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Blade-loading effects on the propagation of unsteady flow and on forcing functions in axial-turbine cascades

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Abstract. — This article investigates the effect of tangential blade loading on the propagation of time-dependent pressure disturbances due to potential-flow interaction and viscous-wake interaction from upstream blade rows in axial-turbine-blade rotor cascades. Results are obtained by modeling the effects of the stator viscous wake and the stator-rotor potential-flow field on the rotor flow field. A computer program is used to calculate the unsteady flows in the rotor passages. The amplitudes for the two types of interaction are based on a review of available experimental and computational data. We study the propagation of the isolated potential-flow interaction (no viscous-wake interaction), of the isolated viscous wake interaction (no potential-flow interaction), and of the combination of interactions. We examine the propagation of both interactions in three rotor cascades of high, medium and low tangential-loading coefficient (1.2, 1.0 and 0.8 respectively) for typical values of reduced frequency. The discussion uses as example a stator-to-rotor-pitch ratio $R = 2$. We investigate the differences when the stator-to-rotor pitch ratio is decreased (to $R = 1$) and increased (to $R = 4$). We offer new explanations of the mechanisms of generation of unsteady forces on the blades and study the effects of tangential blade loading and of axial gap between blade rows on the time-dependent forces acting on the blades. The potential-flow field of the rotor-leading-edge region cuts the potential-flow field of the upstream stator, and distorts and cuts the wake centerlines. The potential-flow field cut into the rotor passage propagates downstream as a potential-flow disturbance superimposed on the rotor flow field. The direction and decay rate of this interaction are determined respectively by the stator-outlet flow angle and by the stator-cascade pitch. The cut wake is sheared into the rotor passage and it results in a region of increased unsteady pressure upstream of the wake centerline and a region of decreased unsteady pressure downstream of the wake centerline. The wake shearing is more pronounced in highly-loaded cascades, and for lower stator outlet-flow angles. The potential flow interaction dominates the unsteadiness for high values of $R$ and the wake interaction dominates the unsteadiness for low values of $R$. The above explanations can be used to determine locations of local unsteady-pressure regions, and the shape of the unsteady forcing functions.
Nomenclature.

\( a \)  
as acoustic velocity (Eq. (4))

\( B \)  
amplitude of potential variation (Eq. (6))

\( b \)  
axial chord of rotor or stator cascade

\( C_L \)  
tangential loading (lift) coefficient (Eq. (10))

\( c \)  
local absolute-flow velocity (Eq. (3))

\( c_0 \)  
rotor-inlet total sonic velocity (Eq. (11))

\( c_n \)  
stator-outlet absolute-flow averaged velocity

\( D \)  
wake amplitude, fraction of \( c_n \) (Eq. (3))

\( F \)  
dimensional force (Eq. (11))

\( F' \)  
non-dimensional force (Eq. (11))

\( j \)  
\( \equiv \sqrt{-1} \) (Eq. (6))

\( M \)  
Mach number

\( N \)  
number of blades (Eq. (1))

\( R \)  
\( \equiv N_{rb}/N_{sb} \) stator-to-rotor-pitch ratio

\( S \)  
pitch of a cascade

\( T_z \)  
moment (in the \( z \) direction) (Eq. (11))

\( t \)  
time (nondimensionalized by \( y/S_{sb} \))

\( u \)  
velocity component in the \( z \) direction

\( V \)  
circumferential component of velocity

\( v \)  
velocity component in the \( y \) direction

\( W \)  
characteristic width of the wake, expressed as fraction of \( S_{sb} \) (Eq. (3))

\( (x, y, z) \)  
cartesian coordinates (in subscripts also)

\( \alpha \)  
flow angle

\( \alpha_n \)  
angle of wake propagation at inlet, along \( c_n \)

\( \Delta \epsilon, \epsilon \)  
angles locating the potential (Eqs. (7, 8))

\( \delta \)  
perturbation operator (on \( p, u \) and \( v \))

\( \zeta \)  
parameter for the \( y \) period of \( \Phi \) (Eq. (6))

\( \xi \)  
parameter for the \( z \) decay of \( \Phi \) (Eq. (6))

\( \rho \)  
relative total density at rotor inlet

\( \Phi \)  
velocity potential defined by (Eq. (5))

\( \omega \)  
rotor passing frequency (Eq. (2))

\( \bar{\omega} \)  
reduced frequency parameter (Eq. (2))

Subscripts

\( \text{av} \)  
average value over one stator pitch

\( \text{i,o} \)  
cascade inlet, outlet respectively

\( \text{ip} \)  
flow property for potential-flow model

\( \text{iw} \)  
flow property in the wake

\( \text{p} \)  
pressure side

\( \text{q} \)  
suction (s) or pressure (p) side (Eq. (3))

\( \text{rb} \)  
rotor blade row

\( \text{s} \)  
suction side

\( \text{sb} \)  
stator blade row

\( \phi \)  
perturbation value due to potential (Eq. (5))
In the preliminary design of turbines stage-design parameters are converted to velocity diagrams at various radii. Starting from the velocity diagrams and past design expertise, designers have a lot of freedom on the choice of the ratio of the number of rotor blades \( N_{rb} \) to the number of stator blades \( N_{sb} \), which affects the pitches \( S_{rb} \) and \( S_{sb} \) and is equal to the stator-to-rotor-pitch ratio \( R \), defined by:

\[
R \equiv \frac{N_{rb}}{N_{sb}} = \frac{S_{sb}}{S_{rb}} \tag{1}
\]

The effect of \( R \) on geometry is illustrated in figure 1. The velocity diagrams, the rotors, and the nondimensional geometry of the upstream stator in figures 1a and 1b are identical, but \( R \) is varied by changing the value of \( S_{rb} \). Engine designers try to "detune" the forced response of the cascades by choosing unequal numbers of blades in stators and rotors. Economic considerations force designs to higher values of \( R \), but this results in larger disturbances entering the rotor cascade and it will probably lead to larger unsteady forces acting on the rotor. For a given rotor size and velocity diagrams, \( R \) is inversely proportional to the reduced frequency parameter, defined by:

\[
\tilde{\omega} \equiv \frac{\omega \cdot b_{rb}}{c_x} = \frac{2\pi \cdot V_{rb} \cdot b_{rb}}{S_{rb} \cdot c_x} \cdot \frac{1}{R} \tag{2}
\]

The sources of two-dimensional flow unsteadiness between the rotor and the stator are: viscous velocity wakes shed from the trailing edge of the stator, generated by the boundary layer along the stator surfaces; inviscid potential-flow interaction because of the relative motion of the lifting surfaces; vortices shed at the stator trailing edge; flutter of both cascades; and the effect of flow changes due to cooling flows in high-pressure high-temperature turbine stages. In figure 1 the velocity wake is illustrated in the dotted region starting at the stator trailing edges; the potential-flow interaction is illustrated as a sinusoidal variation in potential with a maximum at the stator trailing edge. As \( R \) increases the potential-flow interaction relative to the rotor increases, and its frequency and decay rate decrease. As \( R \) increases the amplitude of the velocity defect in the wake remains about constant, and one may observe a modest increase.

Fig.1. — The effect of \( R \) on the stage geometry. The velocity diagrams and the downstream rotors (flow from left to right) are identical. a) \( R = S_{sb}/S_{rb} = 4.0 \). b) \( R = S_{sb}/S_{rb} = 1.0 \).
in the width of the wake because of the increased Reynolds numbers and larger lengths over which the boundary layers develop on the larger stators.

In the study of blade-vibration problems one can find the natural frequencies by setting the forcing functions equal to zero. In order to quantify forced response problems (vibratory stresses and displacements), one needs to provide an accurate estimate of the forcing functions acting on the cascades. This is true whether the solution is obtained by simple or complex geometric models, and whether it is obtained by exact or approximate, energy or classical methods. The purpose of this paper is to provide some insights on the magnitude of the forcing functions acting on the cascades, and to investigate the mechanisms by which these forces are generated in the rotor passages for various geometries.

**Background.**

In past investigations [1, 2, 3] we computed the amplitude of the harmonics of the spatially analyzed unsteady forces on rotor blades as a function of $R$ for a series of typical turbine cascades. We considered the effect of the viscous wake interaction and of the potential flow interaction, neglecting the effects of vortex shedding (which result in much higher harmonics of much lower amplitude in the forcing functions) and assuming infinitely rigid blades without cooling. For these studies (past and the present one) we have used Giles’ [4, 5] computer program UNSFLO (an Euler solver for the two-dimensional, unsteady, compressible, inviscid flow around stator-rotor blades) to compute the flow field and the forces for different stator-exit (nozzle) angles $\alpha_n$. This program was chosen because it can handle arbitrary values of $R$ with reasonable CPU and storage requirements due to a novel “tilting” of the time domain. In the computational field the problem has been simplified by considering the effects of the stator disturbances on the rotor blades in the rotor-relative frame. The accuracy of the computations has been proven in the past by Giles [4] and Korakianitis [1, 2, 6, 7]. These checks involved comparisons of the results of calculations using UNSFLO with the results of four steady and unsteady-flow cases of known theoretical or experimental output. This paper examines the details of the mechanism of propagation of the viscous wake and the potential-flow interaction as three rotor cascades of various tangential-lift coefficients move past typical stator cascades by examining details of the flow fields at six relative positions of rotors and stators. We match observations of details of the flow fields with the shape of the forcing functions and outline the effects of tangential-lift coefficient and stator outlet-flow angle on the unsteady flows and forcing functions.

**The rotor-inlet boundary.**

Computational results have been obtained by considering the rotor unsteady flow fields in the rotor-relative frames. The two disturbances from the stator (viscous wake and potential-flow interaction) have been modeled as inputs to the computational rotor-inlet boundary. This simplification provides accurate computational results only if one is extremely careful to specify the correct boundary conditions to the problem. Details of the following derivations have been published by Korakianitis [1, 2] and Giles [5]. A few important equations have been included here for clarity and completeness because they are essential to understanding the model of the rotor-inlet boundary, and the following results and discussion.

The axial gap between blade rows in modern engines is between 0.2 and 0.5 of the axial chords. Narrower gaps result in shorter engines but increase the unsteadiness between the blade rows. Throughout these investigations we have used a stator-rotor axial gap nondimensionalized with the rotor axial chord $b_{rb}$ equal to 0.30.
The velocity disturbance is characterized by the maximum amplitude of the velocity defect $D$, expressed as a fraction of the undisturbed velocity, and by the “width” $W$ of the velocity defect (see Eq. (3)). Most velocity wakes observed in experimental data have velocity distributions which resemble Gaussian distributions. The width of the velocity defect is characterized by the corresponding characteristic width of the Gaussian distribution that would best fit the velocity data. In the following this width is expressed as a fraction of the pitch of the blade cascade that generates the wake. For the wake model we assumed that: the velocity defect is a Gaussian distribution; in the stator frame the flow vectors in the wake are parallel to the undisturbed flow (a velocity deficit with no angle variation); the static pressure is constant across the wake; and the total enthalpy is constant across the wake. The above are modeled by:

$$u_{w,q} = \left[ 1 - D \cdot \exp \frac{1}{2} \left( \frac{\tan(\alpha_n) x - y}{S_{ab} W_q} \right)^2 \right] c_n \cos(\alpha_n)$$

$$v_{w,q} = \left[ 1 - D \cdot \exp \frac{1}{2} \left( \frac{\tan(\alpha_n) x - y}{S_{ab} W_q} \right)^2 \right] c_n \sin(\alpha_n)$$

(3)

where $q$ is either s (for the suction surface) or p (for the pressure surface). The way this wake function varies from one computational rotor passage to the next, and an extensive literature review of experimental data on wake amplitudes and widths has been published in [1, 2]. Based on these experimental data we chose as representative wake amplitude $D = 0.10$ of $c_n$, and characteristic widths $W_p = W_s = 0.15$ of the rotor pitch $S_{rb}$ for the axial gap $d = 0.30$.

The model for the potential-flow disturbance was developed by observing experimentally measured and computed static pressure fields of various turbine-blade cascades. These indicate that across the line of the trailing edges of the cascades there is a variation of static pressure with maxima at (or very near) the trailing edges and minima at (or very near) the middle of the passage. The exact location and shape of the pressure variation depends on the geometric shape of the passage. The pressure variation is nearly sinusoidal and the amplitude of the pressure disturbance decays very fast with distance downstream. Numerous examples of this static pressure variation have been reviewed in the experimental data listed in [1,2]. For example, measured cascade data such as those shown in figure 6 of [8], figure 4 of [9], the turbine nozzle cascade investigated by Boletis and Sieverding [10], (shown in Fig. 7 of that Ref.) and by Sieverding, Van Hove and Boletis [11], (shown in Fig. 6 of that Ref.) show the near-sinusoidal shape and the rapid decay of the potential-flow interaction.

The potential-flow model (in the stator frame) is derived as a two-dimensional, linear, isentropic, irrotational perturbation to uniform flow (see p. 198 of the text by Liepmann and Roshko [12]):

$$(u^2 - a^2) \frac{\partial u}{\partial x} + (v^2 - a^2) \frac{\partial v}{\partial y} + u v \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = 0$$

(4)

The velocity-potential relationship is defined by:

$$\frac{\partial \Phi}{\partial x} = u \quad \frac{\partial \Phi}{\partial y} = v$$

(5)

For subsonic flows one expects that the potential-flow interaction is periodic in the $y$ (circumferential or tangential) direction and that it decays exponentially in the $x$ (axial) direction. Thus the general solution of equation (4) is of the form:

$$\Phi(x, y) = B \cdot \exp(j \zeta y + \xi x)$$

(6)
where $B$ is the amplitude, $\xi$ governs the decay and $\zeta$ dictates the periodicity of the potential $\Phi$. Substituting these in equation (4) we derive solutions for the corresponding velocity perturbations $\delta u$ and $\delta v$. The latter are coupled with the conditions of no variation in entropy and total enthalpy to give a cosinusoidal pressure perturbation $\delta p$ (later unsteady flow figures show the resulting shape of $\delta p$ in the rotor frame) of the form:

$$\delta p = \Delta p_\phi \cos \left\{ 2\pi (\epsilon + \Delta \epsilon) - \tan^{-1} \left[ \frac{\tan(\alpha_n)}{\sqrt{1 - M^2}} \right] \right\}$$

$$\Delta p_\phi = \frac{pcM \Delta v_\phi}{\sqrt{1 - M^2}}$$

(7)

where $M$ is given by $M = \sqrt{M_x^2 + M_y^2}$, $M_x$ and $M_y$ are the axial and circumferential Mach numbers in the stator frame, and the phases $\epsilon$ and $\Delta \epsilon$ at any location are given by:

$$\epsilon = \frac{y - \tan(\alpha_{ip}) z}{S_{sb}}$$

(8)

$$\Delta \epsilon = \frac{1}{2\pi} \tan^{-1} \left[ \frac{\tan(\alpha_n)}{\sqrt{1 - M^2}} + \frac{[\tan(\alpha_{ip}) - \tan(\alpha_n)] x_{te}}{S_{sb}} \right]$$

$\Delta \epsilon$ is a phase-shifting constant which locates the maximum amplitude in the pressure disturbance due to the potential-flow field at the stator trailing edges. $\Delta v_\phi$ is the maximum perturbation in circumferential velocity $v$ at the inlet boundary.

The velocity perturbations are functions of the axial distance from the inlet boundary $x_{te} = (x - x_{inl})$, where $x_{inl}$ denotes the axial location of the computational inlet boundary and $x_{te}$ denotes the axial location of the stator trailing edges. $\tan(\alpha_{ip})$ is the direction of propagation of the potential field given by:

$$\tan(\alpha_{ip}) = -\frac{M_x M_y}{1 - M_x^2}.$$

(9)

Typical values for the amplitudes of $\Delta p_\phi$ and the corresponding $\Delta v_\phi$ have been chosen [1, 2] by investigating the pressure and velocity fluctuations from experimental measurements and from published computations across the line of the trailing edges of numerous typical subsonic turbine-stator cascades, and across other lines a little downstream (5% and 10% of the axial chord) parallel to the line of the trailing edges. These measurements indicated that across the line of the trailing edges the static pressure fluctuations were typically between 4% and 4.5% of the average pressure. The corresponding values of $\Delta v_\phi$ for values of $V_{rb}$ typical of modern engines were between 4.52% and 5.31% of $c_n$. In the computations presented in this paper we used $\Delta p_\phi = 0.04$.

The combined disturbance of the potential-flow interaction and the wake interaction from the upstream stator at its outlet boundary is found by adding the values of the two disturbances. These are input at the computational rotor-inlet boundary via a coordinate transformation for $V_{rb}$.

Results.

Wilson [13] shows that a measure of cascade tangential loading (tangential force divided by tangentially-projected blade area divided by outlet dynamic head) is the incompressible tangential-loading coefficient, given by:

$$C_L = 2 \frac{S}{b} \cos^2 \alpha_0 [\tan(\alpha_1) - \tan(\alpha_o)]$$

(10)
Fig. 2. — Steady-flow performance of cascades $C_{08}$, $C_{10}$ and $C_{12}$. Nondimensional pressure contours (with inlet pressure) of increment 0.02. Cascade design parameters are included in table I.

For lightly, intermediately, and highly loaded cascades $C_L$ is approximately 0.8, 1.0, and 1.2 respectively. Results have been obtained for high and low $\alpha_n$ for the three cascades ($C_{08}$, $C_{10}$ and $C_{12}$), whose design parameters are outlined in table I and the steady-flow performance is shown in figure 2. The first has been designed with the blade design method described in [14], and the latter have been designed with the blade design method described in [6].
The dimensional forces per unit length of blade span (such as $F_x$ in the $x$ direction) are related to the nondimensional forces ($F'_x$) by:

$$F'_x = \frac{F_x}{z}$$

(The moment $T'_x$ is taken about the leading edge, and it is divided by $b_{rb}^2$).

These forces and the moment are generated by the combination of the potential and wake interactions and they vary as the rotor cascade moves past one full stator-cascade pitch. As the rotor moves past a series of stator pitches the forces and moment exhibit a periodic pattern. In order to study these patterns we chose to study the effects of the wake and potential-flow interactions on cascade $C_{12}$ moving past a stator that has double the pitch of the rotor ($R = 2$). In production engines designers would normally try to choose unequal integers for all blade rows, and $R$ would not be an integer. This unlikely geometry is useful for the purposes of this paper because it provides a symmetry that facilitates the discussion (see later figures), because it is in a range where both types of interaction are of comparable magnitude so that neither one dominates the generation of the unsteady part of the force, and because it is close to the value of $R$ of many turbine stages in production engines. The origin (zero) of the nondimensional period of the unsteadiness in each case corresponds to the non-dimensional time $t = y/S_{sb}$ and $y$ location at which the centerline of a stator velocity wake touches the leading edge of a rotor blade (numbered blade 0 in the following figures). The end of the period corresponds to the velocity wake from the next stator blade touching the leading edge of the same rotor blade. The figures of unsteady flow fields show the rotor-relative frame, with the stator cascade (its inlet boundary) moving towards the negative $y$ direction as time increases.

Figure 3 shows the interactions on rotor cascade $C_{12}$ for $R = 2.0$ in plots of unsteady pressure contours of increment 0.005 in the passages at time $t = y/S_{sb} = 0.1$. The upstream
The potential-flow interaction is shown on the left to propagate from the trailing edge of
the stator (large values of positive unsteady pressure at \( x = 0.30 \)) into the stator-rotor gap
and the rotor passage. The intensity of the potential-flow interaction of the stator is rapidly
decreasing downstream, it is cut by the potential-flow field of the leading-edge region of the
advancing rotor, and after a portion of the potential-flow interaction of the stator is cut into
the rotor passage it propagates downstream into the rotor.

The wake interaction is illustrated in the middle as unsteady pressure contours of increment
0.005 and vorticity contours. The reduced velocity of the wake results in two parallel vorticity

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Table 1. — Some information on the sample cascades. The corresponding reduced frequencies
can be evaluated using equation (2) (note: nondimensionally \( b_{rb} = 1 \) and \( c_x = M_1 \cos \alpha_i \))

<table>
<thead>
<tr>
<th>Cascade</th>
<th>( C_0 )</th>
<th>( C_{10} )</th>
<th>( C_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>inlet flow angle ( \alpha_i )</td>
<td>40.00°</td>
<td>40.00°</td>
<td>50.00°</td>
</tr>
<tr>
<td>outlet flow angle ( \alpha_o ) (&lt; 0)</td>
<td>60.00°</td>
<td>60.00°</td>
<td>60.00°</td>
</tr>
<tr>
<td>stagger angle (&lt; 0)</td>
<td>32.00°</td>
<td>35.00°</td>
<td>30.00°</td>
</tr>
<tr>
<td>loading coefficient ( C_L )</td>
<td>0.80</td>
<td>1.00</td>
<td>1.20</td>
</tr>
<tr>
<td>cascade solidity ( S_{rb}/b_{rb} )</td>
<td>0.6223</td>
<td>0.7779</td>
<td>0.8208</td>
</tr>
<tr>
<td>inlet Mach number ( M_1 )</td>
<td>0.334</td>
<td>0.405</td>
<td>0.502</td>
</tr>
<tr>
<td>outlet Mach number ( M_o )</td>
<td>0.800</td>
<td>0.800</td>
<td>0.800</td>
</tr>
<tr>
<td>steady-flow x-force ( F'_{x,av} )</td>
<td>0.1132</td>
<td>0.1179</td>
<td>0.1000</td>
</tr>
<tr>
<td>steady-flow y-force ( F'_{y,av} )</td>
<td>0.1330</td>
<td>0.1945</td>
<td>0.2296</td>
</tr>
<tr>
<td>steady-flow z-moment ( T'_{z,av} )</td>
<td>0.0853</td>
<td>0.1050</td>
<td>0.1153</td>
</tr>
<tr>
<td>high nozzle angle ( \alpha_n )</td>
<td>74.49°</td>
<td>70.94°</td>
<td>70.14°</td>
</tr>
<tr>
<td>( V_{rb} ) for high ( \alpha_n )</td>
<td>0.7026</td>
<td>0.6300</td>
<td>0.5000</td>
</tr>
<tr>
<td>low nozzle angle ( \alpha_n )</td>
<td>66.75°</td>
<td>58.85°</td>
<td>63.20°</td>
</tr>
<tr>
<td>( V_{rb} ) for low ( \alpha_n )</td>
<td>0.3783</td>
<td>0.2499</td>
<td>0.2498</td>
</tr>
</tbody>
</table>
streams traveling along the wake. These are shown as thicker dashed and dotted lines, identifying positive and negative vorticity streams respectively, propagating in the rotor flow field along the nozzle angle $\alpha_n$ in the rotor-relative frame. The wake centerlines are perturbed by the advanced potential-flow field of the leading edge of the rotor, which eventually cuts a wake segment between the suction side of the rotor blade that has cut the wake and the pressure side of the next rotor blade. The wake segment results in two counter-rotating recirculating flow patterns, one rotating counterclockwise downstream of the wake centerline, and one rotating clockwise upstream of the wake centerline in the passage. These are illustrated in figure 4, which shows unsteady flow vectors superimposed on entropy contours and on unsteady pressure contours of increment 0.005 in a smaller region of figure 3b. The entropy contours are shown in figure 4 as thicker dashed lines and point the location of the centerline of the chopped and sheared wake between blades 0 and 1 at time $t = 0.1$ (or between blades [-1] and [0] at time $t = 0.6$). The recirculating flow patterns result in vorticity contours which in turn cause the local increases and decreases in pressure due to the wake. The result is a region of increased unsteady pressure upstream of the wake centerline and a a region of decreased unsteady pressure downstream of the wake centerline, which can be identified by comparing figures 3 and 4.

The unsteady pressure contours of the combined wake and potential-flow interaction are illustrated in figure 3c superimposed on thicker dashed lines of entropy contours to identify the wake centerlines. The complex regions of increased and decreased unsteady pressure in figure 3c are now easily interpreted as the combination of unsteadiness due to the potential-flow interaction and the wake interaction.

The relative importance of the wake and the potential on the generation of unsteady pressure contours and unsteady forcing functions changes with $R$. This is illustrated in figure 5, which on the left shows the combined interaction on cascade $C_{12}$ for $R = 1$ and on the right shows the combined interaction on cascade $C_{12}$ for $R = 4$ at time $t = y/S_{ab} = 0.0$, when the wake
Fig. 5. — Unsteady flow fields with both interactions on cascade $C_{12}$ for high $\alpha_n$ and at time $t = y/S_{sb} = 0.0$. Left the wake interaction dominates for $R = 1$. Right the potential-flow interaction dominates for $R = 4$.

centerlines impinge at the leading edge of rotor blade 0. It is clear that: for $R = 1$ the potential-flow interaction decays very fast, so that it only affects the leading-edge regions of the rotor flow field (the unsteadiness is dominated by the wake interaction); and for $R = 4$ the potential-flow interaction dominates the unsteady flow field engulfing the rotor and dwarfing the unsteady pressure due to the wake.

The resulting forcing functions on cascade $C_{12}$ are shown in figure 6. On the left the forcing functions for the isolated potential, wake, and both interactions are shown for $R = 2$ (corresponding to Fig. 3). On the right the forcing functions for both interactions are shown for $R = 1, 2, 4$ (corresponding to Figs. 5, 3c, and the bottom row of Fig. 7 below).

Figures 7 and 8 show the propagation of unsteady pressure contours of increment 0.005 superimposed on entropy contours for the high and low nozzle angles respectively for cascades $C_{08}$ (top), $C_{10}$ (center) and $C_{12}$ (bottom) of the figures. The unsteady pressure contours of increment 0.005 are shown superimposed on entropy contours for times $t = 0.000$, [0.167], 0.333, [0.500], 0.667, and [0.833]. The square brackets indicate that one should consider the blades with numbers in square brackets in the appropriate plot. The rotor is always shown at the same location; the wake and the potential-flow interaction of the upstream stator are shown moving at the rotor-inlet boundary towards the negative $y$ values with increasing non-dimensional time at $z = -0.30$. The blades are numbered starting from blade 0, which is the blade at the leading edge of which the centerline of a wake impinges at time $t = 0$. The corresponding forcing functions for the high and low nozzle angles are shown in figure 9.

**Discussion.**

Using the above explanations of the mechanisms of generation of unsteady pressure regions due to the wake and due to the potential we can identify (and with insight predict for other geometries) the regions of influence of the wake and the potential-flow interactions in figures 7
and 8, and the resulting shape of the unsteady forcing functions in figure 9. The shape of $F_x'$, $F_y'$ and $T_z'$ in the figures indicates that the spatially analyzed Fourier harmonics of the unsteady forces are dominated by the first harmonic amplitude, and that the amplitude of the second harmonic is an order of magnitude smaller than the first harmonic for $R = 2$. This agrees with the conclusions from our past work.

**Potential-flow interaction.**—The explanation of the potential-flow interaction in this paper extends our previous interpretations. In [1, 3] we considered two limiting cases of rotor velocity (first infinitely small $V_{rb}$, and second a higher $V_{rb}$ resulting in just subsonic flow fields) to show that the potential interaction is originated at the rotor-inlet boundary but it does not "propagate" in the stationary sense in the rotor passages because it is affected by the rotor velocity. Here we show some additional details of the temporal variation of the potential-flow interaction.

The effect of the potential-flow field of the stator on the potential-flow field of the rotor can be seen by the existence of the unsteady pressure lines in figure 3a. The effect of the potential-flow field of the rotor bends the stator-wake centerlines near the leading edge of the rotors in figures 3b, 3c, 7 and 8. The weaker potential-flow field of the rotor blades propagates upstream, distorts, and eventually cuts the stronger potential-flow field of the stator as shown in figure 3a. Figures 3a, 7 and 8 show the positive unsteady pressure at the trailing edges of the stators, the negative unsteady pressure between the trailing edges of the stators, and the rapid decay of the potential-flow interaction downstream of the stator trailing edge. As the rotor passage moves it cuts the potential flow field of the stator into two regions: one upstream region, still attached to the stator; and one downstream region, which is traversing with the rotor. The latter propagates downstream according to the potential-flow disturbances (in this study from Eqs. (7) and (8)) transformed in the rotor-relative frame and superimposed on the rotor steady-flow field shown in figure 2, while continuously decaying. One can observe a
Fig. 7. — The propagation of unsteady flow fields with both interactions for high $\alpha_n$. Unsteady pressure contours of increment 0.005 superimposed on entropy contours of increment 2.0. Top for $C_{08}$, center for $C_{10}$, bottom for $C_{12}$. Left for $t = 0.000$ and $[t = 0.500]$, middle for $t = 0.333$ and $[t = 0.667]$, and right for $t = 0.667$ and $[t = 0.167]$. 
Fig. 8. — The propagation of unsteady flow fields with both interactions for low $\alpha_0$. Unsteady pressure contours of increment 0.005 superimposed on entropy contours of increment 2.0. Top for $C_{0s}$, center for $C_{10}$, bottom for $C_{12}$. Left for $t = 0.000$ and $[t = 0.500]$, middle for $t = 0.333$ and $[t = 0.833]$, and right for $t = 0.667$ and $[t = 0.167]$. 
direct correspondence between the space and time location of the maxima and minima in the unsteady pressure fields in figure 3a and the increases or decreases from the average forces in figure 6a respectively.

The potential-flow interaction from the stator extends into the rotor cascade passages and large portions of it enter the cascade when the direction of propagation of the potential is aimed near the center of the rotor passage. The potential flow field of the stator is cut by the advancing potential flow field of the rotor. After it is cut it moves downstream according to equation (7) as a potential-flow disturbance in the rotor-relative frame.

Given the rotor axial chord, inlet- and outlet-flow angles, and a value for \( R \), lower values of \( C_L \) result in lower values of \( S_{eb} \) (Eq. (10)). This in turn affects both the frequency (with respect to the rotor) and the decay rate of the potential-flow interaction in the rotor cascade. Simple velocity-diagram considerations also indicate that higher \( \alpha_n \) result in higher rotor circumferential velocities \( V_{rb} \), which result in smaller (more-negative) values of angle of propagation of the stator potential-flow field into the rotor \( \alpha_{ip} \). The effects of these arguments are evident in figures 7 and 8: the stator potential-flow fields decay fastest in cascade \( C_{08} \) and slowest in cascade \( C_{12} \), and the potential-flow fields from the stators of lower values of \( \alpha_n \) propagate at more negative angles with respect to the horizontal into the rotor-relative frames.

Wake interaction.— The explanation of the wake interaction in this paper extends previous interpretations. Meyer [15] and Lefort [16] considered the cutting of wakes by the rotor, and Smith [17] introduced the idea of distinct wake segments in the rotor passages. Hodson [18] discussed the rotation of the cut wake segments and the effects of the lower momentum fluid in the wake region, and Korakianitis [1, 3] used the idea of recirculating flows to show how they resulted in “suction” and “impingement” of unsteady-flow (low-momentum) jets that generate
suction and pressure regions in the turbine passages. In [3] we stated that "...the temporal variation of these instantaneous pressure maxima and minima, which act in addition to the local pressure in steady flow, generate the unsteady forces on the blade..." Here we show and explain some new and additional details of the temporal variation of the wake interaction.

The wakes are first bent by the potential flow field of the rotor as shown in the region $(X,Y) = (-0.1,1.5)$ in figure 3. As the leading edge in the rotor stagnation region interacts with the lower-momentum fluid in the wake (for example near points $(X,Y) = (0.0,0.0)$ and $(X,Y) = (0.0,1.7)$), recirculating-flow patterns are established in the stagnation region of the leading edge of the rotor as shown in the region $(X,Y) = (0.0,1.1)$ in figure 4. These recirculating flow patterns are generated as the wake is being cut; once generated they result in a counter-clockwise rotating unsteady-flow pattern downstream of the wake centerline, and a clockwise rotating unsteady-flow pattern upstream of the wake centerline. The wakes are cut by the passing rotor into individual segments that are acting in each passage. After the stator wake is cut to produce a segment of a wake in the rotor passage, the two ends of the wake segment travel at the local speeds: the portion attached to the pressure side moves downstream into the passage much slower than the portion attached to the suction side, because the local flow velocities are higher on the suction side of the cascade. At the same time lower momentum fluid moves from the wake end near the pressure side to the wake end near the suction side. The last two phenomena cause a thinner wake on the pressure side, a thicker wake on the suction side, and a counter-clockwise rotation of the centerline of the wake as it moves through the passage. The initial vorticity is conserved: before the wake is cut, for example near $(X,Y) = (-0.2,0.2)$, there is no unsteady pressure on either side of the wake; after the wake is cut, for example near $(X,Y) = (0.3,0.3)$, $(X,Y) = (0.3,0.7)$ and $(X,Y) = (0.8,1.1)$ there is substantial positive unsteady pressure on the upstream side of the wake and negative unsteady pressure on the downstream side of the wake (near $(X,Y) = (0.7,0.4)$). These vortices start to form as the leading edge of the rotor shears into the downstream side of the wake, generating the positive unsteady pressures upstream of the wake centerline and the negative unsteady pressures downstream of the wake centerline. The vorticity of the two unsteady-flow counter-rotating vortical-flow patterns that flank the wake centerline are equal and opposite. The two vortex patterns act in opposite directions. The upstream clockwise-rotating vortical flow pattern causes a local increase in pressure, and the downstream counter-clockwise-rotating vortical flow pattern causes a local decrease in pressure. As the vortices move downstream and out of the rotor cascade the wake is sheared, distorted and enlarged, while the amplitude of the unsteady pressure maximum or minimum is decreased and its region of influence increased. One can observe a direct correspondence between the space and time location of the maxima and minima in the unsteady pressure fields in figure 3b and 3c and the increases or decreases from the average forces in figure 6a respectively.

The cutting of the wake by the rotor generates two recirculating-flow regions, one upstream and the other downstream of the wake centerline. The unsteady forces from the wake interaction are generated by the positive unsteady pressure generated in the vortical pattern upstream of the wake centerline and by the negative unsteady pressure generated in the vortical pattern downstream of the wake centerline.

Figures 7 and 8 indicate that the wakes from lower nozzle angles act for a shorter part of the stator period in the rotor cascade, because they are already further downstream in the passage by the time they are cut by a rotor leading-edge region. The residence times of the wakes inside the rotor cascade are longer for the higher $\alpha_v$; and there are better opportunities for unsteady pressure maxima and/or minima to align themselves at opposite sides of the blades (and thus increase or decrease the resultant unsteady forces). The loading diagrams of figure 2 indicate that cascades with higher tangential-lift coefficient $C_L$ have higher suction-surface
and lower pressure-surface velocities. The centerlines of the cut wakes attached to the suction and pressure sides of the passages respectively travel at the local flow velocities downstream. Therefore, as indicated in figures 7 and 8, the wakes are sheared more: in cascades with higher values of $C_L$; and when they enter from lower $\alpha_0$.

Although many parameters affect the results, the above mechanisms are confirmed by observing the results from all the cascades associated with this and our past studies.

**Combined potential-flow and wake interactions.**—The corresponding unsteady pressure maxima and minima due to the combined interaction are interpreted as the combined (additive) effect of the two interactions described above. The magnitude of the unsteadiness of the force is increasing or decreasing accordingly at different times in figure 9. The average values of $F'$ are higher for higher $C_L$; the shape and magnitude of both $F'_u$ and $F'_v$ are influenced by the stagger (setting) angle of the cascades.

The value of $R$ has a significant effect on which type of interaction dominates the unsteady flow field. For low values of $R$ (less than 1.5) the stator potential-flow interaction decays very fast downstream so that it hardly affects the flow in the rotor cascade. The unsteady pressure contours inside the cascade passage are due to the wake interaction only. For high values of $R$ (greater than 2.5) the stator potential-flow interaction decays very slowly downstream and it dominates the flow in the rotor cascade.

**Other considerations.**—The accuracy of steady-flow cascade computations has been proven almost exact with grids as coarse as 60 x 15. (Using Hobson's transonic turbine cascade no. 2, cascade data from [18], comparison in [6].) All computations shown in this paper have been performed on an 100 x 40 grid. This grid results in minimum numerical diffusion, as illustrated by the very small spreading of the wake entropy contours between the computational inlet boundary and the region of influence of the potential-flow field of the rotor in figures 7 and 8. Finer grids would not have changed the results significantly, since they would hardly change the location of the unsteady-pressure disturbances from the two interactions. Coarser grids would diffuse the wakes more, thus spreading somewhat their region of influence of unsteady-pressure disturbances. This would affect the magnitudes of unsteadiness of the forcing functions, but it would not change their general shape, or the explanations of the mechanisms of generation of unsteady forces discussed above.

**Conclusions.**

The unsteady forces on two-dimensional gas-turbine rotors due to potential-flow interaction and viscous-wake interaction from upstream blade rows are computed using a compressible, two-dimensional, inviscid rotor / stator-disturbance interaction program. The viscous wake and the potential-flow interaction from the upstream stator are modeled at the rotor-inlet boundary. Explanations of the mechanism of generation of unsteady forces on the rotor blades are offered by observing the unsteady pressure contours in various combinations of the two interactions.

We conclude the following:

1. The results used in this paper to facilitate the discussion have been compared with the results from numerous other cascades of a variety of loading distributions and geometries. The reasoning presented in this paper is confirmed by all cases we have studied to date.
2. The potential flow field of the rotor cuts into the potential flow field of the stator and
vice versa. After the potential-flow disturbance from the stator is cut into a rotor cascade, it propagates into the relative flow field of the rotor passage as a potential-flow disturbance superimposed on the rotor flow field. Due to the relative size of rotor and stator the unsteadiness from the potential flow field dominates the rotor flow field only at high values of \( R \).

3. Given the rotor axial chord, its inlet- and outlet-flow angles, and a value for \( R \), higher tangential-loading coefficients result in slower decay of the potential-flow interaction from the upstream stator.

4. The potential flow field of the rotor near the leading edge and the leading edge itself cut into the wake. The cutting action generates two counter-rotating vortical patterns flanking the wake-segment centerline in the passage. The vortical pattern upstream of the wake centerline generates an increase in local pressure (and forces acting on the sides of the passage). The vortical pattern downstream of the wake centerline generates a decrease in local pressure (and forces acting on the sides of the passage). The wake-segment ends attached to the sides of the passage travel at the local flow velocities: the wake-segment end near the suction surface moves downstream faster than the wake-segment end near the pressure surface; and the centerline of the wake segment rotates counter-clockwise as it propagates downstream. Due to the relative size of rotor and stator the unsteadiness from the vortex patterns generated by the wake dominate the rotor flow field at low values of \( R \).

5. The wake is sheared more at higher values of tangential-loading coefficient and at lower values of nozzle angles.

6. The resulting unsteady forces on the blades are generated by the combined (additive) interaction of the two disturbances.

7. For low values of stator-to-rotor-pitch ratio (\( R \approx 1 \)) the unsteady forces are dominated by the wake interaction. For high values of stator-to-rotor-pitch ratio (\( R > 3 \)) the unsteady forces are dominated by the potential-flow interaction. For intermediate values of stator-to-rotor-pitch ratio (\( R \approx 2 \)) the unsteady forces are affected by both wake interaction and potential-flow interaction.

8. Since the mechanisms and locations of generation of unsteady pressure distributions due to the potential-flow and wake interactions are different, designers could use the above arguments to influence the shape of the unsteady forcing functions and minimize the magnitude of the unsteadiness by varying the stator-to-rotor pitch ratio and the axial gap between blade rows.

9. Although the above have been derived assuming an upstream stator influencing a downstream rotor, the conclusions can also be used for an upstream rotor influencing a downstream stator.

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References


