Effect of arterio-venous heat exchange on breast temperature profile

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Abstract. — The role of arterial and venous blood thermal interaction on surface tissue temperature of female breast is investigated. The analytical results of the physical model for arterio-venous counter-current heat exchange are presented graphically. They indicate in general that the central breast temperature is lower than the rest of the breast. Also, the results shed light on the role of deep tissues physiological condition on the surface temperature distribution of female breast.

Nomenclature.

q	convective heat flow per unit volume of tissues, W/mm^3,
C	specific heat of blood, J/g C,
$\dot{m}$	blood perfusion rate per unit area of tissues, g/mm^2 s,
U	overall heat transfer coefficient, W/mm^2 °C,
A	blood vessels area per unit volume of tissues, mm^{-1},
T_a	arterial temperature at any arbitrary position, °C,
T_v	venous temperature at any arbitrary position, °C,
T_{tissue}	deep tissues' temperature, C,
X	dimensionless arterial temperature = $(T_a - T_\infty)/(T_0 - T_\infty)$,
Y	dimensionless venous temperature = $(T_v - T_\infty)(T_0 - T_\infty)$,
Z	dimensionless deep tissues' temperature = $(T_{tissue} - T_\infty)/(T_0 - T_\infty)$,
R	breast radius (72 mm),
θ	angular position of an arbitrary control surface,
M_a	dimensionless arterial conductance = $(UA)_a R/\dot{m}C$,
M_v	dimensionless venous conductance = $(UA)_v R/\dot{m}C$,
M_t	dimensionless arterio-venous conductance = $(UA)_t R/\dot{m}C$,
T_∞	surrounding ambient temperature, C,
T_0	normal body temperature, C.

Introduction.

The woman's breast is a highly perfused mammary gland, and the mechanism of arterial and venous blood thermal interaction plays a great role in thermal energy balance of the breast.

Several investigators [1-5] developed a one-dimensional analytical model for the counter-current heat exchange mechanism in animals and they found that the effect of ambient temperature on the thermal control system of the animal may produce changes in blood flow, blood vessel diameter, and blood flow distribution. Also, they concluded that the blood passing any given plane in opposite directions carries with it the small local blood tissue temperature difference that existed above and below this plane.

Analytical model.

The thermal analysis of the physical model shown in figure 1, incorporates the following assumptions and simplifications :

i) the temperature of the arterial and venous flows vary with angular position in the flow direction only ;

ii) there are only two major blood flow routes representing venous and arterial vascularization as shown in figures 1 and 2 ;

iii) the effect of metabolic heat generation change for normal and malignant tissues has been accounted for by studying the research problem at different constant deep tissues temperature ;

iv) the temperature distribution of the breast is not changing with time.

Considering the control volumes shown in figure 1, the following energy equation for both venous and arterial heat flows can be deduced :

for $0 < \theta < 90$

$$\frac{1}{r} \dot{m}C \frac{\mathrm{d}T_{\mathrm{v}}}{\mathrm{d}\theta} - (UA)_{\mathrm{i}} (T_{\mathrm{a}} - T_{\mathrm{v}}) + (UA)_{\mathrm{v}} (T_{\mathrm{v}} - T_{\infty}) \frac{r_2}{r} = 0 \quad (1)$$

$$\frac{1}{r} \dot{m}C \frac{\mathrm{d}T_{\mathrm{a}}}{\mathrm{d}\theta} + (UA)_{\mathrm{i}} (T_{\mathrm{a}} - T_{\mathrm{v}}) + (UA)_{\mathrm{a}} (T_{\mathrm{a}} - T_{\mathrm{tissue}}) \frac{r_1}{r} = 0 \,. \quad (2)$$

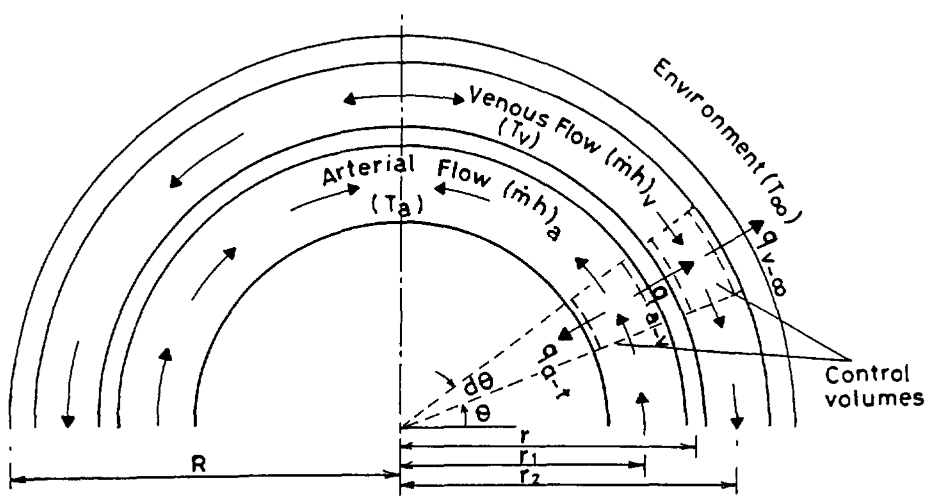

Fig. 1. — Physical model for counter-current heat exchange in woman's breast.

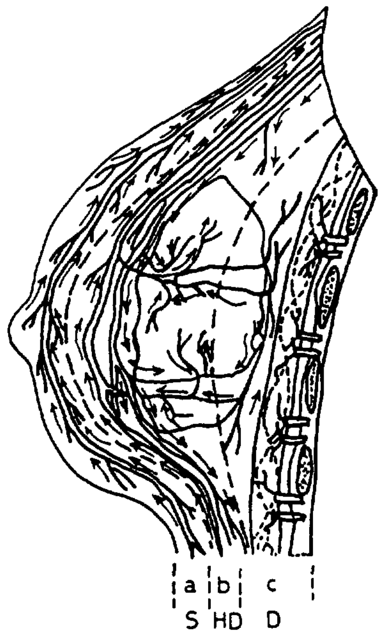

Fig. 2. — Scheme of distribution layers of the mammary vascularization [6] : a) superficial vascular tree, mainly due to a venous origin ; b) half-deep vascular tree, formed by arterial and venous vessels ; c) deep vascular tree, formed by arterial and venous vessels.

Where, r_1 = radial distance between center of the breast and center line of the arterial flow, r_2 = radial distance between center of the breast and center line of the venous flow, $r = \frac{1}{2}(r_1 + r_2)$ = radial distance between center of the breast and the imaginary separating line of venous flow from arterial flow.

By introducing the dimensionless arterial, venous and arteriovenous conductances (M_a, M_v, M_l) and the dimensionless arterial, venous and deep tissues temperatures (X, Y, Z) and the following boundary conditions :

$$\begin{array}{lll} \text{at} & \theta = 0 & T_a = T_0 \\ \text{and at} & \theta = \frac{\pi}{2} & T_a = T_v . \end{array} \tag{3}$$

The following final form of general solution for arterial and venous dimensionless temperatures can be reached :

$$X(\theta) = \frac{\left[(E+F) - \left(\frac{r_1 r}{R^2} M_l M_a + F \right) Z \right] e^{-\left(\frac{1}{2}G + \frac{r}{R} M_l\right)\theta}}{(E+F)\sinh\left(\frac{\pi}{2} C_l - \phi\right)} \sinh\left[\left(\frac{\pi}{2} - \theta \right) C_l - \phi \right] - $$

$$- \sqrt{\frac{r}{R}\frac{M_l}{H}} F Z \frac{e^{\left(\frac{1}{2}G + \frac{r}{R} M_l\right)\left(\frac{\pi}{2} - \theta\right)}}{(E+F)\sinh\left(\frac{\pi}{2} C_l - \phi\right)} \sinh(C_l \theta) + \frac{\left(r_1 \frac{r}{R^2} M_l M_a + F \right)}{(E+F)} Z \tag{4}$$

$$Y(\theta) = $$

$$= -\frac{e^{-\left(\frac{1}{2}G+\frac{r}{R}M_i\right)\theta}\left[(E+F)-\left(\frac{r_1 r}{R^2}M_i M_a+F\right)Z\right]\cosh\left[\left(\frac{\pi}{2}-\theta\right)C_1+\psi-\phi\right]}{(E+F)\sinh\left(\frac{\pi}{2}C_1-\phi\right)} -$$

$$-\frac{e^{\left(\frac{1}{2}G+\frac{r}{R}M_i\right)\left(\frac{\pi}{2}-\theta\right)}\sqrt{\frac{r}{R}\frac{M_i}{H}}FZ\cosh(\psi-C_1\theta)}{(E+F)\sinh\left(\frac{\pi}{2}C_1-\phi\right)}+\frac{\frac{r_1 r}{R^2}M_i M_a}{(E+F)}Z \quad (5)$$

where

$$C_1 = \sqrt{\left(H^2+4\frac{r^2}{R^2}M_i^2\right)/4}$$

$$G = \frac{r_1}{R}M_a + \frac{r_2}{R}M_v$$

$$H = \frac{r_2}{R}M_v - \frac{r_1}{r}M_a$$

$$E = \frac{r}{R}M_i\left[\frac{r_1}{r}M_a + \frac{r_2}{R}M_v\right]$$

$$F = \frac{r_1 r_2}{R^2}M_a M_v$$

$$\cosh\phi = \frac{\left(\frac{r}{R}M_i+\frac{1}{2}H\right)}{\sqrt{\left(\frac{r}{R}M_i H\right)}}$$

$$\cosh\psi = \frac{C_1}{\frac{r}{R}M_i} \ \& \ \sinh\psi = \frac{\frac{1}{2}H}{\frac{r}{R}M_i}$$

Based on reference [7], the following order of magnitude of the dimensionless conductances will be considered to study its effect on the arterio-venous counter-current heat exchange and consequently on the breast surface temperature distribution : $M_v = M_a$ (M_a ranges between 0.1 and 0.2 depending on the size of arteries), M_i/M_a = 25 % to 100 % (depending on the degree of the arterio-venous counter-current thermal interaction).

Discussion of results.

Figures 3, 4 and 5 are representing the graphical plots of equations (4) and (5) for $M_a = 0.1$, 0.15 and 0.2, respectively. Each figure is considering different physiological condition of deep tissues metabolic heat generation rate for various degrees of thermal interaction.

The venous blood temperature has been considered representative of breast surface temperature since it is very near to it.

The results indicate in general that the central surface breast temperature (@θ = 90° ; areola and periareolar region) is lower than for the rest of the breast. This result is well established by contact thermography patterns of the normal woman's breast [8-15].

CASE I : $M_a = M_v = 0.1$. — Figure 3 shows the effect of arterio-venous counter-current heat exchange on breast surface temperture for different cases of deep tissue temperature. It can be noted that the venous blood temperature at the breast bottom circumference is directly related to the breast core deep tissues temperature. This can be attributed to the breast thermal regulation. Going up through the breast surface from $\theta = 0°$ to $\theta = 90°$, the venous blood temperature decreases and the arterial blood temperature increases slightly up to a final equilibrium temperature in the central area of the breast. Comparing the variation of both arterial and venous blood temperature with the tissue angular position, one can deduce that the amount of heat conducted to the superficial tissues is completely dissipated mainly by heat convection to the surroundings and slightly by counter-current heat exchange to the arteries. The latter amount along with the amount of heat conducted to the arteries are dissipated completely to the deep tissues. This former amount of heat is comparably large with respect to

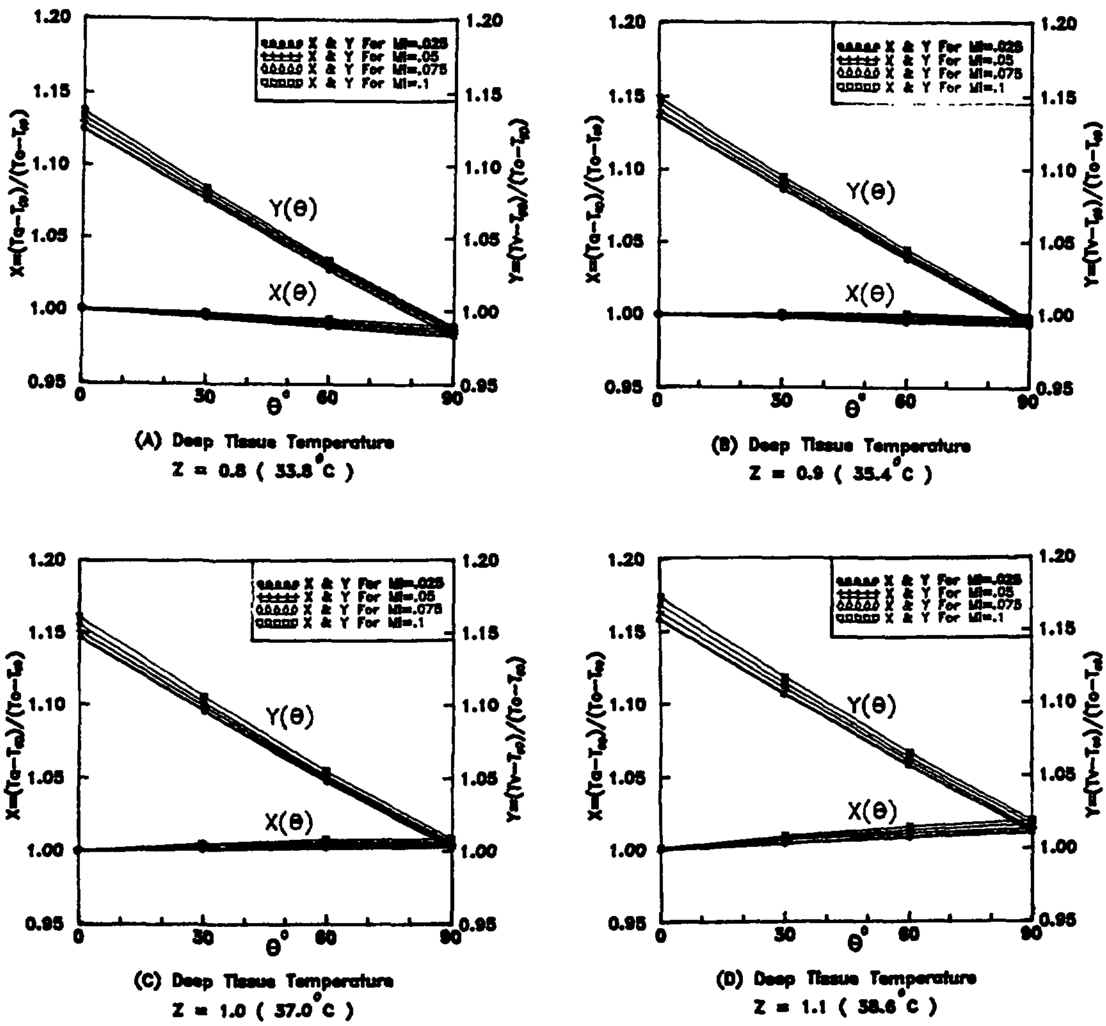

Fig. 3. — Effect of arterial and venous blood thermal interaction on surface tissue temperature for different physiological conditions of breast deep tissues (dimensionless conductances $M_a = M_v = 0.1$ and variable arterio-venous dimensionless conductance (M_1)).

the amount of heat conducted to the arteries. The very slight increase in temperature profile of both arterial and venous blood is directly related to the increase of arterio-venous thermal interaction from 25 % to 100 % of that corresponding to arteries and veins conductances.

Figures 3C and 3D indicate that for any deep tissue temperature equal to or higher than normal body temperature of 37 °C, the final equilibrium temperature between arteries and veins at the central region of the breast will be higher than normal body temperature (i.e., $Y(90°) > 1$). This will not be the case for a normal woman breast as indicated by figures 3A and 3B. The maximum difference between arterial venous blood temperature is 2.75 °C at the breast circumference superficial tissues based on $T_0 = 37$ °C and $T_\infty = 21$ °C (i.e., $Y(0°) - X(0°) = 0.173$).

CASE II : $M_a = M_v = 0.15$. — The effect of increasing arterial and venous blood dimensionless conductances to 0.15 on the breast surface temperature profile is shown in figure 4. The

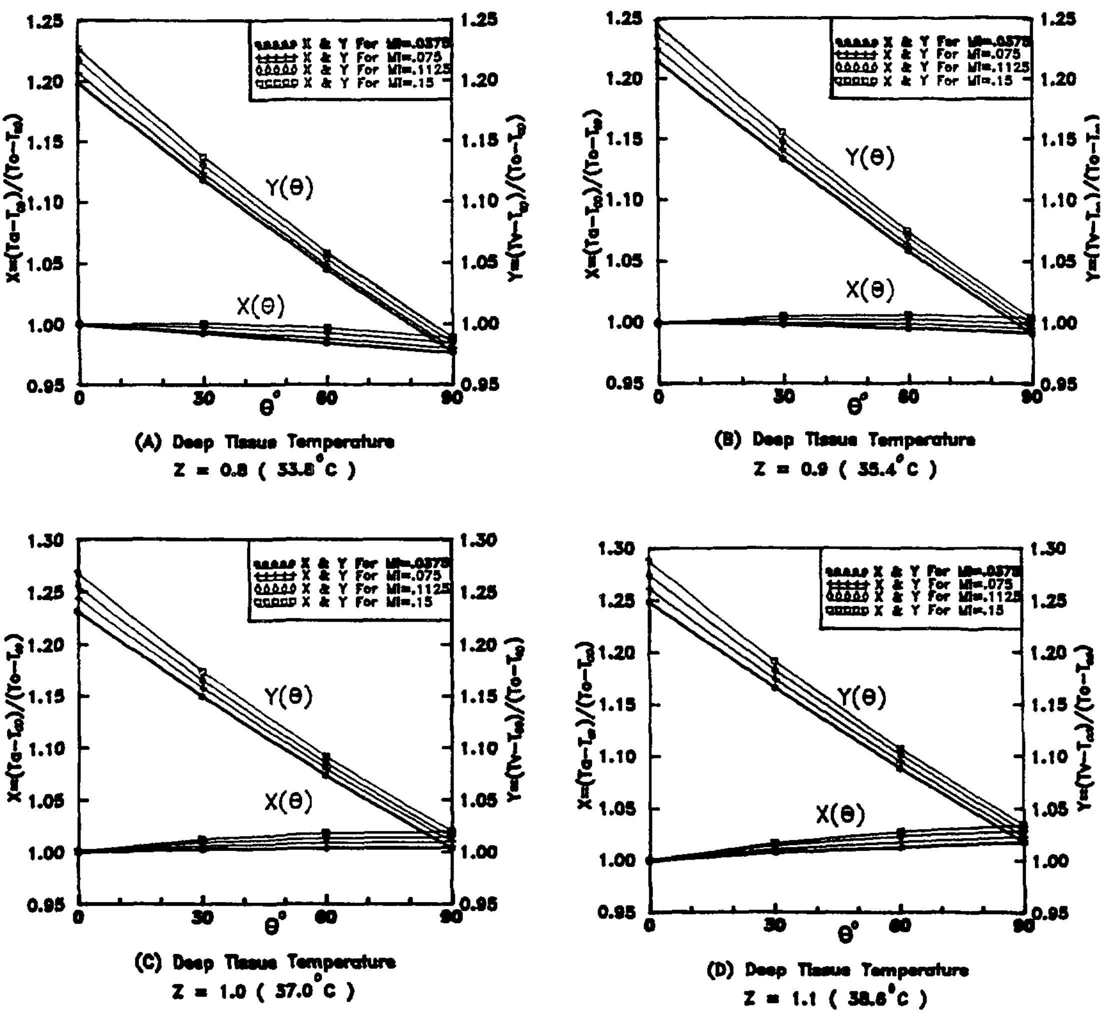

Fig. 4. — Effect of arterial and venous blood thermal interaction on surface tissue temperature for different physiological conditions of breast deep tissues (dimensionless conductances $M_a = M_v = 0.15$ and variable arterio-venous dimensionless conductance (M_i)).

heat dissipated to the deep tissues and the heat conducted through the arterial blood flow is completely due to the counter-current heat exchange between veins and arteries. The variation of both arterial and venous dimensionless conductance results in increasing both the arterial and venous blood temperature profile by an approximate value of 0.08 and 0.18 °C, respectively (Fig. 4A). Reaching an abnormal physiological condition of deep tissues temperature of 38.6 °C (i.e., $Z = 1.1$) (Fig. 4D), results in an increase of temperature difference between arteries and veins to about 4.6 °C at $\theta = 0°$ (i.e., $Y(0°) - X(0°) = 0.287$).

CASE III : $M_a = M_v = 0.2$. — The amount of counter-current heat transferred between veins and arteries increases with the increase of both arterial and venous dimensionless conductances (M_a & M_v) as shown in figure 5. The dissipation of heat conducted through veins by arterio-venous blood thermal interaction increases to about 23.5 % of heat dissipated to the

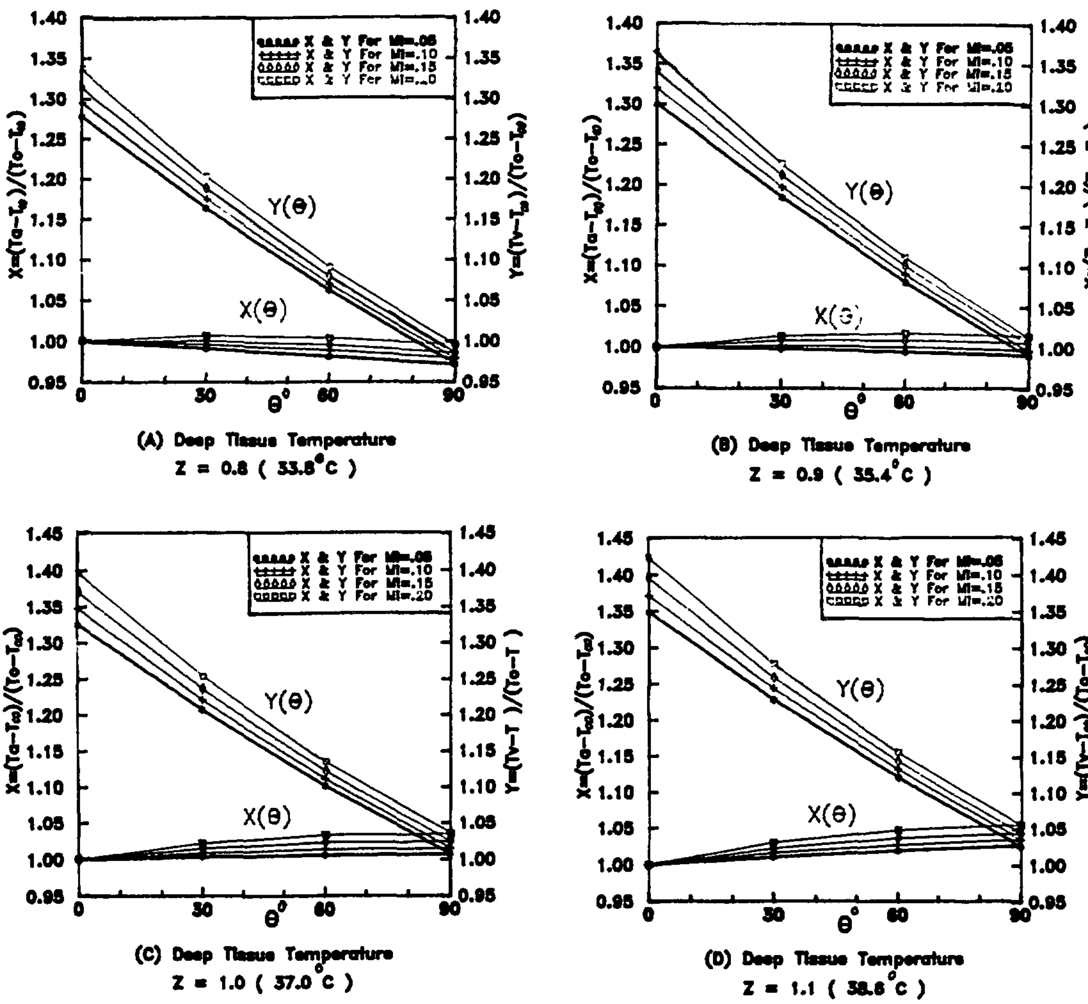

Fig. 5. — Effect of arterial and venous blood thermal interaction on surface tissue temperature for different physiological conditions of breast deep tissues (dimensionless conductances $M_a = M_v = 0.20$ and variable arterio-venous dimensionless conductance (M_i)).

surroundings. There is an increase in the breast central region temperature ranging between 0.24 and 0.88 °C above normal body temperature for various deep tissue temperatures of figure 5B through 5D. Also, the difference between arterial and venous blood temperature increases up to 6.8 °C (i.e., $Y(0°) - X(0°) = 0.424$) above normal body temperature at the breast circumference superficial tissues. For this case, the amount of heat transferred from veins to arteries at breast bottom is estimated to be five times the amount of heat transferred from deep tissues to arteries.

Conclusions.

1. The amount of counter-current heat exchange between veins and arteries is smaller than the amount of heat convected from venous blood flow to the surroundings.

2. The increase of the arterial and venous blood dimensionless conductances due to any physiological condition results in an increase of breast surface temperature.

3. The central region of the breast temperature is lower than T_0 whenever deep tissue temperature is lower, and *vice versa*.

4. The counter-current heat exchange between veins and arteries is the dominant mode of heat transfer inside the breast for higher values of arterial and venous dimensionless conductance.

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