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Critical current simulation in granular superconductors above the percolation threshold

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Résumé. — Dans la limite de champ magnétique nul et de cohérence de phase du paramètre d'ordre supraconducteur, le courant supraconducteur maximal dans un réseau est limité par les jonctions intergranulaires qui se comportent comme des jonctions Josephson. Nous analysons les problèmes de percolation spécifiques aux réseaux de jonctions et du mélange de grains normaux et supraconducteurs. Nous donnons des exemples bidimensionnels et tridimensionnels ; après moyenne sur les configurations et analyse en taille finie, nous montrons que les lois de puissance valables au voisinage du seuil de percolation s'étendent sur un grand domaine au-delà du seuil de percolation, à deux et trois dimensions. Nous discutons les autres modèles limitant le courant supraconducteur, ancrage de vortex et fluctuations du paramètre d'ordre.

Abstract. — In the phase-coherent regime without applied external magnetic field, the critical superconducting current is limited by intragranular junctions which behave like Josephson junctions. We study the percolation aspects specific to lattices of such junctions and/or the mixing of superconductor with normal grains by averaging over configurations. We illustrate on 2 and 3 dimensional examples. The power laws valid near the percolation threshold are valid well above it, in two and three dimensions. We discuss the other models limiting the superconducting current, the vortex creep and superconducting order parameter fluctuations.

Introduction.

Electrical conduction without losses is one of the major technological challenges offered by the new superconducting ceramics [1, 2]. This problem interests also the fundamental physicists, among the famoust ones [3-6]. The new superconducting ceramics are perovskite materials exhibiting a strong polytypism [7]. The superconducting coherence length is very short, of the order of interatomic distances ; this is related to the nature of the superconducting transition, differing from the standard BCS mechanism [3, 4]. At present time, the interpretation of critical currents relies on the phenomenological anisotropic Ginzburg Landau theory.

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Materials obtained by sintering are inhomogeneous and crystallites may resemble either to interconnected needles, either to parallelepipedic microcrystallites aggregated during sintering; their size is of order of a micron. Intergranular grain boundaries build « weak links », rather well described as Josephson junctions [9, 10]. Their conducting behavior differs strongly from the corresponding bulk materials: the voltage-current \((V-I)\) characteristics exhibit two plateaus, the high-temperature one corresponding to the superconducting phase transition of grains, the lower-temperature one to the onset of intergranular coherence [11, 12]. One of the facts to which we contribute here is the possible existence of an upper bounded superconducting critical current without losses (at \(V = 0\), below the lowest critical temperature even in a disordered medium. The limitations of the supercurrent and the origin of losses are developed in the following.

1. Migration of vortex lines.

The migration of vortex lines across the sample perpendicularly to the current flow creates potential differences which are dissipated by Joule effect [13-15]. The vortex lines created by the penetration of external magnetic field, or the current-induced field trap a flux quantum \(\phi_0 = h/2 e\). At low temperature, the vortex lattice is pinned on the defects of the material (parasitic phases inclusions, grain boundaries) and the migration process is activated. The phase transition from a frozen regime of weakly dissipating vortices to a strongly dissipative vortex « liquid » has been studied by [16, 17]; this transition is found at a temperature well below the superconducting transition. Several authors proposed « phase diagrams » exhibiting these several conduction regimes [16, 5, 18, 19]. We should also mention the work of Rzchowski et al. [20] where the pinning of vortices on a lattice of junctions.

Finlayson et al. [21] proposed an interesting model relating the dissipation from vortices to the grain size; they showed that

\[
j_c(T) = j_c(0)(1 - T/T_c)^p, \tag{1}
\]

\(p\) varying between 1 and 3.

2. Order parameter fluctuations.

Intergranular phase fluctuations of the order parameter induce also dissipation. The smaller the coherence length is, the more important are the fluctuations [5, 11, 22]. By assuming that the fluctuations are slowly varying in space, the superconductor is equivalent to the magnetic X-Y model [23]. In two dimensions, as well a planar lattice of Josephson junctions as a superconducting film, as well vortices as phase fluctuations should exhibit the Kosterlitz-Thouless transition [24] with a logarithmic singularity; it has been recently observed by Ying and Kwok [25] on the \(V-I\) characteristic.

Both preceding contributions are of the same intrinsic origin, related to the creation of vortices and antivortices; the first is concerned with the dynamics of these defects.

3. Limitation of critical current by disorder.

Mixtures of superconducting and non-superconducting phases during the elaboration of materials, as well as the stoichiometry defects (in peculiar, oxygen) which are the most sensitive on the surface — and boundaries of grains, the texture of grains [26] may severely limit the maximal superconducting current within a sample. This constitutes a percolation
problem which has been initially studied by Deutscher [27]. One obtains a power law for the maximal supercurrent as a function of the defect concentration

$$J_c = \begin{cases} A (p_c - p)^\varepsilon, & p \leq p_c \\ 0, & p > p_c \end{cases}$$

(2)

The exposant \(\varepsilon\) varies with the dimension of space and the model limiting the current: Rhyner et Blatter [28], Mannhart et al. [26] and ourself [29, 30] gave an upper bound of the maximal superconducting critical current able to flow across a lattice of distributed Josephson junctions in zero field:

$$J_{ij} = \begin{cases} I_{ij}^m \sin (\phi_i - \phi_j) & \text{if } |I_{ij}| < |I_{ij}^m| \\ (V_{ij}/R_{ij}) & \text{else}. \end{cases}$$

(3)

We suppose that each grain is homogeneous and connects without losses the junctions located at its boundary. According to (3), \( |j_{ij}| \leq |I_{ij}^m| \). The algebraic current concerning each grain vanishes. In that way, we may give an upper bound for the superconducting current able to flow without losses through the sample

$$I = \sum_{\text{max}} I_{ij} \quad -I_{ij}^m \leq I_{ij} \leq I_{ij}^m$$

$$I_i = \sum_j I_{ij} = 0.$$

(4)

This constitutes a cut across the sample. This is a linear programming problem, solved by the simplex method or transfer matrices by Rhyner et al. [28]. We solved this problem by the Ford-Fulkerson theorem [31], which builds the maximal flux through a lattice with integer and bounded «capacities». This numerical solution is valid whatever the dimensionality of space, and it is valid even far of the percolation threshold. We studied Josephson junctions distributed according to

$$I_{ij}^m = 0 \text{ or } 1$$

(5)

which accounts for bad Josephson junctions due to disorientation of stoechiometry defects, as well as the problem of mixtures of normal and superconducting grains, or vacuum [30]. In case of a non-superconducting grain, we drop all junctions at its boundary. We used pipe-shaped samples, with periodic boundary conditions on the lateral edges, and we placed two electrodes at both ends of the pipe. The main results of this model are:

i) the maximal critical supercurrent is bounded by a critical cut between both electrodes, which exhibits the minimal transport capacity. This macroscopic property is not a volume average, but over a critical interface.

ii) With distributions (3), our problem is equivalent to find out the maximal number of self-avoiding walks across the sample; this gives a strong analogy to polymer problems. These walks are strongly correlated owing to the excluded volume property. The maximal current and the cut are unique, even if there exist several configurations leading to the same result for the maximal current.

iii) Numerical simulations over hundred random generated configurations with fixed defects concentration can be interpretated with power laws, as far as the maximal critical current is concerned, over a broad concentration range. We show in figure 1 one sample of superconducting sample with textured superconducting and normal grains. The percolation
Fig. 1. — Maximal flux on a $6 \times 6$ superconducting checkerboard; active paths: bold; normal grains: dotted.

Fig. 2. — Maximal current density on a $10 \times 10$ checkerboard of superconducting and normal grains; $c$ is the fraction of normal conducting grains. We averaged each concentration over 100 configurations.

Fig. 3. — Maximal flow on a $10 \times 10$ checkerboard of superconducting grains, with junction decimation; $c$ is the proportion of defective junctions; we averaged over one hundred configurations for each configuration.
threshold $c^*$ is strongly dependent on the connectivity model we adopt (directional percolation or not). In figure 2 we represent the critical current density for a distribution of normal and superconducting grains textured on a checkerboard. In figure 3, we represent the critical current density for a distribution of Josephson junctions with maximal capacity 0 or 1.

The electrical conduction problem in granular materials with coexisting supra- and normal-conducting phases has been treated by [32]; by lowering the temperature, the system exhibits several crossover regimes and textures.

Conclusion.

The topic of our work is a sensitive problem, which mobilises the efforts of many experimental and theoretical physicists. Even in the anisotropic Landau-Ginzburg framework, the Josephson junction network model requires several concepts of Physics, percolation, phase transitions, fluctuations, vortex dynamics.

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