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Spectral impedance boundary condition (SIBC) method for a spherical perfect electric conductor (PEC) with an uniform biisotropic coating

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Abstract. — An analysis for electromagnetic response of a PEC of spherical shape having an uniform concentrical coating of biisotropic medium is rigorously formulated by constructing spherical vector wavefunctions. For the prescribed radius and thickness of the coating, an anisotropic impedance boundary condition at the outer surface of the coating is developed in the spectral domain, which is useful for solving the scattering problem of the biisotropically coated spherical PEC structure.

1. Introduction.

It has been known that surface impedance conditions can be very helpful in simplifying the analytical or numerical solution of wave problems involving complex structures [1-3]. They are great topics of research in electromagnetics, where surface boundaries are encountered.

On the other hand, with the advancement of polymer synthesis techniques, reciprocal chiral media have become possible to be realized and utilized for their novel features in microwave- and millimeter-wave frequency ranges. Among the numerous potential applications of a chiral medium, the utilization for radar-absorbing layer is an excellent example. Biisotropic media, so-called nonreciprocal chiral media, need four constitutive parameters (instead of three for reciprocal chiral media) to describe their macroscopic electromagnetic phenomena, and provide more flexibility to design artificial materials having the desired electromagnetic properties. Therefore, more and more attention is attracted to the area of electromagnetic wave interaction with this class of medium. In this context, one should mention the following valuable works: field decomposition [4], waveguide structure [5], biisotropic mixtures [6], duality transformation [7] and plane wave reflection [8].

It is an imperative task to analyze the properties of the layered coating in order to understand and control the radar cross section (RCS) of the coated objects. As would be expected, the RCS and other scattering characteristics of scatterers are profoundly influenced by the shape and the electrical size of these scattering bodies. Owing to great mathematical difficulties, exact
theoretical analysis is impossible for actual shapes. For this reason, recourse is usually made to
the case where the scattering object is represented by certain geometrical shape which is
amenable to rigorous mathematical treatment. Although none of these idealized shapes
duplicate the form of an actual scatterer, they do provide models for qualitatively studying the
influence of the shape and the electrical size of the scatterer on the scattering characteristics.
Considering these aspects, in this manuscript we propose the SIBC method to study the
electromagnetic response of a perfectly conducting sphere with a concentric coating consisting
of homogeneous biisotropic medium. By constructing a pair of spherical vector wavefunctions, the
formulation of this problem is rigorously carried out in the spectral domain. Having the
total tangential fields inside the biisotropic coating expanded in the spectral domain we define,
the spectral impedance matrix interrelating the tangential components of the electric and
magnetic field is derived at the outer surface of the coating, which is independent of the
properties of the exterior medium. Numerical computations are presented to demonstrate the
general validity, reliability, and suitability of the proposed spectral impedance boundary
condition in solving the scattering problem for biisotropically coated spherical PEC structures.

2. Preliminaries.

The geometry configuration of the problem to be considered is shown in figure 1. With respect
to the spherical coordinate system \((r, \theta, \varphi)\), whose origin is coincident with the center of the
spherical conductor, we represent the perfectly conducting structure by a sphere of radius
\(a\). A homogeneous biisotropic medium is concentrically coated on the surface of the PEC
sphere. The constitutive relations for the biisotropic coating, which is bounded by an outer
surface \(r = b\), are usually characterized as (Here, the conventional harmonic \(\exp(-i\omega t)\) time
dependence is adopted.)

\[
\begin{align*}
\mathbf{D} &= \varepsilon \mathbf{E} + \alpha \mathbf{H} \quad (1a) \\
\mathbf{B} &= \beta \mathbf{E} + \mu \mathbf{H} \quad (1b)
\end{align*}
\]

where \(\varepsilon, \mu\) are the permittivity and permeability scalars, respectively, and \(\alpha\) and
\(\beta\) are the magnetoelectric pseudo-scalars. The surrounding medium is taken to be homo-

![Fig. 1. — Cross section of a PEC sphere with a concentrical coating consisting of homogeneous biisotropic medium.](image-url)
eneous with permittivity \( \varepsilon_0 \) and permeability \( \mu_0 \) (these two quantities may be frequency-dependent complex numbers accounting for electric and magnetic losses [9]). It is known that the eigenmodes of propagation wave in biisotropic medium, left- and right-handed circularly polarized (LCP and RCP), travel with two unequal characteristic wavenumbers given by \(^{(1)}\)

\[
k = \omega \left\{ (\alpha \varepsilon - (\alpha + \beta)^2/4)^{1/2} \mp i (\alpha - \beta)/2 \right\}.
\]

From a mathematical point of view, this pair of wavenumbers and their accompanying eigenmodes become the reason for expanding the electromagnetic fields into LCP and RCP parts.

We are primarily interested in the relationship between the tangential components of the electric and magnetic fields at the outer surface \( r = b \) of the coating for a general exciting source in the external region. Thus, we first examine the general forms of the fields within the coating (i.e., \( a < r < b \)).

Proceeding from the source-free Maxwell’s equations in association with the above mentioned constitutive relations, the vector field equations inside the biisotropic coating are yielded

\[
\nabla \times \nabla \times \left\{ \frac{E}{H} \right\} + i \omega (\alpha - \beta) \nabla \times \left\{ \frac{E}{H} \right\} = \omega^2 (\alpha \beta - \varepsilon \mu) \left\{ \frac{E}{H} \right\} = 0.
\]

Equation (2) will now be solved in terms of a dual of spherical vector wavefunctions we try to introduce. For this purpose, we review the two conventional orthogonal vector functions \( M_{mn}(k) \) and \( N_{mn}(k) \) which are usually defined as

\[
M_{mn}(k) = \nabla \times [\psi_{mn}(k) \hat{e}_j] = \frac{1}{k} \nabla \times N_{mn}(k)
\]

\[
N_{mn}(k) = \frac{1}{k} \nabla \times \nabla \times [\psi_{mn}(k) \hat{e}_j] = \frac{1}{k} \nabla \times M_{mn}(k)
\]

where \( k \) is an as yet undetermined wave number, \( \hat{e}_j \) is the unit vector in the \( j \) (\( j = r, \theta, \phi \)) direction. \( \psi_{mn}(k) \) is the generating function given by

\[
\psi_{mn}(k) = z_n(kr) P^m_n(\cos \theta) e^{im\phi}
\]

where \( z_n(kr) \) represents any of the spherical Bessel functions, \( j_n(kr), n_n(kr), h_n^{(1)}(kr) \), or \( h_n^{(2)}(kr) \) of order \( n \), and \( P^m_n(\cos \theta) \) is the associated Legendre function of the first kind with degree \( n \) and order \( m \). It is apparent that neither the traditional vector wavefunction \( M_{mn}(k) \) or \( N_{mn}(k) \) is the solution to (2). However, it is possible to construct linear combinations of \( M_{mn}(k) \) and \( N_{mn}(k) \) which do satisfy these vector wave equations [10]. These combination vectors \( V_{mn}(k) \) and \( W_{mn}(k) \) are such that

\[
\begin{align*}
\{ V_{mn}(k) \} &= \frac{M_{mn}(k) \pm N_{mn}(k)}{\sqrt{2}} = \frac{e^{im\phi}}{2} \left\{ \pm n(n + 1) P^m_n(\cos \theta) \frac{z_n(kr)}{kr} \hat{e}_j \right. \\
&\quad \left. + \left[ \frac{im}{\sin \theta} P^m_n(\cos \theta) z_n(kr) \pm \frac{\partial P^m_n(\cos \theta)}{\partial \theta} \frac{\partial}{\partial r} [rz_n(kr)] \hat{e}_\theta \right. \\
&\quad \left. \pm \frac{im}{kr \sin \theta} P^m_n(\cos \theta) \frac{\partial}{\partial r} [rz_n(kr)] \pm \frac{\partial P^m_n(\cos \theta)}{\partial \theta} z_n(kr) \hat{e}_\phi \right\}
\end{align*}
\]

\(^{(1)}\) Since the traditional harmonic time dependence \( \exp(-i\omega t) \) adopted here is different from that used in reference [4], some modification should be made to the formulae presented in reference [4], not only for wavenumber formulae but also for wave impedance identity used in the following analysis.
and \( V_{mn}(k) \) and \( W_{mn}(k) \) now satisfy
\[
V_{mn}(k) = \frac{1}{k} \nabla \times V_{mn}(k), \tag{6a}
\]
\[
W_{mn}(k) = -\frac{1}{k} \nabla \times W_{mn}(k). \tag{6b}
\]

It should be noted that for manipulation simplicity, the definition of the generating function \( \psi_{mn}(k) \) for \( V_{mn}(k) \) and \( W_{mn}(k) \) introduced here are different from that appeared in [10]. Similar to what have been pointed out and used by Jaggard and Liu [11] for reciprocal chiral media, our spherical vector wave functions \( V_{mn}(k_+ \rangle \) and \( W_{mn}(k_- \rangle \) correspond to the LCP and RCP eigenmodes of the biisotropic medium, respectively. So, any travelling electromagnetic wave in the biisotropic medium described by (1a) and (1b) can be expanded in terms of its LCP and RCP parts, and expressed in the form
\[
E = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[ a_{mn}' V_{mn}(k_+) + b_{mn}' W_{mn}(k_-) \right] \tag{7}
\]
where the superscript \( q = 1, 2, 3, 4 \) represents \( z_n(kr) = j_n(kr), n_n(kr), h_n^{(1)}(kr), \) and \( h_n^{(2)}(kr), \) respectively.


According to the last statement in the previous section, and since both inward and outward travelling waves exist in the biisotropic coating, the electromagnetic fields in the coating can be represented in terms of the above mentioned spherical vector wave functions in the spectral domain, similar to what we have done in the circular cylindrical coordinate system [12] (this spectral domain is composed of both discrete angular wave numbers \( m \) and discrete azimuthal wave numbers \( n \))
\[
\begin{bmatrix} E \\ H \end{bmatrix} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left[ \begin{bmatrix} 1 \\ -i/\eta_+ \end{bmatrix} a_{mn} V_{mn}^{(1)}(k_+) + \begin{bmatrix} 1 \\ i/\eta_- \end{bmatrix} b_{mn} W_{mn}^{(1)}(k_-) + \begin{bmatrix} 1 \\ -i/\eta_+ \end{bmatrix} c_{mn} V_{mn}^{(3)}(k_+) + \begin{bmatrix} 1 \\ i/\eta_- \end{bmatrix} d_{mn} W_{mn}^{(3)}(k_-) \right] \tag{8}
\]
where the wave impedances for LCP and RCP eigenmodes are
\[
\eta_{\pm} = \sqrt{\frac{\mu}{\varepsilon} - \left( \frac{\alpha + \beta}{2\varepsilon} \right)^2} = \pm i \left( \frac{\alpha + \beta}{2\varepsilon} \right). \tag{9}
\]

Being a perfectly electric conductor, the innermost sphere entails the boundary conditions at \( r = a \) as \( E_\theta = 0 \) and \( E_\varphi = 0 \), which lead to the relationship between the spectral coefficients \( a_{mn}, b_{mn} \) and \( c_{mn}, d_{mn} \)
\[
\begin{bmatrix} c_{mn} \\ d_{mn} \end{bmatrix} = \frac{1}{\Delta_{mn}} \begin{bmatrix} e_{mn} & f_{mn} \\ g_{mn} & h_{mn} \end{bmatrix} \begin{bmatrix} a_{mn} \\ b_{mn} \end{bmatrix} \tag{10}
\]
where
\[
\begin{align*}
\Delta_{mn} &= \left[ m^2 p^2 + (\partial p)^2 \right] (h_- \partial h_+ - h_+ \partial h_-) + 2 i mp \partial p (h_+ h_- + \partial h_+ \partial h_-), \tag{11a} \\
e_{mn} &= \left[ m^2 p^2 j_+^2 + (\partial p)^2 \partial j_+ \right] (\partial h_- - h_-) - i mp \partial p (j_+ + \partial j_+) (h_- + \partial h_-), \tag{11b}
\end{align*}
\]
\( f_{mn} = m^2 p^2 j_{-} (\Delta h_{-} - h_{-}) + i m p \partial \phi \Delta h_{-} (\partial j_{-} - j_{-}) - i (\partial p)^2 (k_{-} a)^2 , \)

\( g_{mn} = (\partial p)^2 \partial j_{+} (\Delta h_{+} - h_{+}) + i m p \partial h_{+} (\partial j_{+} - j_{+}) - i m^2 p^2 (k_{+} a)^2 , \)

\( h_{mn} = [m^2 p^2 + (\partial p)^2] (h_{+} \partial j_{-} - j_{-} \partial h_{+} + j_{+} h_{+}) - 2 i m p \partial \phi (\partial j_{-} \partial h_{+} + j_{+} h_{+}) , \)

will the notations

\[ j_{\pm} = j_{\pm}(k_{\pm} a) \quad \text{and} \quad \partial j_{\pm} = \frac{1}{k_{\pm} a} \frac{\partial}{\partial r} [r j_{\pm}(k_{\pm} r)] \Big|_{r=a} \]

\[ h_{\pm} = h_{\pm}^{(1)}(k_{\pm} a) \quad \text{and} \quad \partial h_{\pm} = \frac{1}{k_{\pm} a} \frac{\partial}{\partial r} [r h_{\pm}^{(1)}(k_{\pm} r)] \Big|_{r=a} \]

\[ p = \frac{P_{n}^m(\cos \theta)}{\sin \theta} \quad \text{and} \quad \partial p = \frac{\partial P_{n}^m(\cos \theta)}{\partial \theta} \]

Here the Wronskian relationship

\[ W[h_{\pm}, j_{\pm}] = \partial j_{\pm} h_{\pm} - \partial h_{\pm} j_{\pm} = -i l/(k_{\pm} a)^2 \]

has been employed.

Then, substituting the relevant quantities appeared in (8) from (10) and (5), we can rewrite the spectral representations of tangential electromagnetic field components at the surface \( r = b \) as

\[
\begin{align*}
\{ E_{\theta}(r = b) \} &= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\phi}}{\sqrt{2}} \left\{ E_{\theta mn} \right\} \\
\{ E_{\phi}(r = b) \} &= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\phi}}{\sqrt{2}} \left\{ E_{\phi mn} \right\} \\
\{ H_{\theta}(r = b) \} &= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\phi}}{\sqrt{2}} \left\{ H_{\theta mn} \right\} \\
\{ H_{\phi}(r = b) \} &= \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\phi}}{\sqrt{2}} \left\{ H_{\phi mn} \right\}.
\end{align*}
\]

where

\[ P_{mn}^e = \frac{i m}{\sin \theta} p \hat{j}_{+} + \partial p \frac{\partial \hat{j}_{+}}{\partial \theta} \frac{e_{mn}}{\Delta_{mn}} \left( \frac{i m}{\sin \theta} p h_{+} + \partial p \partial h_{+} \right) + \frac{g_{mn}}{\Delta_{mn}} \left( \frac{i m}{\sin \theta} p h_{-} - \partial p \partial h_{-} \right) \]

\( Q_{mn}^e \) is derived from \( P_{mn}^e \) by replacing \( \hat{j}_{+} \) by \( \hat{j}_{-} \), \( \partial \hat{j}_{+} \) by \( - \partial \hat{j}_{-} \) and \( e_{mn} \) by \( f_{mn} \), \( g_{mn} \) by \( h_{mn} \). \( R_{mn}^e \) is obtained from \( P_{mn}^e \) by replacement of \( \hat{j}_{+}(h_{+}) \) by \( \partial \hat{j}_{+}(\partial h_{+}) \), \( \partial \hat{j}_{+}(\partial h_{+}) \) by \( - \hat{j}_{+}(-h_{+}) \) and \( h_{-} \) by \( - \partial h_{-} \), \( \partial h_{-} \) by \( h_{-} \). \( T_{mn}^e \) is derived from \( P_{mn}^e \) by the following substitution \( \hat{j}_{+} \) by \( \hat{j}_{-} \), \( \partial \hat{j}_{+} \) by \( - \hat{j}_{-} \), \( \partial \hat{j}_{-} \) by \( - \hat{j}_{+} \), \( e_{mn}(g_{mn}) \) by \( f_{mn}(h_{mn}) \), \( h_{+} \) by \( \partial h_{+} \), \( \partial h_{+} \) by \( \partial h_{+} \), \( \partial h_{-} \) by \( h_{-} \), and \( \partial h_{-} \) by \( h_{-} \). And \( P_{mn}^h \), \( Q_{mn}^h \), \( R_{mn}^h \) and \( T_{mn}^h \) can be obtained from \( P_{mn}^e \), \( Q_{mn}^e \), \( R_{mn}^e \) and \( T_{mn}^e \), respectively, by dividing the terms with \( k_{+} \) by \( i \eta_{+} \) and terms with \( k_{-} \) by \( - i \eta_{-} \). In the above notations, short bars below the spherical Bessel functions and their derivatives denote these functions are evaluated at \( r = b \).

Up to now, without any difficulty we can deduce that the spectral tangential field components at the outer surface of the coating are interrelated by the following identity description in which we are particularly interested

\[
\begin{bmatrix} E_{\theta mn} \\ E_{\phi mn} \end{bmatrix} = \mathbf{v}_{mn} \begin{bmatrix} H_{\theta mn} \\ H_{\phi mn} \end{bmatrix}
\]
where the $2 \times 2$ spectral impedance matrix $\mathbf{\bar{V}}_{mn}$ is defined as

\[
\mathbf{\bar{V}}_{mn} = \frac{1}{\Delta_{mn}^h} \begin{bmatrix}
P_{mn}^e T_{mn}^h - Q_{mn}^e R_{mn}^h & Q_{mn}^e P_{mn}^h - P_{mn}^e Q_{mn}^h \\
R_{mn}^e T_{mn}^h - T_{mn}^e R_{mn}^h & T_{mn}^e P_{mn}^h - R_{mn}^e Q_{mn}^h
\end{bmatrix}
\]

(16)

with $\Delta_{mn}^h = P_{mn}^h T_{mn}^h - Q_{mn}^h R_{mn}^h$.

It is necessary to indicate that the four elements of the impedance matrix $\mathbf{\bar{V}}_{mn}$ are functions of both the azimuthal order $n$ and the angular order number $m$. Consequently, the impedance condition in the form expressed as (15) cannot be directly applied to the total tangential field components.


To illustrate the reliability and facilities of the SIBC presented here for the biisotropic coating of a spherical conductor, we first compute the normalized RCS of such a structure with a uniform coating of a chiral reciprocal medium ($\alpha = - \beta$ in the constitutive relations (1a) and (1b)).

For the symmetry of the scattered spherical configuration, we will let the incident wave propagate in the $z$-axis direction. Any plane wave illuminating the scatterer along the $z$-axis may be decomposed as the $x$-polarized and $y$-polarized incident waves [13]. By the symmetry of $x$-axis and $y$-axis, we consider the case where a plane $x$-polarized wave is incident upon the chiral coated PEC sphere without losing any generality. This $x$-polarized incident wave can easily be expressed as

\[
E_i^x = E_0 \exp(ik_0 z)
\]

\[
\begin{bmatrix}
E_i^\theta \\
E_i^\varphi
\end{bmatrix} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\varphi}}{\sqrt{2}} \begin{bmatrix}
E_{\theta mn}^i \\
E_{\varphi mn}^i
\end{bmatrix}
\]

(17a)

\[
H_i^x = E_0 \exp(ik_0 z) / \eta_0
\]

\[
\begin{bmatrix}
H_i^\theta \\
H_i^\varphi
\end{bmatrix} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{e^{im\varphi}}{\sqrt{2}} \begin{bmatrix}
H_{\theta mn}^i \\
H_{\varphi mn}^i
\end{bmatrix}
\]

(17b)

with $\eta_0$ representing the wave impedance of free space. After some algebra, we have

\[
E_{\theta mn}^i = \begin{cases}
- \sqrt{2} E_0 [1 + \exp(-2i\varphi)] a_n [J_n^2(k_0 r) P_n^1(\cos \theta)] & m = 1, n \neq 0 \\
+ i J_n^1(k_0 r) P_n^1(\cos \theta) / (2k_0 r \sin \theta) & m = 1, n \neq 0 \\
0 & \text{others}
\end{cases}
\]

\[
E_{\varphi mn}^i = \begin{cases}
- \sqrt{2} E_0 [1 - \exp(-2i\varphi)] a_n [J_n^2(k_0 r) P_n^1(\cos \theta)] & m = 1, n \neq 0 \\
+ i J_n^1(k_0 r) P_n^1(\cos \theta) / (2k_0 r \sin \theta) & m = 1, n \neq 0 \\
0 & \text{others}
\end{cases}
\]

\[
H_{\theta mn}^i = \begin{cases}
- \sqrt{2} E_0 [1 - \exp(-2i\varphi)] a_n [J_n^2(k_0 r) P_n^1(\cos \theta)] & m = 1, n \neq 0 \\
+ i J_n^1(k_0 r) P_n^1(\cos \theta) / (2 \omega \mu_0 r \sin \theta) & m = 1, n \neq 0 \\
0 & \text{others}
\end{cases}
\]
where \( a_n = (2n + 1) i^n [n(n + 1)] \) and \( \tilde{J}_n(k_0 r) = k_0 r j_n(k_0 r) \). Expanding the four tangential components of the scattered field \( E_\theta', E_\varphi', H_\theta', H_\varphi' \) in the form similar to (13), and considering the constraint conditions imposed by Maxwell's equations on the scattered field, we have

\[
H_{\varphi mn} + \frac{1}{k_0^2 r^2} \frac{\partial}{\partial \theta} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta H_{\varphi mn}) \right] = \\
= \frac{1}{k_0^2 r^2} \frac{\partial^2}{\partial \theta \partial \varphi} \left( \frac{1}{\sin \theta} H_{\theta mn} \right) + \frac{1}{i \omega \mu_0 r} \frac{\partial}{\partial r} (r E_{\varphi mn}) , \quad (18a)
\]

\[
E_{\varphi mn} + \frac{1}{k_0^2 r^2} \frac{\partial}{\partial \theta} \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta E_{\varphi mn}) \right] = \\
= \frac{1}{k_0^2 r^2} \frac{\partial^2}{\partial \theta \partial \varphi} \left( \frac{1}{\sin \theta} E_{\theta mn} \right) + \frac{i}{\omega \epsilon_0 r} \frac{\partial}{\partial r} (r H_{\theta mn}) . \quad (18b)
\]

Recalling the spectral impedance matrix \( \tilde{Z}_{mn} \), and assuming the spectral tangential components of the scattered field have the expressions (according to the separate variable solution to the Laplace equation in the spherical coordinate system [13])

\[
\begin{bmatrix}
E_{\theta mn} \\
E_{\varphi mn} \\
H_{\theta mn} \\
H_{\varphi mn}
\end{bmatrix} = 
\begin{bmatrix}
E_{\theta mn}^{\infty} \tilde{J}_n(k_0 r) P_n^m(\cos \theta) \exp(i m \varphi) \\
E_{\varphi mn}^{\infty} \tilde{J}_n(k_0 r) P_n^m(\cos \theta) \exp(i m \varphi) \\
H_{\theta mn}^{\infty} \tilde{J}_n(k_0 r) P_n^m(\cos \theta) \exp(i m \varphi) \\
H_{\varphi mn}^{\infty} \tilde{J}_n(k_0 r) P_n^m(\cos \theta) \exp(i m \varphi)
\end{bmatrix}
\]

the amplitude of the spectral tangential components of the scattered field at \( r = b \) may be related to those of the incident wave with the aid of (18)

\[
E_{\theta mn}^{\infty} = E_{\varphi mn}^{\infty} = H_{\theta mn}^{\infty} = H_{\varphi mn}^{\infty} = 0 \quad \text{for} \quad m \neq 1
\]

while

\[
\begin{bmatrix}
E_{\theta 11}^{\infty} \\
H_{\theta 11}^{\infty}
\end{bmatrix} = 
\begin{bmatrix}
s_{11}^{11} & s_{12}^{11} \\
s_{21}^{11} & s_{22}^{11}
\end{bmatrix}^{-1} 
\begin{bmatrix}
e_{11}^{1} \\
e_{21}^{1}
\end{bmatrix}
\]

with

\[
s_{11}^{11} = 1 - \nu_{11}^{12} \alpha_{12} , \\
s_{12}^{11} = - (\nu_{11}^{11} + \nu_{12}^{12} \alpha_{12} ) , \\
s_{21}^{11} = \beta_{11} - \nu_{11}^{21} \alpha_{21} , \\
s_{22}^{11} = \beta_{21} - \nu_{11}^{22} \alpha_{22} , \\
e_{11}^{1} = \nu_{11}^{11} H_{\theta 11}(r = b) + \nu_{12}^{11} H_{\varphi 11}(r = b) - E_{\theta 11}(r = b) , \\
e_{21}^{1} = \nu_{11}^{21} H_{\theta 11}(r = b) + \nu_{12}^{21} H_{\varphi 11}(r = b) - E_{\varphi 11}(r = b) ,
\]
and
\[ \alpha_{1n} = \frac{k_0^2 r^2}{i \omega \mu_0 r [k_0^2 r^2 - n(n+1)]} \left[ i \hat{J}_n(k_0 r) \right]'_{r = b}, \]
\[ \alpha_{2n} = \frac{P_{n1}^1(\cos \theta)}{[k_0^2 r^2 - n(n+1)]} \left[ P_n^1(\cos \theta) \right]'_{r = b}, \]
\[ \beta_{1n} = - \alpha_{2n}, \]
\[ \beta_{2n} = \frac{ik_0^2 r^2}{\omega \varepsilon_0 r [k_0^2 r^2 - n(n+1)]} \left[ i \hat{J}_n(k_0 r) \right]'_{r = b}. \]

We define the RCS of such a structure as
\[ A_s = \lim_{r \to \infty} \frac{4 \pi r^2 |E_s(r)|^2 + \eta_0^2 |H_s(r)|^2}{|E_s(r)|^2}, \]
and normalize it with respect to the RCS of a perfectly conducting sphere of equal size with the chiral coating removed. In figure 2, the computational results are pictured, and for comparison purpose, data derived from field expansion method are also plotted. Good agreement is seen in these calculations. For low frequency \( \omega < \omega_0 \), there exists a minimum value of the normalized RCS, which may owe to the interference from the scattering of the front and back surfaces of the coating. As the frequency of the illuminating wave increases further, the backscattering cross-section is monotonically decrease. This phenomenon may result from the fact that the incident wave is absorbed by the chiral coating which has an increase in electrical thickness as the frequency increases.

To completely verify both the suitability and facilities of the proposed SIBC method in solving the scattering of a general biisotropically coated spherical PEC structure, in figure 3 we present the computed variation of the normalized RCS of a PEC sphere with a biisotropic coating against the normalized frequency, both by the approach we propose here and by field expansion method. Similar variation tendency appeared in figure 2 and figure 3, as well as the good agreement between the results derived by the SIBC method and those by the field expansion method presented in figure 3, leads to the conclusion that the spectral impedance condition is suitable and reliable to implement the scattering problem of the biisotropically coated PEC spherical structure.

It should be mentioned that in the process of computing the scattering problems both in figure 2 and in figure 3, the spectral impedance boundary condition method is universally time saving, comparing with the field expansion method.

5. Conclusion and discussion.

With the prolific application of complex electromagnetic materials in microwave- and millimeter wave system, it is an urgent task to investigate the interaction between electromagnetic waves and these media in order to determine whether and how using these materials would provide better solutions to current engineering problems. A good example may resort to a chiral medium, a new artificial material, used as a RCS reduction coating [11]. In this paper, the impedance boundary condition at the outer surface of the coating is rigorously formulated in the spectral domain for a PEC sphere with a homogeneous coating comprised of biisotropic medium. As we have shown, the spectral impedance matrix presented here not only
Fig. 2. — Normalized backscattering radar cross-section of chiral coated spherical conductor versus the normalized frequency ($\omega - \omega_0/\omega_0$) with $f_0 = 10$ GHz, $a = 2.0 \lambda_0$, $b = 2.1 \lambda_0$, and the chiral medium is characterized as $r/e_0 = 4.0$, $\mu/\mu_0 = 1.0$, $\alpha = -\beta = 0.005 \times i \mu$ mho. Dots indicate the exact results calculated from a traditional series expansion of the fields, while the full line represent the ones computed using the SIBC method we propose.

Fig. 3. — Normalized backscattering radar cross-section of a PEC sphere coated with a biisotropic medium versus the normalized frequency ($\omega - \omega_0/\omega_0$) with $f_0 = 10$ GHz, $a = 2.0 \lambda_0$, $b = 2.1 \lambda_0$, and the biisotropic coating is characterized as $r/e_0 = 4.0$, $\mu/\mu_0 = 1.0$, $\alpha = 0.003 \times i \mu$ mho and $\beta = -0.008 \times i \mu$ mho. Dots indicate the exact results calculated from the field expansion method, while the full line represent the ones computed using the SIBC method we propose.
is independent of the properties of the surrounding medium, but also facilitates the relevant numerical computations. Two numerical examples confirm the general applicability and advantages of the proposed method for spherical geometries. However, in order to correctly exploit this SIBC method, we should note that these formulae cannot be applied to the total tangential field components directly, and the total tangential fields, both for the incident wave and for the scattered wave, must be expanded in the spectral domain.

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References