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Dynamics of Growing Interfaces in a Disordered Medium: The Effect of Lateral Growth

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Abstract. — We study the model of a driven interface in a disordered medium including the KPZ-nonlinearity. The coupled functional renormalization group flow equations for the disorder correlator and the coupling constant associated with the KPZ-nonlinearity possess a strong coupling fixed-point for the interface dimensions \( d = 1, 2 \). In \( d = 1 \) we get the roughness exponent and the dynamical exponent in one-loop approximation respectively as \( \zeta = 0.8615 \) and \( z = 1 \).

The interface growth appears in many different situations such as fluid displacement in porous media [1, 2], domain growth in random magnets [3-6], ballistic deposition [7, 8], growth of Eden clusters [9], crystallization of solid from vapor [10-14]. Driven interfaces between two fluids in disordered media and driven domain walls in random magnets possess a threshold in dependence on the driving force. Below the threshold the interface becomes pinned. The critical behavior of such interfaces above the threshold has been studied recently [15, 16] on the basis of the equation proposed by Koplik and Levine [2] (fluid displacement) and Bruinsma and Aeppli [3] (random-field Ising model)

\[
\mu^{-1} \frac{\partial z}{\partial t} = \gamma \nabla^2 z + F + g(x, z),
\]

where a single-valued function \( z(x, t) \) describes the height profile above a basal plane \( x \), \( \mu \) is the bare mobility, \( F \) is a driving force, \( \gamma \) is the surface tension. \( g(x, z) \) is assumed to be a Gaussian variable with the zero mean \( \langle g(x, z) \rangle = 0 \) and the correlator \( \langle g(x, z) g(x', z') \rangle = \delta^d(x - x') \Delta(z - z') \), \( \Delta(z) \) has a finite width and is assumed to decay exponentially for large \( z \). The Eden model, ballistic deposition, etc. are believed to be described by Kardar, Parisi, Zhang (KPZ) equation [17]

\[
\mu^{-1} \frac{\partial h}{\partial t} = \gamma \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(x, t),
\]

where \( h(x, t) \) \((z = h + \mu F t)\) is the interface height in the co-moving coordinate system, \( \lambda \) is responsible for the lateral growth and \( \eta(x, t) \) is a Gaussian white noise satisfying the condition

\[
\langle \eta(x, t) \eta(x', t') \rangle = 2D \delta^d(x - x') \delta(t - t').
\]
In contrast to the quenched noise $g$, the thermal noise $\eta$ is delta-correlated with respect to $t$. In connection with the equations (1) and (2) there appear the questions i) how the interface dynamics changes if in (2) instead of white noise one uses the quenched randomness $g(x, z)$? and ii) how the lateral growth influences the dynamics of (1)? The quenched randomness can be introduced into (2) by considering the dynamics with a quenched lateral growth. Repeating the derivation of (2) according to [17] by assuming an inhomogeneous lateral growth instead of (2) we get

$$\mu^{-1} \partial z/\partial t = \gamma \nabla^2 z + V(x, z)(1 + (\nabla z)^2)^{1/2}$$

$$= \gamma \nabla^2 z + F(1 + (\nabla z)^2)^{1/2} + g(x, z)(1 + (\nabla z)^2)^{1/2},$$

where $V(x, z) = F + g(x, z)$ is a quenched random variable being responsible for inhomogeneous lateral growth (compare to [17]). In contrast to [17] we write $\mu^{-1}$ in front of $\partial z/\partial t$ on the left-hand side of (4), so $F$ on the right-hand side is a force. Expanding (4) in powers of $(\nabla z)^2$ we get (1) with KPZ-nonlinearity. Comparing (4) after the expansion mentioned with (2) we see that the bare value of $\lambda$ coincides with the driving force $F$. The practical motivation for considering a more general description for the interface dynamics is the anomalous roughness found in recent experiments by Rubio et al. [18] ($\zeta = 0.73$), Horváth et al. [19] ($\zeta = 0.81$), He et al. [20] ($\zeta = 0.65 - 0.91$), and in simulations by Martys et al. [21] ($\zeta = 0.81$), and Nolle et al. [22] ($\zeta = 0.8$). The apparent disagreement between these experiments and KPZ's predictions have stimulated much theoretical interest [23-26].

The aim of this paper is to present the results of the renormalization group analysis of the equation (4) in the vicinity of the threshold. The main result of this analysis is i) equation (4) belongs to a new universality class differing both from that of (2) and (1), and ii) equation (4) possesses a strong coupling fixed-point in space dimensions $d = 1, 2$.

Just from the dimensional analysis we find that the present problem has two relevant coupling constants: $g_1 = (\Delta''(0)/\gamma^2)l^\epsilon$ (l is length) and $g_2 = (\Delta(0)/\gamma^4)l^{\epsilon - d}$ with $\epsilon = 4 - d$. $g_1$ is the expansion parameter of the equation of motion of the interface in disordered medium without lateral growth (1). $g_2$ is analogous to the expansion parameter of the KPZ-equation (2). We find also two combinations of the parameter which have the dimension of the force $F_\lambda \approx \Delta(0)(\lambda/\gamma)l^{2 - d}$ and $F_\Delta \approx (\Delta'(0)/\gamma)l^{2 - d}$. $F_\lambda$ and $F_\Delta$ give rise to an additive shift of the external bare force $F$, i.e. these terms generate a threshold. $F_\Delta$ differs from zero if the correlator $\Delta(z)$ possesses an appropriate singularity at the origin, which if not present in the bare propagator may be generated by the renormalization [15].

In order to carry out a quantitative analysis of (4) we use the path integral method [27]. To do this we rewrite the nonlinear Langevin equation (4) as a Fokker-Planck equation for the probability density $P(z(x), t)$. The conditional probability density $P(z(x), t; z^0(x), t^0)$ averaged over disorder to have the profile $z(x)$ at time $t$ having the profile $z^0(x)$ at time $t^0$ can be represented by path integrals as

$$P(z(x), t; z^0(x), t^0) = \int \int Dp \: Dz e^{-S},$$

$$S = S_0 + S_\eta + S_\lambda,$$

$$S_0 = D \int_{t_0}^{t} dt' \int dx \: p^2(x, t')$$

$$- \int_{t_0}^{t} dt' \int dx \: p(x, t')(\mu^{-1} \dot{z}(x, t') - F - \gamma \nabla^2 z(x, t')),$$
\[ S_i = \frac{1}{2} \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int dx \ p(x, t') \times \sqrt{1 + (\nabla z(x, t'))^2} \Delta(z(x, t') - z(x, t'')) p(x, t'') \sqrt{1 + (\nabla z(x, t''))^2}, \]  

(8)

\[ S_\lambda = \frac{\lambda}{2} \int_{t_0}^{t} dt' \int dx \ p(x, t') (\nabla z(x, t')). \]  

(9)

To study the shift of \( \bar{F} \) and \( \bar{\lambda} \) we consider the average \( \langle g(x, z(x, t)) \rangle \), where \( \langle \ldots \rangle \) is the average over disorder and the overline implies the average over the zero mode (center of mass) of \( z(x) \). These averages can be carried out by using the conditional probability density \( P(z(x), t; z^0(x), t^0) \). The result of computing the latter to the first order in the disorder strength and in the limit \( F \to 0 \) is

\[ \langle g(x, z(x, t)) \rangle = -F_\lambda^\Delta \sqrt{1 + (\nabla z)^2}, \]  

(10)

where \( F_\lambda^\Delta = -\mu \int_0^t dt \int_k G_\lambda^0(k)(t) \Delta'(0+) \), and \( G_\lambda^0(t) = \exp(-\gamma k^2 t) \) is the Fourier transform of the Green function of the diffusion equation. The one-loop computation of \( F_\lambda^\Delta \) yields \( F_\lambda^\Delta = -(1/2) \Delta(0) \lambda \int_k k^{-2} \). Thus \( F_\lambda^\Delta < 0 \) when \( \lambda > 0 \) which means that the KPZ-nonlinearity acts as a driving force. Combining (10) and (4) we see that \( \bar{F} \) and \( \bar{\lambda} \) are shifted in a different way. The shift of \( \lambda \) is two times larger than that of \( \bar{F} \). This difference appears because in order to carry out the average over \( g(x, z(x, t)) \) we have to take the term \( g \sqrt{1 + (\nabla z)^2} \) (see Eq.(4)) in the distribution function \( P(z(x), t; z^0(x), t^0) \), with which the averages should be carried out. Because of the different shift \( \bar{F} \) and \( \bar{\lambda} \) even having the same bare values will become different under the renormalization. From this we conclude that at the threshold, \( \bar{F} - F_c \to 0 \), \( \bar{\lambda} \) could differ from zero. There is a widespread opinion in the literature that the \( \lambda \)-term at the threshold should be zero [16, 23]. The argument used in [16] is based on the following. By dividing the both sides of (4) (without the term \( \gamma \nabla^2 z \)) by \((1 + (\nabla z)^2)^{1/2} \) and expanding in powers of \((\nabla z)^2 \) we get the KPZ-nonlinearity in the form

\[ \mu^{-1} \partial z / \partial t (\nabla z)^2. \]  

(11)

By comparing (11) with the KPZ-nonlinearity \( \lambda(\nabla z)^2 \) one concludes that \( \lambda \) should be identified with the velocity, with the consequence that the latter disappears at the threshold. This argument does not apply to (4), since the factor \((1 + (\nabla z)^2)^{1/2} \) is missed in front of \( \gamma \) in equation (4). Let us consider the quenched average of (11) over disorder and the thermal average over the center of mass of the interface analogous as we have done it above for the quenched force \( g \). To do this we eliminate \( \mu^{-1} \partial z / \partial t \) in (11) according to the equation of motion. It is easy to see that the result of the average of this quantity yields the shift of \( \lambda \) which is two times larger than that of \( \bar{F} \) in accordance with the analysis carried out above. Therefore, at the threshold \( \mu \) in contrast to the conclusion made in [16] \( \lambda \) is not expected to be zero. The quantities \( v \) and \( \lambda \) behave differently also under the multiplicative renormalization. Whereas the velocity given by \( v \simeq \mu(\bar{F} - F_c) \) renormalizes like mobility in accordance with equation (12), \( \lambda \) renormalizes in according to (14).

Already the existence of the coupling constant \( g_2 \) is a signal that the universality class of equation (4) could differ from that of (1). Despite this the coupling constant \( g_2 \) could be irrelevant if i) the fixed-point value of the latter, \( g^*_2 \), is zero, and ii) if the bare value \( g_2 \text{bare} \) is proportional to \( \bar{F} - F_c \) and will disappear at the threshold. This would be the case if the shift of \( \bar{F} \) and \( \lambda \) would be the same. Intuitively we do not see reasons for \( \lambda \) to be zero at the threshold. The bare value of \( \lambda \) is the driving force, which in contrast to equation (1) acts normally to the surface.
From the above discussion we conclude that equation (4) represents a new universality class. The same conclusion was drawn in [28], where the authors studied a model similar to (4) numerically.

We now outline the procedure of evaluating the path integral (5) by Wilson’s shell integration method. First we separate \( z(x,t) = z_<(x,t) + z_+(x,t) \) and \( p(x,t) = p_<(x,t) + p_+(x,t) \), where \( z_+(x,t) \) and \( p_+(x,t) \) only contain Fourier components in the momentum shell \( \Lambda < k < \Lambda_0 \). Then we expand (5) up to the second order in \( S_1 \) and up to the third order in \( S_2 \), carry out the average over \( z_+(x,t) \) and \( p_+(x,t) \) and represent the result of the average as \( e^{-S(z_-,p_-)} \) with renormalized quantities \( \mu, \gamma, \lambda, \Delta(z) \). The flow equations for the quantities under the renormalization are obtained in the one-loop approximation as follows

\[
\begin{align*}
\frac{d}{dt} \ln \mu &= g_1, \\
\frac{d}{dt} \ln \gamma &= -\frac{d-2}{2d} g_2, \\
\frac{d}{dt} \ln \lambda &= -\frac{1}{d} g_2, \\
\frac{d}{dt} \Delta(z) &= \frac{1}{2} \Delta_+^2 - \gamma^2 - \gamma^{-2} \frac{d^2}{dx^2} \left( \frac{1}{2} \Delta_+^2 - \Delta(z) \Delta(0) \right),
\end{align*}
\]

where the factor \( (d/2\pi)^{d/2} \) is absorbed into \( g_1 \) and \( g_2 \) and \( l = 1/\Lambda \). For \( \lambda = 0 \) these equations reproduce the flow equations derived in [15]. In case of the uncorrelated disorder, \( \Delta(z) = \Delta_0 \), equations (13-15) reduce to the equations derived by Nattermann and Renz [29]. It is easy to give the connection of the flow equations (13-15) and that of the KPZ-equation. The number of time integrations associated with each loop being two for quenched disorder reduces to one for the delta-correlated thermal disorder given by (3). Each time integration in the loop bring the factor \( k^{-2} \) resulting in that the critical dimension for the quenched disorder is four and reduces to two in case of thermal disorder (KPZ). The diagram representation of the perturbation series is the same in both cases. To get the right-hand side of the flow equation of \( \gamma \) in the KPZ-case, besides the redefinition of the coupling constant \( g_2 \) (\( g_2 = \Delta(0) \lambda^2 / \gamma^2 k^{2-d} \)) one should multiply the right-hand side of (13) by an factor 1/2. The factor 1/d in (14) changes to zero in the KPZ-case due to compensation of diagrams. The right-hand side of (15) in KPZ-case is obtained by the change \( \Delta(z) \to \frac{1}{2} \Delta(0) \).

The flow equation for the function \( f(x) \) defined by \( \Delta(z) = \gamma^2 l^{2-\epsilon} f(z/l^\epsilon) \) is obtained from (15) as

\[
\frac{d}{dt} f(x) = \left( \epsilon - 2\zeta + \frac{d-2}{d} g_2 \right) f(x) + \zeta x f'(x) + \frac{1}{2} g_2 f^2(x)/f(0) - \frac{d^2}{dx^2} \left( \frac{1}{2} f^2(x) \right) + f''(x) f(0).
\]

The fixed-point solution of (16) for small \( x \) is found in the form

\[
f^*(x) = 1 + f_1^* x + \frac{1}{2} f_2^* x^2 + \ldots
\]
with

\[
    f_1^* = - \left( \epsilon - 2\zeta + \frac{3d - 4}{2d} g_2^* \right)^{1/2},
\]

\[
    f_2^* = (1/3) \left( \epsilon - \zeta + \frac{2d - 2}{d} g_2^* \right).
\]

We note that \( f_2^* \) is the fixed-point value of the coupling constant \( g_1 \). The nonanalyticity of \( f(x) \) at the origin contributes to the flow equation of \( g_2 \) which is obtained by using (12-19) as

\[
    \frac{d}{dt} g_2 = 2\zeta g_2 - \frac{\epsilon}{d} g_2^2
\]

with the fixed-point \( g_2^* = 2d\zeta/\epsilon \). The roughness exponent \( \zeta \) is obtained from demanding that \( f^*(x) \) decays exponentially for large \( x \) [30]. We solved the equation, rhs(15) = 0, and found for \( \zeta = 0.8615 \) (\( d = 1 \)) and \( \zeta = 0.815 \) (\( d = 2 \)) positive exponentially decaying solutions. In fact in solving rhs(15) = 0 numerically we captured the values of \( \zeta \) at which \( f(x) \) decays in most fast way. The same procedure in the case of random Ising model (\( g_2 = 0 \)) yields \( \zeta = \epsilon/3 \) (random fields) and \( \zeta = 0.2083\epsilon \) (random bonds) as it was previously obtained by Fisher [30]. At \( d = 3 \) there is no positive exponentially decaying solution. We expect that for random fields, which we consider here, the bare disorder correlator being positive for all distances remains positive under renormalization [15]. There are also other solutions where \( f^*(x) \) has negative values. The roughness exponents associated with solutions having one node are \( \zeta = 0.548, 0.412, 0.3 \) at \( d = 1, 2, 3 \), respectively.

The critical exponents which we will compute below can be expressed through the roughness exponent \( \zeta \). The fixed-point value of \( g_1 \) is obtained from (19) as

\[
    g_1^* = (\epsilon - 5\zeta + 12\zeta/\epsilon)/3.
\]

The dynamical exponent \( z \) is derived from the relation \( \gamma_\mu t \sim \xi^2 \) and \( t \sim \xi^z \) as follows

\[
    z = 2 - \epsilon/3 + 2\zeta/3 - 2\zeta/\epsilon.
\]

The renormalized quantities \( \mu_i \) and \( \gamma_i \) are solutions of the equations (12, 13) at fixed points \( g_1^* \) and \( g_2^* \). The correlation length \( \xi \) can be introduced as the characteristic length above which the term \( vt \) in the random force \( g(x, v(t) + h(x, t)) \) dominates the term \( h(x, t) \) [15]. Using the relations \( t \sim \xi^2 \) and \( h \sim \xi^\gamma \) we get \( \xi \sim v^{1/(z-\gamma)} \). Combining the power-law dependence of the correlation length on \( v \) and that of the velocity on \( f = F - F_c \), \( v \sim f^\theta \), we get for the correlation length \( \xi \sim f^{-\nu} \), with \( \nu = \theta/(z - \zeta) \). The velocity exponent \( \theta \), \( v \sim f^\theta \), is derived from the relation \( v = \mu_i f \) with \( \mu_i \sim \xi^{-\gamma} \) as \( \theta = 1 - \nu g_1^* \). Combining this expression with the expression for the correlation length exponent \( \nu \) we get the velocity exponent as

\[
    \theta = (-6\epsilon + \epsilon^2 + 6\zeta + \epsilon\zeta)/(6(-\epsilon - \zeta + \epsilon\zeta)).
\]

Substituting the values for \( \theta \) and \( z \) into the above derived expression for the correlation length exponent \( \nu \) we get the latter as follows

\[
    \nu = \epsilon/(2(\epsilon + \zeta - \epsilon\zeta)).
\]

The fixed-point value for \( f_1^* \) is \( f_1^* = (\epsilon - 5\zeta + 8\zeta/\epsilon)1/2 \). By using the numerical values for the roughness exponent \( \zeta \) obtained from the numerical solution of the fixed-point equation for \( f(x) \) we give in Table I the values of the exponents and the fixed-point values of the coupling constants computed for \( d = 1, 2 \).
Table I.— *Numerical values of the critical exponents and the fixed-point values of the coupling constants for different space dimensions.*

<table>
<thead>
<tr>
<th>$d$</th>
<th>$\zeta$</th>
<th>$z$</th>
<th>$\theta$</th>
<th>$\nu$</th>
<th>$g_1^*$</th>
<th>$g_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8615</td>
<td>1</td>
<td>0.163</td>
<td>1.174</td>
<td>0.713</td>
<td>0.574</td>
</tr>
<tr>
<td>2</td>
<td>0.815</td>
<td>1.062</td>
<td>0.208</td>
<td>0.844</td>
<td>0.938</td>
<td>1.63</td>
</tr>
</tbody>
</table>

The renormalization group analysis presented here gives a fixed-point for dimensions $d < 3$, which is probably a strong coupling fixed-point. In case of a weak-coupling fixed-point the fixed-point values of the coupling constants being proportional to $\epsilon = d_c - d$ ($d_c$ is the critical dimension) tend to zero in approaching the critical dimension. In the present case (see the two last columns of Tab. I) the coupling constants increase by increasing the dimensionality. The situation resembles that one in the KPZ-equation where the fixed-point was found below $d = 3/2$. The roughness exponent, $\zeta = 0.8615$ at $d = 1$, lies in the range of experimental findings [18-20] ($\zeta = 0.63 - 0.91$). Because we have to do with a strong coupling fixed-point we should not expect to get a good agreement with the experiment. We note that the fixed-point values of the coupling constants achieve their minimal values ($< 1$) at $d = 1$, so the comparison to the experiment is expected to be the best at $d = 1$.

In conclusion we have considered the dynamics of an interface in a disordered medium taking into account the KPZ-nonlinearity. This model belongs to a new universality class differing from both that of interface in a disordered medium without lateral growth and that of KPZ equation. From the (strong) fixed-point of the coupled functional renormalization group flow equation for the disorder correlator and the flow equation for the coupling constant associated with the $\lambda$-term we have derived the critical exponents for the dimensions $d = 1, 2$.

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References