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The finite difference time-domain (FDTD) method applied to the computation of resonant frequencies and quality factors of open dielectric resonators

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Résumé. — Dans ce travail la méthode des différences finies dans le domaine des temps est appliquée pour analyser des résonateurs diélectriques. Pour calculer les fréquences de résonance et les coefficients de qualité des résonateurs on utilise la méthode de Prony au lieu de la transformée de Fourier ; de cette façon il est possible de réduire le temps de calcul de deux ordres de grandeur. Les résultats obtenus avec la présente méthode tant pour des résonateurs isolés que pour des résonateurs en cavité, sont en très bon accord avec les résultats obtenus par d'autres auteurs ainsi qu'avec les résultats obtenus par la méthode de raccordement modale dans le cas des résonateurs enfermés en cavités.

Abstract. — This paper analyzes dielectric resonators using the FDTD method. Resonant frequencies and quality factors are calculated using Prony's method instead of the classical FFT method; in this way, reductions of up to two orders of magnitude are achieved in computation time. The results obtained for both open and shielded resonators are in good agreement with those reported by other authors, and the results for shielded resonators also agree with those obtained with the mode matching method.

1. Introduction.

Dielectric resonators (DRs) are widely used in a variety of microwave components and subsystems, such as filters and oscillators, because of their high quality factor, low manufacturing cost, temperature stability, miniaturization and compatibility with microwave integrated circuits. A number of rigorous methods have been proposed for characterizing these elements. A review of the most frequently used procedures can be found in [1].

The method of finite differences in the time domain (FDTD), first introduced by Yee [2], is a numerical technique for the direct solution of Maxwell's equations in the time domain. This method has been applied extensively to scattering and coupling problems, and more recently to microstrip, waveguide and eigenvalue problems. The method has important advantages
because it is conceptually simple and easy to implement. Since it is a time-domain method, results for a large frequency bandwidth can be obtained from a single computer simulation.

In previous applications of the FDTD method to cylindrical dielectric resonators and other eigenvalue problems, the step from the time-domain response to the frequency-domain response has been computed by using the Fast Fourier Transform (FFT) [3-6]. However, when this method is used, a large number of time samples are needed for a reliable characterization of the resonance performance of the structure, which means unduly high computation time. The feasibility of using methods other than the FFT in conjunction with time-domain methods — FDTD or transmission-line matrix (TLM), for example — has recently been demonstrated [7-9]. This paper uses Prony's method as an alternative to FFT because it is capable of obtaining resonant frequencies and quality factors from shorter time-domain responses, thus reducing CPU time and memory requirements. Coupling Prony's method with the FDTD technique makes it possible to determine the resonant frequencies and radiation quality factors of shielded and open DRs with azimuthal symmetry with reasonable computation times.

2. Theory.

Consider Maxwell's curl equations in the cylindrical coordinate system \((r, \phi, z)\), in a source-free region of space for linear isotropic media

\[
\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E} \quad (1a)
\]

\[
\frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon} \nabla \times \mathbf{H} \quad (1b)
\]

where \(\mathbf{E}\) is the electric field, \(\mathbf{H}\) the magnetic field, \(\epsilon\) the permittivity of the medium, which is a scalar function of position, and \(\mu_0\) the permeability of the vacuum.

For structures with rotational symmetry, the three-dimensional problem can be reduced to an equivalent two-dimensional problem by using a Fourier series expansion with respect to the angle of azimuth \(\phi\):

\[
\mathbf{E}(r, \phi, z, t) = \sum_{m=0}^{\infty} [e_{m,r}(r, z, t) \cos(m\phi) \mathbf{a}_r + e_{m,\phi}(r, z, t) \sin(m\phi) \mathbf{a}_\phi +
\]

\[
+ e_{m,z}(r, z, t) \cos(m\phi) \mathbf{a}_z] \quad (2a)
\]

\[
\mathbf{H}(r, \phi, z, t) = \sum_{m=0}^{\infty} [h_{m,r}(r, z, t) \sin(m\phi) \mathbf{a}_r + h_{m,\phi}(r, z, t) \cos(m\phi) \mathbf{a}_\phi +
\]

\[
+ h_{m,z}(r, z, t) \sin(m\phi) \mathbf{a}_z]. \quad (2b)
\]

For isotropic media it is sufficient to use in the expansion only one harmonic function, either sine or cosine. The above expansion allows the electromagnetic fields to be classified in families of modes according to the modal index \(m\). In the following theoretical development we shall consider one particular family of modes.

Substituting (2) into (1) (for a specific \(m\)), and expressing (1) in scalar form, we have

\[
\frac{\partial h_r}{\partial t} = \frac{1}{\mu_0} \left( \frac{m}{r} e_z + \frac{\partial e_\phi}{\partial z} \right) \quad (3a)
\]

\[
\frac{\partial h_\phi}{\partial t} = \frac{1}{\mu_0} \left( \frac{\partial e_z}{\partial r} - \frac{\partial e_r}{\partial z} \right) \quad (3b)
\]
THE FINITE DIFFERENCE TIME-DOMAIN (FDTD) METHOD

\[
\frac{\partial h_z}{\partial t} = \frac{1}{\mu_0 r} \left( m e_r - \frac{\partial (r e_\phi)}{\partial r} \right) \quad (3c)
\]
\[
\frac{\partial e_r}{\partial t} = \frac{1}{\varepsilon} \left( \frac{m h_z}{r} - \frac{\partial h_\phi}{\partial r} \right) \quad (3d)
\]
\[
\frac{\partial e_\phi}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial h_r}{\partial z} - \frac{\partial h_z}{\partial r} \right) \quad (3e)
\]
\[
\frac{\partial e_z}{\partial t} = \frac{1}{\varepsilon r} \left( \frac{\partial (r h_\phi)}{\partial r} - m h_z \right). \quad (3f)
\]

For \( m \geq 1 \) the modes are hybrid and they have been denoted by HEM\(_m\), and for \( m = 0 \) the above system is decoupled into two sets of independent modes: the TE\(_0\) modes (with \( e_\phi, h_r \) and \( h_z \) components), and the TM\(_0\) modes (with \( e_r, e_z \) and \( h_\phi \) components).

**DISCRETIZATION.** — By using centered finite-difference expressions for the space and time derivatives, positioning the components of \( e \) and \( h \) about the unit cell of the mesh as is shown in figure 1, and evaluating \( e \) and \( h \) at alternate half-time steps, we can approximate the:

![Fig. 1. — FDTD mesh for HEM\(_m\) modes.](image)

system (3) with the following system of finite-difference equations

\[
h_r^{n+1/2}(i, k + 1/2) = \frac{\Delta t}{\mu_0} \left( \frac{e_\phi^n(i, k + 1) - e_\phi^n(i, k)}{\Delta z} + \frac{m}{i} \frac{\Delta r}{\Delta z} e_z^n(i, k + 1/2) \right) + \\
+ h_r^{n-1/2}(i, k + 1/2) \quad (4a)
\]

\[
h_\phi^{n+1/2}(i + 1/2, k + 1/2) = \frac{\Delta t}{\mu_0} \left( \frac{e_\phi^n(i + 1, k + 1/2) - e_\phi^n(i, k + 1/2)}{\Delta r} - \frac{e_r^n(i + 1/2, k + 1) - e_r^n(i + 1/2, k)}{\Delta z} \right) + h_\phi^{n-1/2}(i + 1/2, k + 1/2) \quad (4b)
\]

\[
h_z^{n+1/2}(i + 1/2, k) = \frac{-\Delta t}{(i + 1/2) \Delta r \mu_0} \left( m e_r^n(i + 1/2, k) + (i + 1) e_\phi^n(i + 1, k) - i e_\phi^n(i, k) \right) + \\
+ h_z^{n-1/2}(i + 1/2, k) \quad (4c)
\]

\[
e_r^{n+1}(i + 1/2, k) = \frac{\Delta t}{\varepsilon (i + 1/2, k)} \left( \frac{m}{(i + 1/2) \Delta r} h_z^{n+1/2}(i + 1/2, k) - \frac{h_\phi^{n+1/2}(i + 1/2, k + 1/2) - h_\phi^{n+1/2}(i + 1/2, k - 1/2)}{\Delta z} \right) + e_r^n(i + 1/2, k) \quad (4d)
\]
where \( i, k \) and \( n \) are the spatial and time indices of the nodal points of the space-time mesh respectively; \( \Delta r \) and \( \Delta z \) are the space increments and \( \Delta t \) is the time step. In this system (4), the new value of the field vector component at any mesh point depends only on its previous value and on the previous values of the components of the other field vector at adjacent points.

**Boundary Conditions.** — The study of DRs enclosed in a cavity requires the use of boundary conditions only for electrically or magnetically conducting walls (in the latter case, to take into account possible symmetries). At the electrically (magnetically) conducting walls the tangential electric (magnetic) and the normal magnetic (electric) field components are maintained at zero.

With open structures, however, difficulties arise because the domain in which the field must be computed is unbounded. A special boundary condition, the so-called absorbing boundary condition, must be used to limit the domain in which the computation is made. Although several different schemes have been developed to overcome the problem of reflections from artificial boundaries, no ideal reflection-free boundary conditions have been proposed. In this work we have used two types of absorbing boundary condition: for laterally open resonators, a first order absorbing condition for cylindrical waves \cite{10}; and for isolated resonators, one based on a parabolic interpolation \cite{11}.

On the axis of symmetry \((r = 0)\) the equations (4) are simplified and we have

i) **HEM\(_m\)** modes

For \( m = 1 \)

\[
e_r(0, z) = h_z(0, z) = 0 \tag{5a}
\]

\[
e_\phi(0, z) = -me_r(0, z) \tag{5b}
\]

\[
h_\phi(0, z) = mh_z(0, z) \tag{5c}
\]

In this case, the condition of \( r = 0 \) can be established either for the components \( e_\phi, e_z \) and \( h_z \) or for the components \( e_r, h_z \) and \( h_\phi \). For example, if the first alternative is taken, from (5) and (3b) we obtain

\[
\frac{\partial h_r}{\partial t} = \frac{1}{\mu_0} \left( \frac{\partial e_\phi}{\partial z} + \frac{\partial e_z}{\partial r} \right) \tag{6}
\]

Discretization of equations (3e) and (6) allows the algorithm to be applied in a simple way to the HEM\(_1\) modes for \( r = 0 \).

For \( m > 1 \)

\[
e_r(0, z) = e_\phi(0, z) = e_z(0, z) = h_r(0, z) = h_\phi(0, z) = h_z(0, z) = 0 \tag{7}
\]

ii) **Modes TE\(_0\) and TM\(_0\)**

\[
e_r(0, z) = e_\phi(0, z) = h_r(0, z) = h_\phi(0, z) = 0 \tag{8}
\]
For the TE modes the axis \( r = 0 \) must be located at the place in the mesh where the components \( e_\phi, yh_r \) are defined, whereas for the TM modes it must be located so that \( e_r \) and \( h_\phi \) are defined.

**APPLICATION OF THE FDTD ALGORITHM TO EIGENVALUE PROBLEMS.** — The following steps are performed when applying the FDTD method to the calculation of the resonant frequencies and quality factors of the different resonant modes.

— The structure under analysis is discretized and the boundary conditions imposed. The choice of the space increments and the time step is determined by the desired accuracy of the solution and by the need to guarantee the stability condition of the algorithm.

— One or several points of the structure are chosen and the desired excitation is applied (non-zero initial conditions or pulsed). One or several points are also selected for which the time response of one or more components of the electromagnetic field is stored. In order to obtain the time-domain response of the structure under analysis, a delayed Gaussian pulse was used for the initial excitation since this has a smooth shape and its spectrum is negligible for high frequencies.

— Finally, in order to obtain the time-domain response, the fields at the nodal points of the mesh are calculated in a time marching manner evaluating the system (4) at each instant of time.

3. **Calculation of resonant frequencies and quality factors.**

The resonant frequencies and the quality factors are calculated from the frequency response, which is obtained from the spectral analysis of the time response, obtained previously with the FDTD method. In earlier applications of FDTD to eigenvalue problems, the results in the frequency domain have usually been obtained by applying the FFT algorithm to the time domain response recorded at a selected observation point of the FDTD mesh. Then, the resonant frequencies have been calculated from the local maximum of the spectrum, and the quality factors from the width of the resonant peaks. There are, however, several inherent performance limitations in the FFT approach, which we shall now describe briefly.

**LIMITATIONS OF THE FFT APPROACH.** — The main limitation of the FFT approach is that of the frequency resolution \( \Delta f \), which is roughly the reciprocal of the observation time, i.e. \( \Delta f = 1/N \Delta t \) where \( \Delta t \) is the time step and \( N \) the total number of iterations used in the FDTD method. A second limitation arises from the windowing of the transient data because the FDTD response is truncated in time. This has the effect of viewing the true time domain response through a rectangular window of duration \( T = N \Delta t \). In the frequency domain this windowing is translated into the convolution of the true spectrum with the function \( \sin(\pi f T)/\pi f T \). As a result of this convolution, the peaks in the spectrum response are widened, the whole spectrum is distorted and some weak signal spectral responses can be masked. Distortion can be reduced and spectral resolution increased only by making the duration of the window longer, that is by increasing simulation time, which in turn leads to an undue increase in CPU time and memory requirements.

In order to improve the accuracy of the FFT algorithm for analyzing lossless structures, the results of the resonant frequencies have been obtained by assuming that in the proximity of the resonant peaks the spectral response has the form \( \sin(\pi f T)/\pi f T \), which is therefore used for interpolation. For structures with finite quality factors, the results have been interpolated by making the assumption that the spectral response in the neighborhood of the resonance of each mode corresponds to that of a resonant RLC circuit.

Attempts to alleviate the limitations of the FFT approach have led to numerous alternative spectral estimation procedures being proposed in applications such as speech processing. For
our research Prony’s method was chosen because it is particularly suitable for the calculation of resonant frequencies and quality factors.

PRONY’S METHOD. — Prony’s method is a technique for modelling sampled data as a linear combination of complex exponentials. Although it is not a spectral estimation technique in the usual sense, it is closely related to the least squares linear prediction methods used for parametric spectral estimation [12, 13].

The FDTD time-domain response of an eigenvalue problem can be expressed in terms of a superposition of the resonant modes

\[ s(n) = \sum_{i=1}^{p} A_i z_i^n \quad \text{for} \quad n = 0, \ldots, N - 1 \]  

with

\[ z_i^n = \exp \left( (\alpha_i + j 2 \pi f_i) n \Delta t \right) \]

where \( s \) is one of the six electromagnetic field components, \( A_i \) the complex modal amplitude, \( \alpha_i \) the damping factor and \( f_i \) the resonant frequency of the \( i \)-th resonant mode, \( N \) the number of data samples, and \( p \) the order of the model (twice the number of resonant modes, since \( s(n) \) is a sequence of real data). The direct solution of (9) is a difficult non-linear least squares problem. An alternative solution is based on Prony’s method, which solves two sequential sets of linear equations with an intermediate polynomial rooting step that concentrates the nonlinearity of the problem.

The key to Prony’s technique is to recognize that (9) is the homogeneous solution to a constant coefficient linear difference equation [12, 13].

\[ \sum_{m=1}^{p} a(m) s(n - m) = -s(n) \quad \text{for} \quad n = p, \ldots, N - 1. \]  

The characteristic polynomial associated with this linear difference equation is

\[ \sum_{m=0}^{p} a(m) z^{p-m} = 0 \]  

which has the complex roots \( z_1, \ldots, z_p \).

Prony’s method can be summarized in three steps. First, by solving (11), exactly if the number \( N \) of data samples \( s(n) \) is equal to \( 2p \) or as a least-squares solution if \( N > 2p \), the polynomial coefficients \( a(m) \) are obtained. Second, the roots of the polynomial (12) are calculated. The damping factors \( \alpha_i \) and the resonant frequencies \( f_i \) can be determined from the roots \( z_i \) as

\[ \alpha_i = \frac{\ln |z_i|}{\Delta t} \]

\[ f_i = \frac{\tan^{-1} \left( \frac{\text{Im} \left( z_i \right)}{\text{Re} \left( z_i \right)} \right)}{2 \pi \Delta t} \]

Finally, the roots computed in the second step are used to solve (9) (as in the first step, either exactly or as a least squares problem) for the \( p \) complex modal amplitudes \( A_1, \ldots, A_p \).

When the damping factor and the resonant frequency have been determined, the quality
factor can easily be obtained as

$$Q_i = \left| \frac{\pi f_i}{\alpha_i} \right|.$$ \hspace{1cm} (15)

Once the parameters of the model represented by equation (9) have been found with a few iterations of the FDTD method, the rest of the time response can be obtained through a process of extrapolation by means of expression (9). The results in the frequency domain can be obtained by applying the FFT as is done in [9], or directly from the $z$ transform of (9).

$$S(z) = \sum_{i=1}^{p} \frac{A_i}{1 - z^{-1}}.$$ \hspace{1cm} (16)

Assuming that all the damping factors are negative, the substitution of $z = \exp(j2\pi f\Delta t)$ in (16) will yield the discrete Fourier transform of (9).

This technique shows good resolution with relatively short data sequences and presents no problems of windowing. The main difficulty with Prony’s method lies in the determination of the order of the model, $p$. If the value of $p$ is very low (less than the number of excited resonant modes), the spectral resolution is poor (some modes are missed). On the other hand, if the value of $p$ is very high, spurious modes appear. One way of distinguishing between real and spurious modes is to apply Prony’s method with the time sequence in reverse order: real modes appear with positive damping factors ($\alpha_i > 0$) whereas spurious modes have negative damping factors ($\alpha_i < 0$) [14]. Therefore, in order to avoid missing solutions, a value of $p$ is chosen that is larger than what is considered necessary, and spurious solutions are eliminated subsequently. Before Prony’s method is applied, the time-domain response is submitted to preprocessing that involves low-pass filtering in order to limit the number of resonant modes and, therefore, the number of parameters to be calculated; the signal was also decimated since the FDTD method usually gives rise to oversampled signals.

4. Results.

In order to check the accuracy of the FDTD method, we have used this method to calculate the resonant frequencies of the four first HEM$_{1\ n}$ modes for a DR enclosed by conducting walls and compared the results with those obtained with the mode matching method (MM) [1]. The results of both methods are shown in figure 2 which shows the variation of the resonant frequencies with the distance, $s$, from the resonator to the top wall of the cavity. The resonant frequencies are independent of $s$ for distances larger than the length of the resonator ($s/\ell > 1$). The agreement of the results is very good although a greater discrepancy is observed for the higher modes since their fields have a larger number of variations. For the MM method, 18 proper modes were used in each waveguide section. The parameters used in the FDTD simulation were $\Delta r = \Delta z = 0.4$ mm and $\Delta t = 0.47$ ps. Resonant frequencies have been calculated by using both the FFT algorithm and Prony’s method. However, the results of the two methods have not been plotted separately because they would be practically indistinguishable for the frequency scale used in this figure.

Figure 3 shows the spectral response obtained by using the FFT and Prony’s algorithm for the first three resonant modes and for the case $s/\ell = 1$ of figure 2. As can be seen, at the sampling point chosen the mode HEM$_{12}$ has an amplitude about 10 times greater than that of the mode HEM$_{13}$. The resonant frequencies of these two modes are quite close so that when the FFT is used with 10 000 FDTD iterations the resonance of mode HEM$_{12}$ is detected quite well but mode HEM$_{13}$ is masked. Moreover, the resonant frequency of mode HEM$_{11}$ is shifted from its actual value. In order to obtain the resonant frequency by means of the FFT with good
Fig. 2. — Comparison of the resonant frequencies of the four lowest HEM_{1n} modes of a cavity DR calculated by the FDTD method and MM method.  \( a = 8.6 \text{ mm}, \quad \ell = 3.8 \text{ mm}, \quad b = 14.4 \text{ mm}, \quad \varepsilon_r = 35.74. \)

Fig. 3. — Spectral response calculated with FDTD-FFT and 10 000, 20 000 and 30 000 FDTD iterations, and with FDTD-PRONY and 5 000 iterations for the first three HEM_{1n} modes of the cavity DR of figure 2 with \( s = \ell. \)

The results for Prony's method were obtained with \( p = 28 \) and 5 000 FDTD iterations, of which only the last 3 200 were used to yield 80 data points.
samples, after resampling with a decimation factor of 40. All these results are shown in numerical form in table I. It can be seen that if the FFT algorithm is used with very long data sequences (65 536 FDTD data samples for this case), it is possible to obtain a spectrum with a negligible effect of the truncation error and a high resolution, yielding results which are the same as those obtained with Prony's method.

The variation of the resonant frequency of a DR as a function of the distance to the lateral wall is shown in figure 4. The flat portions of the response correspond to the resonances of the

Table I. — Resonant frequencies (in GHz) for the first three HEM_{1n} modes of the DR in figure 2 with s = ℓ.

<table>
<thead>
<tr>
<th>Mode</th>
<th>FDTD-FFT 10000 It.</th>
<th>FDTD-FFT 20000 It.</th>
<th>FDTD-FFT 30000 It.</th>
<th>FDTD-FFT 65536 It.</th>
<th>FDTD-PRONY 5000 It.</th>
<th>MM 18 modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>HEM_{11}</td>
<td>4.312</td>
<td>4.265</td>
<td>4.267</td>
<td>4.264</td>
<td>4.261</td>
<td>4.275</td>
</tr>
<tr>
<td>HEM_{12}</td>
<td>5.936</td>
<td>5.935</td>
<td>5.935</td>
<td>5.940</td>
<td>5.940</td>
<td>5.959</td>
</tr>
<tr>
<td>HEM_{13}</td>
<td>-</td>
<td>6.324</td>
<td>6.322</td>
<td>6.321</td>
<td>6.320</td>
<td>6.344</td>
</tr>
</tbody>
</table>

**Fig. 4.** — Comparison of the resonance frequencies of the lowest HEM_{1n} modes of a cavity DR and of a laterally open DR. Results obtained using the FDTD-PRONY method. a = 8.6 mm, ℓ = 3.8 mm, s = ℓ, ε_r = 35.74.
dielectric, and the portions in which the frequency depends on the distance to the lateral conducting wall correspond to the resonances of the cavity [15]. When the lateral conducting wall of the cavity is replaced by absorbing boundary conditions in the same position, the dotted lines shown in the figure are obtained. It can be seen that the resonances of the cavity disappear in this case. The parameters used in the simulation for the FDTD-PRONY method were the same as those employed in figure 2. If the FFT is used instead of Prony's method, more than 60 000 iterations are required to obtain similar results for those parts of the graph where there are several modes whose frequencies are close to one another (e.g. for \( b/a = 3.5 \)).

Figure 5 shows the resonant frequencies and quality factors of the first \( \text{TE}_{0,s} \) modes of an isolated DR as a function of the distance to the absorbing boundary. The results were obtained by the FDTD-PRONY method. The existence of a finite quality factor is due to radiation losses. It can be seen that the distance to the absorbing walls must be at least five times the radius and height of the DR to achieve convergence of the quality factor whereas twice as far is enough for the resonant frequency. Thus, the quality factor is more sensitive than the resonant frequency to errors due to the non-perfect behaviour of the absorbing boundary conditions. The parameters used for the FDTD method were \( \Delta r = 0.29167 \) mm, \( \Delta z = 0.28750 \) mm and \( \Delta t = 0.48 \) ps. Simulation was performed over 3 000 iterations, which was sufficient to obtain good accuracy for the modes calculated. In Prony's method, the model order was \( p = 10 \) and the last 2 000 iterations were used with a decimation factor of 50.

Fig. 5. — Convergence of the resonant frequencies and \( Q \)-factors of the three lowest \( \text{TE}_{0,s} \) modes of an isolated DR as a function of the distance to the absorbing wall. Results obtained using the FDTD-PRONY method. \( a = 5.25 \) mm, \( \ell' = 4.6 \) mm, \( \varepsilon_r = 38 \).
Figure 6 shows the convergence of the resonant frequencies and the quality factors versus the number of FDTD iterations for the same DR as in figure 5. It can be seen that Prony' method provides a reduction in the number of FDTD iterations necessary: for mode TE₀₁ the number of iterations is reduced by a factor of 10 and in the case of the mode TE₀₂ by a factor of 40. The spectral response of the mode TE₀₁ can be seen in figure 7. As can be observed in this figure, using the FFT method with an insufficient number of iterations does not have a dramatic effect on the value of the resonant frequency, however, the error incurred in determining the quality factor may be very great.

Finally, table II compares the results obtained with FDTD and those obtained with other methods [16, 17] and experimental data [18]. It can be seen that the FDTD method provides results with an accuracy similar to those obtained with other rigorous methods.

5. Conclusions.

Coupling the FDTD method and Prony's method allows the resonant frequency values and quality factors of DRs to be obtained with an accuracy similar to that of other rigorous methods. Moreover, the use of Prony's method rather than the FFT provides a considerable reduction in CPU time, which depends on the specific problem that is being considered. The
Table II. — Comparison of the resonant frequencies and Q-factors of the three lowest axis symmetric modes (a), and the three lowest hybrid modes (b) of an isolated DR computed by FDTD-PRONY, by other methods and by measurement. $a = 5.25$ mm, $\ell = 4.6$ mm, $\varepsilon_r = 38$, $x = 5$.

<table>
<thead>
<tr>
<th></th>
<th>TE$_{01}$</th>
<th></th>
<th>TM$_{01}$</th>
<th></th>
<th>TE$_{02}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F(GHz)</td>
<td>Q</td>
<td>F(GHz)</td>
<td>Q</td>
<td>F(GHz)</td>
<td>Q</td>
</tr>
<tr>
<td>Ref.[16]</td>
<td>4.829</td>
<td>45.8</td>
<td>7.524</td>
<td>76.8</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ref.[17]</td>
<td>4.8604</td>
<td>40.819</td>
<td>7.5384</td>
<td>76.921</td>
<td>8.3311</td>
<td>301.02</td>
</tr>
<tr>
<td>Measured[18]</td>
<td>4.85</td>
<td>51</td>
<td>7.60</td>
<td>86</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FDTD</td>
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Reduction varies from one order of magnitude for problems with low quality factors and resonant frequencies that are not very close together up to about two orders of magnitude for modes with medium or high quality factors or with resonant frequencies that are very close to each other.

References