave mode is enhanced by both the aerodynamic and combined aerodynamic-structural detuning when the detuning due to the splitters causes the phase of the unsteady pressure difference on the two reference airfoils to either lag or be nearly in phase with that on the baseline reference airfoil, for example figure 20. However, when the
Fig. 16. — Stability of aerodynamically detuned 24 bladed splitted-rotor, splitters at 50% passage.

Fig. 17. — Stability of aerodynamically detuned 24 bladed splitted-rotor, splitters at 60% passage.

Fig. 18. — Stability of aerodynamic-structurally detuned 12 bladed splitted-rotor, splitters at 50% passage.

Fig. 19. — Stability of aerodynamic-structurally detuned 24 bladed splitted-rotor, splitters at 50% passage.

Aerodynamic detuning results in the unsteady pressure difference phase on the reference airfoils leading that of the baseline reference airfoil, the resulting rotor is more unstable than the baseline (Fig. 21).
Summary and conclusions.

A mathematical model has been developed and utilized to predict the effect of incorporating splitter blades on the torsion mode stability of a rotor operating in an incompressible flow field. The splitter blades, positioned circumferentially in the flow passage between two principal blades, introduce aerodynamic and/or combined aerodynamic-structural detuning into the rotor. The two-dimensional oscillating cascade unsteady aerodynamics, including steady loading effects, were determined by developing a complete first-order unsteady aerodynamic analysis together with an unsteady aerodynamic influence coefficient technique. The torsion mode flutter of both uniformly spaced tuned rotors and detuned rotors were then predicted by incorporating the unsteady aerodynamic influence coefficients into a single-degree-of-freedom aeroelastic model.

The viability of splitters as a passive torsion mode flutter control technique for an aerodynamically loaded rotor operating in an incompressible flow field was then considered, accomplished by applying this model to an unstable baseline 12 and 24 bladed rotors. This study demonstrated that aerodynamic detuning and combined aerodynamic-structural detuning due to incorporating splitters into unstable baseline rotors resulted in stable splitted-rotor configurations.
For splitters with only aerodynamic detuning, the aerodynamic damping and, therefore, the splittered-rotor stability was found to be a function of the splitter circumferential position, being stable for splitters at 40% of the passage but even more unstable than the baseline rotor with splitters at 60% spacing. Combined aerodynamic-structural detuning due to the splitters had a significant effect on rotor stability, resulting in instability splittered-rotors for all splitter circumferential positions.

The aerodynamic detuning creates blade-to-blade differences in the unsteady aerodynamics, thereby affecting the fundamental driving force — the unsteady pressure difference across the airfoil chords. The unstable baseline traveling wave mode was found to be enhanced by the splitters when this detuning causes the phase of the unsteady pressure difference to either lag or be nearly in phase with that on the baseline reference airfoil. However, when the aerodynamic detuning results in the unsteady pressure difference phase leading that of the baseline reference airfoil, the resulting rotor is more unstable than the baseline.

Thus, the improved aerodynamic performance potential of a rotor incorporating splitter blades also offers the potential of increased stability.

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References