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Vortex lines in layered superconductors. II. Pinning and critical currents in high temperature superconductors

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Résumé. — On effectue une présentation qualitative des divers phénomènes qui contrôlent la valeur du courant critique dans les supraconducteurs à haute température. La notion de courant critique qui est utilisée est reliée à un critère de champ électrique non nul, fixé par des considérations expérimentales ou des exigences de rendement pour les applications. On se restreint au problème des courants critiques intragranaires d’origine extrinsèque, qui dépendent de façon complexe des caractéristiques d’ancrage des défauts présents dans le matériau et des propriétés du réseau de vortex. On privilégie la configuration de champ $B//c$ qui est la plus révélatrice à cet égard. On analyse les différences de comportement entre les composés Y(123) et BSCCO (Bi(2212) ou Bi(2223)) en liaison avec leurs degrés d’anisotropie respectifs. Les différents régimes d’ancrage et de « creep » possibles pour ces composés sont examinés en fonction de la température. Les courants critiques obtenus pour BSCCO semblent correspondre à un régime d’ancrage fort, alors que la question reste ouverte pour Y(123). La décroissance en température du courant critique expérimental suscite l’apparition d’un régime de « creep » collectif pour ces deux composés, avec cependant des différences notables sur la position et l’étendue des domaines correspondants. Au voisinage de la ligne d’irréversibilité, la disparition progressive des corrélations entre vortex provoque une chute de l’énergie d’ancrage effective. Dans BSCCO, celle-ci ne semble pas compatible avec l’hypothèse d’un ancrage par des lacunes d’oxygène réparties de façon aléatoire. On donne en conclusion quelques indications concernant les microstructures susceptibles d’améliorer les propriétés d’ancrage des futurs matériaux.

Abstract. — In this article, a qualitative survey is given on the various phenomena which influence the critical current of high temperature superconductors. Critical current is defined as a property related to a non-zero electric field criterion, the level of which is fixed by experimental considerations, or efficiency requirements of applications. The presentation is restricted to extrinsic intragranular critical current, which depends in a complex way on the interplay between the characteristics of pinning centres and the properties of the vortex lattice. The discussion is focussed on the configuration $B//c$, which contains the main elements of this problem. Differences of behaviour between Y(123) and BSCCO (Bi(2212) or Bi(2223)) are analysed in the

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context of their respective anisotropy factors. Possible regimes for pinning and creep are discussed in various temperature domains. From critical current results, a strong pinning regime is found to occur in BSCCO, whereas the pinning strength in Y(123) is still an open question. The thermal decrease of critical current allows a collective creep regime to appear in both materials, but at different temperature ranges. The disappearance of correlation effects near the irreversibility line results in a fall of the effective pinning energy. We show that in BSCCO, the effective pinning energy deduced from experimental results is not in agreement with pinning by randomly dispersed oxygen vacancies. Finally, we shortly describe the microstructures which could allow a more efficient pinning in future materials.

1. Introduction.

For high power/high current applications, large pinning forces are needed inside the superconductor, in order to prevent flux movements and dissipation which is their natural consequence. In most practical situations, Lorentz forces have non-zero components in the (a, b) plane. Intrinsic pinning forces, which were previously described, can only balance the c axis component of the Lorentz force. Thus, extrinsic pinning, relying on crystalline defects and chemical inhomogeneities, is needed to obtain a full immobilization of the vortex lattice (VL).

In superconductors, the behaviour of extrinsic pinning depends on their fundamental parameters ($\lambda$, $\xi$) and the various elastic moduli $C_{ij}$ considered in part I, as well as on the physical and geometrical characteristics of the pinning centres. Thus, electronic anisotropy is again a crucial property, mainly responsible for the differences observed between Y(123) and Bi (or Tl) compounds.

The purpose of this second part is to provide a synthetic view on the relations which occur between pinning behaviour and microstructure in HTS. No considerations will be given for the very important subject of intergranular critical current (governed by weak links), which from a fundamental point of view would not preserve the unity of the questions addressed in this paper.

Extrinsic pinning is a highly complex subject, and a very active research field, both from a theoretical and experimental point of view. The main concepts used in this field have all experienced a significant progress in recent years. Besides, many features observed in the pinning behaviour of high temperature superconductors (HTS) appear today to be specific of this new class of materials. However, some difficulties still prevent an accurate (quantitative) description of this behaviour to be obtained from theoretical models, especially when high critical currents and strong pinning effects are considered.

The general point of view adopted in this paper does not allow to present all the experiments or models which appeared recently. Our purpose is rather to emphasize and to comment on the main conceptual advances which have been realized in the last few years. For classical descriptions of pinning in low temperature superconductors, authoritative reviews may be found in reference [1, 2]. A very detailed analysis of irreversible phenomena in HTS, with more emphasis on macroscopic behaviour, was recently given in reference [3].

In order to get some insights about the most efficient microstructures for various temperature and field conditions, the first step (and the main goal of this paper) is to settle what pinning or creep « regimes » have to be expected in these cases. Three main items have to be determined in this respect:

— the pinning strength: one has to compare the maximal values of the elementary forces which can be produced by each pinning centre with the elastic forces arising from the deformation of the vortex lattice;
— the nature of the magnetic flux displacements (creep) which limit the experimental critical current. These may either correspond to elastic or plastic deformations of the vortex lattice;

— the essential source of the flux displacement: the case in which thermal activation brings only a minor contribution (because the effective pinning barrier is strongly depressed by the Lorentz force) must be distinguished from the opposite one, in which the Lorentz force is much smaller than the maximal slope of the pinning potential. The distinction between these regimes is schematized in figures 1b and 1c which are introduced below.

In section 2, we discuss these different alternatives in the context of high critical current HTS, which are needed for applications. Section 3 is devoted to technical considerations, showing how a practical definition of the critical current has to be related to the various application modes which can be projected for HTS. This section also gives preliminary indications about the occurrence of some particular regimes in the conditions corresponding to the required current densities and dissipation levels. In section 4, we select a few experimental data from the recent literature, which exemplify the results of our previous discussion. The microstructures which seem to be favoured by these considerations are shortly described in the conclusion.

2. Pinning regimes in HTS.

2.1 Pinning strength at \( T = 0 \).

2.1.1 Basic definitions. — Pinning occurs because the VL energy is locally modulated by various defects (« pinning centres ») dispersed in the material. Each modulation \( U_p \) associated with an individual pinning centre \( i \) depends on the distance \( r_{ij} \) between this centre and the nearest flux lines \( j \). For simplicity’s sake, we restrict our discussion to the case where the interaction range \( r_p \) and the defect size are smaller than the VL parameter \( a_0 \). Then one can define a one-body potential \( U_p (r_i) \), where \( r_i \) is the distance of the centre to the nearest flux line (Fig. 1a).

![Fig. 1. — Schematic description of the potential energy of a flux line at a distance \( r_i \) of a particular pinning centre; a) pinning potential \( U_p (r_i) \); b) and c) distortion of the total energy profile by the Lorentz component \( f_L r \). 1b) represents a depinning process in a configuration close to the low temperature critical current, whereas 1c) represents a high temperature depinning process occurring at a low driving force.](image)
Even in this restricted framework, pinning is still a complex problem because flux line interactions influence the equilibrium state of the VL. As stressed in part I, each flux line segment interacts both with other segments of its own flux line and with neighbouring flux lines. Thus a vortex is not free to move exactly to the position defined by the minimum of
$U_p(r_i)$. The equilibrium state of the system results from a minimization of the sum of pinning and elastic energies obtained for a given displacement field $u(r)$ of the VL. The elastic term is perfectly negligible only in case of a uniform $u(r)$, which would correspond to a very special «synchronized» array of pinning centres. Usual pinning centres are randomly distributed, in such a way that a frustration effect prevents a full matching of the VL. However, various regimes may be obtained whether elastic or pinning energy is dominant (at a given scale defined below).

Elementary pinning forces are defined by $f_i(r_i) = -\nabla r_i(U_p)$. The potential well $U_p(r_i)$ is usually assumed to be symmetrical, with a maximal slope at $|r_i| = r_p$ corresponding to the highest possible value $f_p$ of $|f_i(r_i)|$. If the force $f_i$ applied by the VL on the pinning centre is such that $|f_i| = f_p$, the flux line jumps out of the potential well (Fig. 1b). Actually, interactions between flux lines lead several flux lines, constituting a flux «bundle», to jump simultaneously [4]. A high depinning rate might also be obtained in a temperature range $kT = U_0$ with $|f_i| \ll f_p$ (Fig. 1c). This regime is further discussed in the next sections.

A macroscopic current $J$ produces a Lorentz force $J \times B$ per unit volume, which is transferred to the pinning centres by the VL. The zero temperature critical current density $J_c(0)$ is the lowest $J$ value for which some dissipation occurs, whatever is the nature of the process. For type II superconductors in the mixed state, flux displacement is the relevant phenomenon for this threshold. It occurs when the maximal macroscopic pinning force $F_p$ which can be produced by the array of pinning centres is just balanced by the Lorentz force:

$$J_c(0) \times B = -F_p.$$  \hspace{1cm} (1)

2.1.2 Strong pinning limit. — For small flux line interactions, the influence of forces perpendicular to the Lorentz force direction Ox can be neglected. In this limit, we show below that the force $f_i = -f_i$ applied by the VL on a pinning centre is the Lorentz force $V_j J \times B$ acting on a volume $V_j$ surrounding the pinning centre, bounded by the surface $S_j$ defined by the following condition:

$$\sigma_{\alpha\nu} d_{\alpha\nu} + \sigma_{\nu\nu} d_{\nu\nu} = 0$$ \hspace{1cm} (2)

where $\sigma_{\alpha\nu}$ and $\sigma_{\nu\nu}$ are the shear and tilt stresses of the VL along the Ox axis, and $d_{\alpha\nu}, d_{\nu\nu}$ are the related components of the unit vector $d_i$ normal to $S_j$ (Fig. 2). Care has to be exercised because stresses have a non-local relationship with the strain field. Indeed, they basically result from interactions between vortex segments up to distances $\lambda$, (which may be much larger than the distance between pinning centres).

The relation (2) may be viewed as a 3D generalization of simple 1D schemes previously considered for individually pinned flux lines [5], or for the shear limit of flux lines pinned by parallel equidistant planes [6]. It results from the following considerations. The $x$ component $-f_{i\alpha}$ of the total force $-f_i$ applied to $V_j$ through $S_j$ by the VL is:

$$-f_{i\alpha} = \int \int_{S_j} \sigma_i \cdot d_s$$ \hspace{1cm} (2a)

where $\sigma_i$ is a vector with components $\sigma_{\alpha\alpha}, \sigma_{\alpha\nu}$ and $\sigma_{\nu\nu}$. Choosing $S_j$ such as to satisfy (2) leads to:

$$-f_{i\alpha} = \int \int_{S_j} \sigma_{i\alpha}, d_{\alpha\alpha} = \int \int \int_{V_j} (d_{\alpha\alpha}/dx) dv.$$ \hspace{1cm} (2b)
For $B \gg \mu_0 H_{c1}$, the derivative of the magnetic pressure $d\sigma_{xy}/dx$ is the $x$ component of the Lorentz force per unit volume $\mathbf{J} \times \mathbf{B}$. Since $B$ and $J$ vary slowly at a scale given by the distance between pinning centres, the right member of (2b) corresponds to $J B V_i$.

The existence of a closed surface $S_i$ around each pinning centre requires $J$ to be higher than a minimum value $J_m$ which increases with the elastic interaction strength between flux lines. Each point of the space belongs to a particular $V_i$. The macroscopic pinning force $F(J)$ for $J_m < J < J_{c0}(0)$ is just the sum of the elementary modules $|f_i|$. However, even in this case, the randomness of the system and the residual interactions between flux lines combine in such a way that some pinning centres reach their maximal pinning force whereas their neighbours are still in a state $|f_i| < f_{pi}$. As a consequence, $J_{c0}$ is limited below the value $J_{cds}$, the critical current in the ideal « direct summation » (DS) limit, which is defined by:

$$J_{cds} B = \sum_i f_{pi} / \sum_i V_i = n_p \langle f_{pi} \rangle .$$

(3)

Where $\langle f_{pi} \rangle$ is the average value of $f_{pi}$ on pinning centres. It can easily be shown that a non zero pinning force relies on the existence of metastable states for flux bundles. For larger
interactions between flux lines, it is no longer possible to define a closed surface $S_i$ around each pinning centre, even for $J = J_{c0}$ (the condition $J_m \leq J_{c0}$ is no longer satisfied). Nevertheless, maximal pinning forces $f_{pi}$ may still be strong enough to induce local metastable states of the VL, while only a concentration $n_{eff} < n_p$ of pinning centres are simultaneously in such states. Consider a pinning centre at $|r| = 0$. The metastability criterion for individual (« single particle ») pinning is:

$$d|f_r|/du(0) > C$$

with:

$$C = d^2E_{el}/du(0)^2$$

$E_{el}$ is the elastic energy of the VL associated to a displacement field $u(r)$ such that 1) $E_{el}$ is the minimum elastic energy under the condition 2), 2) the state (metastable or not) of pinning centres located at $|r| > 0$ is not modified by the displacement. As long as the above criterion is fulfilled, the pinning regime is strong in the sense that $f_{pi}$ is larger than the random part of elastic forces acting on pinning centres. This regime is shown in figure 3a. Expressions of $C$ for various pinning microstructures have been calculated, however simple expressions can only be obtained in the case of dilute pinning arrays. In such a limit, where displacement fields of neighbouring pinning centres do not overlap, relation (4) is usually referred to as the Labusch criterion [7]. If verified with a sufficiently large margin, it leads to a macroscopic force approximately given by [8]:

$$BJ_{c0}(0) = n_{eff} f_{pi}.$$  

If most of the pinning centres in the material do not fulfill the above criterion (a case schematized in Fig. 3b), metastable states can only be produced by the collective action of many centres on large correlated flux bundles. In this case, interactions between flux lines make $F_{pi}$ much lower than the direct summation limit, as shown by the collective pinning (CP) theory described below. It is important to stress that before the CP concept was settled, the Labusch criterion was believed to define an absolute threshold, below which no macroscopic pinning force could occur (in strong contradiction with experiments).

Fig. 3. — Strong pinning (a) and weak pinning (b) regimes. In a), a small displacement of a flux line (shown by the dotted arrow) results in a variation of the pinning force $\delta f_i$ much larger than the variation $\delta f_t$ of the interaction force with other flux lines, whereas the reverse behaviour occurs in b).
2.1.3 Apparent trends of HTS. — Quite early after the HTS discovery, it was recognized that their very short coherence length could lead to a particular type of pinning, sometimes referred to as « chemical ». In low temperature superconductors (LTS), small scale (≈ 1 nm) chemical inhomogeneities can only produce vanishingly small pinning forces, owing to the order parameter stiffness at scales smaller than \( \xi \) (about 5 nm or more in LTS). In HTS, the order parameter should respond to very small scale defects since \( \xi \) is in the range 0.5 to 2 nm, depending on the crystalline direction.

Besides, unusually high flux creep rates (as compared with the previous LTS results) were obtained at the same time in HTS, pointing to rather low pinning energies, hence to small defects. Finally, a moderate dependence of the critical current versus \( B \) (in the range \( T \ll T_c \)) seemed to result from a finely dispersed array of pinning centres. Both tendencies were in agreement with the chemical pinning hypothesis. Oxygen vacancies (or interstitials), a quasi-universal source of chemical inhomogeneity in HTS, were then considered as the most probable source of pinning in these materials [9, 10].

For the theoretical treatment of such a microstructure, the collective pinning (CP) framework [8] seems to be required by the presumed weakness of the pinning forces. We give below a brief overview of this theory before discussing its relevance in the particular case of HTS.

2.1.4 Collective pinning theory. — For simplicity’s sake, we again restrict our presentation to the case where each pinning centre interacts with only one vortex.

The maximal elementary pinning force produced by the pinning centre \( i \) is defined as before as \( F_{p_i} \). However the maximal pinning force \( F_V \) available in a volume \( V \) is now quite smaller than \( \sum V F_{p_i} \), owing to the frustration effect between these elementary forces. Indeed, the actual elementary forces \( f_i \) acting on the VL are randomly distributed in direction and intensity. With a perfectly rigid VL and a randomly distributed array of pinning centres, the maximal resultant force acting on the VL in a volume \( V \) is given by \( F_V = \langle n_p V \langle f_i^2 \rangle \rangle^{1/2} \), where \( n_p \) is the pinning centre density and \( \langle f_i^2 \rangle \) is the mean square of the elementary forces (averaging has to be performed on volume and pinning centres).

It can be shown that at large scale, the VL rigidity is no longer able to produce the same frustration effect between pinning forces \( F_{V_i} \) acting on large neighbouring volumes \( V_i \). At these scales, a direct summation of \( F_{V_i} \) becomes justified, leading to a macroscopic pinning force which is scale-independent:

\[
J_{\text{CP}} B = - \sum_{V_i} F_{V_i} / \sum V_i
\]  

(7)

whatever is \( V_i \) larger than a limit defined below. However, this relation does not allow the critical current to be estimated since \( F_{V_i} / V_i \) is still unknown. The collective pinning theory relies on the definition of a correlation volume \( V_c \), basically defined as the volume in which the displacement field variations resulting from the pinning forces are smaller than \( r_p \) (this definition will also be used hereafter in other pinning regimes). In the CP theory, \( V_c \) corresponds to the smallest scales above which a direct summation procedure is justified. This theory makes two important hypothesis:

a) at scales smaller than the critical ones, the VL is considered as perfectly rigid;

b) the condition \( n_p V_c \gg 1 \) holds, allowing a statistical treatment of pinning centres inside \( V_c \).

Within these limits, the critical currents is simply defined by:

\[
J_{\text{CP}} B = (W/V_c)^{1/2}
\]  

(8)

\[
W = n_p \langle f_i^2 \rangle
\]  

(9)
Owing to the anisotropies of the vortex lattice and (eventually) of the superconductor, $V_c$ is an ellipsoid for 3D problems. Considering the case $B \parallel c$ in HTS, one has to define a correlation radius $R_{c3}$ in the $(a, b)$ plane and a correlation length $L_{c3}$ along the $c$ axis, such as $V_{c3} = (4/3) \pi R_{c3}^2 L_{c3}$. In this field configuration, strongly anisotropic HTS like Bi(2212) would not have a correlation length $L_{c3}$ (calculated in a 3D CP model) larger than the distance $d$ between the superconducting CuO$_2$ layers in usual conditions [11]. They should behave like 2D systems with $V_{c2} = \pi R_{c2}^2 d$. Other cases of reduced dimensionality occur if the condition $R_{c2,3} > a_0$ is not verified. The domain $R_{c3}$ (calculated) $a_0$ is usually defined as the single vortex limit of the CP model. The actual correlation radius in this case is $R_{c1} = a_0$, thus $V_{c1} = \pi a_0^2 L_{c1}$. For strongly anisotropic HTS with $B \parallel c$, a 0D limit can be defined with $V_{c0} = \pi a_0^2 d$. In this case, the system is constituted by uncorrelated 2D or « pancake » vortices [12].

If larger than their lower limits $a_0$ and $d$, the correlation lengths calculated by the CP theory define the scales at which the elastic energy of the perturbed VL roughly balance the statistical fluctuations of the pinning energy. Therefore, their determination requires knowledge of the pinning centre characteristics and of the elastic response of the VL. This is a complex task, since at small scale the elastic moduli $C_{11}$ and $C_{44}$ are strongly dispersive, and also because the estimation of $\langle \vec{f}_i \rangle$ is very uncertain, except for idealized models which may not be representative of pinning centres present in hard superconductors. However, the CP theory also provides a relation between $J_{c0}$ and $\langle \vec{f}_i \rangle$. Thus, expressions giving the correlation lengths as functions of the critical current and of the elastic constants may be obtained, from which the unknown quantity $\langle \vec{f}_i \rangle$ has been removed [13]. Some examples of this treatment are given in sections 4.1.2 and 4.2.2.

2.1.5 Collective pinning and HTS. — From the above presentation, it appears that $V_c$ is an increasing function of the elastic moduli, whereas $J_c$ is a decreasing one. To study the relevance of CP theory to HTS, one should keep in mind some general characteristics of these materials:

— as stressed in part I, the VL of high-$\kappa$ and anisotropic HTS is quite soft at usual fields for power applications (a few teslas). Thus, unless the pinning potential is very weak, correlation lengths are expected to be small;

— the electronic structure of HTS is very sensitive to disorder: localized states induced by native or irradiation defects, or impurities, have been evidenced by various experiments;

— most HTS samples exhibit large defects when characterized by electron microscopy. In the literature, many examples are found of high critical currents associated with at least one extended defect type such as: dislocations, twin boundaries, micro-precipitates, etc.

These remarks suggest that, whatever is the dominant pinning structure, HTS contain strong pinning centres for which the validity of the CP model is questionable. If plastic deformations of the VL allow some flux shear to occur around strong centres without depinning processes, $J_{c0}$ could still be controlled by a finely dispersed array of weak centres. If plastic deformations are forbidden by the strength of the VL, one should at least consider that both strong and weak pinning systems cooperate to withstand the Lorentz force.

For 3D VL (representative of Y(123)), flow without depinning requires vortex cutting processes (Fig. 4a) because each flux line is pinned by many centres. The stress threshold for such processes is high, at least in LTS. Thus some hybrid pinning regime is possible. For 2D VL (exemplified by Bi(2212) in high fields), the shear stress threshold for plastic flow without depinning (Fig. 4b) decreases rapidly with the distance between pinning centres. In the latter case, a distributed pinning structure is needed to maintain $J_c$ at high levels. Such a
2.2 THERMAL EFFECTS. — Thermal activation allows some characteristic flux volumes to jump over local pinning potential barriers, leading to dissipation and « flux creep » at currents $J$ below the threshold value $J_{\text{c0}}$. The simplest approach of this effect, the Kim-Anderson model [4], assumes 1) current independent values for the jump distance $d_j$ and the jumping volume

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Fig. 4. — Schematic description of flux displacements without depinning ; a) : cutting process of vortex lines in a 3D VL (requiring strong driving forces); reconnections figured by dotted lines (in the left drawing) allow the pinned vortex 1 to move away from the pinned center (right drawing); b) : plastic flow of 2D vortices between pinning centres 1 and 2.

microstructure is probably responsible for the pinning characteristics obtained on very anisotropic Bi and Tl compounds. However the question of its strength still deserves some complementary discussion (see Sect. 4.1.3).
$V_c$, and 2) a triangular shape for the pinning potential well $U_p(r_i)$. Taking $V_c$ as defined in section 2.1.4 for the jumping volume leads to the well-known field-current relation:

$$E = B\Omega d_j \text{sh} \left( \frac{BJV_c r_p}{kT} \right) \exp \left( -\frac{U_0}{kT} \right)$$

(10)

where $\Omega$ is the attempt frequency ($\Omega = 10^{10}$ s$^{-1}$) and $U_0$ is the maximum depth of the potential well: $U_p(r_i) = \text{Min} \left[ U_0 \left( \frac{r_i}{r_p} \right) - 1, 0 \right]$. The jump distance $d_j$ has sometimes been taken as the VL parameter, or as the inter-defect distance.

The assumption of current-independent jumping volume and $d_j$ may only be justified if the essential depinning process for dissipation does not depend on the correlations between the jumping flux unit and its neighbourhood. Otherwise, one should use the concept of «collective creep» (CC) [14], which considers new current-dependent correlation lengths $R_j$ and $L_j$. They define the smallest possible volume $V_j = R_j^3 L_j$ which can jump to a stable configuration when the neighbouring VL is kept at a fixed position. $R_j$ and $L_j$ are $J$-dependent because the stability of the configuration after the jump is determined by the balance between elastic energy $E_{el}$ and Lorentz force work $E_j = JB V_j d_j$. Although built within a different formalism, the vortex glass (VG) models [15] rely on similar considerations.

If the jumping volume $V_c$ as defined in section 2.1.4 is larger than the value of $V_j$ obtained from this balance, the creep process is not strongly influenced by correlations, and relation (10) is valid. This may occur for strong pinning regimes if the temperature is low enough. For weak collective pinning, $V_j$ becomes larger than $V_c$ (the correlated volume of the CP model at $T = 0$) as soon as $T > 0$. Indeed, $V_j$ is a decreasing function of $J$ and, at non-zero temperatures, one has $J < J_{c0}$ in all accessible experimental conditions. Besides, the lower limit of $V_j$ if $T \rightarrow 0$ is $V_c$, since in this limit they are both determined by the same energy balance if $J$ takes its highest possible value $J_{c0}$. As a consequence, the CP regime gives way to CC as soon as $T > 0$.

If strong pinning is a necessary condition for uncorrelated creep to occur, it is by no means a sufficient one. When $J$ decreases, $V_j$ becomes equal to the low temperature jumping volume $V_c$ at some characteristic current $J_{cr}$. For $J < J_{cr}$ one obtains a particular regime in which single particle pinning and collective creep coexist. In other words, collective effects related to creep phenomena should not be taken as evidence for collective pinning.

In the limit $J \rightarrow 0$, CC and VG theories have an important limitation, because they only take into account elastic processes. For decreasing current densities, $U(J) = J^{-a}$ grows until it becomes equal to the threshold $U_{pl}$ for plastic deformations [16-17]. Beyond this threshold, the activation energy of the creep is dominated by these plastic processes and remains finite in the limit $J \rightarrow 0$. In this plastic creep regime, only a weak dependence of $U$ versus $J$ is expected. ($J$ is already smaller than the range in which barrier lowering effects described in 3.3 are important.)

Within the strong pinning scheme, there should exist a high temperature regime in which correlation effects disappear even at low $J$. This may occur if the thermal energy $kT$ is higher than the typical elastic energy involved in an individual jump. An example is given in section 4.1.3, which discusses the creep behaviour of Bi(2212) compounds above 30 K.

2.3 CURRENT DEPENDENCE OF PINNING BARRIERS. — This dependence may result from two distinct mechanisms. The most obvious one results from the reduction of the barrier height experienced by the correlated flux volume, as a result of the Lorentz force work $BJV_c r_i$. It is reminiscent of the field-assisted emission of charge carriers, and should mainly influence the creep rate at high $J/J_{c0}$. It depends sensitively on the shape of $U_p(r_i)$. For the
triangular form assumed in the Kim-Anderson model, the barrier lowering $\delta U(J)$ is simply given by $BJV_c r_p$. For more complex potentials, one should apply the general relation:

$$U_0 - \delta U(J) = U_p(r_{i2}) - U_p(r_{i1}) - BJV_c r_i$$

(11)

where $U_0$ is the potential well depth for $J = 0$ and $r_{i1}(r_{i2})$ is the smallest (largest) value of $r_i$ for which $dU_p/dr_i = BJV_c$ in (11), it is assumed that all pins produce the same pinning potential. A logarithmic form of $U_p(r_i)$ for $r_i \gg r_{i1}$ leads to $U_0 - \delta U = U_0 \ln (J_{00}/J)$, a relation providing a satisfactory interpretation of various experimental results as: transport measurements of dissipation [18], or the exponential decrease of irreversible magnetization $M$ of Y(123) with temperature [19].

The above expressions for $\delta U(J)$ assume that the jumping volume and the jump distance are $J$-independent. They should be restricted to the strong pinning limit (or to the CP regime at $T = 0$), and to current (or temperature) values allowing to neglect correlation effects.

Within the CP model, such correlations are essential. They lead to another source of barrier dependence, $\delta U$ mainly depending on $J$ through $V_j$ and $d_j$. Collective creep (CC) [14] and vortex glass (VG) [15] theories were developed to take this type of phenomena into account. They both lead to a divergent barrier $U = J^{-\alpha}$ for $J \to 0$ (as long as $T$ is lower than a glass temperature $T_G$ defined in the VG model). This results from related divergences of $V_j$ and $d_j$ in the same limit.

The VG model was essentially developed to show the existence of a new phase below $T_G$, with zero dissipation in the limit $J \to 0$ (owing to infinite barriers), at variance from usual models of single particle creep. The CC model intended to give an account of the dissipation rate in various current ranges up to $J_{00}$. Both models gave a satisfactory account for measurements of $M$ on BSCCO from 5 to 26 K [12].

One problem with the CC theory in its present form is that the Lorentz force lowering of potential barriers is ignored: the scale of the flux bundle and the current corresponding to this scale are obtained by equating deformation ($E_{el}$) and pinning ($E_p$) energies with the Lorentz force work $E_l$. Then the barrier which is used for the estimation of the creep rate is again $E_{el} = E_p$, although some reduction below this value should occur. The loss of accuracy resulting from this procedure might be negligible for $J \ll J_{00}$, but is certainly not in the limit $J \to J_{00}$ for which $V_j \to V_c$. As a result, the CC neither provides a precise description of the limit $T \to 0$ in the CP regime, nor of the eventual crossover with the single particle creep regime.

2.4 STRONG PINNING AS A LIMIT OF THE COLLECTIVE PINNING REGIME. — For superconductors characterized by larger ratios of pinning to elastic energies, the correlated volume shrinks to scales at which the pinning disorder can no longer be considered as random. This is obviously the case if $n_p V_c \approx 1$, but some deviations to the CP behaviour should already occur if $\sqrt{(n_p V_c)} \ll n_p V_c$ is not justified.

In the range $n_p V_c \approx 1$, a « modified direct summation » rule seems to be applicable, as revealed by computer simulations [20]. This corresponds to: $J_{00} B = n_p \langle f_i^2 \rangle^{1/2}$. Although $\langle f_i^2 \rangle^{1/2}$ is smaller than the maximal elementary force $f_{pi}$ averaged on pins, such a relation is very close to the DS limit characteristic of strong pinning.

The derivation of $V_c$ from the CP theory for superconductors with known $J_{00}$ and elastic moduli, but characterized by a strong pinning regime, may lead to meaningless correlated volumes $V_c < n_p^{-1}$. Obviously, $n_p^{-1}$ is a lower limit for the actual $V_c$. However this discrepancy does not easily appear, since precise experimental data about the array of pinning centres are not generally available. Whereas model samples of LTS have been prepared and characterized,
equivalent experiments are still lacking for HTS. Only irradiation experiments presently allow a relation between \( J_c \) and \( \eta_p \) to be established.

3. Practical definition of the critical current.

3.1 Critical currents from experiments. — The above section presents the current theoretical view, defining the critical current density \( J_{c0} \) as the highest \( J \) value for which the dissipation rate is strictly zero. However, such a definition leads to \( J_c = 0 \) for any \( T > 0 \), owing to thermal activation effects. This difficulty is not removed in CC or VG models, since the pinning energy remains finite as soon as \( J > 0 \).

On the experimental side, a large majority of results relies on a critical current definition related to a small but non-zero dissipation. For example, transport measurements refer to an electric field criterion \( E_c \), such that the current density is said to be critical when the field \( E_c \) is reached in the sample. Irreversible magnetization measurements imply a criterion \( E_c = (d\phi/dt)/2 \pi R \), where \( \phi \) is the magnetic flux through the sample section \( \pi R^2 \). We use below the notation \( J_{ce} = J(E_c) \) for these critical currents at non-zero dissipation rates. Many experimental situations reported in the literature corresponds to \( 0.1 \mu \text{V/cm} < E_c < 10 \mu \text{V/cm} \).

We first give a very simple presentation of the temperature dependence of \( J_{ce} \), neglecting correlation effects between flux bundles as well as effects related to the \( U_p(R) \) non-linearity. A more complete description of phenomena is made in section 3.3. In the Kim-Anderson model, \( J_{ce} \) is related to the pinning potential through the relation (10), taken in the limit \( B JV_c r_p \gg kT \), which gives:

\[
J_{ce} = \frac{U_0}{B V_c r_p} \left( 1 - \frac{kT}{U_0} \ln \frac{B \Omega d_j}{E_c} \right). \tag{12}
\]

This expression is only valid for a triangular pinning potential. In many experimental situations, \( \ln (B \Omega d_j/E_c) \approx 20 \), therefore \( J_{ce} \ll J_{c0} = U_0/B V_c r_p \), unless \( kT \ll U_0 \) is satisfied. For LTS, \( kT/U_0 \approx 10^{-3} \) for \( T = T_c/2 \) is not unusual. For HTS, \( kT/U_0 \) for \( T = T_c/2 \) is expected to be only in the range \( 10^{-1} \) to \( 10^{-2} \). The steeper fall of \( J_{ce}(T) \) observed in HTS essentially results from this higher ratio. In this case, the above relation becomes invalid near \( T_c \) because \( BJ(E) V_c r_p \ll kT \) at typical electric fields, and backward jumps occur at a significant rate. The electric field is thus a hyperbolic sine function of the current, which can be developed to first order. This approximation corresponds to the « thermally activated flux flow » (TAFF) regime [21], in which the \( J(E) \) characteristic is linear:

\[
J = \frac{kTE}{B^2 \Omega d_j V_c r_p} \exp \left( -\frac{U_0}{kT} \right). \tag{13}
\]

In other words, the characteristic electric field threshold \( E_0 = B \Omega d_j \exp \left( -\frac{U_0}{kT} \right) \) for non-linear effects becomes higher than the electric field \( E_c \) produced by the experiment. In this case, it is useless to define \( J(E_c) \) as a critical current since the resistivity \( \rho_{\text{TAFF}} = B^2 \Omega d_j V_c r_p/kT \) \exp \left( -\frac{U_0}{kT} \right) gives an electric field-independent description of the dissipation rate.

3.2 Critical current criterion in HTS from a technical point of view. — Up to now, critical current criteria related to applications have generally been discussed in the context of liquid helium temperatures [22, 23]. We discuss below a possible adaptation of those to the case of HTS used up to 77 K.
Most high current applications of superconductors require to combine a high current density (10^4 to 10^5 A/cm²) with a low dissipation rate. More precisely, the ratio $K$ between the resistive losses $P$ and the useful power $P_u$ delivered by the equipment must be comparable or lower than for room temperature classical designs. If a loss amount $P_c$ occurs at low temperature, $P$ is given by $P_c \eta$, where $1/\eta$ is the refrigerator efficiency. $P_u$ is typically proportional to the product $IL_s$, where $I$ is the current and $L_s$ the conductor length. Thus:

$$K = \alpha P_c \eta / IL_s = \alpha \eta JEV_s / JV_s = \alpha \eta E$$

where $\alpha$ is a parameter roughly equal for room and low temperature conceptions, $V_s$ is the superconductor volume, and $E$ is the electric field in the superconductor. For normal conductors at room temperature, $K$ is just $\alpha \rho_N J_N$, where $\rho_N$ and $J_N$ are respectively the resistivity and current density in the normal conductor. Acceptable losses in the superconductor must then correspond to the criterion:

$$\eta E \leq \rho_N J_N \quad \text{(15)}$$

High power equipments are usually designed with $\rho_N J_N$ in the range $10^{-3}$ to $10^{-4}$ V/cm. As far as LTS are concerned, helium refrigerators have low efficiencies $1/\eta = 2 \times 10^{-3}$ In this case, the maximal electric field in the superconductor should not exceed 0.1 $\mu$V/cm, which is just the standard chosen for the definition of critical current density in LTS.

If the $E(J)$ characteristic is extremely non-linear, the critical current is not very sensitive to the criterion choice. In HTS, thermal activation reduces (or even cancels) the $E(J)$ non-linearity, and it will be important to define precise standards for $E_c$. In this respect, one has to consider two modes of superconductor utilization:

- Applications in which the superconductor current may be fixed by an external source. In this case, long wires are needed, a requirement which could be difficult to fulfill with Y(123) compounds, owing to weak link problems. BSSCO could be a better candidate to provide such components. For wires, a criterion related to the equipment efficiency is still relevant. However losses occurring at 77 K may be absorbed with $\eta = 0.1$, and a criterion $E_c \approx 10 \, \mu$V/cm is now allowed;

- Applications in which the magnetic flux is trapped in massive superconductor components (superconducting "permanent magnets"). In this case the electric field criterion is no longer determined by efficiency considerations, but by stability problems. For example, a relative drift of the field of about 1 %/hour in a 1 cm² section requires $E_c \approx 10^{-4} \, \mu$V/cm, a much more stringent criterion than above. Y(123) seems to be better choice in this case, because it allows higher pinning energies, hence smaller creep rates, to be obtained at a given $J$.

Owing to the reduced non-linearity of $E(J)$ in HTS, a relative variation of $10^5$ on the field criterion (as found above) is expected to induce a sensible shift of the resulting $J_{cc}$. Thus, various criteria will have to be taken into account, depending on the application type. Fundamental studies at the related dissipation rates are needed to evaluate the future performance of these materials.

If the TAFF regime (where the non-linearity vanishes) extends up to $E_c$, the concept of critical current becomes useless and should give way to some optimal current density, determined by the same considerations as in normal conductors. TAFF regime is not expected to be compatible with trapped flux-type applications; however TAFF resistivities lower than $10^{-8} \, \Omega$ cm could be used at 77 K with wire-type components.

As a conclusion of this section, it appears that present experiments and potential applications of HTS rely on the dissipation level corresponding to some particular values $E_c$ of the electric
field. Qualitative non-equilibrium «phase diagrams» of HTS have been described in the literature [11, 14, 24, 25]. Thus, constant-E scans of these diagrams should allow the identification of the mechanisms which limit $J_{cc}$ in different field or temperature domains. The next section is an attempt to describe the general trends expected in this respect, in various temperature ranges and for moderate fields ($\approx 1$ T).

However the discussion presented below is restricted to the case where individual pinning energies are larger than $kT$. With such an hypothesis, the influence of thermal fluctuations on the pinning behaviour is strongly reduced, as compared to the predictions of [24] (depinning lines related to thermal softening of the pinning potential disappear from the $J_{cc}$ diagram). This point of view is related to our conclusion about the prevalence of strong pinning regimes in HTS. Besides, we do not detail here the dimensional characteristics of the pinning and creep regimes.

### 3.3 Relation with Pinning Regimes of HTS

The elements presented above may now be combined to provide a qualitative description of the phenomena which control $J_{cc}$ when the temperature domain is scanned at a given electric field value.

Two possible scenarios are presented in figure 5. The branch (1) of the $J_{cc}(T)$ curve refers to a strong pinning regime, with a rapid fall of $J_{cc}(T)$ resulting from thermal activation at the level

Fig. 5. — Examples of possible diagrams for the experimental critical current $J_{cc}$ with two different regimes simultaneously shown in the low temperature domain. Branch (1): strong pinning and single particle creep (SPC) with $V_{j}(J_{cc}) < V_{c}$. Branch (2): collective pinning at $T = 0$ and collective creep for $T > 0$. The dashed-dotted portion indicates the $J_{cc}$ range where the $T^{-\alpha}$ variation predicted by the CC theory fails, because $J_{cc} = J_{cr}$. Branch (3): domain in which $J_{cc}$ is controlled by plastic deformations of the VL. Branch (4): TAFF regime. The relative position of crossover points depends on the magnetic field, the electric field criterion, the pinning characteristics and the VL elastic or plastic properties.
of individual pinning centres. This «single particle creep» (SPC) gives way to a CC regime when the bundle volume for collective creep $V_J(J_{ce})$ becomes larger than $V_c$, the correlated volume at $T = 0$ ($V_c \approx n^{1/4}_{\text{eff}}$ for point-like strong pinning centres).

Further decrease of $J_{ce}(T)$ leads to a growing jump energy $U(J_{ce}) \sim J_{ce}^{-\alpha}$, with various $\alpha$ values depending on the dimensional characteristics of the creep process, until $U(J_{ce}) \approx U_{pl}$. The critical current at which this crossover occurs is referred to as $J_{pl}$. At still higher $T$, $J_{ce}$ decreases in relation with the roughly constant activation energy $U_{pl}$ (branch 3 of Fig. 5), until a «pseudo» glass temperature $T_G$ is reached (the regime below $T_G$ is not a true VG owing to the occurrence of plastic processes). For $T > T_G$, the correlated volume decreases rapidly and the activation energy steeply falls below $U_{pl}$. In a single particle pinning scenario, if the criterion for TAFF behaviour with flux bundles of volume $n^{1/4}_{\text{eff}}$ has been reached below $T_G$ (which is the case represented in Fig. 5), $J_{ce}$ is quickly reduced to very small values as soon as $T > T_G$ and then loses its signification in the linear TAFF regime. In this case, the curve $T_G(B)$ appears as an irreversibility line. However, individual pinning energy may be high enough, as compared to $kT_G$, to still maintain some non-linearity at the electric field $E_c$. Thus the irreversibility line may also be a depinning line distinct from $T_G(B)$.

The second scenario represented in figure 5 (branch 2) refers to collective pinning at $T = 0$. In this case, the CC theory applies as soon as barrier lowering effects may be neglected. The $J_{ce}(T)$ curve is thus expected to show a $T$-dependence similar to the first scenario (when the latter is considered in the domain $V_J(J_{ce}) > V_c$). However, in this case, the loss of correlation effects which occurs at $T_G(B)$ immediately induces a reversible behaviour. Thus, indications about the pinning regime may be obtained from the dissipation character at $T > T_G$ [15].

Obviously, the relative position of the different crossover points of figure 5 may change with $B$ and with the characteristics of the VL and the pinning centre array. However, a general presentation of all the possible behaviours is out of the scope of this paper. We rather intend to discuss more precisely the interpretation of $J_{ce}(T)$ for the two compounds Y(123) and BSCCO for which many experimental results are already available.

4. Discussion of specific examples.

4.1 BSCCO COMPOUNDS.

4.1.1 General behaviour of $J_{ce}(B, T)$. — Many studies on non-irradiated single crystals or epitaxial thin films show a critical current density in the range $5 \times 10^5$ to $5 \times 10^6$ A/cm$^2$ at 4.2 K [26-28]. Irradiation allows to increase $J_{ce}$ above $10^7$ A/cm$^2$ [29].

Up to 10 K, $J_{ce}$ is not strongly influenced by magnetic fields. Experiments performed up to 38 T showed $J_{ce} \sim 10^4$ A/cm$^2$ [30]. However $J_{ce}$ decreases strongly with $T$ and becomes extremely sensitive to the magnetic field component $B_c$ (along $c$ axis) above 30 K [31]. At this temperature, the TAFF regime is sometimes already observed for $B_c > 0.1$ T [32], although the irreversibility line may be shifted to higher fields or temperatures by the introduction of strong pinning centres.

4.1.2 Correlation lengths. — The very large anisotropy of BSCCO compounds, combined with the high current densities observed at low temperature, leads to the conclusion that the correlation length $L_c$ for $B \parallel c$ is smaller than the crystal parameter $d$ along the $c$ axis [11]. It means that layers of 2D vortices along $(a, b)$ planes are pinned independently of each other. The next step is to compare the correlation radius $R_{c2}$ to the VL parameter $a_0$, in order to see if
the pinning regime is 0D or 2D for this field configuration. In the CP regime with a short range pinning potential \( r_p \approx \xi_{ab} \), the correlation radius is roughly given by [12]:

\[
R_{c2} \approx \left( \frac{C_{66} \xi_{ab}}{J_{c0} B} \right)^{1/2}
\]

The dispersionless shear modulus \( C_{66} \) is given by \( \phi_0 B / 16 \pi \mu_0 \lambda_{ab}^2 \). With \( \lambda_{ab}/\xi_{ab} \approx 100 \), and \( J_{c0} \) in the range quoted above, \( R_{c2} \) would be smaller than \( a_0 \) up to a few teslas. Since the smallest possible value of \( R_{c2} \) is \( a_0 \), it is concluded that, if CP is the correct approach, the pinning problem is 0D (independently pinned fluxons).

However, in this case, CP theory is only relevant if many pinning centres interact with one single fluxon. Since \( r_p \) is not expected to be much larger than \( \xi_{ab} \), this requires a very high concentration of pinning centres. Only very small scale defects, such as randomly dispersed oxygen vacancies, could satisfy this requirement. Nevertheless, this scenario is difficult to reconcile with pinning energy measurements, as shown in the next section.

4.1.3 The problem of oxygen vacancies. — The discussion on pinning by oxygen vacancies must take into account experimental results related both to pinning energies and to critical current. As stressed above, individual pinning energies are not easily accessible from experiments owing to barrier lowering and collective creep effects. The main point we intend to show in this section is that, if the mean distance \( d_p \) between pinning centres (oxygen vacancies) is such that \( d_p \approx (n_p)^{-1/3} < \xi_{ab} \), pinning energies deduced from experiments performed at \( T > 30 \) K are those of single fluxons. The argument is based on the weakness of the elastic coupling energy involved in small fluxon displacements, which cannot maintain a collective creep regime at high temperatures.

The CC approach in this respect is to define a current-dependent correlation radius \( R_1 \) for the 2D case from the balance between the elastic energy and the Lorentz force work. This leads to \( R_1 \approx R_{c2} (J_{c0} / J)^{5/8} \) [12] (for the correlation length \( R_1 \) measured in the jump direction). Since at 30 K, experimentally accessible currents \( J \) are much smaller than \( J_{c0} \), one may have \( R_1 \approx a_0 \). In this case the pinning energy deduced from creep phenomena would be \( U_1 N_i^p \), with \( U_1 \) the pinning energy of a single fluxon, \( N_i \) the number of fluxons involved in a correlated jump and \( \alpha \) a positive exponent defined in the CC theory. However this approach is only relevant if the probability for an individual jump to lead to a dissipative process is much lower than the occurrence probability of the collective jump. In order to see in which case this condition is satisfied, we consider a volume \( V \), larger than the interaction volume \( \lambda_{ab}^2 \lambda_c \), in which a fluxon experiences an individual jump of length \( d_j \approx 2 r_p \approx 2 \xi_{ab} \). This \( d_j \) value should correspond to the typical distance between pinning potential minima in a highly concentrated pinning array characterized by \( d_p \approx \xi_{ab} \).

The average fluxon velocity \( \langle v \rangle \) in this volume is given by:

\[
\langle v \rangle = N_i^{-1} \sum_i \Omega d_j \exp(-U_p/kT) \sin (f_{ti} r_p/kT)
\]

where \( f_{ti} \) is the projection along the Lorentz force direction of the elastic interaction force acting on the vortex \( i \), and \( N_i \) is the fluxon number inside \( V \). The balance of forces at a macroscopic scale \( V^{1/3} \) requires:

\[
\sum_i f_{ti} = JBV.
\]

We now assume that the jumping fluxon is not close to the boundary of \( V \), and thus cannot interact with fluxons outside of \( V \). As a consequence \( \sum_i f_{ti} \) is constant during the jump process.
However, correlation effects prevail only if \( \langle v \rangle \) is modified during the jump by an amount \( \delta v \) at least of the order of \(- \langle v \rangle / N_j\), reflecting a high probability for the fluxon to undergo a jump back (or near) to its initial position (allowing a non dissipative global process). The key point is that this requirement is compatible with relation (18) only through the non-linearity of the function \( g_i = sh (f_{ti} r_p/kT) \). Indeed, if \( f_{ti} r_p < kT \), then \( \sum_i g_i \approx JBV r_p/kT \) is not modified by the jump, and the mean velocity \( \langle v \rangle \) as defined by (17) is also a constant.

The forces \( f_{ti} \) are related to the typical elastic energy variation resulting from a jump:

\[
E_{elo} = C_L d_j^2/2
\]

which would correspond to a jump from a pinning site located at a minimum of the elastic potential. \( C_L \) is a lumped elastic constant obtained by keeping all the fluxons at fixed positions except the jumping one. It is shown in Appendix that \( E_{elo} \approx 35 \text{ K} \) for \( d_j \approx 2 \xi_{ab} \).

We first assume that the fluxon distribution is not strongly modified by thermal excitations. In this case, the mean square value \( \langle u^2 \rangle \) of the displacement field at the centre of fluxons is of the order of \( d_j^2/2 \). Since \( f_{ti} = 2 E_{elo} u/d_j \), one obtains:

\[
\sqrt{\langle f_{ti}^2 \rangle} = 2 E_{elo} \sqrt{\langle u^2 \rangle/d_j^2} = E_{elo}/d_j \tag{20}
\]

\[
r_p \sqrt{\langle f_{ti}^2 \rangle/kT} \approx (r_p/d_j) E_{elo}/kT . \tag{20bis}
\]

For temperatures high enough to strongly modify the thermal distribution of fluxons, the condition \( kT \gg E_{elo} \) would be valid. In this case, with an average elastic energy per fluxon given by \( E_{elo} \langle u^2 \rangle/d_j^2 \approx kT \), one obtains:

\[
r_p \sqrt{\langle f_{ti}^2 \rangle/kT} \approx (2 r_p/d_j)(E_{elo}/kT)^{1/2} \tag{21}
\]

Both relations (20) and (21) lead to \( r_p \sqrt{\langle f_{ti}^2 \rangle/kT} < 1 \). (In the first case, \( E_{elo} \approx kT \) but \( r_p/d_j < 1 \).) As a consequence, a TAFF regime of uncorrelated fluxons is expected for temperatures such that \( kT \gg E_{elo} \approx 35 \text{ K} \), whatever the value of \( J \).

Considering for instance the results of Palstra et al. [32], it appears that pinning by randomly dispersed oxygen vacancies has to take into account a pinning energy \( U_0 \) in the range of 500 K at \( B = 1 \text{ T} \) and \( T = 35 \text{ K} \). Since observed critical currents also require a high vacancy concentration such that \( \langle n_p \rangle^{-1/3} < \xi_{ab} \), the distance between pinning potential minima, which determines the individual jump distances \( d_j \), should not be larger than \( 2 \xi_{ab} \). Thus, the previous discussion is relevant and \( U_0 \approx 500 \text{ K} \) should represent the pinning energy of a single fluxon. A strong contradiction appears on this point, owing to the very large number (larger than \( 10^3 \)) of oxygen vacancies which would have to interact with one fluxon in order to explain such a \( U_0 \) in a CP model, whereas only 10 are expected to be involved in the fluxon core where the pinning interaction is thought to be significant.

One has also to consider the interpretation in which the measured activation energy would be related to flux displacements controlled by 2D VL dislocations [16]. However such a regime still implies individual pinning energies at least of the same order of magnitude as the measured \( U_0 \) for a sizeable fraction of fluxons, in order to obtain a prevalence of plastic creep over elastic creep related to depinning events (a highly defective VL can only reduce \( C_L \) below its value in a perfect VL). Thus, this interpretation does not release the contradiction with the low pinning energy expected from randomly dispersed oxygen vacancies.

Nevertheless, it is important to stress that within a different hypothesis about the origin of pinning forces, a larger value of \( d_j \) could allow a CC creep model to be valid up to temperatures
higher than 35 K. In such a model, the limitation of $U(J)$ by plastic creep energies $U_{pl}$ related to the displacement of 2D VL dislocations could explain the experimentally observed linearity of the field-current relation $E(J)$ [32].

A natural conclusion is that if oxygen vacancies are mainly responsible for pinning, they cannot be viewed as an uncorrelated disorder at the scale $r_p$. One should rather think about clusters of vacancies, which can be treated as individual defects. Since it has been shown that fluxons are pinned independently of each other at moderate $B$, one has all the ingredients of a strong pinning scenario. Other types of large defects are also expected to lead to a strong single particle pinning regime.

4.2 Y(123) COMPOUND.

4.2.1 General behaviour of $J_{ce}(B, T)$. — In unirradiated Y(123) samples, many studies have shown that $J_{ce}$ remains typically larger than $10^4$ A/cm$^2$ up to 5-10 T [33] and larger than $3 \times 10^5$ A/cm$^2$ up to 20 T [34]. A low field critical current peak is observed, the width and height of which seems to depend on the sample size and microstructure. At low fields, $J_{ce}$ decreases with $T$ roughly as $\exp(-T/T_0)$, with $T_0 = 10-20$ K [35]. At intermediate temperatures (30-70 K), $J_{ce}$ often exhibits a complex $B$-dependence, with two distinct maxima: a zero field centered peak, which becomes quite narrow at high temperatures, and a medium field (1-3 T) broad peak, above which $J_{ce}$ decreases rapidly.

The general trend of the decrease of $J_{ce}$ with field and temperature is much slower than for BSSCO, since critical currents higher than $10^4$ A/cm$^2$ are still observed at 77 K with $B = 1$ T (B $\parallel$ c). However, the irreversibility line $J_{ce} = 0$ still appears at lower field/temperature than the $H_{c2}(T)$ line: generally, $J_{ce} \rightarrow 0$ at a few teslas in the configuration B $\parallel$ c.

4.2.2 Zero temperature correlation length. — The discussion is restricted to the configuration B $\parallel$ c. High critical currents obtained at low temperature in Y(123) again indicate a (calculated) correlation radius $R_{c3} < a_0$ [36]. If the CP theory does apply, it is the so-called « single vortex limit » of this model which has to be considered at moderate fields. However, the anisotropy factor $\Gamma \approx 5$ allows a better coupling between CuO$_2$ planes than in BSSCO. Thus the correlation length is now larger than $d$, as shown by the relation $L_{c1} = (C_{44} r_p / J_{ce} B)^{1/2} \approx 5$-10 nm at $B = 1$ T and with $J_{ce}$ values quoted above. Here we take as before $r_p \approx \xi_{ap}$, and the high wave vector limit $4 \sqrt{3} C_{66} \Gamma^2$ of $C_{44}$ is chosen [37]. More precise evaluations of $C_{44}$ and $r_p$ would not change the conclusion $L_{c1} > d$.

4.2.3 Pinning regime. — The question arises as to whether the $L_{c1}$ value obtained above represents the actual correlation length, or if the latter is larger and corresponds to the average distance $\langle \langle d_i \rangle \rangle$ between strong pinning centres along a vortex line. Indeed there is no clear cut indication that the main pinning microstructure can be considered as random at a scale $L_{c1}$.

Oxygen vacancy rows, following the central plane of Y(123) unit cells, do not appear correlated along the c axis. In principle, they could lead to a pinning disorder compatible with the application of the CP theory at a scale $\approx$ 5-10 nm along the c axis. Oxygen vacancies in the CuO$_2$ planes have also been suggested as a possible source of pinning. However, no precise correlation other than a progressive decrease of pinning energy (related to a higher anisotropy and a reduced $T_c$) has yet been established between critical current density and oxygen deficiency in Y(123) [38].

Extended defects (twin boundaries, dislocations, precipitates) are not expected to induce elementary forces with a random character allowing a statistical treatment at the scale $\approx 5$-10 nm. Thus, they should only be considered as a possible source of strong pinning.
Even without a precise identification of the pinning microstructure, the simple fact that \( J_{\text{ce}} \) is very weakly \( B \)-dependent in the high field range could be viewed as an indication in favour of the 1D CP regime. However, this observation may also be interpreted in a strong pinning model, as shown below. The energy minimization of a vortex line requires:

\[
dE_v/d\langle d_i \rangle + dE_p/d\langle d_i \rangle = 0
\]  

(22)

where \( E_v \) and \( E_p \) are the average deformation and pinning energies of the vortex line per unit length. The average distance \( \langle d_i \rangle \) is connected to the concentration \( n_{\text{eff}} \) of active pinning centres defined in \cite{8} by \( \langle d_i \rangle^{-1} = n_{\text{eff}} a_0 \). If we first assume that the line is extremely soft, in order to justify a direct summation procedure, this means only that very large deformations \( u \) are allowed, up to the limit \( u = a_0 \) where vortices begin to interact. Thus it is clear that the single vortex regime implies some limitation of \( n_{\text{eff}} \) (and consequently of the macroscopic force) by the vortex stiffness, a behaviour often associated with the CP regime. But the latter is only justified if at the scale \( L_{\text{cl}} \), the typical variation \( \delta u \) of \( u \) is of the order of \( r_p \). For \( \delta u \gg r_p \), elastic forces are smaller than maximal pinning forces at the minimum energy configuration of the line, and a strong pinning regime is realized. In this case, a direct summation procedure on active pinning centres works better than the CP relations. In other words, the single vortex limit is compatible with strong pinning as long as \( r_p < \delta u < a_0 \) is justified. In \( \gamma(123) \), \( r_p/a_0 \approx 0.3 \times 10^{-2} \) at \( B = 1 \) T. If \( \delta u = 5 \) nm, this regime may be observed up to \( B \approx 30 \) T as required by experimental results.

At low fields, the \( J_{\text{ce}} \) peak seems to be related to a pinning effect by a large scale microstructure, the efficiency of which disappears at medium or high fields owing to the progressive saturation of available pinning sites. The low field peak becomes usually higher and wider when large scale strong pinning centres are purposely added, for example by irradiation, an effect which is coherent with the above interpretation \cite{39}. Recent experiments on high \( J_{\text{ce}} \) \( \gamma(123) \) with \( Y_2O_3 \) inclusions have been satisfactorily interpreted by assuming that the concentration of active pinning centres increases only as \( B^{1/2} \). The resulting \( J_{\text{ce}} \sim B^{-1/2} \) dependence is in agreement with experimental results \cite{40}.

4.2.4 Creep and experimental critical current \( J_{\text{ce}} \) at non-zero temperatures. — In a strong pinning scheme, the fast decrease of \( J_{\text{ce}} \) with \( T \) may be attributed to the thermal activation of single pinning centres \cite{19}. However, for current densities \( J_{\text{ce}} \) lower than \( J_{\text{cr}} \), only correlated jumps can lead to dissipative processes (as recalled in Sect. 2.2 and 2.3). Thus it is natural to expect that, in some intermediate temperature range, \( J_{\text{ce}} \) exhibits a behaviour related to collective creep. The second maximum of \( J_{\text{ce}} \), usually observed at \( B = 1-3 \) T between 30 and 70 K has been interpreted in this way. It is believed to result from a 1D-3D dimensional crossover, occurring when the correlation length \( L \) becomes of the order of \( a_0 \) \cite{41}.

It would be interesting to consider the \( B \) dependence of \( C_{44} \) as a possible source of this effect: at intermediate fields, interactions between vortices add to the low field term \( C_{44}^{(\text{corr})} \) (related to the stiffness of isolated vortices) the usual dispersive term \( C_{44}^{(1)} \). The lowest field range \( B \), where \( C_{44}^{(1)}(k) \approx C_{44}^{(\text{corr})} \), depends on \( k \). For \( (k_x^2 + k_y^2) \approx (\pi/a_0)^2 \) and \( k_z \approx \pi/L_{\text{cl}} \approx 0.5 \) nm, \( B \) is in the range of 1 to 4 T, which is the field domain where \( J_{\text{ce}} \) decreases with \( B \) at 30 K. At higher temperature, \( B_{\text{cr}} \) is expected to decrease, in agreement with experiments, since \( L \) has to be substituted for \( L_{\text{cl}} \) in order to obtain the essential value of \( k_z \). (The crossover condition is \( a_0^2 = \Gamma^2 L \).)

It should be stressed that a crossover \( R_1 \approx a_0 \) should also occur in this intermediate temperature domain \cite{41}.

In the high field/high temperature range, the rapid decrease of \( J_{\text{ce}} \) with \( T \) and \( B \) is related to the fact that the correlated volume \( V_j \) is strongly reduced by thermal excitations. For instance,
if the irreversibility line $B_{irr}(T)$ is a melting line, $R_1$ is again given by $a_0$. The correlation length $L_1$ along the vortex lines is also reduced below its maximum value in the 30-70 K domain. The effective pinning energy barrier, which varies as a positive power of $V_j$ [14] becomes too small for an irreversible behaviour to be observed. This interpretation is coherent with the rapid variation of the $E(J)$ non-linearity at $B_{irr}(T)$, as shown by experiments devoted to the quest of the vortex glass state [42].

5. Conclusions and characteristics of efficient pinning microstructures.

5.1 Pinning regimes. — For moderate fields (1 – 10 T) as considered in this paper, the single vortex limit seems to be valid for both Y(123) and BSCCO since these compounds allow critical current densities higher than $10^6$ A/cm$^2$ at low temperatures. Whereas the large anisotropy factor $J^2\approx 3000$ of BSCCO leads to a OD-type pinning regime (uncorrelated 2D fluxons), the moderate $J^2\approx 25$ of Y(123) results in a 1D-type behaviour (uncorrelated vortex lines).

— In BSCCO, a strong pinning regime prevails, with maximal elementary forces larger than the random part of elastic forces acting on pinning centres. In Y(123), the stiffness of single vortex lines may lead to collective pinning if the dominant microstructure produces elementary forces with random variations at the scale of 5 to 10 nm. However, the precise source of such a small scale pinning has yet to be identified. Pinning microstructures with characteristic scales larger than 5 to 10 nm along the c axis should result in a strong pinning regime for Y(123).

— At intermediate temperatures, collective effects may be observed on the experimental critical current $J_{ce}$ (in spite of the fact that a strong pinning regime might prevail). These effects allow an increase of the effective pinning energies $U_{eff}$ and a slowdown the decrease of $J_{ce}$ up to the vicinity of the irreversibility line $B_{irr}(T)$. Near $B_{irr}(T)$, the minimum size of flux bundles involved in dissipative jumps is strongly reduced, thus effective pinning barriers may be again of the same order of magnitude as individual pinning energies. If $U_{eff}/kT < 20$, a TAFF behaviour is observed at electric fields $E_c \sim 1$ $\mu$V/cm. This general scheme could be applied to both Y(123) and BSCCO, but with different characteristic temperatures and fields owing to the large difference between the VL elastic module of these compounds. The pinning energy value observed near $B_{irr}(T)$ in BSCCO seems to rule out the hypothesis of randomly dispersed oxygen vacancies as the major source of pinning in presently available samples.

5.2 Optimal microstructure. — Pinning microstructure must be optimized in view of a specific temperature and field domain and by taking into account the VL anisotropy. Up to now, there is no precise prescription on how to increase $U_{eff}$ by collective effects in order to shift the irreversibility line at higher fields or temperatures. Taking only individual pinning into account, one should search for the best compromise between contradictory requirements:

— a reduction of the elastic creep rate, which requires large pinning energies and consequently large pinning centres;
— a reduction of the plastic creep rate (or an increase of the $J_{c0}$ limit for plastic shear of the VL), which requires a high concentration of pinning centres;
— the conservation of a high critical temperature, which requires that perturbed domains associated with pinning centres are separated by distances $d_p \gg \xi_{ab}$ and leave a large percolating volume of defect-free material.

In BSCCO, this optimization is especially challenging since the quasi-2D properties of the VL simultaneously lead to reduced pinning energies and to a low threshold for plastic flux shear. The largest defect dimension measured perpendicularly to the vortex axis must be larger
than \( \varepsilon_{ab} \) in order to benefit from kinetic pinning effects [19]. However, the distance \( d_i \) between intersections of active defects with a particular CuO\(_2\) plane should not be large as compared to \( a_0 \), in order to keep the shear threshold in the range \( 10^4 \) to \( 10^5 \) A/cm\(^2\). For \( B = 1 \) T, this implies \( d_i \approx 50 \) nm. Columnar defects produced by ionic irradiation seem to approach this optimal microstructure [43] and have allowed a significant shift of the irreversibility line. However their efficiency at 77 K has yet to be demonstrated.

In Y(123), plastic flow requires continuous inflation of curved vortex segments between pinned points, or vortex cutting. The first process defines a current density limit in which enters the distance \( d_i \) between pinned points [34]. For \( J_{ce} \) to be in the range \( 10^4 \) to \( 10^5 \) A/cm\(^2\), \( d_i \) has to be smaller than 5-50 \( \mu \)m, a requirement which should be rather easily satisfied. Strongly distorted vortex lines may undergo vortex cutting events, since the cutting energy goes to zero with \( | \cos (\theta) | \), where \( \theta \) is the local angle between vortex lines [45]. However \( \theta \ll 1 \) should result from \( d_i \ll 1 \) \( \mu \)m for current densities quoted just above.

In Y(123), pinning energies benefit from the sizeable vortex line stiffness, which prevents individual 2D-fluxon depinning. Nevertheless the critical depinning length is expected to become shorter than the total pinned length along extended defects [44]. Besides, the kinetic pinning energy saturates for transverse defect sizes much larger than \( \varepsilon_{ab} \). Finally, too large a microstructure scale could result in a lowering of the maximal pinning force because it would require deformations of the VL at scales larger than \( \lambda_{ab} \) or \( \lambda_c \), for which the high local values of elastic moduli would be relevant. From a very qualitative point of view, the above arguments seem to favour typical defect diameters in the range of 30 nm, and inter-defect distances in the range of 100 nm, for quasi-spherical defects in Y(123).

Elaborated models of strong pinning, taking into account the concentration variation of active pinning centres with field, temperature and anisotropy are needed to go beyond these qualitative considerations.

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Appendix.

We calculate the elastic energy produced by the one-component displacement field:

\[
u_x(r) = d_i \sin x' \sin y' \sin z'/x' y' z' \tag{A.1}\]

along the Ox axis, with \( r = (x, y, z) \), \( x' = \pi x/a_0 \), \( y' = \pi y/a_0 \), \( z' = \pi z/d \). For simplicity’s sake, we assume a simple square symmetry of the VL. The Fourier components of \( u_x \) are in the first Brillouin zone with a constant distribution:

\[
v_x(k) = d_i a_0^2 d \tag{A.2}\]

The above displacement field is not optimal for the minimization of elastic energy under the condition:

\[
u_x(K_i) = d_i \delta (|K_i|) \tag{A.3}\]

where \( K_i \) is any vector of the 3D fluxon lattice. Thus, the \( E_{clo} \) value obtained below is an upper estimate of the minimal deformation energy involved in a displacement \( d_i \) of one fluxon, with all others kept at fixed positions.
For small displacements $d_j \approx 2 \xi_{ab}$, linear elasticity theory is a valid approach for this problem since the conditions $\xi_{ab} \ll a_0$ and $\xi_{ab}/d \ll \Gamma$ hold. [44]. The elastic energy $E_{elo}$ may be obtained from the $xx$ element of the elastic matrix $\phi_{\alpha\beta}(k)$ [46]

$$E_{elo} = (1/2) \int_{BZ} [v, (k)]^2 \phi_{xx}(k)/8 \pi^3 d^3k$$  \hspace{1cm} (A.4)

where the summation may be reduced to the first Brillouin zone, taking advantage of the particular form chosen for $u_\alpha(x)$.

For small wavevectors $|k_{\alpha\beta}| < k_{BZ} = \pi/a_0$, $\phi_{xx}$ is given in the continuum approximation by:

$$\phi_{xx} = C_{11}(k) k_x^2 + C_{66} k_y^2 + C_{44}(k) k_z^2.$$  \hspace{1cm} (A.5)

For a precise evaluation of $E_{elo}$, one should take into account the $\phi_{xx}$ periodicity in the $k$ space, which modifies its value near the zone boundary. For the qualitative estimation which is presented below, the above relation may be conserved. We also use the following approximations of the elastic moduli:

$$C_{11}(k) = (B^2/\mu_0 \lambda_{ab}^2)(k_x^2 + k_y^2 + k_z^2/\Gamma^2)^{-1}$$  \hspace{1cm} (A.6)

$$C_{44}(k) = C_{11}(k) \Gamma^2$$  \hspace{1cm} (A.7)

valid in all the Brillouin zone except in a small domain around the zone centre (the contribution $C_{44}^{(cor)}$ to $C_{44}(k)$ is neglected because it does not change the order of magnitude of $C_{44}(k)$, whereas the main energy is related to $C_{11}(k)$ owing to the very large value of $\Gamma^2$ for BSCCO). The usual expression $C_{66} = \mu_0 B/16 \pi \mu_0 \lambda_{ab}^3$ is also taken into account. Combining relations (A.2) and (A.4) to (A.7) leads to:

$$E_{elo} = (d d_j^3 \phi_0 B/2 \mu_0 \lambda_{ab}^2) [1 - \ell(\alpha) + \pi/48]$$  \hspace{1cm} (A.8)

where:

$$\ell(\alpha) = \int_{V_k} [k_x^2/(k_x^2 + k_y^2 + \alpha k_z^2)] d^3k'$$  \hspace{1cm} (A.9)

with $\alpha = a_0^2/\Gamma^2 d^2$, $k_{\alpha\beta} = a_0 k_{\alpha\beta}/2 \pi$, $k_z' = dk_z/2 \pi$ and $V_k$ is the unit volume in the reduced wavevector space centered at $k' = 0$.

From (A.8), it can be easily deduced that an upper estimate of $E_{elo}$ is given by $d_j^2 d \phi_0 B/2 \mu_0 \lambda_{ab}^2$. With $d_j = 2 \xi_{ab}$, $\xi_{ab}/\lambda_{ab} \approx 10^{-2}$ and $d = 1.5 \text{ nm}$, this leads to $E_{elo} \approx 3 \text{ meV}$ or $E_{elo}/k \approx 35 \text{ K}$.

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