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Excitability and Defect-Mediated Turbulence in Nematic Liquid Crystal

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Abstract. — We report the observation of one-arm spiral waves and of turbulence states in a nematic slab subjected to an external forcing. These one-arm spiral waves constitute a close analog to the spiral waves observed in chemical and biological excitable media. Those results were obtained by adding a small vertical magnetic field on the experimental apparatus described in [1], in which double-arm spiral waves were observed in a sample with homeotropic alignment. We also describe a Ginzburg-Landau dynamical model accounting for the main kinds of experimental behavior and patterns obtained. In particular, the experiments and the simulations of this model reveal the appearance of a new instability of the $2\pi$ walls which is associated with the spontaneous formation of $+1,-1$ vortex, with the undoubling of the wavefronts, and which also leads to 2D turbulence.

1. Introduction

The study of excitable spiral waves has been motivated by the problem of cardiac fibrillation. Excitations in the cardiac muscle propagate as wave-fronts, and the break of such fronts is at the origin of the spiral waves which lead to arrhythmia [2,3]. Excitable chemical systems exhibit similar phenomena. In the Belousov-Zhabotinsky reaction for example, chemical oscillations propagate with basically the same properties as the cardiac excitation [4]. Double-arm excitable spiral waves have been recently observed by [1,5] and analyzed in the framework of a Ginzburg-Landau model [1,6], in nematic liquid crystal under the influence of a rotating magnetic field. The existence of one-arm excitable spiral waves was investigated theoretically [7]. In this article, the author considered the case of a tilted, rotating electrical field on the Freederiksz transition. However, the predictions addressed in this study are difficult to realize experimentally.

In this paper, we first intend to describe the new experimental results obtained by adding a small vertical component of the magnetic field to the experimental set-up described in reference [1]. In particular, we observe the formation of one-arm spiral waves corresponding to $2\pi$ Bloch walls.

As we increase the value of the vertical magnetic field, a new instability of the $2\pi$ walls appears, associated to the spontaneous formation of $+1,-1$ vortex and to the replication of
the wavefronts. Generally, this constitutes an example of a two-dimensional, defect-mediated turbulence [8].

Secondly, we discuss and analyze these results, using a Ginzburg-Landau model derived from the first principles of elasticity theory, taking into account the presence of the vertical magnetic field. Finally, we shall also consider the case in which the vertical component of the magnetic field is larger than the horizontal one. We will show that this situation is equivalent to that considered in reference [7].

2. Experimental Results

2.1. Double-Arm Spirals. — The experimental set-up used in this section has been previously described in reference [1] as being free of the vertical magnetic field component. This particular geometry and the curvature of the magnetic flux lines lead to the spontaneous formation of splay-bend Brochard-Leger wall (Ising wall), when the permanent magnets are approaching the lower surface of the nematic slab.

If we now progressively rotate the magnets, starting from a rotation frequency \( \omega = 0 \), the part of the wall situated in the observation field of the microscope, (in which the magnetic field can be considered parallel to the sample glass plates), undergoes a transformation at a particular threshold value \( \omega_{1-B} \) described as a static to dynamic soliton transition in reference [5]. This is understandable as an Ising-Bloch second-order transition [1] of the Brochard-Leger wall. As seen in Figure 1, this transformation leads, in the particular geometry used here, to the spontaneous formation of a +1 defect on the rotation axis of the magnetic field. This formation originates from the opposed component of the magnetic field, perpendicular to the plates, giving rise to the non-degenerated, opposed-tilting direction of the director, far from the rotation axis (Fig. 2).

The formation of this +1 defect is associated with two different Bloch wall segments with opposed spatial rotation of the director projection on the mid-plane of the sample (this projection corresponds to a complex order parameter \( A = n_x + ini_y \)). If the rotation rate \( \omega \) is now maintained constant in the C domain of the phase diagram of Figure 3, the Bloch wall
Fig. 2. — An Ising wall in presence of the inhomogeneous magnetic field. (a) The Ising wall as formed, perpendicular to the in-plane magnetic field: the out-of-plane opposed components of the magnetic field, present in the external regions of the sample, were at the origin of the wall formation. (b) If the magnets undergo a 45° rotation around the optical axis, the degeneracy of the Ising wall upper and lower regions distant from the axis, is removed in an opposed way. This is again associated to the the out-of-plane opposed components of the magnetic field in these two regions.

segments, situated in the upper and lower region of the photograph, respectively, undergo a reverse drift, giving rise to the formation of double-arm spirals around the initial +1 defect. A simple explanation of the drift lies in the shape of the modulus of A in the core of the walls. This modulus is associated with a bump and possesses a minimum value in the center of the wall. In this region, the rotation torque exerted on the tilted molecules is reduced and the rotation is no longer synchronous with the magnetic field. As a result, the direction of the wall drift is defined by the temporal rotation of the field and the spatial rotation of the director across the wall. This effect can be understood as the first localized manifestation of the asynchronous state occurring in the homogeneous regions, with the higher rotation rate threshold \( \omega_a \), when we penetrate the D asynchronous region of Figure 3.

2.2. **One-Arm Spirals**

2.2.1. **Experimental Set-Up and Procedure.** — Figure 4 shows the simple experimental, permanent magnet configuration presently used in order to obtain a vertical component (perpendicular to the plate) added to the parallel one. The Nd-Fe-B magnets are now placed with their magnetization parallel to the supporting plate rotation axis. In this case, the curvature of the flux lines is lower than that used in reference [1] and mentioned Section 2.1. The variable displacement from the center of the magnetic system, constituted by the two magnets, relative to the rotation axis, allows a variable value of the \( h = H_0/H_1 \) ratio applied to the central region of the nematic sample. The value of the field \( H_0 \) is easily varied by the adjustment of the distance between the nematic slab and the upper surface of the magnets. In the first approximation, we can consider that the \( h \) ratio remains constant whenever this distance is varied. As in the previous \( H_1 = 0 \) case, a 5 kHz destabilizing sinusoidal variable electric voltage
Fig 3. — Phase diagram of the Freedericksz transition with a magnetic field $H$, rotating at the frequency $\omega$ and a fixed value of the electric field $E < E_c$. Region A: the homeotropic state $A = 0$ is stable; Region B: the synchronous state in which Ising walls are stable, Region C: the synchronous state in which the moving Bloch walls are stable; Region D: the asynchronous state in which $A$ undergoes a continuous phase slip relative to the magnetic field in the homogeneous domains.

Fig 4. — Experimental apparatus allowing the presence of a vertical component $H_1$, added to the in-plane rotating component $H_0$. This vertical component is obtained from the displacement from the center of the two-magnet system, relative to the rotation axis.
can be applied simultaneously, and the magnetic plate can be rotated at a variable speed $\omega$, ranging from a few tenths to a few tens of Hz.

In the off-centered geometry used here, the initially present Brochard-Leger splay bend wall (associated with the curvature of the magnetic flux line at $\omega = 0$) is now formed beyond the observation polarizing microscope field. Nevertheless, it is possible to use the same procedure, as previously described in the $H_1 = 0$ case in order to get a +1 defect at the Ising-Bloch transition of this wall: a progressive increase of $\omega$ leads to the Bloch transformation and the degeneracy of the I-B transition being removed (by the curvature of the flux lines) in the two opposed ways, in the upper and lower regions of the observation field, a +1 defect is spontaneously formed.

Because of its ability to increase the speed and adjustability of the Bloch transformation, the simultaneous application of the electric field is of particular interest. When formed, it is now possible to bring back the core of the +1 defect and its two surrounding wall branches by moving the sample in the $(x, y)$ plane and in the central part of the microscope field, where a definite $h$ is present.

2.2.2. Experimental Observations. — For a rather high value of $h$, the rotation of the $H_0$ component once more produces the drift, in opposite directions around the +1 vortex, of both the Bloch branches, which assume the form of a double Archimedean spiral. But the finite $h$ value is responsible for a different surface extent of the now non-equivalent homogeneous domains separating the spiral arms (Fig. 5). One of these homogeneous domains is now energetically favored by being better aligned with the oblique rotating field. The progressive decrease of $h$ is associated with an increase of the asymmetry of the double spiral and of the ratio of the favorable over the unfavorable domain extent. In the present version of the experiment, it is not possible to vary $h$ as the field rotates. Furthermore, the quantitative variation of the asymmetry versus $h$ is difficult to measure. On the other hand, the same effect can be obtained by a decrease of the electric field amplitude $E$.

The increase of asymmetry leads from a particular threshold value of the control parameter used to the complete disappearance of the unfavorable region and also to the merging of the two 180° walls into a single 360° one. In this case, we can observe one-arm spirals, as seen in Figure 6. As developed in the theoretical part of this paper, this first observation of the
The formation of one-arm spiral waves observed in the $\gamma_c < \gamma_0 < \gamma_t$ situation. (a), (b), (c) and (d): time evolution of a $360^\circ$ wall spiraling around its tip; (e): typical pattern constituted by left and right one-arm spirals obtained in this excitable regime.

one-arm excitable wave in a homeotropically anchored nematic slab is a partial analogue of the case as predicted in reference [7], of the electric rotating field.

When the electrical field is decreased again from the value which produces stable one-arm spirals in our observation field, an instability occurs, thereby causing the spontaneous breaking of the $360^\circ$ propagating fronts which are associated with the formation of pairs of vortex $\pm 1$. If the electrical field is maintained rather close to the bifurcation, the $360^\circ$ arm break is not complete and only affects the rear concave $180^\circ$ half part of the wall (Fig. 7). As observed with the behavior of the extinction lines in polarized microscopy, two vortices of opposed signs are nucleated, and a new $360^\circ$ front relating them is generated. The previous front is regenerated and the new front is drifted in the opposite direction. The spontaneous generation of vortices is compensated by the annihilation of the last vortices, which is related to the retraction of the propagating waves. Consequently, this process maintains the number of fronts and defects constant, in a rather large sample domain.

When the control parameter is decreased again, the stability of the fronts decreases. Now, they simply break and retract, allowing the formation of the homogeneous synchronous phase.
Fig. 7. — Defect-mediated turbulence in the unstable regime of the walls ($\gamma_0 > \gamma_\varepsilon$). (a), (b), (c), (d) and (e): time evolution of a rounded $2\pi$ wall undergoing the emission of $-2\pi$ fronts. These emitted walls retract and can either disappear completely, or expand again on the form of a closed loop, depending on the way the vortex of opposed topological charge recombine. (f): a low enlargement view of the texture corresponding to this turbulent process.

in the whole observation field.

3. Theoretical Model

Our theoretical approach, similar to those used in references [1,6], assumes that the director is only weakly tilted from the $z$-axis. This condition is satisfied only in the vicinity of the Freedericksz transition. In this case, it is legitimate to neglect back-flow effects. Furthermore, the effects of the magnetic field are considered to be small compared to the effects of the electrical field. The dynamical equation for the director $n$ reads:

$$\gamma_1 n \times n_t = -n \times \frac{\delta F}{\delta n}$$

(1)

where $\gamma_1$ is the rotational viscosity and the Frank free energy reads

$$F = \frac{1}{2} \int d\nu [K_1(\nabla \cdot n)^2 + K_2 (n \cdot \nabla \times n)^2 + K_3 (n \times \nabla \times n)^2$$

$$-\chi_a (H \cdot n)^2 - \Delta (E \cdot n)^2]$$

(2)
Here $H = H_0[\cos(\omega t)\hat{z} + \sin(\omega t)\hat{y}] + H_1\hat{z}$, $E = E\hat{z}$, and $K_1, K_2, K_3$ are the elastic constants. We shall first consider the case where the $H_0 >> H_1$, which corresponds to the previously described experimental work. For the sake of simplicity, we shall also neglect the effects of the anisotropy of elasticity (we set $K_1 = K_2 = K_3$). We have checked numerically that these effects are not significant for the phenomena described here.

For an order parameter, let us take $A(x, y) = X(x, y) + iY(x, y)$, which measures the deviation from the homeotropic states, where $X(x, y)$ and $Y(x, y)$ are defined by $n_x = X(x, y)\cos(\frac{\pi z}{d})$, $n_y = Y(x, y)\cos(\frac{\pi z}{d})$, $n_z = 1 - \frac{n_x^2 + n_y^2}{2}$ and $d$ is the thickness of the sample. At the mid-plane of the cell $z = 0$, therefore this definition of the order parameter is consistent with that given in the introduction. We also assume that, close to the threshold of the Freedericksz transition, only the first mode of buckling is unstable.

Direct replacement of this Ansatz into equation (1) leads, after some algebra similar to the one described in reference [6], to the following equation:

$$
\gamma_1 A_t = \mu A + \gamma \tilde{A} e^{-2\omega t} + K_1 \nabla^2 A - a|A|^2 A + \Gamma_0 e^{-\omega t} \tag{3}
$$

Here $\mu = \frac{\chi_a}{2}H^2 - \epsilon_a E^2 - K_2 \frac{\partial^2}{\partial z^2}$, $\gamma = \frac{\chi_a}{2}H^2$, $a = \frac{1}{2}(K_1) \frac{\partial^2}{\partial z^2}$, $\nabla^2 = \partial_{xx} + \partial_{yy}$, $\Gamma_0 = 4\chi_a H_0 H_1$.

Setting $A \rightarrow \frac{A e^{\omega t}}{\sqrt{\alpha}}$ in equation (3) in order to get rid of the explicit time dependence, we obtain:

$$
\gamma_1 A_t = \mu A + i\Omega A + \gamma \dot{A} + K_1 \nabla^2 A - |A|^2 A + \gamma_0 \tag{4}
$$

Here $\Omega = -\gamma_1 \omega$ and $\gamma_0 = \Gamma_0/\sqrt{\alpha}$.

As seen in the last equation, just as in the case investigated in the theoretical work of reference [9], the presence of the vertical magnetic field simply appears through the constant term $\gamma_0$.

3.1. Excitable One-Arm Spiral Waves. — When $\gamma_0 = 0$, equation (4) admits two spatially homogeneous time-independent, stable solutions. These solutions read:

$$
A_0 = \pm R_0 e^{i\theta_0} \tag{5}
$$

where

$$
\sin(2\Theta_0) = \frac{\Omega}{\gamma} \tag{6}
$$

$$
R_0^2 = \mu + \gamma \cos(2\Theta) \tag{7}
$$

These solutions, to which we shall refer as locked-state, correspond to states in which the director turns at the same frequency as the magnetic field, but with some phase lag due to viscosity. In this regime, $\pi$ Bloch wall are heteroclinic orbits joining these two fixed points [1]. As long as $\gamma_0$ is smaller than a critical value $\gamma_c$, which will be defined later, the phase space diagram of equation (4) contains two stable, fixed points corresponding to the two locked states (Fig. 8a). When $\gamma_0$ reaches the critical value $\gamma_c$, one of these solutions disappears by a saddle node bifurcation (Fig. 8b). In this parameter regime, the phase space diagram is characteristic of an excitable system [7,10]. Our numerical simulations performed in this regime show that the spiral waves are one armed and the arm is constituted of $2\pi$ Bloch wall (Fig. 9).

The value of $\gamma_c$ can be calculated analytically in the following limit:

$$
\gamma_0 \ll \gamma, \quad \gamma \ll \mu, \quad \Omega < \gamma \tag{8}
$$

We obtain that

$$
\gamma_c = \sqrt{2\mu(\gamma - \Omega)} \tag{9}
$$
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**Fig. 8.** (a) Qualitative phase portrait of equation (4) showing two excitable points. When $\gamma_0 < \gamma_c$, there are two stable fixed points corresponding to the locked states, here $\Omega < \gamma$ and $A = X + iY$. (b) Qualitative phase portrait of equation (4) showing one excitable point. When $\gamma_0 > \gamma_c$, there is only one stable fixed point, here $\Omega < \gamma$ and $A = X + iY$.

**Fig. 9.** One-arm spiral wave. Numerical simulation of equation (4) in two dimensions showing the real part of $A$. $\gamma_1 = 1, \mu = 1, \Omega = 0.275, \gamma = 0.3, \gamma_0 = 0.04, K_1 = 1$.

This approximation holds when $\Omega$ is slightly smaller than $\gamma$.

Direct inspection of equation (9), shows that $\gamma_c$ can be reduced by lowering the intensity of the electrical field since $\epsilon_a < 0$. Those results are consistent with those described in Section 2.2.2.

**3.2. DEFECT-MEDIATED TURBULENCE.** — As we increase $\gamma_0$ over a critical value $\gamma_l > \gamma_c$, we observe numerically that the one-arm spiral waves become unstable (Fig. 10). We have found numerically that $\gamma_c = 0.037$ and $\gamma_l = 0.060$ with $\mu = 1, K_1 = 1, \Omega = 0.275, \gamma_1 = 1.0$ and $\gamma = 0.3$. As in the experiment, we observe that this instability arises due to spontaneous undoubling of the wave front on some part of the spiral arms. This undoubling leads to nucleation of vortices, which again becomes the center of spiraling front that grows unstable after a certain time. As seen in our numerical simulation, this process is permanent in time.

This disordered state is characterized by a large topological defect number which fluctuates in a random way around a mean value. This state is an example of defect-mediated turbulence [8].

The phenomenon of undoubling of the wave front, which is at the heart of the instability, has also been shown to exist in one-dimensional space. In this case, our numerical simulation shows that fronts periodically auto-reproduce in time (Fig. 11). Such a phenomenon has been
Fig. 10. — Time evolution of a one-arm spiral wave. The instability of the spiral leads to a defect-mediated turbulent state. Numerical simulation of equation (4) in two dimensions showing the real part of $A$. $\gamma_1 = 1$, $\mu = 1$, $\Omega = 0.275$, $\gamma = 0.3$, $\gamma_0 = 0.063$. $K_1 = 1$. Time evolves in alphabetical order.

Fig. 11. — Self-replicating Bloch walls (undoubling of the wave front). Numerical simulation of equation (4) showing the modulus of $A$ with the same value of parameters as in Figure 4 for a space dimension equal to one. Time evolves in alphabetical order.

understood in terms of an imperfect Ising-Bloch transition [6]. A detailed analytical description of these phenomena will be devoted to another work. We would also like mention that imperfect Ising-Bloch bifurcations of fronts have also been analyzed independently of our work in the context of chemical systems [11]. However, in these articles, the authors did not mention the
existence of self-reproducing Bloch walls. As we increased $\gamma_0$ to a value of 0.07, while the other parameters keep the same values as above, we observe that the density of defects rapidly decreases and eventually goes to zero.

4. The Strongly Tilted Magnetic Field Situation

We now consider the case where $H_1 >> H_0$. In this case, proceeding as in the beginning of Section 3, we obtain the following equation for the order parameter

$$\gamma_1 A_t = \mu A + i\Omega A + K_1 \nabla^2 A - |A|^2 A + \gamma_0$$  \hspace{1cm} (10)

The values of the coefficient of equation (10) are given in Section 3.

This equation, studied in reference [7], is similar to equation (4), except for the absence of a term in $\gamma A$. However, the authors considered there the case of a rotating electrical field. We have found that it is, in practice, easier to create a rotating magnetic field than an electrical one. Our preliminary numerical simulations first confirm the results of reference [7] in which the existence and dynamical stability of the one-arm spiral wave were reported. Secondly, our new simulation of equation (10) and preliminary experimental results show that a transition from a simple rotation of the core of the spiral to a meandering mode occurs when the vertical magnetic field reaches a critical value [10]. We are at present studying this transition more accurately.

5. Conclusion

We have studied the effect of the addition of a weak, vertical component on the horizontal, rotating magnetic field experiment of reference [1]. We have shown that one-arm excitable spiral waves form, and that a defect-mediated turbulence state takes place in a narrow region of the parameter space. Our system is a simple liquid crystal analog of the biological and chemical excitable systems exhibiting one-arm spiral waves. It will give us the opportunity to investigate the influence of external stress (for example, transverse electrical fields) on the stability of these spiral waves, which are at the origin of cardiac fibrillation [4,12]. We also think that similar types of instability, like the undoubling of wave front, could be seen in the chemical excitable systems studied in reference [11].

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