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Minimization of the induction space harmonics of a P.M. synchronous machine using finite element method and penalty method

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Résumé. — L'objet de cet article est de présenter une optimisation de forme d'une machine électrique synchrone à aimants permanents à l'aide d'un logiciel d'élément finis 2D et d'un algorithme d'optimisation. L'une des formes optimales de cette machine est obtenue lorsque les septième et neuvième harmoniques de l'induction magnétique dans l'entrefer sont minimisées.

Abstract. — The purpose of this work is to apply a 2D electromagnetic software using finite element, and a mathematical optimization programming method in order to find the most appropriate geometry of the rotor of a permanent magnet synchronous machine. The optimum design of this machine was considered as being the one obtained when the seventh and the ninth space harmonic of the magnetic flux density signal are minimized.

1. Introduction.

During the last two decades, new developments in finite element method (FEM) and mathematical programming methods have provided engineers with a useful tool by using the computer [1].

The purpose of this paper is to apply a numerical algorithm solving constrained nonlinear optimization in a magnetostatic problem. The optimization method used is the penalty method (P.M.). It will be considered to optimize the rotor shape of a permanent magnet (P.M.) synchronous machine. This machine is a four poles self-starting motor, it can be seen in figure 1 the presence of cage bars in the rotor. The computed magnetic flux distribution is shown in figure 2. This configuration of machine was described by Binns [2].

A discrete Fourier transform analysing the magnetic flux density along the air-gap is employed in the function to be minimized. This technique allows to obtain an optimal solution with a small iteration number. The length of the machine is assumed to be infinite in z-axis, i.e., neglecting end effects.

In order to highlight the accuracy of the proposed method, two important space harmonics of the magnetic flux density along the air-gap for one rotor position were chosen to be reduced.
Fig. 1. — P.M. machine synchronous machine to be optimized.

Fig. 2. — Computed magnetic flux distribution under one pole.


Generally a nonlinear programming problem consists of finding the values of a vector of design variables $x$, that minimize an objective function $F(x)$. These specific values must satisfy the constraint functions $G_i(x)$. Then, the problem to solve is:

Minimize

$$ F(x) \quad x = (x_1, x_2, \ldots, x_n) $$

Subject to

$$ G_i(x) \geq 0 \quad i = 1, 2, \ldots, m \quad (1) $$

And

$$ x_{jl} \leq x_j \leq x_{ju} \quad j = 1, 2, \ldots, n $$
where, $x_{l}$ and $x_{u}$ are, respectively, the lower and the upper bound for the design variable $x$. Therefore, solving an optimization problem involves:

— describing the objective and constraint functions versus the design variables;
— employing an optimization method to find the global minimum of the objective function satisfying the constraints.

In this paper, these two steps will be described.


Different criteria to be minimized by means of a mathematical method can be chosen. The e.m.f. waveform of an electrical machine can be corrected or the torque, power, weight... So, the choice of the criteria to be minimized depends essentially on the electrical machine designers.

The first step consists to define the criteria to minimize. It can be seen in figure 3 that the waveform of the magnetic flux density is principally square. It can be also seen the influence of the stator slots on the magnetic flux density waveform. Consequently the magnetic flux density is composed by odd space harmonics. These harmonics can produced noise, oscillation of the torque, e.m.f... The high order harmonics have unimportant influence on electrical parameters. In this paper, we have chosen to reduce two space harmonics of the magnetic flux density and to increase the fundamental of the magnetic flux density along the air-gap of the P.M. machine for one rotor position. The object of our choice is to:

i) reduce the influence of the slots on the waveform magnetic flux density. The number of slots per pole is equal to nine. So, the nineth space harmonic was chosen to be minimized;

ii) reduce the influence of one space harmonic which is near the nineth space harmonic. So, the seventh space harmonic was prefered to the eleventh space harmonic;

iii) increase the fundamental of the magnetic flux density in order to increase the amplitude of the electrical parameters (flux, e.m.f., power, ... ) of the P.M. synchronous machine.

![Graph](image)

**Fig. 3.** — Computed magnetic flux distribution along the air-gap. Initial geometry.
The second step consists to choose the design variables to be optimized. The choice of these variables is important. On the one hand, we have to choose a minimum of design variables in order to reduce the consuming computer time and to check the sensitivity of them on the criteria to be minimized. On the other hand, we have to choose simple design variables in order to avoid increasing of the machine conception cost. In these assumptions, three design variables were chosen, as:

- $x_1$: angle between two cage bars;
- $x_2$: angle between the $x$-axis and the point $A$;
- $x_3$: angle between the $x$-axis and the point $B$.

These variables are shown in figure 1. The $x_j$ values ($j = 1, 2, 3$) are obtained by minimizing the following function:

$$ F = \frac{1}{2} \left[ \frac{B_7(x)}{B_1(x)} \right]^2 + \frac{1}{2} \left[ \frac{B_9(x)}{B_1(x)} \right]^2 + \frac{1}{2} [B_1(x) - B_s]^2 $$  \hspace{1cm} (2)

So, the three terms on the right side of the objective function allow to:

i) reduce the seventh space harmonic of the magnetic flux density $B_n(x, \theta)$ with respect to the fundamental component of $B_n(x, \theta)$;

ii) reduce the ninth space harmonic of $B_n(x, \theta)$ with respect to the fundamental component of $B_n(x, \theta)$;

iii) increase the fundamental component of $B_n(x, \theta)$ to a specified value $B_s$.

The spectrum analysis of the magnetic flux signal is done by a discrete Fourier transform (DFT), so that:

$$ B_n(x, \theta) = \sum_{j=1}^{n_h} B_j(x) \cos (j \theta + \xi_j) $$  \hspace{1cm} (3)

where $B_j(x)$ is the amplitude of the $j$-th space harmonic of the magnetic flux density, $\xi_j$ the phase angle of the $j$-th space harmonic. So, the DFT allows to choose any space harmonic of the magnetic flux induction to minimize. The $j$-th constraint function is obtained from the technical limits imposed on the $j$-th variable (Eq. (1)). So, the constraint functions can be given as:

$$ G_j(x) = -x_j^2 + (x_{j1} + x_{ju}) x_j - x_{j1} \cdot x_{ju}; \quad j = 1, 2, 3. $$  \hspace{1cm} (4)

That function is shown in figure 4. It is convex and defined so that the hatched part represent the admissible domain of the $j$-th variable.

Fig. 4. — Illustration of the $j$-th constraint function.
The function $F$ involved is a function of the magnet flux induction $B$. Therefore, it is necessary to compute the magnetic state of the system. The equations to be solved for a magnetostatic problem are:

$$\text{Curl}(\mathbf{H}) = \mathbf{J}$$  \hspace{1cm} (5)

$$\text{Div}(\mathbf{B}) = 0$$  \hspace{1cm} (6)

where $\mathbf{H}$ is the magnetic field density, $\mathbf{J}$ the current density vector and $\mathbf{B}$ the magnetic flux density. The vector $\mathbf{B}$ is related to the vector $\mathbf{H}$ by the constitutive relationship of Maxwell’s law:

$$\mathbf{B} = \mu \mathbf{H} + \mathbf{B}_c$$  \hspace{1cm} (7)

$$\mu = \mu_r \cdot \mu_0$$  \hspace{1cm} (8)

where $\mu$ and $\mathbf{B}_c$ are the permeability and the coercitive magnetic flux density of the magnetic materials used, respectively. To compute the magnetic flux density along the air gap of the P.M. synchronous machine, a 2D FEM is employed to solve these Maxwell’s equations.

4. Penalty method.

Mathematical programming techniques employ numerical search procedures for finding the constrained minimum and have a wide selection of optimization algorithms. Generally, the algorithms are based on the following iterative equation:

$$x_{k+1} = x_k + \alpha_k d_k; \quad k = 0, 1, 2, \ldots$$  \hspace{1cm} (9)

where $k$ is the iteration number, and $\alpha_k$ the step size, $d_k$ the search direction and $x_0$ the initial design variable. The search direction $d_k$ can be given as a function of the first and/or the second derivative of $F$ and/or $G_j$.

In engineering design problems, the objective and constraint functions and their gradients can have analytic expressions in terms of design variables. But, usually these functions depend implicitly on the design variables [3]. In this last case, their evaluation is complicated and time consuming. It can be seen from the section 3 that the function $F$ is an implicit function. Consequently, the gradient of the objective function is determined numerically. The first order information required for the methods is obtained by the finite difference approximation:

$$\frac{dF}{dx_j} = \left. \frac{F(x + h \cdot e_j) - F(x - h \cdot e_j)}{2 \cdot h} \right|_{x_0}$$  \hspace{1cm} (10)

where $h$ is a step size and $e_j$ the $j$-th unit vector. The step size should be carefully chosen: if $h$ is too large, a loss of accuracy of the difference approximation will happen and a too small step size may generate meaningless gradients because of subtractive cancellation errors [4]. In this paper, the penalty method is used [5]. This method is based on a sequential unconstrained minimization technique (SUMT). The SUMT algorithm transforms a constrained problem into a sequence of an unconstrained minimization problem, using a quadratic extended interior penalty function formulation. A compound function is introduced as:

$$W(x, r) = F(x) + r \sum_{j=1}^{m} P_j(G_j(x))$$  \hspace{1cm} (11)

where $r$ is a penalty term and the function $P_j$ associated to the $j$-th constraint function is defined as:

$$P_j(G_j(x)) = \begin{cases} 1/G_j(x); & \text{for} \quad G_j(x) \geq \varepsilon \\ 1/\varepsilon \cdot ((G_j(x))^2 - 3 \cdot (G_j(x)/\varepsilon) + 3); & \text{for} \quad G_j(x) \leq \varepsilon \end{cases}$$  \hspace{1cm} (12)

with $r = c \cdot r^{1/2}$
where $\varepsilon$ is the transition parameter and $c$ a positive constant. The function $P_f(G_f(x))$ is thus defined as an interior penalty function in most of the feasible domain. The solution is obtained by minimizing the function $W(x, r)$ for a decreasing sequence of $r$ value using a Newton-like method (BFGS). This method uses the first and the approximate second order derivatives of the penalty terms. The principle of this method is shown in figure 5.

![Diagram](image)

**Fig. 5. — Principle of the interior penalty method.**

5. Numerical results.

One iteration of the optimization process consists to compute the magnetic flux density by applying a software using FEM. Then, to compute the spectrum analysis of the magnetic flux density by using a discrete Fourier transform. Then, to determine the objective, constraint functions and their gradients. Finally, the penalty method is employed to calculate another vector of design variables. The optimization process stops when the global minimum of the objective function is reached. The programming sequence represented in figure 6, where $k$ is the iteration number, shows the iterative process for the minimization of the problem ($P$).

Solving equation (10) requires two computations of the magnetic flux density for two different positions. By using the « Rubber Mesh » technique [6] the finite elements topology of the modelled machine is kept constant during these computations. In fact, the finite elements topology is preserved during one iteration of the optimization process.

Only one half of the tetrapolar P.M. synchronous machine was used. The magnets are considered to be isotropic. These characteristics are :

$$B_t = 1.07 \text{ T} ; \quad \mu_r = 1.06 ; \quad H_c = 811 \text{ KA/m}.$$

The diameter of the shaft is 7.6 mm. The diameter of the rotor is 78 mm. The total diameter of the P.M. machine is 98.06 mm. The width of the air-gap is 1.84 mm. The width of the permanent magnet is 8.62 mm. The depth of the permanent magnet is 31.1 mm. The magnetic flux density $B_s$ is specified to 0.69 T. The number of finite elements used for the modelling of the machine was 2 282 elements of second order.
The initial spectrum analysis of the magnetic flux density is shown in figure 7. As presented in figures 8 and 9, the seventh and ninth space harmonic of the magnetic flux density decreased substantially and its fundamental increased to $B_s$. Table I points out that one optimal solution was obtain after only six iterations. The calculation time on an Apollo series 400 workstation for this optimization problem is also given in table I. It can be seen that only 22 mn was necessary to obtain one optimal solution.

6. Conclusions.
In this paper, the accuracy of the penalty method in association with a numerical field calculation for the design of a P.M. synchronous machine in order to reduce two space harmonics of the magnetic flux density for one position of the rotor was proved. The
Fig. 7. — Spectrum analysis of the normal flux density before optimization.

Fig. 8. — Spectrum analysis of the normal magnetic flux density after optimization.
Fig. 9. — Computed magnetic flux distribution along the air-gap. Final geometry.

Table I. — *Numerical results of the optimization of the P.M. synchronous machine.*

<table>
<thead>
<tr>
<th>Initial value of $x$ (deg.)</th>
<th>Initial value of $F$</th>
<th>Initial values of the 1st, 7th, 9th space harmonic (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1 = 10$</td>
<td></td>
<td>$B_1 = 0.612$</td>
</tr>
<tr>
<td>$x_2 = 33$</td>
<td>$0.28 \times 10^{-1}$</td>
<td>$B_7 = 0.11$</td>
</tr>
<tr>
<td>$x_3 = 37$</td>
<td></td>
<td>$B_9 = 0.082$</td>
</tr>
<tr>
<td>Final value of $x$ (deg.)</td>
<td>Final value of $F$</td>
<td>Final value of the 1st, 7th, 9th space harmonic (T)</td>
</tr>
<tr>
<td>$x_1 = 10.170$</td>
<td></td>
<td>$B_1 = 0.688$</td>
</tr>
<tr>
<td>$x_2 = 29.188$</td>
<td>$0.11 \times 10^{-2}$</td>
<td>$B_7 = 0.03$</td>
</tr>
<tr>
<td>$x_3 = 42.779$</td>
<td></td>
<td>$B_9 = 0.0077$</td>
</tr>
</tbody>
</table>
The optimization programming method used was described. The methodology employed was described. Thus, the good results obtained for the design of this machine are derived from:

1) the use of a discrete Fourier transform permitting to choose any space harmonic of the magnetic flux density to minimize;

2) the use of judicious parameters in the rotor of the P.M. synchronous machine.

The methodology described can be apply for the design of electrical machines by considering the rotor movement. After this work, two papers [6, 7] were published in order to prove the validity of the association of a 2D finite element package with a penalty method when the rotor movement is considered. The computation time necessary in order to optimize the shape of this machine was shown to be low. By considering the rotor movement the computation time is more important. An analytical model of electrical machines can be used during the optimization process [8]. But the difficulty to obtain a good analytical model permits us to think that the association of an optimization method with an electromagnetic software using FEM is a useful tool for the design of P.M. synchronous machines in spite of an important calculation time.

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References


