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Short Communication

Different Transmission Regimes of Two Quantum Point Contacts in a Magnetic Field

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Abstract. — We calculate how the Aharonov-Bohm transmission oscillations of a cavity formed by two quantum point contacts superimpose on those induced by a magnetic field in a single point contact. The series of two Quantum Point Contacts (QPCs) amplifies in an irregular, noisy way the magnetic field induced transmission oscillations of a single QPC. The total transmission, given by the Airy formula, shows a great sensitivity to the values of the parameters of the barrier representing the QPC. However the system cannot exhibit a deterministic chaotic behaviour: the Airy formula corresponds to a linear map in the complex plane of the transmitted amplitude. We have a coherent transport in a structure with two characteristic lengths: the length $L$ of the cavity and that of its walls, $L_0$. When these lengths are close each other, the transmission as a function of the field shows a disordered look. On the contrary if $L$ is much larger than $L_0$, the transmission oscillations of the single quantum point contact, due to phase vortices trapped within the walls, simply modulate and make more regular the real Aharonov-Bohm oscillations of the cavity.

There is experimental evidence of oscillations in the two terminal magnetoresistance both in a single Quantum Point Contact (QPC) [1] and in two QPCs in series [2] formed in a two-Dimensional Electron Gas (2DEG). All these oscillations are periodic in the magnetic field strength and have been discussed in terms of Aharonov-Bohm effect, supposing to be in the validity regime of the Landauer conductance formula [3, 4].

Let us begin to consider a single QPC whose transmission properties in a magnetic field, perpendicular to the 2DEG, have been studied in the simple model of a barrier interrupting a quantum wire [5–8]. The transmission oscillations appear at the opening of a magneto-electric subband, when the subband bottom is just at the Fermi energy of the 2DEG. These oscillations can be also described in terms of creation or annihilation of a phase vortex-antivortex pair of the edge state wave function scattered by the barrier [9]: the trapping of a diamagnetic vortex within the barrier goes with an oscillation of the transmission.

If the edge states of only one subband propagate along the wire confining potential, we get
the following analytical expression [7] for the edge state amplitude $r$ reflected by a square barrier of width $L_0$ and height $U_0$

$$r = \frac{-iuA \sin(kaL_0)}{a \cos(kaL_0) + (u - 1) \sin(kaL_0)} = \sqrt{R_0} e^{i\eta_0} \tag{1}$$

where

$$u = \frac{(1 + \Omega_c^2)U_0}{2k^2}; \quad a = \sqrt{(u - 1)^2 - A^2u^2}$$

We assume a parabolic confining potential $m\omega_0^2x^2/2$ while $\Omega_c = \omega_c\omega_0$ indicates the cyclotron frequency $\omega_c = eB/mc$ in units of $\omega_0$; $k$ is the wave vector of the plane wave representing the edge state along $y$ axis and

$$A(k) = \exp\left(-\frac{2\Omega_c^2k^2}{(1 + \Omega_c^2)^{3/2}}\right)$$

is the overlap integral between the transverse parts, along the $z$ axis, of the entering and exiting edge states belonging to the lowest subband. From now on, we measure the energy in units of $\hbar\omega_0$ and the lengths in units of $\sqrt{\hbar/2m\omega_0}$. Obviously the transmission coefficient $T_0$ for this simple model of QPC is $T_0 = 1 - R_0$.

Let us now consider a cavity formed by a double barrier, obtained putting in series two QPCS $a$ and $b$, along the rectilinear channel. Let the two barriers be at a distance $L$ apart, so large that the amplitudes of the transmitted edge states $t_a, t_b$ are independent. If only one subband propagates along the channel, the amplitude of an edge state emerging from the cavity is given by

$$\frac{t_at_b}{1 - r_ar_be^{2ikL}} \tag{2}$$

where $r_a$ and $r_b$ are the reflected amplitudes. Therefore the cavity transmission coefficient $T_{ab}$ is given by the Airy formula

$$T_{ab} = \frac{(1 - R_a)(1 - R_b)}{1 + R_aR_b - 2\sqrt{R_aR_b}\cos\phi} \tag{3}$$

with

$$r_a = \sqrt{R_a} e^{i\eta_a} \quad r_b = \sqrt{R_b} e^{i\eta_b}$$

and

$$\phi = \eta_a + \eta_b + 2kL.$$  

We discuss elsewhere [10] how the Airy formula still holds when the edge states of many different subbands propagate along the longitudinal confining potential.

The phase accumulated by the wave function along the corresponding classical path within the cavity can be calculated approximatively in terms of the magnetic flux $\Phi$ through the area bounded by this path, measured in flux quanta, as:

$$\phi' = 2\pi \frac{e}{\hbar c} \Phi$$

The classical path surrounds the area $S = 2\pi_0L$, where
\[ x_0 = -\frac{k\Omega_c}{1 + \Omega_c^2} \]

is the center of the incoming edge state (with \( k > 0 \)). The evaluation of the magnetic flux \( \Phi \) through \( S \) gives

\[ \Phi = 2Lx_0(\sqrt{\frac{\hbar}{2m\omega}})^2B = 2kL\frac{\Omega_c^2}{1 + \Omega_c^2} \frac{\hbar c}{e}. \]

If we neglect the phase shifts due to the reflections on the walls we have \( \phi = 2kL \) and

\[ \phi' = \phi \frac{\Omega_c^2}{1 + \Omega_c^2}. \]

So, the two phases \( \phi \) and \( \phi' \) coincide at high magnetic field, when \( \Omega_c \gg 1 \). The area \( S \) bounded by the classical path behaves as an excluded area because the current-carrying magnetic wave function are mostly localized at the edges of the cavity. Therefore the oscillations driven by the phase \( \phi \), that is proportional to the magnetic flux through \( S \), are Aharonov-Bohm oscillations. Their origin is the same of those appearing in rings threaded by a magnetic flux.

On the contrary, for a single QPC there is not a simple and direct relation between the phase \( kaL_0 \) and the flux of the magnetic field across the area of the barrier \( [9] \). The oscillations of \( T_0 \) are driven by \( kaL_0 \), but this phase, that is nearly linear in the field strength, is not proportional to a magnetic flux through any excluded area.

These different behaviours of the phases \( kL \) and \( kaL_0 \) distinguish the real Aharonov-Bohm transmission oscillations due to the two QPCs cavity from those of the single QPC.

Now we study how these effects mix each other in the cavity total transmission, by substituting the reflected amplitude of equation (1) in the Airy formula of equation (2). The transmitted amplitude of a single QPC is

\[ t = i\sqrt{1 - R_0 e^{i\pi\omega t}}. \]

In Figure 1 the transmission oscillations of a single QPC are compared with those of two identical QPCs in series. The oscillations of a single QPC appear to be much more regular than those of the series of two QPCs. When we form a cavity with two square barriers, the single barrier transmission oscillations are amplified while their amplitude varies in a more irregular way with the field. Also, there is a large sensitivity of \( T_{ab} \) as a function of \( \Omega_c \) on the choice of the parameters. Figure 2 shows what happens if the length \( L \) of the cavity is slightly varied: the transmission oscillations change drastically in a small interval of values of \( \Omega \).

The question arises if this system could exhibit a deterministic chaotic behaviour as a consequence of the non linear relationship between the transmission of the whole cavity in terms of that of the walls given in equations (2) and (3). However the Airy formula can be obtained as a limit of a linear map and so no chaotic behaviour is possible. In fact the multiple reflection can be represented by the following recursive relation for the amplitude \( t_N \) transmitted after \( N \) reflections.

\[ t_{N+1} = (\nu^2 e^{2iCL})t_N + t^2. \]

Obviously

\[ \lim_{N \to \infty} t_N = t_{ab} \]
Fig. 1. — The transmission of a single square barrier versus $\Omega_c$ (upper part of the Figure) is compared (lower part of the figure) with that of two of them at a distance $L = 50$ apart.

Fig. 2. — The transmission $T_{ab}$ of the cavity delimitated by two square barrier versus $\Omega_c$ for two values of the cavity length $L$ very close each other shows its high sensitivity with respect to this parameter.
with $T_{ab} = |t_{ab}|^2$. The real and the imaginary parts of $t_N$ can be considered the components of a two dimensional vector $t_N = (\text{Re} t_N, \text{Im} t_N)$ that satisfies the recursive relation:

$$t_{N+1} = \mathcal{A} t_N + b$$

where

$$b = ((1-R_0) \cos \eta_0, (1-R_0) \sin \eta_0)$$

$$\mathcal{A} = \begin{pmatrix} R_0 \cos \phi & -R_0 \sin \phi \\ R_0 \sin \phi & R_0 \cos \phi \end{pmatrix}$$

with

$$\phi = 2\eta_0 + 2kL.$$  

This system of difference equations are the Hopf equations [11] linearized about the point $b$. A bifurcation can arise if the eigenvalues of the matrix

$$\begin{pmatrix} R_0 \cos \phi - 1 & -R_0 \sin \phi \\ R_0 \sin \phi & R_0 \cos \phi - 1 \end{pmatrix}$$

have a positive real part. In this case a circle is the limit cycle for the Hopf bifurcation. But our eigenvalues are
\[ \lambda_{\pm} = R_0 \cos \phi - 1 \pm t R_0 \sin \phi \]

so that they have always a negative real part. Increasing the field, the pair of conjugate eigenvalues never crosses the imaginary axis. Starting from an initial point \( t^2 \), the dynamics described by equation (4) tends always to a fixed point in the complex plane of the transmitted amplitude \( t_{ab} \).

In conclusion we note that, despite of the irregularities introduced by the cavity on the total transmission calculated in our model, genuine Aharonov-Bohm oscillations have been measured in real devices [2]. The separation between the magnetic field induced transmission oscillations of a single QPC and those of a series of two QPCs can be achieved when the length \( L \) of the cavity is much larger than the length \( L_0 \) of a single QPC. Figure 3 shows what happens when \( L \) is 30 and 100 times larger than \( L_0 \). When \( L = 100L_0 \) the Aharonov-Bohm oscillations of the cavity are modulated by those of the single barrier that arise from the trapping of the wave function phase diamagnetic vortices under one of the two barriers. In our opinion the experiments [2] refer to the regime \( L \gg L_0 \).

In summary we have used the Airy formula to calculate the transmission of a series of two QPCs in a transverse magnetic field, representing the single QPC by a square barrier. The cavity can amplify in irregular way the oscillations transmission of the single QPC introducing some noise; besides slight variations of the parameters cause large modifications in the oscillations. We have excluded the possible appearance of deterministic chaos analyzing the Airy formula in terms of a couple of linear recursive equations. In the complex plane of the transmitted amplitude these equations describe how the initial point \( t^2 \) goes in \( t_{ab} \), the amplitude transmitted after infinite reflections within the cavity. The evolution always ends in a fixed point.

This behaviour can change due to the competition between the two characteristic phases of the system \( kL \) and \( kaL_0 \). In fact the Aharonov-Bohm oscillations can stand up as a distinct character of the cavity in a magnetic field when the length of the cavity is much larger than the width of its walls. In this limit the transmission oscillations of a single QPC induced by the magnetic field appear as a slow amplitude modulation of the real Aharonov-Bohm oscillations.

References