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To cite this version:

HAL Id: jpa-00249108
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Submitted on 1 Jan 1994

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Integral mathematical model of magnetic fluid ejector

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(Received 16 July 1993, revised 17 November 1993, accepted 3 December 1993)

Abstract. — On the basis of the ideal fluid approach the magnetic fluid ejector, the main part of ejector heat exchanger, was considered. Mathematical model of the ejector was build with the aid of integral laws of mass and momentum conservation. The conditions under which the magnetic fluid ejector is able to function were determined.

Magnetic fluid heat exchanger of ejector type was proposed in [1,2]. Its scheme is presented in figure 1. It has two circuits: cooler and heater. Main part of this heat exchanger is a magnetic fluid ejector working in the following way. The fluid from cooler circuit, having temperature \( T_i < T_C \) (\( T_C \) is Curie temperature of disperse material of magnetic fluid), is involved by magnetic force into the region of uniform magnetic field. It entrains the fluid from heater circuit which has temperature \( T_c > T_C \). The “hot” and “cold” fluids are mixed in the region of uniform magnetic field. At the exit from the field the fluid has temperature \( T \) (\( T_i < T < T_c \)). The advance of this type of a magnetic fluid heat exchanger in comparison with ordinary one [3] is that both heater and cooler are outside of magnetic field. Thus, the construction of magnetic system does not impose the limits on that of heat exchangers.

The purpose of this work is determination of the range of temperatures, properties of magnetic fluid and geometrical parameters of ejector in which the heat exchanger regarded is able to function.

Let us consider at first more simple model problem. Suppose that the inlet and the outlet of the mixing chamber are not connected hydrodynamically (Fig. 2). Magnetic fluids with temperatures \( T_i \) and \( T_c \) flow into the region considered from two different volumes and, after mixing, flow out to surroundings. Any influence of gravity is neglected. It is presumed also that the fluid is ideal, i.e. inviscid and incompressible. Such formulation of the problem permits, we suppose, to reveal main hydrodynamical principles of the magnetic fluid ejector without missing generality.

We shall build a mathematical model of the ejector on the foundation of integral laws of
mass and momentum conservation:

\[
\int_{\Sigma} \rho u_n d\sigma = 0, \quad (1)
\]

\[
\int_{\Sigma} \rho uu_n d\sigma = \int_V F_d\tau + \int_{\Sigma} P_n d\sigma. \quad (2)
\]

Here \( \rho \) is the fluid density, \( u \) is the fluid velocity, \( p \) is the fluid pressure, \( F \) is the force acting on the mass unit of the fluid, \( V \) and \( \Sigma \) are the control volume of the fluid and its surface, respectively, \( n \) is the external normal to the surface of the control volume.
Let us write general equations (1, 2) for the case of the fluid volume bounded by the mixing chamber walls and planes I and II (Fig. 2). We assume that the plane I is inside of the uniform magnetic field and the plane II is outside of it. We also suppose that in the regarded volume the complete mixing of the flows takes place. It is of course very rough approximation. However, only in this case the volume force in equation (2) can be easily calculated and has the following form

\[ S \mu_0 \int_0^H M(T, H) \, dH, \]

where \( S \) is the cross-section of the mixing chamber, \( \mu_0 \) is magnetic permeability of vacuum, \( H \) is magnetic field strength, \( M \) is magnetization of magnetic fluid, \( T \) is temperature of fluid at the exit from the region of uniform magnetic field. Real force acting on the regarded volume of the fluid will be greater because of non-completeness of the mixing and viscous stress. Trying to save the simplicity of integral model we shall take into account the deflection of the model from the ideal conditions by multiplying the force on a parameter \( \alpha \). It is impossible to calculate this parameter in the frame of integral model. For such calculation it is necessary to consider the flow in the mixing chamber in detail with the aid of numerical methods. It will be the subject of the further investigation but in this paper we shall suppose the parameter \( \alpha \) to be given and consider its influence on the functioning of the device regarded.

Thus, instead of (1) and (2) we have

\[ u_i S_i + u_e S_e = u S, \quad (3) \]

\[ (p_e + \rho u_e^2) S_e + (p_i + \rho u_i^2) S_i - (p_0 + \rho u^2) S = \alpha S \mu_0 \int_0^H M(T, H) \, dH, \quad (4) \]

where indexes i and e denote parameters of ejecting and ejected flows, parameters without indices are related with the exit from the mixing chamber, \( S_i \) and \( S_e \) are cross-sections of ejecting and ejected channel, respectively. Write Bernoulli’s equation for ejecting and ejected flows:

\[ p_e + \rho u_e^2/2 - \mu_0 \int_0^H M(T_e, H) \, dH = p_0 = 0, \quad (5) \]

\[ p_i + \rho u_i^2/2 - \mu_0 \int_0^H M(T_i, H) \, dH = p_0 = 0, \quad (6) \]

where \( p_0 \) is the pressure of the surrounding which is presumed to be equal zero. It is evident that we can write the following equation:

\[ p_i = p_e. \quad (7) \]

It was already mentioned above that the complete mixing was assumed to take place in the region of uniform magnetic field. We also neglect the energy dissipation in the result of the flows mixing. The magnetocaloric effect is also supposed to be negligible. Thus we can obtain the temperature of fluid after the mixing from the heat balance:

\[ T = \frac{G_e T_e + G_i T_i}{G_e + G_i}, \quad (8) \]

where \( G_i, \ G_e \) are volumetric flow rates of ejecting and ejected flows, respectively.
Thus, the integral model of magnetic fluid ejector is described by the set of equations (3-8). To analyze it let us represent this set of equations in the dimensionless form. In order to make it let us introduce the following dimensionless parameter:

\[ w = \frac{(W^2(T) - W^2(T_e))}{(W^2(T_i) - W^2(T_e))}, \]
\[ \omega = \frac{W^2(T_e)}{W^2(T_i)}, \]  

(9)

where

\[ W^2(T) = \frac{(2/\rho)\mu_0}{\int_0^H M(T, H)dH}. \]  

(10)

All velocities must be divided by the magnitude \( U_0 = \sqrt{W^2(T_i) - W^2(T_e)} \) having the dimension of velocity. Introduce also the dimensionless temperature

\[ t = \frac{(T - T_i)}{(T_e - T_i)} = \frac{G_e}{(G_e + G_i)}. \]  

(11)

After appropriate transformations we have the following set of equations:

\[ Av^4 + Bv^2 + C = 0, \]  

(12)

\[ v_i^2 = 2v^2 + w_1 - 2g + 1, \]  

(13)

\[ v_e^2 = v_i^2 - 1, \]  

(14)

\[ t = (1 - g)v_e/v, \]  

(15)

where

\[ A = 8g^2 - (1 - 4g)^2, \]
\[ B = 4g^2(w_1 - 2g + 1) - 2\omega(1 - 4g), \]
\[ C = -\omega^2, \]
\[ \omega = (w_1 - 2g + 1)(1 - 2g) - (1 - g)^2, \]
\[ w_1 = \alpha w + (\alpha - 1)\omega/(1 - \omega). \]

Here \( g = S_i/S, \) \( S = S_i + S_e \) and \( v_i, v_e, v \) are dimensionless velocities of respective flows.

If the parameter \( w \) is regarded as independent, one can find \( t \) as function of \( w \) from the equations (12-15). It may be shown that the function \( t(w) \) is determined in the interval:

\[ 0 \leq w \leq w_m = \frac{2g(1 - g)(1 - \omega) - (\alpha - 1)\omega}{\alpha(1 - \omega)} \]

On the other hand, accounting \( t \) as independent parameter, one can determine \( w(t) \) from the equation (9). It should be noted that the concrete form of the function \( t(w) \) is determined by the only parameter \( g \), while the function \( w(t) \) depends on the temperature range, properties of magnetic fluid and the magnetic field strength; in other words, it is defined by the function \( M(T, H) \) in the given temperature range.
It functions $t(w)$ and $w(t)$ are drawn on the coordinate plane $t - w$, a cross-point of these curves corresponds to a solution of the set of equations (9, 12-15). For more simple analysis of this set of equations let us divide temperature interval $[T_1, T_e]$ into two ones: $[T_1, T]$ and $[T, T_e]$. Then we assume that magnetization is linear with temperature on each of these intervals. Thus, the dependence $w(t)$ becomes more simple and takes the following form:

$$w = \frac{k(1-t)}{t + k(1-t)},$$

where

$$k = K^+/K^-, \quad K^+ = -\mu_0 \int_0^H (\Delta M/\Delta T)^+ dH, \quad K^- = -\mu_0 \int_0^H (\Delta M/\Delta T)^- dH,$$

$$(\Delta M/\Delta T)^+ = (M(T_e) - M(T_1))/(T_e - T),$$

$$(\Delta M/\Delta T)^- = (M(T) - M(T_1))/(T - T_1).$$

Some functions $t(w)$ and $w(t)$ for the case of $\alpha=1$ are presented in figure 3. One can see that corresponding curves either have no cross-points or have two of those. It can be easy shown (See Appendix) that the solution corresponding to less value of $t$ is unstable and, therefore, is not physical. On the other hand, the point of the curves crossing which is upper with respect to temperature of the exit flow corresponds to a stable solution.

![Figure 3](image_url)

**Fig. 3.** — Dependence $t(w)$ and $w(t)$ for $\alpha=1$. $t(w): 1) g = 0.9$; 2) $g = 0.5$; 3) $g = 0.1$. $w(t): 4) k = 0.1$; 5) $k = 0.5$; 6) $k = 1$.

One can also conclude from the same figure that the set of equation (9, 12-15) has not a solution if the parameter $k$ is greater than some value $k^*$, which depends on the ratio $g$ of the ejecting channel cross-section to that of the mixing chamber. If $t_1(w)$ denotes the function reversed to $w(t)$, the value $k^*$ will be found from the condition that the minimum of function
\[ f(w) = t_1(w) - t(w) \text{ equals zero.} \]

This condition leads to the following set of equations:

\[ t(w) = t_1(w), \quad t'(w) = t'_1(w), \quad (17) \]

from which the dependence \( k^*(g) \) is determined numerically. This dependence for different values of parameter \( \alpha \) and \( \varepsilon = 0.1 \) is presented in figure 4. It is evident that the region under the curve \( k^*(g) \) corresponds to the solution existence region for the set of equation (9, 12 - 15).

It is clear from the figure that the greater parameter \( \alpha \) the less region of functioning of the device regarded.

![Dependence \( k^* \) on the ratio \( g \) of the nozzle cross-section to that of the mixing chamber. \( \varepsilon = 0.1; 1) \alpha = 1, 2) \alpha = 1.2, 3) \alpha = 2. \)](image)

Thus, we have a criterion for choice of a magnetic fluid suitable for use as working medium of magnetic fluid ejector. For this purpose it is necessary to establish experimentally the dependence \( M(T, H), \) or \( \mu_0 \int_0^H M(T, H) dH, \) in the given temperature range and determine the dimensionless function \( w(t) \) according to equation (9). From the last dependence and equation (16) one can find \( k \) by the square least method. Knowing this value of \( k \) and the determined above dependence \( k^*(g) \) one can establish whether a solution of set of equations (9, 12-15) exists for the given magnetic fluid, temperature range and magnetic field strength. And if it does, one can determine the range of parameters \( g, \alpha \) and \( \varepsilon \) in which the solution exists. As it was mentioned above \( \alpha \) can be calculated from detail consideration of the flow in the mixing chamber of the magnetic fluid ejector.
Appendix.

Let us show that the solution of set of equations (9, 12-15) corresponding to less magnitude of \( t \) is unstable. The given set of equations may be written in the following general form:

\[
t = f(w; g, \alpha, \omega),
\]

\[
w = q(t; k).
\]

Let \( t_0, \ w_0 \) be one of solutions of this set of equations. Let further dimensionless temperature of exit flow change in any way and acquire new magnitude \( t = t_0 + \delta t \ (|\delta t| \ll 1) \). Then it follows from (19) that \( w \) changes in the following way:

\[
w = q(t_0 + \delta t; k) \approx q(t_0; k) + q'(t_0) \delta t = w_0 + q'(t_0) \delta t.
\]

\( \delta t \) denotes differentiation with respect to the variable denoted by the subscript. The change of \( w \) causes in its turn the further alteration of \( t \) which may be calculated from equation (18):

\[
\Delta t = f(w_0 + q'(t_0) \delta t) - t_0 \approx f'_w q'(t_0) \delta t.
\]

Evidently, for the stability of the solution it is necessary that \( |\Delta t| < |\delta t| \), which is equivalent to the following:

\[
|f'_w q'(t_0)| < 1.
\]

Let us consider further the function \( F(t) = q(t; k) - f^{-1}(t; g) \), where \( f^{-1} \) denotes a function reversed to \( f \). One can see that if \( t_1 \) and \( t_2 \) are solutions of the set of equations (18, 19) and \( t_1 < t_2 \), the following relations take place:

\[
F(t_1) = F(t_2) = 0,
\]

\[
F(t) > 0, \text{ if } t < t_1 \text{ or } t > t_2,
\]

\[
F(t) < 0, \text{ if } t_1 < t < t_2.
\]

The last two inequalities lead to the following:

\[
F'(t_1) < 0, \ F'(t_2) > 0.
\]

Substituting the function \( F(t) \) into (26), we have for \( t = t_1 \).

\[
q't' - (f^{-1})' = q't' - 1/f'_w < 0.
\]

One can see from figure 3 that \( q't' < 0 \) and \( f'_w < 0 \) everywhere. Taking into account the last inequalities, we obtain from (27) that for \( t = t_1 \),

\[
|q't'f'_w| > 1.
\]

Exactly we have for \( t = t_2 > t_1 \).

\[
|q't'f'_w| < 1.
\]
Thus, the solution corresponding to less magnitude of $t$ is unstable with respect to small disturbances. On the other hand, the solution corresponding to the greater value of $t$ is stable with respect to such disturbances. It easy to show that under final disturbances $\delta t$ the solution is stable while $t_2 + \delta t > t_1$.

References