Optical elements for interferometry of atoms with J > 1

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Optical elements for interferometry of atoms with $J > 1$

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Abstract. — We have studied how velocity-selective coherent population trapping (VSCPT), adiabatic population transfer (AT), and Raman transitions can be used to manipulate atoms. In particular, we have made calculations for all three methods for J to J transitions ($J > 1$) in atoms interacting with two counterpropagating beams of opposite circular polarization ($\sigma^{\pm}$ configuration). The momentum exchanged between the atoms and the light increases with J thus increasing the separation produced by beamsplitters and the angle of deflectors based on these techniques.

1. Introduction.

The creation of coherent superpositions of atomic momentum states or the transfer of population between them by interaction with light, has been achieved by several techniques, for example: velocity-selective coherent population trapping [1-3], adiabatic population transfer [4-7], and Raman transitions [8-12]. These techniques can be combined in various ways to act as beamsplitters and coherent deflectors in atomic interferometers. We have considered how these optical elements for matter waves made from light act on a transition between two levels of the same total angular momentum J. Once the equations for the coupling of the different M_J-states by the light field have been written down, including atomic momentum and the changes produced by spontaneous emission, then all the three methods can be calculated. We have used the generalized optical Bloch equations introduced by Aspect *et al.* [2] for calculations of VSCPT for the simplest case of a $J = 1$ to 1 transition and the work of Papoff *et al.* [3] on $J = 3/2$ to $1/2$ and $J = 2$ to 1. The latter work shows that, for higher angular momenta ($J > 1$) the dark states used in VSCPT are intrinsically leaky, but this can be overcome. We have extended these calculations to $J = 2$ to 2 and 3 to 3 with the aim of obtaining larger momentum differences and hence larger separations in an interferometer. A momentum transfer of 8 times the photon momentum ($8\hbar k$) has already been demonstrated by adiabatic transfer (AT) in the $F = 4$ ground state hyperfine level of caesium [7]. In this paper we show (Sect. 3) that AT can also be affected by leakage. AT is a robust method suitable for deflecting atoms and, under certain circumstances, for recombining states, alternatively, a Raman transition can be used for higher J levels in an analogous way to that demonstrated by Kasevich and Chu [8, 9] for two levels (Sect. 4).

In all of the calculations we consider counterpropagating beams with σ^+ and σ^- polarizations as used in the first VSCPT experiments [1]. This gives light linearly polarized perpendicular to the laser beams. The axis of this field depends on the relative phase of the two beams and rotates through 2π over a distance along the laser beams equal to λ, the wavelength of the light. Both the VSCPT and AT mechanisms require the existence of so-called dark states which do not interact with the light. For $\Delta J = 0$ transitions there is one such state; this is most obvious in the basis with quantization axis along the local electric field, in which only π-transitions are excited. Taking this to be the Ox axis then the dark state is $|M_x = 0>$ since $< J, M_x = 0|\hat{J}_x|J, M_x = 0 >= 0$. In the z-basis the dark state is a symmetric superposition of different M_z states which is the eigenvector of $\hat{J}_x^2$ corresponding to eigenvalue $M_x = 0$. Atoms in this light field scatter light until they eventually fall into the dark state where they remain trapped. This simple picture shows how coherent population trapping arises but not why it depends on velocity. As shown in reference [1] for a $J = 1$ to 1 transition the effect of the light is to link $|p - \hbar k, M_z = -1 >$ to $|p + \hbar k, M_z = +1 >$ by stimulated Raman transitions to form a Λ-structure. For $p = 0$ both states have the same energy and form a stationary dark-state. No stationary state exists for $J > 1$ since the states connected by the light are not exactly degenerate and the finite lifetime of the dark states limits the velocity-selection in CPT to only a little below the recoil limit. (For $J = 2$ the inverted-W (IW) structure formed from $|p, M_z = 0 >$ and $| p \pm 2\hbar k,\ M_z = \pm 2 >$ cannot produce a completely non-absorbing state because even for $p = 0$ the kinetic energy gives a energy difference of 4 times the recoil energy $E_{\mathrm{R}} = \dfrac{(\hbar k)^2}{2m}$; the leakage is proportional to this separation). It has been proposed by Ol'Shanii [13] to stop the leakage in VSCPT by applying a static electric field. The Stark shift has a tensor term proportional to M_J^2 and so in principle could be used to compensate for the kinetic energy differences. Unfortunately a d.c. field would not work for the alkalies which we consider here, but the a.c. Stark shift (light shift) from a far-off-resonance laser can compensate for the recoil energy to allow the formation of long-lived trapping states for $J > 1$.

To build a complete interferometer starting from beamsplitting by a VSCPT interaction requires some technique for forming the paths into a closed loop and for transferring components of the wavepacket into the same state so that they can interfere. Adiabatic transfer is a robust technique which would be very well-suited for this [4, 6, 7]. AT can only be used as a deflector/mirror if it is possible to act on each arm of the interferometer separately otherwise a Raman pulse or evanescent-wave mirror would be used. We have adapted our calculation of VSCPT of atoms with a $J = 2$ to 2 transition interacting with light in a $\sigma^{\pm}$ configuration to include the case of AT and Raman transitions in which the $\sigma^{\pm}$ beams are pulsed on, either in sequence (AT) or together (Raman).

2. Calculation of VSCPT.

We consider an atom with a $J = 2$ to 2 transition interacting with a pair of laser beams counterpropagating along z direction with circular polarization, σ^+ and σ^-, as shown in figure 1. The light couples different states together forming a W-structure, for which no trapping state

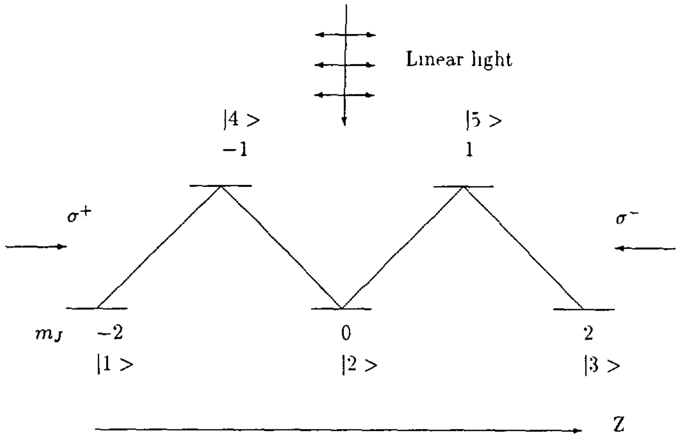


Fig. 1. — The configuration of laser interaction with an inverted W (IW) atomic transition.

is formed, and an inverted W-structure (IW) for which the interaction Hamiltonian is

$$\begin{aligned} H_{\rm int} &= \sum_p \hbar\Omega_1 |e_{-1}, p> < g_{-2}, p - \hbar k| e^{-i\omega_l t} + \sum_p \hbar\Omega_2 |e_{-1}, p> < g_0, p + \hbar k| e^{-i\omega_l t} \\ &+ \sum_p \hbar\Omega_1 |e_1, p> < g_0, p - \hbar k| e^{-i\omega_l t} + \sum_p \hbar\Omega_2 |e_1, p> < g_2, p + \hbar k| e^{-i\omega_l t} \\ &+ \text{H.C.} \end{aligned} \qquad (1)$$

where k is the magnitude of the wavevector of the light and ω_l is its frequency. $\Omega_{1,2}$ are the Rabi frequencies of the beams (for simplicity we have put all Clebsch-Gordan coefficients equal). The superposition of the states corresponding to the dark state is

$$| \Psi_{\rm d} >= c_{-2} e^{-i\frac{E_{-2}}{\hbar}t} |g_{-2}, p - 2\hbar k> + c_0 e^{-i\frac{E_0}{\hbar}t} |g_0, p> + c_2 e^{-i\frac{E_2}{\hbar}t} |g_2, p + 2\hbar k> \qquad (2)$$

For $| \Psi_{\rm d} >$ to be a non-absorbing state, $H_{\rm int} \mid \Psi_{\rm d} >= 0$ i.e.

$$\hbar\Omega_1 c_{-2} e^{-i\frac{E_{-2}}{\hbar}t} |e_{-1}, p - \hbar k> + \hbar\Omega_2 c_0 e^{-i\frac{E_0}{\hbar}t} |e_{-1}, p - \hbar k>= 0 \qquad (3)$$

and

$$\hbar\Omega_1 c_0 e^{-i\frac{E_0}{\hbar}t} |e_1, p + \hbar k> + \hbar\Omega_2 c_2 e^{-i\frac{E_2}{\hbar}t} |e_1, p + \hbar k>= 0$$

where the energies of the states are

$$E_{-2} = \frac{(p - 2\hbar k)^2}{2m}, \qquad E_0 = \frac{p^2}{2m}, \qquad E_2 = \frac{(p + 2\hbar k)^2}{2m}$$

The exponential terms in equation (3) are not equal to each other for any momentum, even $p = 0$, and the coefficients $c_{\pm 2,0}$ are not time independent. This means that no completely non-absorbing state is formed – the dark state is always leaky. In order to restore the degeneracy, it has been proposed to use a d.c. Stark shift to compensate for energy differences since both depend on M_J^2 [13]. This can not be applied to alkali atoms because it has the same sign as the kinetic energy shift in any $s_{1/2}$ ground state [14-17]. Moreover the tensor term is very small since it arises only from mixing caused by the spin-dipolar hyperfine interaction. The summation over excited states which leads to an effective operator acting on the ground state for a d.c. field does not apply in the case of the a.c. Stark shift (light shift) from a laser which resolves the hyperfine structure of the upper state. Only a very moderate intensity is required to give light shifts which compensate for the recoil energy and the sign of the a.c. Stark shift is determined by the detuning of laser frequency. For the $J = 2$ to 2 transition, we consider an additional laser beam propagating along the x-direction which is linearly polarized along the z-direction so that the shifts of the magnetic sublevels in the ground level are

$$\Delta_{M_z} = -\frac{\Omega_0^2}{2\delta_0}(C_{M_z}^{M_z})^2 = -\frac{\Omega_0^2 M_z^2}{12\delta_0} \tag{4}$$

where Ω_0 is the Rabi frequency of this far-off resonant beam, δ_0 is the absolute value of the detuning and the negative sign arises for red detuning. The recoil energy shift is cancelled when $\Omega_0 = \sqrt{6\frac{\hbar k^2}{m}\delta_0}$ so that when the momentum is zero, $E_{-2'} = E_{0'} = E_{2'} = 0$ and a stationary trapping state may be formed. The excitation by the additional compensating beam is discussed below.

To make a comparison between the two cases of trapping states with and without leakiness we derive the equation for motion of atoms (see Appendix) from the Liouville equation

$$\dot{\sigma} = -\frac{i}{\hbar}[H, \sigma] + \left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_{\mathrm{sp.}}$$

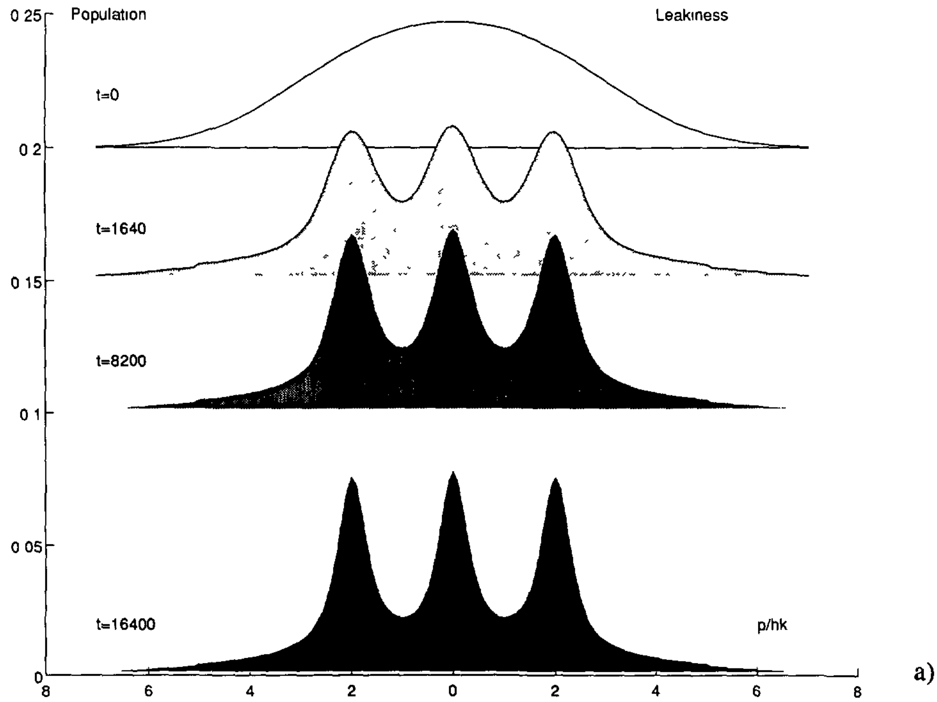
In the numerical calculations of these generalized optical Bloch equations with external and internal degrees of freedom we used the parameters for the 794.7-nm transition of ^{87}Rb atom, $5\mathrm{S}_{1/2}\mathrm{F}_\mathrm{g} = 2 \Leftrightarrow 5\mathrm{P}_{1/2}\mathrm{F}_\mathrm{e} = 2$.

Let us first consider the case of leakiness shown in figure 2a. The initial momentum distribution of the atoms was Gaussian with a FWHM of $6\hbar k$. When time is long enough, for example $t > 16,000$ atomic lifetimes, the initial Gaussian distribution evolves into stable distribution comprising of a background and three broad peaks centered $\pm 2\hbar k$ and 0. The limiting width of about $\hbar k/5$ is determined by the balance between the optical pumping rate and the leakage (see Eq. (12) of Ref. [3]). The evolution of momentum distribution is much faster when the leakiness is repaired, as shown in figure 2b. The absolute height of the peaks increase as the peaks become narrower, i.e. more and more atoms fall into the dark state and stay there.

In figure 3 we plot the population of the excited state against time for the atoms with zero momentum. When the trapping state is leaky, there are always some atoms interacting with the laser and a steady state is established in which the leakage is equal to the rate into the dark state. This equilibrium depends on the parameter α_r – the ratio of the atom-laser interaction to the kinetic energy – which itself depends on the Rabi frequency, spontaneous emission rate, and the recoil frequency shift [3] $\alpha_\mathrm{r} = \frac{\hbar(\Omega_{1,2})^2}{4\Gamma E_\mathrm{R}}$ (in all of the calculations we set $\alpha_\mathrm{r} > 1$ so that three peaks centered at $\pm 2\hbar k$ and 0 appear). In contrast, when a completely non-absorbing state is formed the excited state population tends to be zero.

These results are similar for a $J = 3$ to 3 transition as shown in figure 4 which evolves on at a similar rate to $J = 2$ to 2, however for a given element the leakiness of the states increases with J.

In these calculations we have omitted the excitation caused by the additional compensating beam which necessarily interacts with the atoms. However if it is far-off resonance it causes



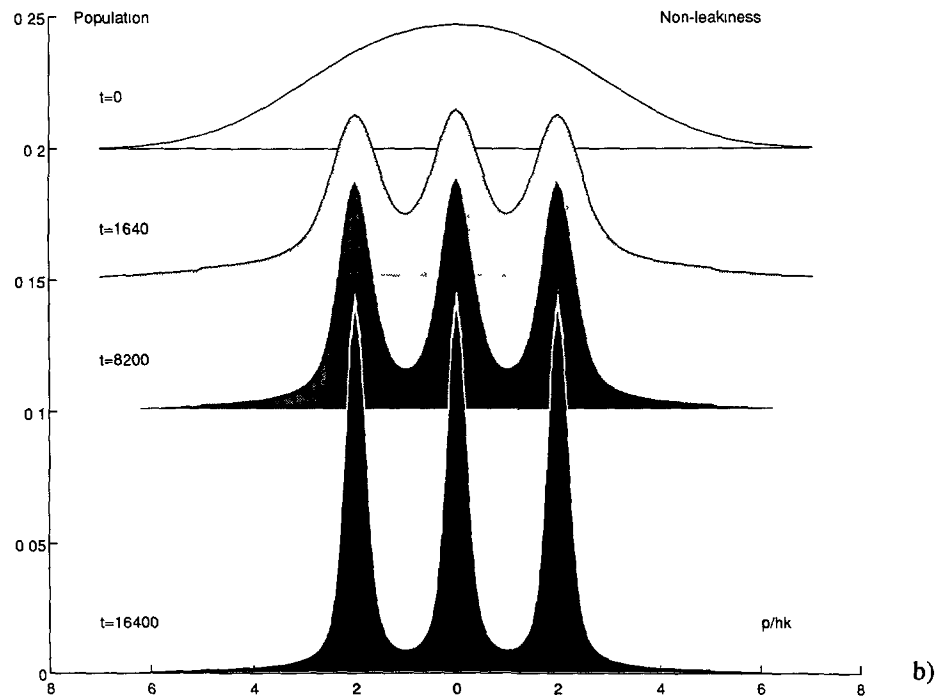


Fig. 2. — The evolution of the atomic-momentum distribution of ^{87}Rb with leakiness (a) and without leakiness (b). The evolution times are 0, 1640, 8200, 16400 times of lifetime respectively. $\Omega_1 = \Omega_2 = 0.15\Gamma$, where Γ represents spontaneous emission rate.

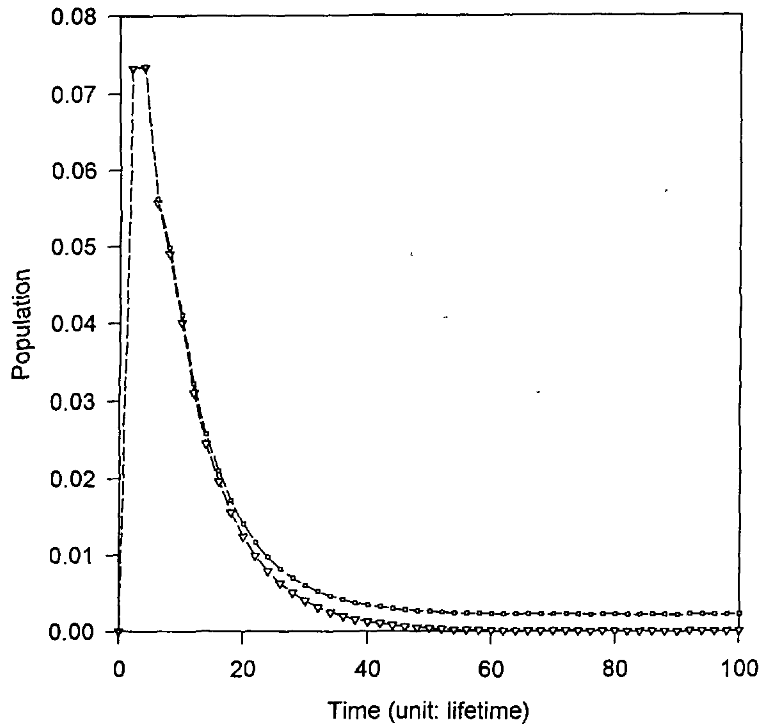


Fig. 3. — The population in the excited state of ^{87}Rb with leakiness for curve (□) and without leakiness for curve (▽) when only the atoms with zero momentum are considered. $\Omega_1 = \Omega_2 = 0.15\Gamma$, $t = 100\Gamma^{-1}$

negligible excitation as can be seen by considering the orders of magnitudes of the processes involved. The light shift is given approximately by Ω^2/δ and must be of the order the recoil frequency, $\omega_r = E_r/\hbar$. The excitation rate is thus of order $\frac{\Omega^2\Gamma}{\delta^2} \simeq \frac{\omega_r\Gamma}{\delta}$ This indicates that large detunings are desirable however when detuning is much greater than hyperfine structure splittings of either level the shifts of each M_F level tend to the same value, as already stated, thus the minimum excitation rate produced by the compensating light is of order $\omega_r\Gamma/\Delta\nu_e$ where $\Delta\nu_e$ is the h.f.s. in the excited state. For the D1-line of ^{85}Rb which has $F = 2$, $F = 3$ levels in both ground and excited states this gives a rate of $10^{-5}\Gamma$. For the D1-line of ^{87}Rb and ^{133}Cs the situation would be more favourable. The use of the D1-lines also reduces the effects of the interaction of the $\sigma_\pm$ beams on the $\Delta F = 0$ transitions with other transitions. There are two effects to consider: there is off-resonant excitation which is of order $\Omega^2\Gamma/\Delta\nu_e{}^2$ $\sim 10^{-6}\Gamma$ under typical conditions for VSCPT (e.g. parameters used in Fig. 2). There is also an a.c. Stark shift of order $\Omega^2/\Delta\nu_e$ which gives negligible leakiness in VSCPT. (N.B.: These considerations do not apply in the case of metastable rare gases). This gives plenty of time for population to evolve into distinct states. The limitation imposed by spontaneous emission is worse for lighter atoms since although the evolution time to a given momentum distribution varies, roughly as mass m, there are two factors which make things worse for lighter atoms; the recoil shift which must be compensated increases as $1/m$, for constant wavelength, and the hyperfine structure decreases ($\propto$ atomic number, Z, approximately).

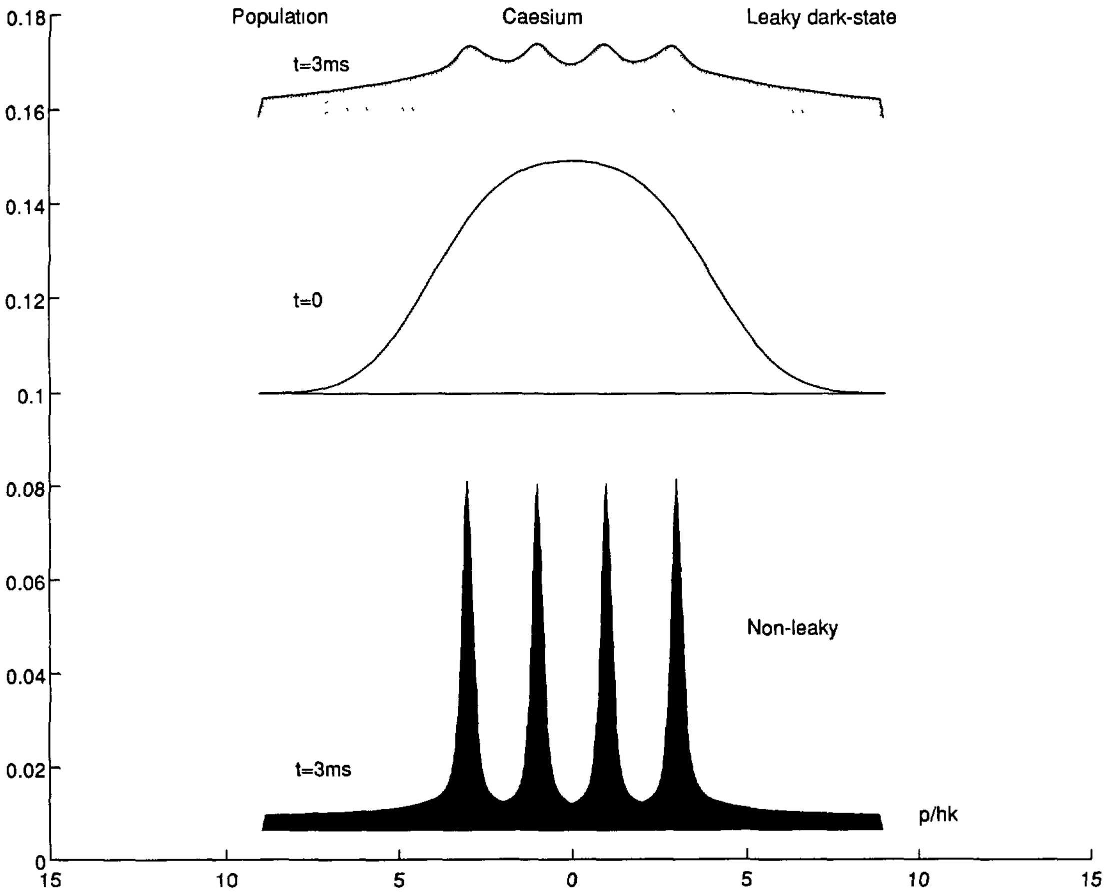


Fig. 4. — The evolution of the atomic-momentum distribution of caesium with leakiness and without leakiness. $\Omega_1 = \Omega_2 = 0.8\Gamma$, $t = 3$ ms.

Further work needs to be carried out to consider the effect of forces.on the trapping. For the $J = 1$ to 1 transition there is no force in a σ^+ and σ^- configuration [3], but VSCPT can be obtained by linearly polarized beams [1] where there is Sisyphus cooling [18]. For $J > 1$ there is a weak force even in the σ^+ and σ^- situation [19] although it is also worth considering other configurations.

3. Adiabatic population transfer.

In the last section we have discussed the momentum splitting of atoms using dark state. In this section we briefly discuss the method of using a dark state to realize another element of atom optics: an atomic deflector or mirror. Since Marte *et al.* [4] proposed the idea of adiabatic population transfer of atoms using dark state, experiments have been done with caesium [5, 7] and with helium [6]. When the AT is extended from a Λ structure to a IW structure or more complicated structure, we need to consider the leakage of the dark state which may decrease the transfer efficiency.

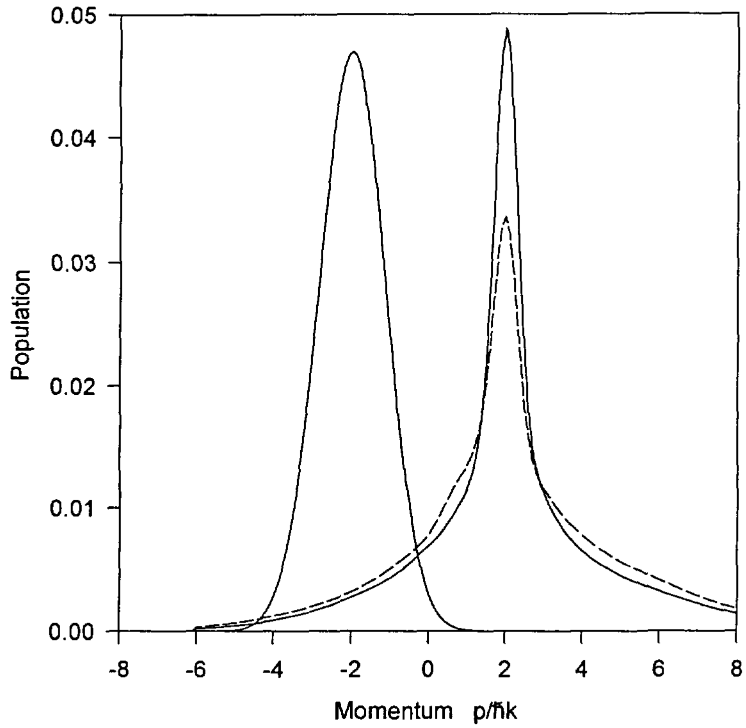


Fig. 5. — The adiabatic population transfer of atoms from ground state $|g_{-2}, p - 2\hbar k>$ to $|g_2, p + 2\hbar k>$. Two Gaussian pulses are used of FWHM $T = 50\Gamma^{-1}$, maximum Rabi frequency $\Omega_1 = \Omega_2 = 1.0\Gamma$ and detuning, $\delta = 0$. The σ^- pulse precedes the σ^+ pulse by 1.2 T. The left solid line shows the initial momentum distribution, and the right solid (dash) line indicates the final distribution for the case without (with) leakiness.

There are two conditions for which adiabatic population transfer is achieved, these are

$$(i) \quad \Omega_{1,2}T \gg 1, \qquad \Omega_{1,2} \gg \Gamma$$

$$(ii) \quad \Omega_{1,2} \ll \Gamma, \qquad \Omega_{1,2}^2 T \gg \Gamma$$

When condition (i) is valid, the laser intensity is strong and overcomes the leakage because the recoil shift is usually much less than Γ, and the initial and final momentum distributions are very similar. However AT does not necessarily require a large Rabi frequency because the atoms are in the dark state and Rabi oscillation is not vital (unlike the Raman transitions described in the next section). For condition (ii), the leakage may cause loss during the transfer and it should be repaired as in the last section. In this case of low Rabi frequency, the AT may be accompanied by further cooling of the atoms through simultaneous VSCPT.

The dark state present in AT (and VSCPT) for the IW structure, is

$$|\Psi_d> = -\frac{\Omega_2}{\Omega_1}K_0|g_{-2}, p - 2\hbar k> + K_0|g_0, p> - \frac{\Omega_1}{\Omega_2}K_0|g_2, p + 2\hbar k> \tag{5}$$

where

$$K_0^2 = \frac{\Omega_1^2\Omega_2^2}{\Omega_1^2 + \Omega_2^2}$$

We have assumed that degeneracy has been restored so the exponential terms are omitted. Consider atoms initially prepared in $|g_{-2}, p - 2\hbar k>$ subjected to two Gaussian pulses of light, one with σ^--polarization and Rabi frequency Ω_2 followed by a σ^+-pulse of strength Ω_1, partially overlapping with the first. The dark state evolves continuously from the initial state to $|g_2, p + 2\hbar k>$, and during this time the atoms trapping in this state can narrow the momentum distribution. The momentum evolution resulting from AT is shown in figure 5, calculated using the equations established in previous section with Ω_1 and Ω_2 varying in time.

4. Raman transitions.

In the work at Stanford University [8-12], sequences of Raman transitions were used to cool atoms and to form an interferometer. A $\frac{\pi}{2}$-pulse produced a coherent superposition of states with momentum differing by $2\hbar k$ (as in VSCPT for $J = 1$) and then π-pulses were used to transfer momentum to the atoms to deflect them in the interferometer or for cooling. A large beamsplitting angle was achieved by a $\frac{\pi}{2}$-pulse followed by up to 10 π-pulses and a similar process could be used to increase the momentum separation of the states emerging from a beamsplitter based on VSCPT. Indeed, cumulative momentum transfer by repeated AT was

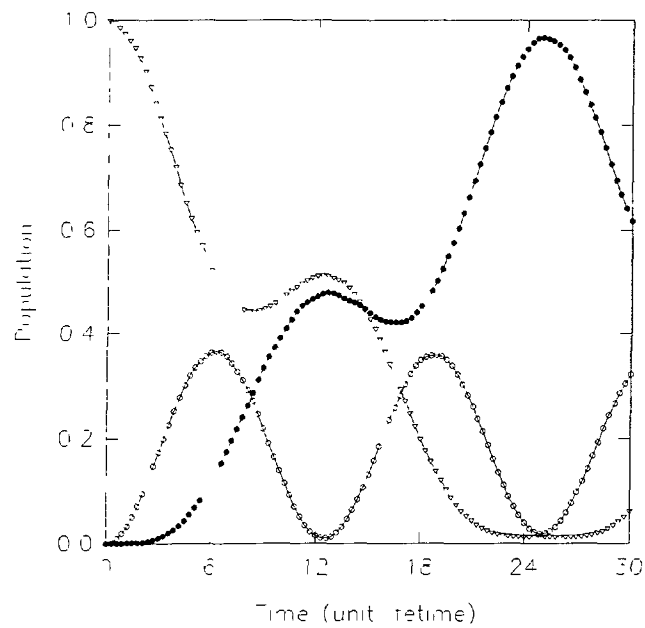


Fig. 6. — Raman π pulse excitation. The atoms are initially prepared in ground state $|g_{-2}, p-2\hbar k>$. If the duration of two rectangular pulses is selected around 25 times of atomic lifetime, the atoms in the ground state $|g_{-2}, p-2\hbar k>$ and $|g_2, p+2\hbar k>$ may be totally reversed simultaneously. $\Omega_1 = \Omega_2 = 7.0\Gamma$; the detuning is 80Γ. ∇, $\circ$, $\bullet$ show the population in the states $|g_{-2}, p-2\hbar k>$, $|g_0, p>$, and $|g_2, p+2\hbar k>$ respectively.

demonstrated in reference [6]. However for the present purpose we consider only a beam splitter using only VSCPT which gives a splitting of $2J\hbar k$ and simultaneously cools the atoms so that atoms of interferometer are well separated (equivalent to pre-cooling by multiple Raman pulses, then a $\frac{\pi}{2}$-pulse followed by some π-pulses). However there is a feature of the Raman interferometer which is more complicated to achieve in other ways, namely the simultaneous reversal of the momentum in both arms by a π-pulse (alternatives which act on both arms separately are AT, evanescent-wave mirror(s) plus gravity).

The concept of a π-pulse can be extended to higher J states as shown in figure 6, which plots Rabi oscillation of the population in the lower level as they evolve in a far-off resonance $\sigma^{\pm}$ light field. In figure 6, in order to realize the inversion of population or strong Rabi oscillation and suppress spontaneous emission, we have to take large Rabi frequency and detuning so leakage is not significant because the recoil shift is small compared with Rabi frequency and detuning. Initially all atoms are in $|M_J = -2, -2\hbar k>$ and at a later time greater than 95% are transfered to $|M_J = +2, +2\hbar k>$. This is not a one-way process, as in AT, and for a pulse of the same time atoms initially in $M_J = +2$ would be coherently transferred in the reverse direction. The calculations were carried out using the same programme as in section 2 and 3; this included spontaneous emission but the detuning was large enough to make its effects small. In practice this process would also work best on the D1-line, as mentioned in [5, 7] where one can achieve sufficiently large detunings without running into the neighbouring transitions and avoid off-resonant excitation.

5. Conclusion.

We have shown that for J to J transitions VSCPT can act as a beam splitter with a spread in output momentum of $2J\hbar k$ and how it might be subsequently recombined. In particular we have shown how the effects of the leakage which occurs for $J > 1$ can be suppressed. Larger splittings have been obtained [12] using sequences of Raman pulses ($\frac{\pi}{2}$-pulse followed by π-pulses) but at the expense of more complexity.

Acknowledgements.

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Appendix.

Generalized optical Bloch equations.

The total Hamiltonian of laser and atomic system for $J = 2$ to 2 transition has two parts

$$H = H_{\text{int}} + H_{\text{atom}}$$

where H_{int} is interaction Hamiltonian shown in equation (1), H_{atom} is the atomic Hamiltonian. When no linear compensating light is applied, H_{atom} is free atomic Hamiltonian,

$$H_{\text{atom}} = \frac{p^2}{2m} + \hbar\omega_0 |\mathrm{e}_{-1}><\mathrm{e}_{-1}| + \hbar\omega_0 |\mathrm{e}_1><\mathrm{e}_1|$$

where ω_0 is atomic resonant frequency. When linear compensating light is applied the H_{atom} is

$$H_{\text{atom}} = \frac{p^2}{2m} + \hbar\omega_{a1}|\mathrm{e}_{-1}><\mathrm{e}_{-1}| + \hbar\omega_{a1}|\mathrm{e}_1><\mathrm{e}_1| + \hbar\omega_{a2}|\mathrm{g}_0><\mathrm{g}_0|$$

where $\omega_{a1} = \omega_0 + \frac{5\hbar k^2}{2m}$, and, $\omega_{a2} = \frac{4\hbar k^2}{2m}$, because the $|M_J| = 2$ ground state levels have been taken as having zero energy to avoid having negative energies. The total atomic Hamiltonian may be expressed in matrix in the basis of $|\mathrm{g}_{-2}, p - 2\hbar k>$, $|\mathrm{g}_0, p>$, $|\mathrm{g}_{+2}, p + 2\hbar k>$, $|\mathrm{e}_{-1}, p - \hbar k>$, $|\mathrm{e}_1, p + \hbar k>$

$$H = \begin{bmatrix} \frac{(p-2\hbar k)^2}{2m} & 0 & 0 & \hbar\Omega_1 \mathrm{e}^{\imath\omega_1 t} & 0 \\ 0 & \frac{p^2}{2m} + \hbar\omega_{a2} & 0 & \hbar\Omega_2 \mathrm{e}^{\imath\omega_1 t} & \hbar\Omega_1 \mathrm{e}^{\imath\omega_1 t} \\ 0 & 0 & \frac{(p+2\hbar k)^2}{2m} & 0 & \hbar\Omega_2 \mathrm{e}^{\imath\omega_1 t} \\ \hbar\Omega_1 \mathrm{e}^{-\imath\omega_1 t} & \hbar\Omega_2 \mathrm{e}^{-\imath\omega_1 t} & 0 & \frac{(p-\hbar k)^2}{2m} + \hbar\omega_{a1} & 0 \\ 0 & \hbar\Omega_1 \mathrm{e}^{-\imath\omega_1 t} & \hbar\Omega_2 \mathrm{e}^{-\imath\omega_1 t} & 0 & \frac{(p+\hbar k)^2}{2m} + \hbar\omega_{a1} \end{bmatrix}$$

The spontaneous emission terms are

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_{\mathrm{sp.}} = \frac{\Gamma}{2} \begin{bmatrix} \bar{\sigma}_{-1-1}(p\prime) & \bar{\sigma}_{-11}(p) & 0 & -\sigma_{-2-1} & -\sigma_{-21} \\ \bar{\sigma}^*_{-11}(p) & \bar{\sigma}_{-1-1}(p\prime\prime) + \bar{\sigma}_{11}(p\prime) & \bar{\sigma}_{-11}(p) & -\sigma_{0-1} & -\sigma_{01} \\ 0 & \bar{\sigma}^*_{-11}(p) & \bar{\sigma}_{11}(p\prime\prime) & -\sigma_{2-1} & -\sigma_{21} \\ -\sigma_{-1-2} & -\sigma_{-10} & -\sigma_{-12} & -2\sigma_{-1-1} & -2\sigma_{-11} \\ -\sigma_{1-2} & -\sigma_{10} & -\sigma_{12} & -2\sigma_{1-1} & -2\sigma_{11} \end{bmatrix}$$

Here, $\sigma_{\imath\jmath}$ is density matrix element.

$$\bar{\sigma}_{\imath\imath}(p\prime) = \int_{-\hbar k}^{\hbar k} \mathrm{d}u H(u) \sigma_{\imath\imath}(p - \hbar k + u)$$

$$\bar{\sigma}_{\imath\imath}(p\prime\prime) = \int_{-\hbar k}^{\hbar k} \mathrm{d}u H(u) \sigma_{\imath\imath}(p + \hbar k + u)$$

$$\bar{\sigma}_{\imath\jmath}(p) = \int_{-\hbar k}^{\hbar k} \mathrm{d}u H(u) \sigma_{\imath\jmath}(p + u), (i \neq j)$$

$$H(u) = \frac{3}{8\hbar k}\left(1 + \frac{u^2}{\hbar^2 k^2}\right)$$

Note added in proof:

The use of a.c. Stark shift to compensate for leakiness has also been considered theoretically by Ol'shanii. He has extended his previous work [13] to the $^5\mathrm{D}_4 \Longleftrightarrow {}^5\mathrm{F}_4$ transition of Fe [20].

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