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A Model of Chevrons in Smectic C* Liquid Crystals: Uniform and Twisted “Soliton” States

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Résumé. — On discute un modèle simplifié de la structure en chevron observée dans les cristaux liquides smectiques ferroélectriques en cellules minces, basé sur une légère généralisation de l'approximation “uniaxiale” (ou “nématique”) de l'élasticité d'orientation. Ce modèle permet de réconcilier la nature continue des couches avec les conditions de raccordement postulées par Clark et prend en compte de fortes rotations de la projection du directeur sur le plan des couches. On montre que lorsque l'angle d'inclinaison des couches est faible devant l'angle du cône moléculaire, la structure des états “uniformes” est donnée par la solution de type “soliton” découverte par Nakagawa. Dans la même limite, on étudie les états “tordus” par un calcul de perturbation complété par des arguments énergétiques plus qualitatifs. Ces deux approches suggèrent l'existence de distributions asymétriques de la torsion du directeur moléculaire similaires à celles observées dans des expériences récentes: dans ces états, la torsion se localiserait dans une demi-épaisseur de cellule, l'augmentation d'énergie étant compensée par une réduction de l'énergie de cœur du chevron.

Abstract. — We consider a simplified model of the chevron structure observed in thin cells of ferroelectric smectic liquid crystals, based on a generalization of the “uniaxial” (or “nematic”) approximation of orientation elasticity. This model allows for the conciliation of the continuous nature of the layers with the matching conditions postulated by Clark, and takes large rotations of the c-director into account. When the layer tilt angle is small compared to the molecular cone angle, we show that the structure of the “uniform” states reduces to the “soliton” solution found by Nakagawa. Within the same limit, “twisted” states are investigated by a perturbation calculation and by qualitative energy considerations. The results suggest the existence of asymmetrical c-director distributions similar to those observed in recent experiments: in these states, the c-director splay remains mainly localized in one half of the cell, the increase of splay energy being compensated by a reduction of the “folding energy” localized in the chevron core.

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1. Introduction

The discovery of smectic ferroelectric liquid crystals by Meyer *et al.* [1] opened up a new field in the physics of liquid crystals and also new possibilities for the development of high speed display techniques. The first step in this direction was achieved by Clark and Lagerwall [2], and by Handschy [3] who noticed that, despite the winding tendency of the molecules associated to their chirality, a macroscopic polarization could be stabilized by confining these compounds in very thin cells under planar alignment. These so-called "surface-stabilized ferroelectric liquid crystals" (SSFLC) exhibited a bistability of the macroscopic polarization [3] combined with very short switching times, and were thus expected to provide a new generation of display devices. Since these pioneering works, the layer structure and the distribution of orientation of the molecular director in thin cells of chiral smectics under planar alignment have given rise to various experimental [4, 14] and theoretical studies [14, 21]. The understanding of this question is of crucial interest for the development of devices combining high display speed and good control of the grey-scale levels [23]. In addition, from a fundamental point of view, the statics and dynamics of SSFLC pose interesting problems of non-linear physics. For instance, the switching of the macroscopic polarization under an external field seems to involve the propagation of "walls" of solitonic nature [24]. As noticed by Nakagawa [17], typical soliton solutions of non linear equations [25] are also involved in the static layer structure.

Concerning this structure, different experiments [4-8] showed that the initial geometry taken into account in which the layers were assumed to be planar and perpendicular to the cell plates ("bookshelf" geometry, Fig. 1a) was inadequate. The information obtained by high resolution X-ray and optical response measurements suggests a more complicated structure called "chevron" (Fig. 1b). In this geometry, the layers tilt and bend in the middle plane of the cell. This phenomenon is usually attributed to a layer thinning phenomenon induced by a progressive tilting of the molecules. More precisely, the smectic C* bookshelf structure

Fig. 1. — In the cooling process of a cholesteric liquid crystal, a "bookshelf" structure (a) can be formed in the smectic A phase. After further cooling, the transition towards a smectic C* state (molecules tilted relatively to the layers at an angle Ω_0) induces a layer shrinking effect that is compensated by chevron formation (b). This structure is characterized by a layer tilt angle $\theta(x)$ that varies between two values $-\theta_0$ and $+\theta_0$. It can also be formed directly if the smectic A phase does not exist with $\theta_0 \ll \Omega_0$. In both cases, the layer tilt angle $\theta(x)$ is coupled with the molecular rotation (c) measured by the angle $\phi(x)$.

is usually obtained by cooling a cholesteric phase under planar orientation. Very often, a bookshelf structure is formed but initially in the smectic A state (see Fig. 1a). In a second step, a A-C* transition occurs, in which the molecules tilt, the director remaining free to rotate on a cone of angle Ω_0 that depends on the temperature. This phenomenon is accompanied by a layer shrinking effect, the layer thickness being reduced by a factor of order $\Omega_0^2/2$ in the small-angle limit. Owing to the conservation of the number of layers and also presumably of anchoring phenomena on the glass plate, the apparent thickness a_y in the plane of the cell should remain unchanged. The only way to accommodate this constraint without too much motion of the molecules near the glass plate is to form the chevron structure suggested in Figure 1b, where the layer tilt effect ($\approx \theta_0^2/2$) compensates the effect of the molecular tilt ($\approx \Omega_0^2/2$) for a layer tilt angle $\theta_0 = \Omega_0$. In fact, things are more complicated and experiments show that these two angles are generally not equal, with $\theta_0 < \Omega_0$. This difference is usually rather small (a few degrees) for compounds possessing a smectic A phase, but a chevron formation with $\theta_0 \ll \Omega_0$ is very often observed when a direct transition between cholesteric and smectic C* phases occurs.

Different models of the chevron structure in smectics C* are now available. Clark [6, 7] postulated a simple solution assuming discontinuities of the layers at the chevron tip, but keeping continuity of the molecular director (see Fig. 2a). This results in the appearance of a "selective pre-tilt" of the c-director that becomes inclined with respect to the cell plane at an angle $\pm\Phi_0$ given by:

$$\sin(\Phi_0) = \theta_0/\Omega_0 \quad (1)$$

This value of the "c-pretilt" can satisfy both the continuity of the molecular director at the

Fig. 2. — Depending on the surface properties and on the cell thickness, two main kinds of states can be imagined in the context of Clark's model: a) "uniform" states in which the electric polarizations $\mathbf{p}$ at the boundaries are nearly parallel and b) "twisted" states in which they are nearly antiparallel. In the latter case, the c-director field exhibits a splay localized in one half of the cell. In both cases, a "selective pretilt" of the c-director is expected at the boundaries to be equal to $\phi = \pm \text{Arcsin}(\theta_0/\Omega_0)$ when the molecules are parallel to the cell plates. c): A slight uniform splay distributed in the whole cell can also be imagined when this selective pretilt is not satisfied.

chevron folding, and the perfect boundary conditions postulated by Clark (molecules parallel to the boundaries). This description has later been extended to take into account the effect of an external applied electric field [26], but the existence of a layer discontinuity is somewhat in contradiction with the current description of smectic elasticity. This is why Nakagawa [16] proposed a continuous model based on the minimization of the elastic energy associated with dilatation, bending of the layers and rotation on the molecular cone. This approach leads to replace the discontinuity by a localized folding associated with a stationary "soliton-like" solution. Nakagawa also suggested the existence of a similar solution in the case of chevrons in the smectic A phase induced by a strain exerted perpendicularly to the layers. Such a solution has been used by Limat and Prost [27] to interpret recent observations of chevrons in smectic A phases performed by Takanishi *et al.* [28]. They attributed this strain to a T -dependent mismatch between the natural layer thickness in the bulk and that imposed at the boundaries and emphasized the importance of critical effects involved by the proximity of the nematic-smectic A transition. These effects have been more carefully discussed by Kralj and Sluckin in the formalism of the Landau-de Gennes theory [29].

In the case of chevrons in smectics C*, Nakagawa progressively refined his model [18–20] by including new effects such as higher order non-linearities associated with layer tilt, two-dimensional dependence, effect of an electric field combined with possibly large values of the tilt angle.... Similar work has also been developed very recently by De Meyere *et al.* [21], who postulated a somewhat different expression for the elastic energy involving a cone angle variation. In both studies, the complexity of Euler equations obtained in the most general cases finally required the use of numerical simulations.

In the present paper we reconsider the problem of chevron formation in smectic C* liquid crystals. Our model is presented in Section 1. Its complexity is intermediate between the first expression suggested by Nakagawa and its more recent versions solved by numerical methods. In this way, we become able to discuss a few points not clarified yet, while avoiding the use of numerical assistance. In particular, we have added to his first model the possibility of a difference between the layer tilt angle and the cone angle, and also the possibility of strong rotations for the c-director. We have tried to specify as much as possible the nature of the constants involved in the gradient terms, especially their possible relations with the value of the cone angle. These considerations lead us to introduce two phenomenological parameters m and n that measure the "degree of departure" from the "uniaxial" or "nematic" approximation (see definitions in Section 2). More precisely, we show in Section 2 that Clark's solution is in fact correct only in the uniaxial "nematic" limit of the model ($m = n = 1$), Nakagawa's solution being relevant in another limit that includes the case $\theta_0 \ll \Omega_0$ mentioned above. In the latter case, a continuous "soliton" solution can be built but the pretilt of the c-director induced by the coupling between the layer curvature and the c-director rotation does not necessarily fit the boundary conditions assumed by Clark (molecules parallel to the plate). This discrepancy can be removed by adding to the solution a slight uniform splay of the c-director in the bulk of the cell, or by restricting the discussion to a particular value of the second coefficient $n = m$. In this case, the "selective pretilt" of the c-director induced by the layer folding coincides with that expected from the boundary conditions and is equal to that deduced from equation (1).

In Section 3, we turn our attention to the case of "twisted states" observed in recent experiments [13,14]. These states appear when the boundaries favor opposite polarization at the boundaries and thus a rotation of the c-director practically equal to π between the two boundaries (Fig. 2c). In fact they exhibit a peculiar localization of the c-director splay in one half of the cell. Such a structure can be understood in the context of the discontinuous solution of Clark (Fig. 2c) but, to our knowledge, no continuous model is available. We show that energy considerations may favor this structure in comparison with a uniform splay; the

cost induced by the splay localization is then compensated by a reduction of the folding region thickness. We also give a first-order solution that leads to this “half splay” chevron, the rotation of the *c*-director at the chevron tip being taken as a small perturbation parameter. For simplicity we limit our study to the case $n = m$ in which the selective pretilt of the *c*-director is defined unambiguously and to the limit $\theta_0/\Omega_0 \ll 1$. This condition allows us to simplify the non-linearities introduced by the finite rotations of the *c*-director since, the gradient of this rotation remains small in the chevron core.

2. Model - Equilibrium Equations

2.1. GENERAL EXPRESSION. — Our notation and the geometry of the problem are depicted in Fig. 1. As in Nakagawa's approach [17], the free energy localized at a position x involves the competition of two terms associated with layer thickness variations, and with orientational effects respectively:

$$f = \frac{B}{2} \left[\frac{\theta_0^2}{2} - \frac{\theta^2}{2} \right]^2 + \frac{K}{2} \left[\left(\frac{d\theta}{dx} \right)^2 + n\Omega_0^2 \left(\frac{d\Phi}{dx} \right)^2 - 2m\Omega_0 \left(\frac{d\theta}{dx} \right) \frac{d}{dx} (\sin \Phi) \right] + Kq\Omega_0 \frac{d}{dx} (\cos \Phi) \quad (2)$$

In this expression θ and Φ denote the local values of the layer tilt angle, and the rotation of the *c*-director respectively (see Figs. 1b and 1c). B denotes the elastic modulus that moderates the variations of the layer thickness, and K is the elastic constant associated with the bending of the layers, that is in turn coupled with the rotation of the *c*-director. Ω_0 is the molecular cone angle, and θ_0 is the layer tilt angle imposed by the shrinking of the layers, which may in turn also differ from Ω_0 although its temperature dependence is certainly correlated with that of Ω_0 (more detail will follow). The parameter q takes the chirality of the compound into account, m and n are the two constants mentioned in the introduction. We now discuss in more detail the approximation and the physics underlying this model.

2.2. COMPRESSIBILITY TERM. — The “compressibility” term (involving the B -modulus) was, in fact, first introduced in the discussion of undulation instabilities of smectics induced by a dilative stress [30,31] (case of homeotropic alignment). It can be justified by using a “rod-like” picture of the molecules. In a cooling process, we can assume that both the length $l(T)$ and the tilt $\Omega_0(T)$ of the “rods” may vary. As a result, the “natural” thickness of the layers should vary as $a_N = l(T) \cos[\Omega_0(T)]$ that one can approximate by $a_N \approx 1[1 - \Omega_0^2/2]$ in the small tilt angle limit. In fact, this relationship is presumably more complicated, the rod-like picture being certainly oversimplified. A more realistic expression that can be justified with a less naive “ellipsoid-like” picture is $a_N \approx 1[1 - \alpha\Omega_0^2/2]$ in which α is a phenomenological parameter which we expect to remain smaller than 1. If one now would consider an ideal “bookshelf” structure free from defects (Fig. 1a), in which the layer thickness a_B imposed at the boundaries differs from a_N , the energy stored per unit volume of the cell would be given by:

$$f_{\text{comp}} = \frac{1}{2} B \left[\frac{a_B - a_N}{a_N} \right]^2 \quad (3)$$

The imposed thickness a_B can be thought of as an initial thickness $a_N(T_0)$ at which the cooling could have began, or as a particular thickness imposed at the boundaries by microscopic

surface effects. The tilt of the layers allows then the system to reduce this stored energy, the effective thickness becoming $a_{\text{eff}} = a_B \cos(\theta) \approx a_B[1 - \theta^2/2]$ instead of a_B . This relationship is a simple geometrical consequence of the fact that in the absence of dislocations, the apparent thickness projected on the z -direction (see Fig. 1) must remain independent of x because of the conservation of the number of layers. After replacing a_B by a_{eff} in equation (3) one can easily check that this expression reduces to the compressional term involved in equation (1) with

$$\frac{\theta_0^2}{2} \approx \alpha \frac{l(T)}{a_B} \frac{\Omega_0(T)^2}{2} + \frac{a_B - l(T)}{l(T)} \quad (4)$$

the modulus B being in fact replaced with an “effective” modulus $B' \approx B(a_B/l)^2(1 + \alpha\Omega^2)$ (see Appendix A). If one identifies a_B with an initial layer thickness at which the cooling began (the most natural choice is the layer thickness formed when the cholesteric phase is replaced by the smectic C* phase, or by the intermediate smectic A phase if it exists), this relationship expresses the fact that the layer tilt compensates the thickness layer variation. This variation contains a cone angle contribution and an intrinsic variation associated with the variations of the molecular length. Both the parameter α and the existence of a molecular length contribution could explain the fact that θ_0 is always smaller than Ω_0 . It should also be noted that this molecular length effect was in fact that invoked in our previous paper on the formation of chevrons in smectic A phases. Recent measurements suggest that this molecular length contribution is most often negative for smectic C or C* phases, but could be positive in smectic A phases near the nematic-smectic A transition.

2.3. GRADIENT CONTRIBUTIONS. — If we now consider the gradient contributions in equation (2), the simplest justification consists in starting from the “uniaxial” approximation built on a “nematic-like” hypothesis [3, 15, 16]. In this approximation, the bending of the layers, as well as the rotation of the c -director, are reduced to gradients of the molecular director $\mathbf{n}$:

$$\begin{aligned} f_{\text{or}} = & \frac{1}{2} K_1 (\nabla \cdot \mathbf{n})^2 \\ & + \frac{1}{2} K_2 [\mathbf{n} \cdot (\nabla \times \mathbf{n}) - q_{\parallel}]^2 \\ & + \frac{1}{2} K_3 [\mathbf{n} \times (\nabla \times \mathbf{n}) - q_{\perp}(\mathbf{a} \times \mathbf{c})]^2 \end{aligned} \quad (5)$$

where K_1 , K_2 and K_3 are three Frank constants associated with splay, twist and bend deformations, respectively. The chirality is taken into account by a spontaneous twist $q_{\parallel}$ and a spontaneous bend $q_{\perp}(\mathbf{a} \times \mathbf{c})$ in which $\mathbf{a}$ and $\mathbf{c}$ denote the unit vector normal to the layer and the c -director, respectively (see Fig. 1). This energy can be expressed in terms of the three angles involved in Figure 1 by using the approximate expressions in the limit of small tilt angles θ and Ω_0 :

$$\mathbf{a} \approx \mathbf{e}_z + \theta \mathbf{e}_x \quad (6a)$$

$$\mathbf{c} \approx \mathbf{e}_x \sin(\phi) + \mathbf{e}_y \cos(\phi) \quad (6b)$$

$$\mathbf{n} \approx \mathbf{e}_z + [\theta - \Omega_0 \sin(\phi)] \mathbf{e}_x + \Omega_0 \cos(\phi) \mathbf{e}_y \quad (6c)$$

At second order in θ , Ω_0 and their gradients, one gets:

$$\begin{aligned}
f_{or} = & \frac{1}{2}K_1 \left\{ \frac{d}{dx}[\theta - \Omega_0 \sin(\phi)] \right\}^2 + \frac{1}{2}K_2 \left\{ \frac{d}{dx}[\Omega_0 \cos(\phi)] + q_{\parallel} \right\}^2 \\
& + \frac{1}{2}K_3 \left\{ \Omega_0 \cos(\phi) \frac{d}{dx}[\Omega_0 \cos(\phi)] - q_{\perp} \cos(\phi) \right\}^2 \\
& + \frac{1}{2}K_3 \left\{ [\theta - \Omega_0 \sin(\phi)] \frac{d}{dx}[\Omega_0 \cos(\phi)] + q_{\perp} \sin(\phi) \right\}^2
\end{aligned} \quad (7)$$

where, as suggested by Figure 1, we have assumed a one-dimensional dependence. In fact, the last two terms (bend contributions) contain fourth-order contributions that must be removed for the consistency of the expression. In addition, we have decided to simplify f_B a little more by assuming $K_1 = K_2 = K_0$. After a few calculations, equation (7) finally reduces to

$$\begin{aligned}
f_{or} = & \frac{1}{2}K_0 \left[\left(\frac{d\theta}{dx} \right)^2 + \Omega_0^2 \left(\frac{d\theta}{dx} \right)^2 - 2 \frac{d\theta}{dx} \frac{d}{dx} \{ \Omega_0 \sin(\phi) \} \right] \\
& + K_0 q_{\parallel} \frac{d}{dx} \{ \Omega_0 \cos(\phi) \} + K_3 q_{\perp} \frac{d}{dx} \{ \Omega_0 \cos(\phi) [\theta \sin(\phi) - \Omega_0] \} \\
& + \frac{1}{2}(K_0 q_{\parallel}^2 + K_3 q_{\perp}^2)
\end{aligned} \quad (8)$$

The last contribution is a constant that can be dropped without loss of generality. For simplicity we have decided to neglect the remaining $q_{\perp}$ contribution associated with the spontaneous bend. Finally, for reasons that will appear more clearly in the next section, we have decided to extend the generality of equation (8) by modifying the respective weights of the remaining terms:

$$\begin{aligned}
f_{or} = & \frac{1}{2} \left[K_{\theta\theta} \left(\frac{d\theta}{dx} \right)^2 + K_{\phi\phi} \Omega_0^2 \left(\frac{d\phi}{dx} \right)^2 - 2K_{\theta\phi} \frac{d\theta}{dx} \frac{d}{dx} \{ \Omega_0 \sin(\phi) \} \right] \\
& + K_{\phi} q_{\parallel} \frac{d}{dx} \{ \Omega_0 \cos(\phi) \}
\end{aligned} \quad (9)$$

where the first three K -moduli must verify $K_{\theta\theta}K_{\phi\phi} - K_{\theta\phi}^2 > 0$. One can easily check that this expression reduces to the orientational contribution postulated in equation (2) with $K = K_{\theta\theta}$, $m = K_{\theta\phi}/K_{\theta\theta}$, $n = K_{\phi\phi}/K_{\theta\theta}$ and $q = K_{\phi}q_{\parallel}/K$. In what follows, we shall assume that m and n are both positive and satisfy $m \leq n \leq 1$. The generalization made in equation (9) means that we leave the "nematic" approximation and really enter the "smectic" description. It is however to be noted that this description is not the most general one. The most general expression would consist in starting from the general Dahl-Lagerwall expression of smectic C* elasticity [32]. After assuming arbitrary relationships between the involved coefficients, equation (9) would appear as a special case of this description. This approach can be found in the work of de Meyere *et al.* [21] to which the reader is referred for more detail.

2.4. EQUILIBRIUM EQUATIONS. — The chevron shape and the c-director field are now to be deduced from equation (2) by minimizing the free energy integrated over x across the sample. The standard Euler-Lagrange formula leads to the following equilibrium equations:

$$\delta_{xx} - \frac{m}{p} [\sin(\phi)]_{xx} = -\frac{B}{2K\theta_0^2} \delta(1 - \delta^2) \quad (10a)$$

$$\phi_{xx} = \frac{mp}{n} \delta_{xx} \cos(\phi) \quad (10b)$$

where we have introduced the normalized quantities:

$$\delta(x) = \theta/\theta_0 \quad p = \theta_0/\Omega_0 \quad (11)$$

These equations are reminiscent of Nakagawa's equations [17] but are complicated with new non-linearities contained in the circular functions. These circular functions result from the fact that we have allowed finite rotations of the *c*-director while Nakagawa implicitly assumed that these rotations were small enough to linearize $\sin(\phi)$ in equation (2). We also note that the chirality has disappeared from the bulk equilibrium equations because the integral of the chiral term retained in equation (2) can be transformed into a surface contribution.

Usually, these bulk equilibrium equations must be completed with appropriate boundary conditions written at the surface of the cell plates ($x = \pm h/2$, h cell thickness). For smectic C* liquid crystals, these conditions combine the layer tilt and the *c*-director direction and it can become rather complicated to specify and to handle them. For simplicity we will assume that the typical size of the chevron core is small compared to the cell thickness, and that we can begin to seek a solution extending up to infinity, the bulk energy f vanishing when x becomes large. In a second step, we will check whether this solution satisfies the condition initially imagined by Clark, i.e., $n_x = 0$ at infinity and we shall eventually introduce modifications of the solution in order to match this condition.

2.5. PARTICULAR CASES. — At this stage it is worth mentioning a few particular cases of our model, that will be important for the discussion of the solutions found in Sections (2) and (3):

- i) the “nematic” approximation is recovered when $m = n = 1$,
- ii) a “quasi-nematic” case is obtained when $n = m^2$, where the orientational energy is in fact of “nematic” nature but with an effective cone angle equal to $\Omega = m\Omega_0$,
- iii) when $n = m$, f_{or} can be viewed as the sum of a “nematic interaction” described by a Franck constant equal to mK and of a phenomenological extra-contribution of the layer bending energy $\frac{1}{2}(1-m)K(\frac{d\theta}{dx})^2$, that takes into account the existence of the density modulation which appears in the smectic C* phase. As this case is the simplest extension in the smectic field of the nematic description, we can call it a “pseudo-smectic” approximation.

3. “Uniform” States: From Nakagawa's Soliton to Clark's Discontinuity

3.1. SYMMETRICAL “SOLITON” SOLUTION. — First, it is instructive to consider the limit $\phi \ll 1$. In this limit, one can simplify equations (10) which reduce to:

$$\delta_{xx} - \frac{m}{p} \phi_{xx} \approx -\frac{B}{2K\theta_0^2} \delta(1 - \delta^2) \quad (12a)$$

$$\phi_{xx} \approx \frac{mp}{n} \delta_{xx} \quad (12b)$$

These two equations are those studied by Nakagawa. Using equation (12b), one can eliminate Φ from equation (10a) and obtain a new equation for θ , the structure of which is often encountered in various fields of (non-linear) physics:

$$(1 - \frac{m^2}{n})\delta_{xx} = -\frac{B}{2K\theta_0^2}\delta(1 - \delta^2) \quad (13)$$

As a first step, we can solve equations (13) and (12b) if assume that the compressional and orientational energies vanish at infinity. This leads to two coupled "kink" solutions

$$\theta(x) = \theta_0\delta(x) = \theta_0 \tanh \left[\frac{x\theta_0}{2\lambda\sqrt{1 - m^2/n}} \right] \quad (14a)$$

and

$$\phi(x) = \frac{mp}{n} \tanh \left[\frac{x\theta_0}{2\lambda\sqrt{1 - m^2/n}} \right] \quad (14b)$$

where $\lambda = \sqrt{K/B}$ denotes the "penetration length" built on the two moduli K and B . This solution is the Nakagawa solution mentioned above, but we can now specify the validity conditions of this solution, at least in the context of our model. The ϕ rotations can be assumed to remain small provided that

$$\frac{mp}{n} = \frac{K_{\theta\phi}}{K_{\phi\phi}} \frac{\theta_0}{\Omega_0} \ll 1 \quad (15)$$

This solution is obviously not satisfied in general: we can expect that, very often, $K_{\theta\phi}$ and $K_{\phi\phi}$ will remain of the same order of magnitude. In addition, experiments show that, for compounds possessing a smectic A phase, θ_0 is indeed smaller than Ω_0 , but the difference does not exceed 10%. However, the limit $m \frac{\theta_0}{\Omega} \ll 1$ is significant because it can be solved exactly and allows one a physical discussion. In addition, it could apply for particular compounds that do not possess a smectic A phase. As mentioned in Section 1, it has been observed in this case that θ_0 is often much smaller than Ω_0 , and so solution (14) could be relevant for these compounds.

This chevron structure is very similar to that involved in smectic A phases and discussed in a previous paper [27]. Basically, this solution constitutes a "grain boundary" or "wall" [33] separating two regions of uniform but opposite values of both the tilt of the layers and "selective pretilt" of the *c*-director. Just as in the smectic A case, this phenomenon allows the system to localize the compression energy in a very thin region of typical thickness:

$$\lambda_w = 2 \frac{\lambda}{\theta_0} \sqrt{1 - m^2/n} \quad (16)$$

with again $\lambda = \sqrt{K/B}$. Assuming that λ is of molecular size (30 Å), θ_0 of order 30°, one gets $\lambda_w \approx 0.01 \mu\text{m}$, a size much smaller than the usual cell thicknesses (3 to 100 μm). As a result, the energy stored by unit of cell surface F_{ch} is much smaller than that of an ideal bookshelf structure F_{bo} satisfying the thickness imposed by the boundaries. This can be checked by integrating f over the cell thickness for both configurations. One gets

$$F_{\text{ch}} = \int_{-\infty}^{+\infty} f(x)dx = 2 \int_{-\infty}^{+\infty} \frac{B}{8} (\theta_0^2 - \theta^2)^2 dx \quad (17)$$

from Hamilton principle and after neglecting the surface terms. Making use of equation (14), one obtains after a straightforward calculation

$$F_{\text{ch}} = \frac{2}{3} \sqrt{KB} \sqrt{1 - m^2/n} \theta_0^3 \quad (18)$$

to be compared with the ideal bookshelf structure

$$F_{bo} = \frac{1}{8} B h \theta_0^4 \quad (19)$$

where h denotes the cell thickness. The ratio F_{ch}/F_{bo} is thus of order $\lambda_w/h \approx 5 \times 10^{-3}$ for $\lambda_w \approx 0.01 \mu\text{m}$ and $h \approx 2 \mu\text{m}$. As mentioned above, the chevron structure allows the system to match the boundary and bulk properties at a very low energy cost.

3.2. CONNECTIONS WITH CLARK'S SOLUTION. — The previous soliton solution can be viewed as a continuous equivalent of Clark's model [3] in which the discontinuity surface has been replaced by a "wall" of thickness λ_w . This wall separates two regions of uniform but opposite values of both the tilt of the layers and "selective pretilt" of the c -director. More precisely, if one forgets the detail of the solution in the chevron core, one has:

$$\theta(x) = \theta_0 \text{ and } \Phi(x) = \frac{m}{n} \frac{\theta_0}{\Omega_0} \quad \text{for } x > 0 \quad (20a)$$

and

$$\theta(x) = -\theta_0 \text{ and } \Phi(x) = -\frac{m}{n} \frac{\theta_0}{\Omega_0} \quad \text{for } x < 0 \quad (20b)$$

This looks like the structure imagined by Clark (in the limit $\sin \phi \approx \phi$), except that the "macroscopic" continuity of the director $\mathbf{n}$ at $x = 0$ and the boundary conditions ($n_x = 0$ at infinity) are not necessarily satisfied. This will appear more clearly after calculating $\mathbf{n}$ from equation (6) at first order in the ratio $p = \theta_0/\Omega_0$:

$$\mathbf{n} \approx \mathbf{e}_z + \Omega_0 \left[\mathbf{e}_y + p \left[1 - \frac{m}{n} \right] \tanh \left[\frac{x}{\lambda_w} \right] \mathbf{e}_x \right] \quad (21)$$

Depending on the values of the parameters m and n , different situations can be thought of:

1) in the "nematic" limit $m = n = 1$, the solution reduces exactly to Clark's solution. The layers are discontinuous at the chevron core, while the molecular director is continuous. In addition, the distribution of $\mathbf{n}$ is uniform across the whole sample and planar ($n_x = 0$). At the boundaries, the molecules are parallel to the cell plates. The solution is then compatible with the strong anchoring conditions initially imagined by Clark.

2) in the "quasi-nematic" case $n = m^2$, we have again a discontinuity surface in the chevron plane but, here, both the layers and the molecular director are discontinuous. In addition, the boundary condition $n_x = 0$ is not satisfied.

3) in the "pseudo-smectic" limit $m = n$, both the layers and the molecular director become continuous at the "microscopic" level. The molecular director remains continuous and in addition uniform in the whole cell, and satisfies the boundary condition $n_x = 0$. Macroscopically, the solution is perfectly equivalent to Clark's solution with an apparent discontinuity of the layers and of the c -director equal to those obtained in the "nematic" limit.

4) in the general case, the solution is "microscopically" continuous and "macroscopically" discontinuous at the chevron core for both the layers and the molecular director. The "macroscopic" picture coincides in fact with the "quasi-nematic" case with in particular the problem of the boundaries where $n_x \neq 0$. This discrepancy can be easily corrected after noting that equation (12b) in fact admits more general solutions

$$\phi(x) = \frac{mp}{n} \tanh \left[\frac{x}{\lambda_w} \right] + 2p \left[1 - \frac{m}{n} \right] \frac{x}{h} \quad (22)$$

Fig. 3. — Typical distribution of the molecular rotation angle $\phi(x)$ obtained in the “nematic limit” (continuous line, $m = n = 1$), in the “pseudo-smectic” case (dashed line, $m = n$) and in the general case (dotted line, $m/n = 0.8$, $\lambda_w/h = 0.003$), when the ratio $p = \theta_0/\Omega_0$ is small. The first approximation leads to the discontinuous Clark’s solution and the second one to a continuous solution macroscopically equivalent to Clark’s solution. In the third case, a uniform splay analogous to that shown in Figure (2c) must be superposed to the “soliton” solution even when the molecules are parallel to cell plates at the boundaries.

with again $\lambda = \sqrt{\frac{K}{B}}$ and $p = \theta_0/\Omega_0$. The pretilt of the **c**-director at the boundaries $\phi = \pm p$ is then recovered by adding to our previous solution a splay of the **c**-director uniformly distributed in the whole cell thickness. This uniform splay disappears in the “pseudo-smectic” limit $m = n$ and of course in the “nematic” limit $m = n = 1$. The three cases are shown in Figure (3).

3.3. CLARK’S LIMIT FOR “STRONG” MOLECULAR ROTATIONS. — The previous Subsections 3.1 and 3.2 were limited to the case where $p \ll 1$, i.e. to weak “molecular rotation” (rotation of the **c**-director) in the chevron core. The case of strong rotations seems impossible to be solved although perturbation expansions over p are possible. In the present section we shall not solve this complicated problem but we only prove that Clark’s distribution remains the correct solution minimizing the free energy in the “nematic” limit ($n = m = 1$), and that its natural generalization also seems to apply in the “quasi-nematic” case ($n = m^2$).

This can easily be shown by noting that, when $m^2 = n$, the bulk free energy (i.e., after the surface term has been removed) can be rewritten as:

$$f = \frac{1}{2}K \left[\left\{ \frac{d}{dx} [\theta - \Omega \sin(\phi)] \right\}^2 + \left\{ \frac{d}{dx} [\Omega \cos(\phi)] \right\}^2 \right] + \frac{B}{8} [\theta_0^2 - \theta^2]^2 \quad (23)$$

where $\Omega = m\Omega_0$ is the “effective” cone angle defined in Section 1. A trivial discontinuous solution ensures that $f = 0$ everywhere and thus minimizes the whole free energy:

$$\theta(x) = \theta_0 \text{ and } \sin[\Phi(x)] = \frac{\theta_0}{\Omega} \quad \text{for } x > 0 \quad (24a)$$

$$\theta(x) = -\theta_0 \text{ and } \sin[\Phi(x)] = -\frac{\theta_0}{\Omega} \quad \text{for } x < 0 \quad (24b)$$

In the case where $m = 1$ and thus $\Omega = \Omega_0$, this distribution coincides with Clark’s solution.

The continuity of the molecular director is thus satisfied in the chevron plane and also the boundary condition $n_x = 0$. This can be checked again in this case by calculating the $\mathbf{n}$ coordinates from equation (6c):

$$\mathbf{n} \approx \mathbf{e}_z + \theta_0 \left[1 - \frac{1}{m^2}\right] \mathbf{e}_x + \Omega_0 \left[1 - \frac{1}{m^2}\right] \mathbf{e}_y \quad \text{for } x > 0 \quad (25a)$$

$$\mathbf{n} \approx \mathbf{e}_z - \theta_0 \left[1 - \frac{1}{m}\right] \mathbf{e}_x + \Omega_0 \left[1 - \frac{1}{m}\right] \mathbf{e}_y \quad \text{for } x < 0 \quad (25b)$$

Just as in the small p case, the n_x continuity and the boundary condition $n_x = 0$ are not satisfied in the "quasi-nematic" limit ($m \neq 1$).

3.4. PSEUDO-SMECTIC LIMIT. — In conclusion to this part, Clark's solution is exactly recovered only in the "nematic limit" $m = n = 1$ of our model whatever the values taken by the other parameters (p can be small as well as finite). There is obviously no compound exhibiting both a smectic ordering and an ideal nematic elastic behavior. Thus, Clark's approximation is presumably never exactly verified. However, this approximation has been proved useful in the qualitative discussion of various experimental situations. Our main finding is that among the possible values taken by p , m and n , those verifying $m = n$ ("pseudo-smectic" limit) and $p \ll 1$ allow one to recover "macroscopically" Clark's approximation, the "microscopic" nature of the layers remaining, however, continuous. We then suggest that this new simple description could also be useful in the physical discussions of chevron properties in smectic C*, especially when energy considerations are required (the limit of Eq. (18) when $m = n = 1$ is clearly unphysical). In the next section, we illustrate this idea by the problem of asymmetrical states observed in twisted chevron configurations.

4. "Twisted" States

4.1. QUALITATIVE DISCUSSION. — In the "twisted" geometry, we have to solve equations (10) again but with new boundary conditions. The "twisted" states occur when the properties of the surfaces favor antiparallel orientation of the polarization and thus of the $\mathbf{p}$ -vector at the two boundaries. This implies a rotation of the $\mathbf{c}$ -director of order π distributed in the whole cell thickness. Typical values of ϕ at the boundaries in the small p limit could be (see Fig. 2(c)): $\phi(-h/2) = -p$, and $\phi(h/2) = -p + \pi$. Intuitively, one would expect a solution combining a uniform splay of the $\mathbf{c}$ -director with a "soliton wall", in a way similar to that involved in the previous section when the boundary conditions did not exactly match the wall properties (see Eq. (22)). Repeated experiments show however that the true solution is more complicated, the splay remaining in fact localized in one half of the cell. Here we propose a simple explanation of this observation in the limit $p \ll 1$ of the "pseudo-smectic" model ($m = n$).

The localization of the $\mathbf{c}$ -director splay in one half of the sample is somewhat surprising because it is intuitively not favored by energy considerations. This can be made more quantitative by calculating the free energy per unit of cell surface associated with a ϕ gradient of order π/h uniformly distributed across the cell thickness

$$F_{sp}(h) = \frac{1}{2} m K \Omega_0^2 h \left(\frac{\pi}{h}\right)^2 \quad (29)$$

If the rotation of the $\mathbf{c}$ -director now occurs in one half of the cell, instead of being spread over the whole thickness, the energy is to be modified by replacing h with $h/2$. This leads to an increase of the free energy that becomes equal to $F_{sp}(h/2) = m K \Omega_0^2 h \left(\frac{\pi}{h}\right)^2 = 2F_{sp}(h)$.

Fig. 4. — In the qualitative discussion of the continuous solutions describing a twisted state, two basic solutions can be imagined: a) a roughly “symmetrical” solution in which $\phi(0)$ is close to $\pi/2$, and b) an “asymmetrical” state in which $\phi(0)$ is close to zero (or at least of order $p = \theta_0/\Omega_0$). Energetically, the modifications of the “wall” structure located near $x = 0$ dominate the increase of energy associated with the splay localization in one half of the cell. This results in a selection of the “asymmetrical” state (see Section 3.1).

Although its energy is the larger, the second solution is experimentally selected. This means that there is another contribution to the free energy in the energy balance that must be taken into account. This contribution is presumably the energy of the chevron itself, i.e. of the “wall” located at $x = 0$. We then have to understand how this contribution could be modified by the c -director rotation field.

Schematically, we can imagine two possible distributions. In the first state, suggested in Fig. (4a) the molecular rotation is nearly uniformly distributed over the cell thickness. The value reached by ϕ at $x = 0$ is nearly equal to $\pi/2$: $\phi(0) = \phi_0 = \pi/2 - p \approx \pi/2$. In the second state (see Fig. 4b), the rotation is mainly located in the half space $x > 0$, so that ϕ_0 is presumably close to zero or at least of order $p \ll 1$: $\phi(0) = \phi_0 = -p \approx 0$. This angle ϕ_0 will be the central parameter of our approach. The dependence of the “wall” energy upon ϕ_0 can be roughly estimated by starting from the equilibrium equations (10). In the small p limit, the variations of ϕ in the chevron core can be expected to remain small. We can thus develop the circular functions as $\cos(\phi) \approx c_0$ and $\sin(\phi) \approx s_0 + \psi c_0$ with $c_0 = \cos(\phi_0)$, $s_0 = \sin(\phi_0)$ and $\psi = \phi - \phi_0$. It is to be noted that the difference of degree between the two developments allows us to keep the consistency of the energy expression (Eq. (2)). We then obtain the set of equations:

$$\delta_{xx} - \frac{m}{p} c_0 \psi_{xx} \approx -\frac{B}{2K\theta_0^2} \delta(1 - \delta^2) \quad (30a)$$

$$\psi_{xx} \approx pc_0 \delta_{xx} \quad (30b)$$

Terms proportional to higher powers of the ψ derivatives (such as $[\psi^2]_{xx}$) have been neglected in the small p limit. After calculations similar to those involved in Section 3.1, we have obtained the following approximate equation governing the reduced layer tilt angle $\delta(x)$:

$$[1 - mc_0^2]\delta_{xx} \approx -\frac{B}{2K\theta_0^2}\delta(1 - \delta^2) \quad (31)$$

At the lowest order in p , the local molecular rotation modifies the “wall” thickness that is in fact given by

$$\lambda_w = 2\frac{\lambda}{\theta_0}\sqrt{1 - m\cos^2\phi_0} \quad (32)$$

In turn, the wall energy will also be modified. This can be shown after repeating the calculations involved in Section 3.1, that finally lead to:

$$F_{ch} = \frac{2}{3}\sqrt{KB}\sqrt{1 - m\cos^2\phi_0}\theta_0^3 \quad (33)$$

From equation (33) we see that the wall energy should be reduced if $\phi_0 = 0$ instead of $\phi_0 = \pi/2$. We thus have a competition between two effects: the localization of the splay in one half of the cell increases the free energy by a factor of order

$$\Delta F_+ = \frac{1}{2}mK\Omega_0^2\frac{\pi^2}{h} \quad (34)$$

while this localization allows us to keep ϕ_0 close to zero and reduces the energy by a factor of order:

$$\Delta F_- = -\frac{2}{3}\sqrt{KB}[1 - \sqrt{1 - m}]\theta_0^3 \quad (35)$$

The respective weight of these two opposite effects can be evaluated by considering the typical ratio:

$$\rho = -\frac{\Delta F_-}{\Delta F_+} = \frac{8}{3\pi^2}\frac{\theta_0^2}{\Omega_0^2}\frac{h}{\lambda_w(0)}\frac{\sqrt{1 - m}}{m}[1 - \sqrt{1 - m}] \quad (36)$$

where $\lambda_w(0)$ denotes the wall thickness obtained for $\phi_0 = 0$. In most cases the ratio $\frac{h}{\lambda_w(0)}$ will dominate this expression and the asymmetrical splay distribution should be selected. This is the fundamental mechanism proposed in the present paper.

4.2. A PERTURBATION APPROACH. — The reader may have noticed a problem in the previous section: equations (30a) and (30b) do not lead to a splay localization in one half of the cell, but to symmetrical solutions analogous to equation (22). In the present section, we show that this localization of the splay is to be related with non-linearities of the circular functions that are missing in equations (30a) and (30b). The effect of these non-linearities is essentially that a slight asymmetry of the soliton wall can give rise to a complete asymmetry of the large scale splay distribution. Again, for the sake of simplicity, we limit our study to the “pseudo-smectic” case ($m = n$) in the limit $p = \frac{\theta_0}{\Omega_0} \ll 1$.

In the “pseudo-smectic” limit, equation (10) reads:

$$\delta_{XX} - \frac{m}{p}[\sin(\phi)]_{XX} = -2(1 - m)\delta(1 - \delta^2) \quad (37a)$$

$$\phi_{XX} = p\delta_{XX}\cos(\phi) \quad (37b)$$

where we have defined a normalized coordinate $X = x/\lambda_w$ with again $\lambda_w = 2(\lambda/\theta_0)\sqrt{1-m^2/n}$. We now develop the circular functions in the vicinity of ϕ_0 as: $\cos(\phi) \approx c_0 - s_0\psi$ and $\sin(\phi) \approx s_0 + c_0\psi - \frac{s_0}{2}\psi^2$, with again $\psi = \phi - \phi_0$, $c_0 = \cos(\phi_0)$ and $s_0 = \sin(\phi_0)$. After a few calculations, equation (37) read:

$$[1 - mc_0^2]\delta_{XX} + 2(1 - m)\delta(1 - \delta^2) = -\frac{m}{p}\frac{s_0}{2}(\psi^2)_{XX} - mc_0s_0\psi\delta_{XX} \quad (38a)$$

$$\psi_{XX} - pc_0\delta_{XX} = -ps_0\psi\delta_{XX} \quad (38b)$$

Of course, far from the chevron core, the development of the circular functions becomes irrelevant. We can, however, expect that equations (37) and (38) can be matched to the same uniform states when $X \gg 1$ and $X \ll -1$, at the accuracy of our calculations.

In a second step, we assume that the solution is very close to that used in the description of the uniform states, but with just a slight lack of symmetry measured by the angle $\phi(0) = \phi_0$. More precisely, ϕ_0 slightly differs from 0 but remains much smaller than p . This assumption considerably limits the validity of the present calculation but allows us to keep reasonably simple expressions. We then consider $\varepsilon = \text{tg}(\phi_0)$ as a small parameter in equation (38) and develop the solution over powers of ε :

$$\delta(X) = \delta^{(0)}(X) + \varepsilon\delta^{(1)}(X) + \dots \quad (39a)$$

$$\psi(X) = \psi^{(0)}(X) + \varepsilon\psi^{(1)}(X) + \dots \quad (39b)$$

This leads to the following set of equations:

$$\delta_{XX}^{(0)} + 2\delta^{(0)}[1 - \delta^{(0)2}] = 0 \quad (40a)$$

$$\psi_{XX}^{(0)} - p\delta_{XX}^{(0)} = 0 \quad (40b)$$

$$[1 - m]\{\delta_{XX}^{(1)} + 2\delta^{(1)}[1 - 3\delta^{(0)2}]\} = -m\delta_{XX}^{(0)}\psi^{(0)} - \frac{m}{2p}([\psi^{(0)}]^2)_{XX} \quad (40c)$$

$$\psi_{XX}^{(1)} - p\delta_{XX}^{(1)} = -p\delta_{XX}^{(0)}\psi^{(0)} \quad (40d)$$

As could be expected, the lowest order equations lead to the familiar solution discussed in Section 2: $\psi^{(0)}(x) = p\delta^{(0)}(x) = p \tanh [X]$. Using this "base state" in equations (40c) and (40d) allows one to calculate the higher order contributions $\delta^{(1)}$ and $\psi^{(1)}$, the integration constants and the parameter ϕ_0 being deduced from the boundary conditions: $\phi = -p$ for $X = -h/2\lambda^*$ and $\phi = -p + \pi$ for $X = h/2\lambda^*$. This calculation is detailed in Appendix B, and here we just give a summary of the result.

Even at order ε , the exact shape of the chevron $\delta(X)$, as well as the splay field $\Phi(X)$, cannot be calculated explicitly, but it is possible to get the combination $\phi(X) - p\delta(X)$ in the following form

$$\begin{aligned} \frac{\phi(X) - p\delta(X)}{\phi_0} \approx & [1 + 2\frac{\lambda_w}{h}X] \left[1 - \frac{2}{3}p^2\text{Log}(2) \right] \\ & - \frac{p^2}{3} \left[2\frac{\lambda_w}{h}X + \tanh^2(X) \right] \\ & + \frac{2}{3}p^2[\text{Log}(2 \cosh(X)) + X] \end{aligned} \quad (41)$$

Fig. 5. — A typical plot of the quantity $\phi - p\delta = \phi - \theta/\Omega_0$ in the limit $\lambda_w/hp^2 \ll p \ll 1$, λ_w/h being the ratio of the “chevron thickness” to the cell thickness. The localization of the c-director splay in one half of the cell is well recovered by our perturbation calculation (parameters: $p = 0.1$, $h/\lambda_w = 10^4$).

where the “asymmetry” parameter ϕ_0 is equal to

$$\phi_0 \approx \frac{3\pi}{2} \frac{\lambda_w}{h} \frac{1}{p^2} [1 + o(p, \lambda_w/hp^2)] \quad (42)$$

and the validity condition of our calculation $\phi_0 \ll p \ll 1$ is given by:

$$\frac{3\pi}{2} \frac{\lambda_w}{h} \frac{1}{p^2} \ll p \ll 1 \quad (43)$$

These conditions are very restrictive but not completely unphysical. For instance, they are roughly satisfied with the orders of magnitude: $p = 0.1$, $\lambda_w = 0.01 \mu\text{m}$ and $h = 100 \mu\text{m}$. A typical plot of the quantity $\phi(X) - p\delta(X)$ is given in Figure 5: one can check that the slight asymmetry introduced in the “wall” structure gives rise to an asymmetrical splay distribution of the asymptotic state observed for large X that allows the “macroscopic twist” $\phi(h/2) - \phi(-h/2) \approx \pi$ to be matched. On this example, this twist (or c-director splay, depending on the selected phrase) is mainly localized in the half space $X > 0$.

4.3. DISCUSSION. — Strictly speaking, we did not really prove the existence of twisted states exhibiting a localization of the splay in one half cell. However, we could give a possible explanation of the occurrence of these states observed experimentally and calculate such a solution explicitly in the limit case $\lambda^*/hp^2 \ll p \ll 1$. Our suggestion is that the most “natural” realization of a twisted state would be obtained as the superposition of a chevron structure and of a splay of the c-director uniformly distributed in the whole cell thickness. Unfortunately, such a structure implies a direction of the c-director in the middle plane of the cell that increases the energy of the “wall” localized in this plane. A state of lower energy that is also compatible with a macroscopic twist of the sample is obtained by keeping the chevron structure involved in the “uniform states” with just a slight deviation of the c-director in the vicinity of the wall. This slight “symmetry breaking” leads to a large scale asymmetry of the splay distribution similar to that observed in the experiments.

5. Conclusion

In summary we have discussed a model of chevrons in smectic C* liquid crystals based on a competition between a compressibility term and orientational contributions to the free energy. This model is certainly oversimplified for various reasons. For instance, the compressibility term as well as the orientational gradients are approximated in the small tilt angle limit, i.e., for small values of both the layer tilt angle and the molecular tilt angle. In addition, the orientational energy is not written in its most general form, and electrical contributions have been completely ignored. However, because of its relative simplicity it allowed us to clarify a few points concerning the chevron structure:

1) We have shown that an appropriate choice of the elastic constants allows the layer continuity to be preserved while the structure imagined by Clark is recovered at a macroscopic level.

2) On the opposite hand, this choice of elastic constants being somewhat arbitrary, deviations to Clark's predictions are to be expected. For instance, the selective pretilt of the *c*-director induced by the central "wall" does not necessarily match that expected from the boundary conditions. In most cases, a slight splay gradient uniformly distributed in the whole cell thickness should accompany the chevron formation.

3) Although we have restricted our study to the case of small molecular rotation gradients, the model allows one to treat large rotations distributed across the whole cell thickness. We were then able to discuss the "twisted states" observed in recent experiments and to interpret the splay localization in one half of the cell. Basically, we have shown that this asymmetrical localization increases the energy of the large scale splay field but sensibly reduces the energy of the central "wall". This effect is clearly related to the introduction of additional non-linearities in the problem that take into account the possibility of large rotations of the *c*-director.

The occurrence of asymmetrical twisted states resulting from a non-linear coupling between the layer tilt and the *c*-director orientation is reminiscent of a similar effect recently discussed by Lubensky and MacKintosh [35] in the context of the "rippled" phases of liquid bilayers. Their model based on a continuum Landau theory contains coupling terms between the layer curvature and the *c*-director orientation and leads in a particular range of temperature to the occurrence of asymmetric ripples. The *c*-director distribution in these ripples looks like that discussed in Section (2).

We suggest that the model studied in the present paper could be successfully applied to the description of other chevron properties. In a first step, the effect of an external electric field on the static structure could be studied easily with appropriate generalizations of the equilibrium equations. In a second step, the dynamics of the polarization in the switching process could be considered. It would also be interesting to check eventually by a numerical approach that our conclusions hold for more general descriptions of smectic elasticity: finite *p* case, more general forms of orientational terms, electrical contributions, etc.

Finally, we mention that a numerical analysis of the model discussed in the present paper has been recently performed by Dupont [36]. These computations confirm our analysis of the twisted states and lead in particular to the observation of asymmetrical states with a splay localization in one half of the cells.

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Appendix A

Layer Dilation Energy

We start from the simplest approximation of the layer dilation-compression energy:

$$f_{\text{comp}} = \frac{1}{2} B \left[\frac{a_{\text{eff}} - a_N}{a_N} \right]^2 \quad (\text{A.1})$$

where $a_N = l[T](1 - \alpha\Omega_0^2/2)$ is the "natural" layer thickness and $a_{\text{eff}} = a_B[1 - \theta^2/2]$ is the effective thickness reached when the layers rotate by an angle θ . Using these expressions, equation (A.1) becomes:

$$f_{\text{comp}} = \frac{1}{2} B \left(\frac{a_B}{l} \right)^2 \left(1 - \alpha \frac{\Omega_0^2}{2} \right)^{-2} \left[\frac{a_B - 1}{a_B} + \alpha \left(\frac{1}{a_B} \right) \frac{\Omega^2}{2} - \frac{\theta^2}{2} \right]^2 \quad (\text{A.2})$$

which corresponds to the form assumed in equation (2), θ_0 being given by equation (4a), the compressional modulus B being redefined as:

$$B' = B \left(\frac{a_B}{l} \right)^2 \left(1 - \alpha \frac{\Omega_0^2}{2} \right)^{-2} \approx B \left(\frac{a_B}{l} \right)^2 \left[\left(1 + \alpha \frac{\Omega_0^2}{2} \right) \right] \quad (\text{A.3})$$

Appendix B

Perturbation Calculations

After using the zero-order solution of equations (40a) and (40b), $\Psi^{(0)}(x) = p\delta^{(0)}(x) = p \tanh[X]$ in equations (40c) and (40d), one obtains the following expressions at order 1:

$$\psi^{(1)} - p\delta^{(1)} = AX - \frac{p^2}{3} \tanh^2[X] + \frac{2}{3} p^2 \text{Log} [\text{Cosh} [X]] \quad (\text{B.1a})$$

$$\delta^{(1)} = g[X] \quad (\text{B.1b})$$

where A is an integration constant and $g(X)$ is a function satisfying the equation:

$$g''(X) + 2[1 - 3 \tanh^2(X)]g(\xi) = -mp^2[1 - \tanh^2(X)][1 - 5 \tanh^2(X)] \quad (\text{B.2})$$

This equation admits a localized solution which vanishes at infinity. Its exact treatment is however very complicated and here we replace this study by that of the simpler problem obtained by replacing $\tanh^2(\xi)$ with 1 for $|\xi| > 1$, and with 0 for $|\xi| < 1$. One is then faced with the equations:

$$g''(\xi) - 4g(\xi) = 0 \quad \text{for } |\xi| > 1 \quad (\text{B.3a})$$

and

$$g''(\xi) + 2g(\xi) = -mp^2 \quad \text{for } |\xi| < 1 \quad (\text{B.3b})$$

This system admits a localized symmetrical solution given by:

$$g(\xi) = -\frac{mp^2}{2}[1 - \beta \cos(\sqrt{2}\xi)] \quad \text{for } |\xi| < 1 \quad (\text{B.4a})$$

and

$$g(\xi) = \frac{mp^2}{2}\beta \frac{\sin(\sqrt{2})}{\sqrt{2}} \exp(-|\xi - 1|) \quad \text{for } |\xi| < 1 \quad (\text{B.4b})$$

where $\beta = \cos(\sqrt{2}) - \sin(\sqrt{2})/\sqrt{2}$. This solution is, of course, not that of the initial problem but supports the results given in the text: equation (B.2) can be expected to admit a symmetrical solution that vanishes at infinity. We also note the fact that $g(0) \neq 0$, which implies that the chevron shape becomes asymmetrical with respect to the plane $x = 0$.

From equation (B.1a), we can now deduce the asymptotic behavior of ϕ for large X :

$$\phi \approx p + \phi_0[1 - \frac{p^2}{3}[1 + 2 \text{Ln}(2)] + [A + \frac{2}{3}p^2]X] \quad \text{for } X \gg 1 \quad (\text{B.5a})$$

$$\phi \approx -p + \phi_0[1 - \frac{p^2}{3}[1 + 2 \text{Ln}(2)] + [A - \frac{p^2}{3}p^2]X] \quad \text{for } X \ll -1 \quad (\text{B.5b})$$

As stated in the text, the slight asymmetry introduced in the "wall" structure gives rise to an asymmetrical splay distribution of the asymptotic state observed for large X . We have now to match this state with the boundary conditions: $\phi = -p$ for $X = -h/2\lambda^*$ and $\phi = -p + \pi$ for $X = h/2\lambda^*$. This is achieved when A and ϕ_0 are equal to:

$$A = \frac{2}{3}p^2 + 2\frac{\lambda_w}{h}[1 - \frac{p^2}{3}[1 + 2 \text{Ln}(2)]] \quad (\text{B.6a})$$

$$\phi_0 = \frac{3\pi \lambda_w}{2h} \frac{1}{p^2} \frac{1 - 2\frac{p}{\pi}}{1 + \frac{\lambda_w}{h}[\frac{3}{p^2} - 1 - 2 \text{Ln}(2)]} \quad (\text{B.6b})$$

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