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Microwave scattering by cylindrical dielectric scatterers buried in a half-space using the moment method

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Résumé. — Cet article analyse la diffraction électromagnétique dans un demi-espace. Les reconstructions diélectriques d'objets inconnus ensevelis dans le demi-espace sont obtenues par la solution de l'équation intégrale, relative au problème de la diffraction inverse bidimensionnelle, avec la méthode des moments. Une transformation pseudo-inverse est utilisée pour obtenir une solution avec la norme minimum. Un procédé basé sur plusieurs angles d'incidence est proposé ; il est très efficace du point de vue des calculs. L'article présente quelques résultats préliminaires obtenus considérant des milieux de propagation affectés par des pertes et des données affectées par un bruit gaussien.

Abstract. — Electromagnetic scattering in a half-space geometry is dealt with in this article. Dielectric reconstructions of unknown buried objects are achieved by means of a moment-method solution to the integral-equation formulation of the 2D inverse scattering problem. Pseudoinversion transformation is used to obtain a minimum-norm solution. A computationally efficient multi-incidence process is proposed, and some preliminary results for lossy propagation media and data affected by Gaussian noise are reported.

1. Introduction.

Electromagnetic scattering by objects, buried in multilayered media, has been widely studied in the last decade by many scientists of the electromagnetic community, in particular, the direct scattering problem has been extensively investigated. This problem lies in predicting electromagnetic energy distribution when the physical and geometrical characteristics of a given configuration and the incidental electromagnetic field are known [1-5]. On the contrary, the inverse scattering problem, i.e., the problem of retrieving a scene by using the information represented by scattered electromagnetic energy, has so far not been faced in many works. Nevertheless, microwave imaging of buried objects is of growing interest in many geophysical and civil-engineering fields, e.g., for detection of cables and plastic pipes. This apparent discrepancy is justified by the difficulties that are usually encountered in developing algorithms able to suitably model physical scattering phenomena in order to obtain realistic reconstructions of the dielectric permittivities and of the electric conductivities of unknown buried objects.
Among previous approaches to retrieval of objects buried in multilayered media, we cite some works describing the use of radar for identification of nonmetallic bodies [6, 7], and an interesting methodology using diffraction tomography, which is based on a technique previously proposed for medical applications. Such methodology is utilized for the imaging of buried cylindrical inhomogeneties [8]. Other approaches aimed to achieve geophysical introspection [9, 10].

In parallel, microwave imaging in infinite media has been widely investigated in recent years [11, 12], and some approaches based on integral equations [13-17] appear to exhibit interesting features. However, the best results have so far been obtained by Fourier diffraction techniques [18].

In the present paper, an approach to microwave imaging of two-dimensional half-space geometrical configurations is proposed. An inverse-scattering formulation is developed in terms of Green functions, and the obtained integral equations, with the products of the scattering potential by the total electric field as unknown terms, are numerically solved by the moment method [19]. In the past, several approaches adopted the moment method for many different imaging applications. 3D biological targets were reconstructed by Ghodgaonkar et al. [16]. The polarization currents of 1D and 2D structures were retrieved by Ney et al. [17], who used the pseudoinversion transformation [20] to overcome ill-conditioning problems. Finally, the authors of the present paper proposed a multiview approach to localization, shaping and reconstruction of penetrable objects [15]. In this paper, a multi-illumination-angle imaging process is described that is more efficient than the single-illumination one, thanks to the invariance of the Green matrix to the incident electric field vector. The possibility of computing the pseudoinverse matrix off line (before the imaging process starts) and once for all, allows one to fix optimal working conditions (dimensions and shape of the investigation domain; number of discretizations; number, compactness, and locations of measurement points; frequency; etc.) and to perform an on-line computation in a short time.

The paper is organized into a theoretical and an experimental section. The latter includes results and a discussion of several preliminary numerical simulations for simple scattering objects in a lossy half-space. Reconstruction distortions versus noise have been evaluated, and the advantages of the multi-illumination approach have been assessed, using different values of the dielectric parameters.

2. Mathematical model.

Let \( \pi \) be the boundary plane at the interface between two homogeneous lossy media (Fig. 1), characterized by the following complex dielectric permittivities:

\[
\varepsilon_1 = \varepsilon_0 \quad \varepsilon_\tau_1 = \varepsilon_0 \left[ \varepsilon_{r1} - j (\omega \varepsilon_0)^{-1} \sigma_1 \right] \quad \text{(region 1)}
\]

and

\[
\varepsilon_2 = \varepsilon_0 \quad \varepsilon_\tau_2 = \varepsilon_0 \left[ \varepsilon_{r2} - j (\omega \varepsilon_0)^{-1} \sigma_2 \right] \quad \text{(region 2)}.
\]

Region 2 contains a fixed test region, A, where the cross-section of an infinite dielectric cylinder is assumed to be present. The position and the shape of the cylinder and the distribution of the dielectric parameters of its cross-section are unknown. However, we consider a cylindrical geometry, that is, the dielectric properties of the scatterer are invariant along a coordinate direction (the \( z \) direction in Fig. 1). Since the position of the scatterer is unknown, we model the whole investigation area using an unknown complex dielectric permittivity:

\[
\varepsilon_A(x) = \varepsilon_0 \quad \varepsilon_{rA}(x) = \varepsilon_0 \left[ \varepsilon_{rA}(x) - j (\omega \varepsilon_0)^{-1} \sigma_A(x) \right] \quad x \in A
\]

where \( \mathbf{x} = (x, y) \). Moreover, we assume \( \mu_1 = \mu_2 = \mu_0 \).
A set of incident electric fields is considered in region 1. In particular, we assume TM waves, that is, the incident field vectors are polarized with the electric field vectors along the z direction (cylinder axis), and the propagation vectors lie on the plane orthogonal to the object axis. Under these assumptions, the electric field vectors reflected by the plane \( \pi \), the transmitted electric field vectors, and the electric field vectors scattered by the inhomogeneities present inside region A are all parallel to the cylinder axis. As a consequence, we have to solve a two-dimensional scalar inverse-scattering problem. The following equations describe the electromagnetic field problems in the two regions:

**Region 1:**

\[
\left[ \nabla_i^2 + \tilde{k}_i^2 \right] E_{\text{tot}}^{1\ i}(x) = 0 \quad y > 0, \quad i = 1, \ldots, I \tag{4}
\]

\[
E_{\text{tot}}^{1\ i}(x) = \left[ E_{\text{inc}}^{1\ i}(x) + E_{\text{ref}}^{1\ i}(x) + E_{\text{scat}}^{1\ i}(x) \right] \quad y > 0, \quad i = 1, \ldots, I \tag{5}
\]

\[
E_{\text{tot}}^{1\ i}(x) = E_{\text{tot}}^{2\ i}(x) \quad x \in \pi \tag{6}
\]

\[
|x| H_{\text{tot}}^{1\ i}(x) < K \quad y > 0, \quad i = 1, \ldots, I \quad (K \text{ real constant}) \tag{7}
\]

\[
|x| E_{\text{tot}}^{1\ i}(x) < K \quad y > 0, \quad i = 1, \ldots, I \quad (K \text{ real constant}) \tag{8}
\]

\[
\lim_{|x| \to \infty} |x| \left[ k \mu_1 x \times H_{\text{tot}}^{1\ i}(x) + k_1 E_{\text{tot}}^{1\ i}(x) \right] = 0 \quad y > 0, \quad i = 1, \ldots, I \tag{9}
\]

\[
\lim_{|x| \to \infty} |x| \left[ k_1 x \times E_{\text{tot}}^{1\ i}(x) + \omega \mu_1 H_{\text{tot}}^{1\ i}(x) \right] = 0 \quad y > 0, \quad i = 1, \ldots, I \tag{10}
\]
Region 2:

\[ \left[ \nabla^2 + \frac{k_0^2}{k_A^2} \right] E_{tot}^{z_1}(x) = \left( k_A^2 - k_0^2 \right) E_{tot}^{z_1}(x) \quad y = 0, \quad i = 1, \ldots, I \]  \hspace{1cm} (11)

\[ E_{tot}^{z_1}(x) = \left[ E_{trans}^{z_1}(x) + E_{scatt}^{z_1}(x) \right] \quad y = 0, \quad i = 1, \ldots, I \]  \hspace{1cm} (12)

\[ E_{tot}^{z_1}(x) = E_{11}^{z_1}(x) \quad x \in \pi \]  \hspace{1cm} (13)

\[ \begin{cases} |x| E_{tot}^{z_1}(x) < K & y < 0, \quad i = 1, \ldots, I \quad (K \text{ real constant}) \end{cases} \]  \hspace{1cm} (14)

\[ \begin{cases} |x| H_{tot}^{z_2}(x) < K & y < 0, \quad i = 1, \ldots, I \quad (K \text{ real constant}) \end{cases} \]  \hspace{1cm} (15)

\[ \lim_{|x| \to \infty} |x| [\omega \mu_2 x \times H_{tot}^{z_2}(x) + k_2 E_{tot}^{z_1}(x)] = 0 \quad y < 0, \quad i = 1, \ldots, I \]  \hspace{1cm} (16)

\[ \lim_{|x| \to \infty} |x| [k_2 x \times E_{tot}^{z_1}(x) + \omega \mu_2 H_{tot}^{z_2}(x)] = 0 \quad y < 0, \quad i = 1, \ldots, I . \]  \hspace{1cm} (17)

The meanings of the involved quantities can be easily deduced from figure 1. Moreover, equations (7)-(10) and (14)-(17) define Sommerfeld's radiation conditions [21] to must be satisfied, as the field solution must represent an outward propagating wave. After defining a set of equivalent current densities:

\[ J_{eq}^i(x) = j(\omega \mu_2)^{-1} (k_A^2 - k_0^2) E_{tot}^{z_1}(x) \quad i = 1, \ldots, I \]  \hspace{1cm} (18)

equations (11) can be rewritten as:

\[ \left[ \nabla^2 + \frac{k_0^2}{k_A^2} \right] E_{tot}^{z_1}(x) = j \omega \mu_2 J_{eq}^i(x) \quad i = 1, \ldots, N . \]  \hspace{1cm} (19)

If we fix a set of \( M \) measurement points, \( \{x_1, \ldots, x_m, \ldots, x_M\} \), inside region 1, we can write the following relations for the scattered electric field in that region (solution of Eqs. (4)-(6), with the imposed radiation conditions (7)-(10)):

\[ j \omega \mu_2 \int_{A} J_{eq}^i(x') G^{(21)}(x_m/x') \, ds = E_{scatt}^{z_1}(x_m) \quad m = 1, \ldots, M ; \quad i = 1, \ldots, I \]  \hspace{1cm} (20)

where the Green function for the half-space geometry [22] can be expressed as:

\[ G^{(21)}(x/x') = \frac{-j}{4 \omega \mu_2 \pi} \int_{-\infty}^{+\infty} \frac{2}{\xi_1 + \xi_2} \exp \{ j y [x - x'] \} \exp \{ - \xi_1 y + \xi_2 y' \} \, dy \]  \hspace{1cm} (21)

where \( \xi_1 = (\gamma^2 - k_1^2)^{1/2} \) and \( \xi_2 = (\gamma^2 - k_2^2)^{1/2} \), with \( \text{Im} \{ \xi_1 \} \geq 0 \) and \( \text{Im} \{ \xi_2 \} \geq 0 \). By applying the moment method [19], using piecewise basis functions, we obtain the following linear systems of algebraic equations:

\[ [G] [J_{eq}'] = [E_{scatt}^{z_1}] \quad i = 1, \ldots, I \]  \hspace{1cm} (22)

where:

\[ [E_{scatt}^{z_1}] = [E_{scatt}^{z_1}(x_1), \ldots, E_{scatt}^{z_1}(x_m), \ldots, E_{scatt}^{z_1}(x_N)]^T \quad i = 1, \ldots, I ; \]  \hspace{1cm} (23)

\[ [J_{eq}'] = [J_{eq}^i(x_1), \ldots, J_{eq}^i(x_m), \ldots, J_{eq}^i(x_N)]^T \quad i = 1, \ldots, I ; \]  \hspace{1cm} (24)

where \( \{x_1, \ldots, x_m, \ldots, x_N\} \) indicate the centers of the \( N \) subdomains of region A. \([G]\) is a Green matrix that is independent of the incident electric field, and that can be computed once for all, independent of the number of illuminations used (i.e., I). The arrays \([J_{eq}']\),
$i = 1, \ldots, I$, can be estimated by using a pseudoinversion transformation [20, 12] in order to overcome ill-conditioning problems [23], following a procedure associated with previous imaging methods for free space [17, 14]. The pseudosolution is obtained by minimizing the following norms:

$$\| [G][J'_{eq}] - [E_{scat}^{i,1}] \|$$
and
$$\| [J'_{eq}] \|$$ \quad $i = 1, \ldots, I$ \quad (25)

and can be expressed as:

$$[J'_{eq}] = [G]^{+} [E_{scat}^{i,1}] \quad i = 1, \ldots, I$$ \quad (26)

where $[G]^{+}$ represents the pseudoinverse matrix of matrix $[G]$. It can be derived once for all and off line (independently of $I$).

Once the distribution of the equivalent current density inside $A$ has been determined (in an approximate way), the dielectric properties of the investigation area are deduced from equation (18) in discretized form:

$$[J'_{eq}] = [K][E_{tot}^{i,2}] \quad i = 1, \ldots, I$$ \quad (27)

where $[K]$ is a diagonal matrix ($N \times N$) whose generic element is given by:

$$k_{ij} = j (\omega \mu^{-1} - k_{0}^{2} - k_{0}^{2}(x_{i})) \delta_{ij}$$ \quad (28)

and $\delta_{ij}$ indicates the Kronecker function. In relation (27), the arrays $[E_{tot}^{i,2}]$ are given by:

$$[E_{tot}^{i,2}] = [E_{tot}^{i,2}(x_{1}), \ldots, E_{tot}^{i,2}(x_{n}), \ldots, E_{tot}^{i,2}(x_{N})]^{T} \quad i = 1, \ldots, I$$ \quad (29)

where the values $E_{tot}^{i,2}(x_{n})$ can be expressed (according to equations (11)-(13) and the imposed radiation conditions (14)-(17)) as [22]:

$$E_{tot}^{i,2}(x_{n}) = j \omega \mu_{2} \sum_{h=1}^{N} \int_{A_{h}} J'_{eq}(x') G^{(22)}(x_{n}/x') \, ds_{n} + E_{inc}^{i,2}(x_{n}) \quad n = 1, \ldots, N ; \quad i = 1, \ldots, I .$$ \quad (30)

In this relation, $G^{(22)}(x_{n}/x')$ is given by [22]:

$$G^{(22)}(x_{n}/x') = \frac{j}{4 \omega \mu_{2} \pi} \int_{-\infty}^{+\infty} \frac{2}{\xi_{1} + \xi_{2}} \exp \{j \gamma [x - x'] \} \exp \{- \xi_{1} y + \xi_{2} y' \} \, d\gamma$$ \quad (31)

where $\xi_{1} = (\gamma^{2} - k_{1}^{2})^{1/2}$ and $\xi_{2} = (\gamma^{2} - k_{2}^{2})^{1/2}$, with $\text{Im} \{ \xi_{1} \} \equiv 0$ and $\text{Im} \{ \xi_{2} \} \equiv 0$. By averaging the values obtained for each illumination angle, from relation (27) we deduce the wavenumber in the generic $n$-th cell:

$$k_{0}^{2}(x_{n}) = k_{2}^{2} + j (\omega \mu_{2})^{-1} \sum_{i=1}^{I} J'_{eq}(x_{n}) E_{tot}^{i,2}(x_{n}) .$$ \quad (32)

The electric conductivity and the dielectric permittivity in each cell can be derived from relation (30), in a straightforward way.


The proposed approach has been tested via some numerical simulations. Let us consider figure 2. A uniform unit plane wave is incident on the interface plane $\pi$, with an
angle $\theta_i$. As a consequence, the incident, reflected, and transmitted waves in regions 1 and 2 are given by:

$$E^{\text{inc}}_1(x) = \exp \left\{ -jk_1 [x \sin \theta_i - y \cos \theta_i] \right\} \quad y > 0 ,$$  

$$E^{\text{ref}}_1(x) = R \exp \left\{ -jk_1 [x \sin \theta_r + y \cos \theta_r] \right\} \quad y > 0 ,$$  

and

$$E^{\text{trans}}_1(x) = T \exp \left\{ -jk_2 [x \sin \theta_t - y \cos \theta_t] \right\} \quad y < 0 ,$$

where

$$R = \frac{1 - n}{1 + n}$$  

$$T = \frac{2}{1 + n}$$  

and

$$n = \left( \frac{\varepsilon_2}{\varepsilon_1} \right)^{1/2} \cos \theta_i \cos \theta_i$$

The investigation domain is assumed to be a square whose side is equal to $5 \lambda_1 2/9$, where $\lambda_1$ is the wavelength in the upper medium. The domain is partitioned into 25 equal square subdomains whose areas are equal to $\lambda_1 2/45$. The distance between the center of the investigation domain and the plane $\pi$ is equal to $\lambda_1 / \sqrt{5}$. In order to collect the information on the scattering phenomena, an observation domain, made up of 25 equally spaced measurement points, is arranged on a line placed in the upper medium, perpendicular to the cylinder axis, and parallel to the plane $\pi$. The distance between the probing line and the plane $\pi$ is $\lambda_1 / 6 \sqrt{5}$, and the distance between two points is $\lambda_1 / 10 \sqrt{5}$. A homogeneous square scattering object is placed in a whole subarea and illuminated by the transmitted wave in the lower.
medium. The scattered electric field at each point of the observation domain is computed by solving the electric field integral equation by a procedure analogous to the Richmond one [24] for direct TM scattering in an infinite medium. To each obtained value we add a noise term, simulating computational and measurement errors. These terms are added to the noiseless scattered field values by adding random arrays, \([E_n^s], i = 1, \ldots, N,\) to the left-hand side of relation (22):

\[
[G][I_{eq}^i] = [E_{scat}^{1 \ldots 1}] + [E_n^s] \quad i = 1, \ldots, I
\]

(39)

where the \(M\) elements of the arrays \([E_n^s]\) are given by \((2M \times I)\) independent sequences (real and imaginary parts) of Gaussian variables characterized by zero mean values and standard deviations obtained, in a straightforward way, after fixing the signal-to-noise ratio. This ratio can be defined as:

\[
S/N = 20 \log \frac{|E_{scat}^{1 \ldots 1}|}{|E_n^s|}
\]

(40)

For the first simulations, we considered lossless media \((\varepsilon_r = 1.0 + j0.0\) and \(\varepsilon_r = 5.0 + j0.0)\) and noiseless input data \((S/N = \infty)\). The scattering object had a dielectric permittivity equal to \(\varepsilon_r = 10.0 + j0.0\). A single-illumination \((I = 1)\) with normal incidence \((\theta_i = 0)\) was assumed. Moreover, in order to quantify the quality of the dielectric reconstruction, some parameters were introduced:

\[
\alpha_1 = \frac{|J_{eq}^*(x_n)|}{|J_{eq}(x_n)|} - |J_{eq}(x_n)|
\]

(41)

\[
\alpha_2 = \max_i \frac{|J_{eq}^*(x_i)|}{|J_{eq}(x_i)|} \quad i \neq n
\]

(42)

\[
\alpha_3 = \frac{|\varepsilon_n^* - \varepsilon_n|}{\varepsilon_n}
\]

(43)

where \(|J_{eq}(x_n)|, |J_{eq}^*(x_n)|, \varepsilon_n, \varepsilon_n^*|\) stand for the amplitude of the electric current density and of the dielectric permittivity (both original and reconstructed), respectively, inside the \(n\)-th cell (indicated on the abscissa) containing the cross-section of the cylindrical object. The parameters \(\alpha_1\) and \(\alpha_3\) represent the percentage errors on the reconstructions of the equivalent current density and of the dielectric permittivity, and \(\alpha_2\) refers to the reconstruction of the background, and indicates a wrong location of the scatterer. Figure 3 shows the behaviours of these parameters when the scatterer's cross-section occupies cells 1, 8, 13, 17 and 22. As we expected, the errors increase as the depth increases. Despite the infinite signal-to-noise ratio, errors of the order of 50% affect the reconstruction of cell 22, while a very good reconstruction is achieved for cell 1 (errors smaller than 5%).

In other simulations, we considered a scattering object characterized by a complex dielectric permittivity \(\varepsilon_r = 1.0 + j10.0\) (idealized lossy scatterer). In order to evaluate the reconstruction of the electric conductivity for this object, figure 3 also gives the plot of the parameter \(\alpha_4,\) defined as:

\[
\alpha_4 = \frac{|\sigma_n^* - \sigma_n|}{\sigma_n}
\]

(44)

where \(\sigma_n\) and \(\sigma_n^*\) stand for the original and the reconstructed electric conductivities inside the \(n\)-th cell containing the scatterer's cross-section.
Moreover, a lossy propagation medium was considered. Figure 3 presents the reconstruction results for the same significant cells. In this case, a dissipative lower medium characterized by \( \varepsilon_2 = 5.0 + j0.2 \) was assumed.

The plots in figure 3 point out an improvement in the reconstruction results for the lossy lower medium. This is due to the fact that, in these noiseless simulations, the lossy medium attenuates scattering data; the attenuation increases with the depth of the contribution cell. Moreover the incident field (transmitted wave) is also attenuated in proportion with the depth; as a consequence, more information on each discretization cell is contained in the input data. The following results, on noisy environments are less satisfactory.

Noise effects are taken into account in figures 4 and 5, where the parameters \( \alpha_1 \), \( \alpha_2 \) and \( \alpha_3 \) are plotted as different values of the signal-to-noise ratio (S/N = 10, 20, 30, 40, 50, 60, 80 dB).

![Fig. 3. — Plots of the coefficients \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) for cells \( n = 1, 8, 13, 17, 22 \) (\( \varepsilon_r \) : object relative dielectric permittivity; \( \varepsilon_{t_2} \) : relative dielectric permittivity of region 2 ; \( l = 1 \), \( \theta_l = 0.0 \) (rad); S/N = \( \infty \)).](image)

![Fig. 4. — Plots of the coefficients \( \alpha_1 \), \( \alpha_2 \), and \( \alpha_3 \) for cells \( n = 1, 8, 13 \) (object : \( \varepsilon_r = 10.0 + j0.0 \); \( \varepsilon_{t_2} = 5.0 + j0.2 \); \( l = 1 \), \( \theta_l = 0.0 \) (rad); S/N = 10, 20, 40, 60, 80 dB).](image)
MICROWAVE SCATTERING BY BURIED DIELECTRIC CYLINDERS

Fig. 5.—Plots of the coefficients $\alpha_1$, $\alpha_2$, and $\alpha_3$ for cells $n = 17, 22$ (object: $\varepsilon_r = 10.0 + j0.0$; $\bar{\varepsilon}_{\alpha2} = 5.0 + j0.2$; $\theta_i = 0.0$ (rad); $S/N = 10, 20, 40, 60, 80$ dB).

For $S/N = 60, 70, 80$ dB, for the presence of the scattering cross-section in cells $1, 8, 13$ (Fig. 4) and $17$ and $22$ (Fig. 5). As can be seen, even in noisy environments, a good dielectric reconstruction can be reached for cell 1, while, for the other cells, the errors increase as the signal-to-noise ratio decreases. For $S/N = 10$ dB, we sometimes obtain a value of $\alpha_2$ greater than 100 percent, that is, for some cells, the amplitudes of the reconstructed equivalent current densities are larger than that of the full cell.

Finally, a multi-incidence process was considered. In particular, 5 incidence angles ($I = 5$) were used: $\theta_i = 0.0, 0.25, 0.5, 0.75, 1.0$ rad. The reconstructed values were

Fig. 6.—Histograms of the reconstructed relative dielectric permittivities for the cells of region A (scatterer contained in cell 17: $\varepsilon_r = 10.0 + j0.0$; region 2: $\bar{\varepsilon}_{\alpha2} = 5.0 + j0.2$; $S/N = 20$ dB). (a) $I = 1, \theta_i = 0.0$ (rad); (b) $I = 5, \theta_i = 0.0, 0.25, 0.5, 0.75, 1.0$ (rad).
averaged, without an appreciable increase in the computational load, due to the invariance of the Green matrix, as previously pointed out. Figures 6-9 give the histograms of the reconstructed relative dielectric permittivities inside all the cells of the investigation area, for a single-illumination angle (black towers (a)) and for 5 illumination angles (white towers (b)). The scatterer cross-section ($\vec{e}_0 = 10.0 + j 0.0$) was placed in cell 17 in all cases, and we assumed $S/N = 20$ dB (Fig. 6), 40 dB (Fig. 7), 60 dB (Fig. 8), and 80 dB (Fig. 9). As can be

Fig. 7. — Histograms of the reconstructed relative dielectric permittivities for the cells of region A (scatterer contained in cell 17: $\vec{e}^* = 10.0 + j 0.0$; region 2: $\vec{e}_2 = 5.0 + j 0.2$; $S/N = 40$ dB). (a) $1 = 1$, $\theta_i = 0.0$ (rad); (b) $1 = 5$, $\theta_i = 0.0, 0.25, 0.5, 0.75, 1.0$ (rad).

Fig. 8. — Histograms of the reconstructed relative dielectric permittivities for the cells of region A (scatterer contained in cell 17: $\vec{e}^* = 10.0 + j 0.0$; region 2: $\vec{e}_2 = 5.0 + j 0.2$; $S/N = 60$ dB). (a) $1 = 1$, $\theta_i = 0.0$ (rad); (b) $1 = 5$, $\theta_i = 0.0, 0.25, 0.5, 0.75, 1.0$ (rad).
Fig. 9. — Histograms of the reconstructed relative dielectric permittivities for the cells of region A (scatterer contained in cell 17: \( \varepsilon^* = 10.0 + j0.0 \); region 2: \( \varepsilon^* = 5.0 + j0.2 \); \( S/N = 80 \, \text{dB} \)). (a) \( I = 1, \; \theta_i = 0.0 \) (rad); (b) \( I = 5, \; \theta_i = 0.0, 0.25, 0.5, 0.75, 1.0 \) (rad).

seen, by using 5 incidences only for the lowest \( S/N \) value (20 dB) we obtained a better reconstruction of the object cross-section. For the other \( S/N \) values, the reconstructions were equal or worse. This is an expected result, as in free-space microwave imaging, the multiview process usually produces a spatial low-pass effect on the scatterer reconstruction [15]. Nevertheless, a better reconstruction of the background (empty cells) was achieved by the multi-incidence process for all the \( S/N \) values. This allowed us to obtain a more accurate reconstruction of the object shape, and to reduce the possibilities for artifacts and wrong locations.

4. Conclusion.

An approach to microwave imaging of infinite dielectric cylinders by means of interrogating microwaves has been proposed. A TM-wave incident electric field is used to illuminate an investigation domain containing the cross-section of a cylinder. A half-space geometry is considered, and both a lossy and a lossless lower medium are assumed. Starting from an integral-equation formulation of the inverse-scattering problem, a numerical solution for the equivalent current density is obtained by using the moment method and a pseudoinverse transformation.

An efficient (in terms of computation) multi-illumination process has been described. Preliminary simulation results suggest the possibility of further developing this methodology, which exhibits interesting features (e.g., the possibility of performing once and for all, and off line the pseudoinverse matrix, independent of the number of incident electric fields). This allows one to fix optimal working conditions. No constraints (except for practical ones) are imposed on the shape and subdivision of the investigation domain and on the number and locations of the measurement points (redundant data can be used). Moreover, since no first-order approximations are utilized, the method can be employed also for imaging of strong scatterers.
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