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An exact analytical model for predicting electromagnetic fields produced by current transients

R. S. Shi, J. C. Sabonnadière and A. Darcherif

Laboratoire d'Electrotechnique de Grenoble, ENSIEG, BP 46, 38402 St Martin d'Heres, France

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Abstract. — A novel time domain analytical method for predicting EM (electromagnetic) field transients resulting from power line switching operations is presented in this paper. This method which is directly derived from EM field theory of a power transmission line, allows the field transients to be accurately and efficiently calculated. It can be proved that conventional numerical methods for computing transmission line transient EM fields are in fact degenerative approximate models of this analytical method.

1. Introduction.

The calculation of transient EM fields produced by switching operations is becoming more and more important in the area of EM compatibility. Up to now, the methods currently in use to do this calculation can be categorized into two types: Hertzian dipole approach and travelling wave decomposition approach. It is noted that both of them are numerical approaches.

The well known Hertzian dipole approach [1, 2] is to segment a transmission line into many short Hertzian dipoles. The EM field radiated by this line is then the sum of all contributions from each constituent dipole. However, this approach is valid only if any Hertzian dipole is short enough for the current to be considered constant along its length and if the distance between the observation point and any Hertzian dipole is much greater than the dipole size. Due to this, the computational time required for an accurate result is considerable, especially for the case where the observation point is near the line.

The second approach, on the other hand, is to resolve the travelling current signal into a series of known functions. The resultant field by the full travelling current signal is a simple superposition over all these functions. The ramp approach [3] is a typical example where the travelling current signal is approximated by ramp basis functions. It should be noted that this approach is entirely different from the Hertzian dipole approach in nature. Although the
ramp approach is valid for both far-point and near-point, the accuracy and computational time depend heavily on the number of ramp functions.

It should be emphasized that the two aforementioned approaches are both numerical approaches and so are computationally heavy especially if accurate results are required. Strictly speaking, the analytical solution of the transient fields does exist, but conventional analysis fails to overcome some mathematical difficulties in solving magnetic vector potential corresponding to the transient current. This is why the solutions can only be obtained by the numerical approaches.

This paper explains how the mathematical difficulties are overcome. This is followed by the development of an analytical model for calculating power transmission line EM field transients. Neither the line nor the travelling current signal requires to be resolved. Since the analytical model which is directly derived from the EM field theory is very simple, exact solutions can be easily obtained with little computational effort. Further, the analytical method allows the EM fields to be expressed as an explicit function of time and space coordinates, which greatly helps interpret the field transient phenomena.

2. Transient current model.

The Bergeron’s method is used to determine the nodal voltages and branch currents in a network under the assumption that the transmission line is lossless and the line’s distributed inductance and capacitance is independent of frequency. The proposed transient current model allows the travelling waves and the current distributions along the line to be found from the nodal voltage and branch current at each extremity of the transmission line.

The voltage and current wave propagation along a lossless line shown in figure 1 are related to the line’s distributed inductance and capacitance by the following equations:

\[-\frac{\partial u(x, t)}{\partial x} = L \frac{\partial i(x, t)}{\partial t} \quad \text{and} \quad -\frac{\partial i(x, t)}{\partial x} = C \frac{\partial u(x, t)}{\partial t} \quad (1)\]

The general solutions of equation (1) are:

\[i(x, t) = i^*_g(x - ct) + i^*_t(x + ct) \quad \text{and} \quad u(x, t) = u^*_g(x - ct) + u^*_t(x + ct) \quad (2)\]

with \[u^*_g(x - ct) = Z_c i^*_g(x - ct) \quad \text{and} \quad u^*_t(x + ct) = -Z_c i^*_t(x + ct)\]

where \(i^*_g\) and \(u^*_g\) are waves travelling at the velocity \(c\) in the forward direction; \(i^*_t\) and \(u^*_t\) waves travelling at the velocity \(c\) in the backward direction and \(Z_c\) and \(c\) the surge impedance and velocity of propagation. Letting \(x = 0\) and substituting \((t - \xi/c)\) and \((t + \xi/c)\) for \(t\) into equation (2), respectively, we obtain:

\[i^*_g(\xi - ct) = \frac{1}{2} \left[ u \left( 0, t - \frac{\xi}{c} \right) / Z_c + i \left( 0, t - \frac{\xi}{c} \right) \right] \]

and

\[i^*_t(\xi + ct) = -\frac{1}{2} \left[ u \left( 0, t + \frac{\xi}{c} \right) / Z_c - i \left( 0, t + \frac{\xi}{c} \right) \right]. \quad (3)\]

Thus, the current at any instant and at any point of the line can be represented by:

\[i(\xi, t) = i^*_g(\xi - ct) + i^*_t(\xi + ct) = i_g(t - \xi/c) + i_t(t + \xi/c). \quad (4)\]

Expressions (3) and (4) show that the current distribution can be obtained from the nodal voltage and branch current at the extremity of the line. In the following analysis, these
expressions will be used to determine the magnetic vector potential at an observation point generated by the transient current flowing in the transmission line.

3. Exact transient electromagnetic field model.

Figure 2 shows the configuration of a line being studied. The line is assumed to be a cylinder shape whose radius is small in comparison with the minimum wavelength of the transient current so that it can be regarded as a filament. The line lies at Z-axis from the original point, and its length is $L$. The coordinate of the observation point is $P(x, y, z)$ in a cartesian coordinate system.

3.1 MAGNETIC FIELD MODEL. — Since the line is approximated by a filament, the $x$-axis and $y$-axis components of the magnetic vector potential $A(t)$ can be neglected, and the $z$-axis component of $A(t)$ created by the current $i(\xi, t)$ flowing in the line at the point $P$ can be given by:

$$A_z(t) = \frac{\mu_0}{4\pi} \int_0^L \left[ i(\xi, t - R(\xi)/c/R(\xi)) \right] d\xi$$

(5)

where $i(\xi, t - R(\xi)/c)$ is the current at the time $[t - R(\xi)/c]$ and at the position $z = \xi$ on the line. The term $R(\xi)/c$ accounts for the propagation delay from the source position $\xi$ to the field point $P(x, y, z)$. It must be noted that the current $i(\xi, t - R(\xi)/c) = 0$ when $(t-R(\xi)/c) \leq 0$. Substitution of $t$ by $(t-R(\xi)/c)$ into equation (4) yields:

$$i\left(\xi, t - \frac{R(\xi)}{c}\right) = i_g\left(t - \frac{\xi + R(\xi)}{c}\right) + i_f\left(t - \frac{-\xi + R(\xi)}{c}\right).$$

(6)

With this relation, equation (5) can be written as:

$$A_z(t) = \frac{\mu_0}{4\pi} \int_0^L \left[ i_g\left(t - \frac{\xi + R(\xi)}{c}\right) + i_f\left(t - \frac{-\xi + R(\xi)}{c}\right) \right] \frac{d\xi}{R(\xi)}. $$

(7)

It is convenient to resolve the term $A_z(t)$ into 2 terms $(\mu_0/4\pi)A_g(t)$ and $(\mu_0/4\pi)A_f(t)$ where $A_g(t)$ and $A_f(t)$ are defined as

$$A_g(t) = \int_0^L i_g\left(t - \frac{\xi + R(\xi)}{c}\right) \frac{d\xi}{R(\xi)}$$

and

$$A_f(t) = \int_0^L i_f\left(t - \frac{\xi + R(\xi)}{c}\right) \frac{d\xi}{R(\xi)}. $$

(8)
Thus, the three components of the magnetic field \( \mathbf{H}(t) \) can be obtained from the curl of the magnetic vector potential \( \mathbf{A}(t) \):

\[
H_x(t) = (1/\mu_0) \partial A_z(t)/\partial y ; \quad H_y(t) = - (1/\mu_0) \partial A_z(t)/\partial x \quad \text{and} \quad H_z(t) = 0 . \tag{9}
\]

It should be noted that equation (8) does not allow an analytical solution of \( A_g(t) \) and \( A_f(t) \) to be found because the currents \( i_g \) and \( i_f \) are arbitrary functions and cannot be written in an analytical form. This is why most studies in literature try to approximate \( i_g \) and \( i_f \) by bi-exponential functions [2], or ramp functions [3], etc. In fact, calculating the \( x \) and \( y \) components of \( \mathbf{H}(t) \) does not require \( A_g(t) \) and \( A_f(t) \) but their derivatives with respect to the coordinate variables \( x \) and \( y \). After some algebraic manipulation, \( \partial A_g(t)/\partial y \), \( \partial A_f(t)/\partial y \) can be expressed by the following analytical forms:

\[
\frac{\partial A_g(t)}{\partial y} = y \left[ \frac{i_G \left( t - \frac{\xi + R(\xi)}{c} \right)}{R(\xi)[R(\xi) + \xi - z]} \right]^L_0 \quad \text{and} \quad \frac{\partial A_f(t)}{\partial y} = y \left[ \frac{i_f \left( t - \frac{-\xi + R(\xi)}{c} \right)}{R(\xi)[-R(\xi) + \xi - z]} \right]^L_0 \tag{10}
\]

After substituting equation (10) into equation (9), we can easily obtain a simple analytical solution for magnetic fields:

\[
H_x(t) = - \frac{y}{4 \pi} [I_0(t) + I_L(t)] \quad \text{and} \quad H_y(t) = \frac{x}{4 \pi} [I_0(t) + I_L(t)] \tag{11}
\]

with

\[
I_0(t) = M_{ag} i_g(t - R(0)/c) + M_{af} i_f(t - R(0)/c)
\]

and

\[
I_L(t) = M_{bg} i_g(t - (R(L) + L)/c) + M_{bf} i_f(t - (R(L) - L)/c)
\]

where

\[
M_{ag} = 1.0/\{R(0)(R(0) - z)\} ; \quad M_{bg} = - 1.0/\{R(L)(R(L) + L - z)\} ; \\
M_{af} = - 1.0/\{R(0)(R(0) + z)\} ; \quad M_{bf} = 1.0/\{R(L)(R(L) - L + z)\} .
\]

Since equation (11) is obtained without any approximation for \( R(\xi) \) and \( \mathbf{H}(t) \) is obtained directly from the vector potential \( \mathbf{A}(t) \), it will be valid not only in near-field but also in far-field. The expressions above show that the radiated magnetic fields depend only on the currents at the ends of the transmission line, and that \( M_{ag} \) and \( M_{bg} \) are magnetic field coefficients of the forward travelling wave at each end of the line and \( M_{af} \) and \( M_{bf} \) are those of the backward travelling wave.

3.2 ELECTRIC FIELD MODEL. — Once the magnetic fields are determined, the electric fields \( \mathbf{E}(t) \) can be found by integrating the curl of \( \mathbf{H}(t) \) over time, and dividing the result by the permittivity of free space \( (\varepsilon_0) \). Thus, the three components of \( \mathbf{E}(t) \) can be obtained:

\[
E_x(t) = \frac{1}{\varepsilon_0} \int - \frac{\partial H_y(t)}{\partial z} \, dt ; \quad E_y(t) = \frac{1}{\varepsilon_0} \int \frac{\partial H_z(t)}{\partial z} \, dt
\]

and

\[
E_z(t) = \frac{1}{\varepsilon_0} \int \left[ \frac{\partial H_y(t)}{\partial x} - \frac{\partial H_x(t)}{\partial y} \right] \, dt . \tag{13}
\]
Substituting $H_y(t)$, $H_x(t)$ described by equation (11) into equation (13) yields the final expressions of the three components as follows

$$E_x(t) = -\frac{x}{4\pi \varepsilon_0 c} \left[ \frac{F_0(t)}{R^3(0)} - \frac{F_L(t)}{R^3(L)} - \frac{z}{R(0)} I_0(t) + \frac{L - z}{R(L)} I_L(t) \right]$$

$$E_y(t) = -\frac{y}{4\pi \varepsilon_0 c} \left[ \frac{F_0(t)}{R^3(0)} - \frac{F_L(t)}{R^3(L)} - \frac{z}{R(0)} I_0(t) + \frac{L - z}{R(L)} I_L(t) \right]$$

$$E_z(t) = -\frac{1}{4\pi \varepsilon_0 c} \left[ \frac{zF_0(t)}{R^3(0)} + \frac{(L - z) F_L(t)}{R^3(L)} + \frac{\rho^2}{R(0)} I_0(t) + \frac{\rho^2}{R(L)} I_L(t) \right]$$

where

$$F_0(t) = c \int \left[ i_z(t - \frac{R(0)}{c}) + i_f(t - \frac{R(0)}{c}) \right] dt$$

and

$$F_L(t) = c \int \left[ i_z(t - \frac{R(L)}{c} + \frac{L}{c}) + i_f(t - \frac{R(L) - L}{c}) \right] dt .$$

According to equations (3) and (4) which relate the nodal voltage and branch current with the travelling current waves, equations (12) and (17) can be expressed alternatively by:

$$I_0(t) = (1/\rho^2) \left[ u(0, t - R(0)/c) / R(0) + \frac{z}{R(0)} i(0, t - R(0)/c) \right]$$

$$I_L(t) = (-1/\rho^2) \left[ u(L, t - R(L)/c) / R(L) + \frac{z - L}{R(L)} i(L, t - R(L)/c) \right]$$

$$F_0(t) = c \int i(0, t - R(0)/c) dt \quad \text{and} \quad F_L(t) = c \int i(L, t - R(L)/c) dt .$$

Equations (18)-(20) show that the radiated EM fields due to a transmission line are only functions of the nodal voltages and branch currents at two ends of the line. Consequently, the proposed approach eases and speeds up the calculation of transmission line EM field transients.

**Conclusion.**

A new time domain analytical model for predicting transient EM fields has been developed in this paper. The model is not only exact and computationally efficient, but also gives significant insight into the physical mechanism of transmission line EM field transients. It is shown that the EM fields generated by a transmission line are only functions of the nodal voltages and branch currents at two ends of the line. We can mathematically prove that the two numerical approaches (i.e. Hertzian approach and ramp approach) are in fact the approximate degenerative models of this analytical approach. The future work is to provide numerical examples which compare the proposed approach with the two numerical approaches. It will be found when the number of ramp functions is chosen as infinite or the length of dipoles as infinitesimal, the EM fields calculated by the two numerical approaches are those obtained by this proposed approach.
References

