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Dynamic friction of self-affine surfaces

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Résumé. — Nous analysons la dépendance en vitesse du frottement entre deux solides limités par une surface rugueuse auto-affine comme celle d'une surface de fracture. Le solide supérieur est soumis soit à la gravité, soit à une raideur élastique externe, et est entraîné à vitesse horizontale constante V sur le solide inférieur fixe. A faible vitesse, le coefficient de friction apparent, est constant. A forte vitesse, le coefficient de friction apparent devient inversement proportionnel à la vitesse. Cette dépendance peut être reliée à la variation du temps de contact moyen, provoquée par l'apparition de sauts au cours du glissement. La transition entre ces deux régimes est caractérisée par une vitesse qui dépend de la géométrie des surfaces et de la force normale moyenne. Dans le cas d'un simple chargement par gravité, la dépendance en vitesse de la friction apparente, pour les fortes vitesses, est en $1/V^2$, où V est la vitesse de glissement imposée. Dans le cas d'une raideur élastique externe, on peut montrer l'existence à forte vitesse de deux régimes : l'un identique au cas gravitaire caractérisé par une dépendance en $1/V^2$, l'autre caractérisé par une dépendance en $1/V$. Cette deuxième transition est aussi caractérisée par une vitesse qui dépend de la rugosité et de la raideur élastique. La distribution statistique des distances de vols balistiques révèle le caractère auto-affine des surfaces de contact.

Abstract. — We investigate the velocity dependence of the friction between two rigid blocks limited by a self-affine surface such as the one generated by a crack. The upper solid is subjected either to gravity or to an external elastic stiffness, and is driven horizontally at constant velocity, V , while the lower solid is fixed. For low velocities, the apparent friction coefficient is constant. For high velocities, the apparent friction is shown to display a velocity weakening. The weakening can be related to the variation of the mean contact time due to the occurrence of jumps during the motions. The cross-over between these two regimes corresponds to a characteristic velocity which depends on the geometry of the surfaces and on the mean normal force. In the case of simple gravity loading, the velocity dependence of the apparent friction at high velocities is proportional to $1/V^2$ where V is the imposed tangential velocity. In the case of external elastic stiffness, two velocity weakening regimes can be identified, the first is identical to the gravity case with a $1/V^2$ dependence, the second appears at higher velocities and is characterized by a $1/V$ variation. The characteristic velocity of this second cross-over depends on the roughness and the elastic stiffness. The statistical distribution of ballistic flight distances is analysed, and is shown to reveal in all cases the self-affinity of the contacting surfaces.

Introduction.

The study of surfaces in moving contact is a very important technological field. The detailed physical understanding of the processes involved during contact is still an active area of research with important implications in terms of friction, striction, wear, and lubrication [1, 2]. Most solid surfaces, natural or hand made, are characterized by a roughness which is scale dependent. This has been observed for molecular surfaces [2-4], crack surfaces [5], and at much larger scale for surfaces of geological faults [6]. A lot of attention has been paid to characterize such statistical aspects of the geometry of crack surfaces both experimentally and through models. It has been shown in particular that fracture surfaces could very generally be characterized as self-affine, with a roughness exponent which seems about constant (close to 0.8 for three dimensional cracks) [5, 6]. While the geometric aspect has been very extensively considered, the physical consequences of such a property have not yet received the attention that they deserve. The microgeometry of those surfaces is supposed to be of major importance for the contact properties, since the true area of contact takes place at the asperity level and depends on the statistical distribution of those asperities. In term of friction, only a few microscale models integrates the roughness distribution [7], and those are limited to the early stages of the development of frictional sliding. There is ample experimental evidences [8] that the friction force between two fully sliding macroscopic solids in contact is dependent on the sliding velocity. In such a statement the « friction » has to be considered at least in the sense of an *apparent macroscopic friction*, and velocity dependence often results from a complex inhomogeneous local distribution of contact forces, or from a « chaotic » temporal sequence of instantaneous friction forces. Indeed such slip rate dependence has long been advocated for the stick-slip motion of two interacting elastic surfaces with frictionally slipping surfaces. The relation between the local instantaneous contact law and the resulting macroscopic apparent friction is a far-reaching fundamental problem, with potential applications ranging from geophysics [9] to mechanics [10]. As an example of the richness of microscopic to macroscopic transition, we can mention recent studies on discrete models by Carlson and Langer [11]. Depending on various constitutive parameters of the model, and in particular on the driving velocity, periodic « stick-slip » [12], chaotic motion [11-13] or smooth macroscopic apparent friction with an inhomogeneous spatial structure [14] can be found.

One possible source for an apparent velocity dependent friction lies in the potential importance of the normal degree of freedom in sliding friction. In the pioneer work of Tolstoi [15], the real area of contact along which shear strength can be transmitted is shown to depend crucially on the evolution of the separation between the two contacting bodies during the sliding. There has been experimental evidences that an increase of the sliding speed increases the mean upward vertical components of the motion as a result of vertical jumps of the slider. In turn, the asperities of the upper slider surface collide with those of the underlying surface, with an higher impact velocity [15]. This can be observed through the dependence of the amplitude of the normal natural vibrations of the slider — which depends on the contact stiffness and mass of the slider — as a function of the sliding speed. Such experimental observations have also been reported for lubricated surfaces [16]. During the jumps the friction force is obviously nul, and the temporal average of the friction force will display a velocity dependence. Furthermore if sufficiently heavy external damping of the tangential vibrations suppress the vibrations, it failed to affect the velocity weakening effect. Tolstoi *et al.* [17] report direct observations of the normal displacements of the slider — carried out at low driving speeds — showing forward movements of the undamped slider in synchronism with the upward jumps. This emphasize the fundamental role played by the occurrence of

vertical jumps of the slider and the normal stiffness of the contacting asperities in sliding friction.

This basic problem has been recently addressed [18] within the context of the seismic friction, and very recently, a renormalization-type approach [19] has been proposed to account for this phenomenon in the case of a simple geometry of the two contacting surfaces. When vertical motion is involved, the geometry of the contacting surfaces is essential, and in particular, the statistical aspects of the roughness should be accounted for in a precise way. The model described here, even though very simplistic in the description of local contacts, focuses on the relation between the geometry of the contacting surfaces and the spatio-temporal distribution of the vertical jumps. The consequences of the self-affine geometry of the two contacting surfaces — like fracture surfaces — are specifically investigated. Two extreme cases are considered for normal loading : in the first, we assume a simple gravity loading ; in the second an external elastic coupling. After a short presentation of the model, results for the two cases of normal loading are presented followed by conclusions and prospects.

2. Presentation of the model.

Let us consider two solids, each of them being limited by a self-affine rough surface with a well defined roughness exponent ζ . Two different exponents could be considered, but only one matters, in a range of scales, and without loss of generality the two exponents will be assumed to be equal. The mean surfaces are horizontal planes. Both solids are assumed to be rigid (undeformable). The lower solid is fixed, whereas the upper one is moved horizontally (x -axis) at a constant velocity V . The motion of the upper solid is restricted vertically (z -axis) by the contact with the lower solid and by a normal force which results from either simple gravity loading or an external elastic coupling. Finally, no other degree of freedom is allowed, and in particular rotations are not considered in this study. Thus, only two cases occur : either the upper body undergoes jumps, or the surfaces remain in contact. In the latter case, the contact will be possible at one single point (the probability that the contact occurs simultaneously at two distant points is zero). If $z_1(x)$ and $z_2(x)$ are the roughness profiles of the two solids, one can define the contact profile as $z(x) = \max_{x'} (z_1(x' + x) - z_2(x'))$. Thus the interactions between two self-affine rough surfaces, for a horizontal displacement x , are captured in the inequality $z \geq z(x)$. It should be noted that the function $z(x)$ does not correspond to the contact of one single asperity on the opposite surface. On the contrary, as the displacement x is varied, the contact point on either surface fluctuates very strongly. It can even be shown in particular that the trajectory of the contacting point is a Levy flight. Let us underline that the profile $z(x)$ is defined as the minimum vertical uplift required for a sliding displacement x of the slider. *It does not represent a physical profile to be found on one of the facing surfaces in general.*

As discussed in detail below, the function $z(x)$ can be shown to be self-affine with the same roughness exponent as the surfaces. The problem turns out to be strictly equivalent to that of a point particle moving along a rough self-affine profile $z(x)$ in $1 + 1$ dimensions, as sketched in figure 1. This property causes a major simplification of the problem.

The frictional behavior at each local contact point is assumed to fulfill Coulomb's law, independently of the sliding velocity, with a friction coefficient μ . In the discrete case, the macroscopic horizontal friction force F is thus a simple function of the orientation of the facet in contact. If s is the slope of the facet, F amounts to

$$F = N \frac{(s - \mu)}{(1 + \mu s)} = N \phi(s) \quad (1)$$

where N is the normal force exerted on the upper solid. In the case with only gravity loading, $N = mg$, while for the case with an external vertical elastic coupling, $N = mg + k(z - z_0)$,

Fig. 1. — Example of a trajectory (bold line) of one point on the upper solid together with the underlying quasistatic lower limit $z(x)$ (border of the shadowed region).

where z_0 is a reference vertical level, and k the elastic stiffness. F is counted algebraically in equation (1) so that a negative force means that it opposes the prescribed motion.

In the case of a quasistatic motion (infinitesimal V), the macroscopic friction coefficient will thus simply be equal to $F(x)/N(x) = \phi(dz(x)/dx)$. As the velocity V increases, the upper solid will undergo jumps when the contact surface can not be followed. Such a situation arises each time the slope decreases from one facet to the next.

In the definition of the normal force given above, the gravity effect can be absorbed in the redefinition of $z_0 \rightarrow z_0 - mg/k$. Depending on z_0 and k , two extreme cases can be clearly distinguished :

i) first, the normal force does not vary much with the vertical displacement of the slider : i.e. $kA \ll \bar{N}$ where $\bar{N}$ is the average normal force : $\bar{N} = mg - kz_0$, and A the standard deviation of $z(x)$ over the system size. In such a case, we are in a situation similar to that with a gravity acceleration, $B = \bar{N}/m$. This occurs in particular for $z_0 \ll z_{\min}$, where $z_{\min}$ is the minimum height of the z -profile ;

ii) second, the change induced in the normal load by the vertical displacements is large compared to the body force, this situation typically arises when $z_{\min} = z_0$ and $kA \gg \bar{N}$.

Intermediate cases can be regarded as the second case weighted by the probability of finding heights greater than the reference one z_0 . In the following we shall only consider the two generic cases : case A, for which the normal force results from a simple gravity loading ; and case B, for which the normal force results from an external elastic coupling.

The numerical simulation performed consists of the following steps :

i) generation of the self-affine contact profile $z(x)$, with the prescribed roughness exponent, using an algorithm of Voss [20, 21]. The self-affine exponent of the generated profile was checked independently ;

ii) construction of the trajectory of the upper solid, for an imposed velocity V . An example of such a construction is displayed in figure 1. In this second stage, the friction force is computed, as well as the average contact time, and the distribution of ballistic flight distances.

The second step is repeated for a series of velocities, and the entire computation is averaged over a number of realizations. The dependence on the system size is analyzed through the introduction of a reduced velocity, and thus we will essentially present below the results in terms of velocity, at fixed overall roughness and system size. Typically, the profile contained 2^{16} points. Conversely, the number of realizations was kept rather small (at most 200).

3. Influence of the size of the contacting solid.

Self-affine surfaces are characterized by a roughness which is intrinsically scale dependent. More precisely, the height fluctuation of the surface, defined as the standard deviation A , scales as a power-law of the system size, with an exponent ζ . If the y -axis is defined as the direction perpendicular to the sliding direction in the mean surface plane, size effects may be expected as the surface is extended along the y direction in the characteristics of $z(x)$. When the width of the surface (along the y axis) increases, the range of variation of height is increased, and therefore, the mean level of contact $(1/L) \int_0^L z(x) dx$ increases. However, the

most interesting feature of $z(x)$ is not its average level, but rather its standard deviation, A . In particular, the latter controls the distribution of angle of facets in contact and therefore the shape of the parabolic ballistic flight (initial slope), and the mean contact time.

We generated a series of $512 \times L_y$ self-affine surfaces with an exponent $\zeta = 0.8$ using an algorithm due to Voss [20, 21]. The width of the rectangles, L_y , was varied from 2 to 512 (square). The resulting surfaces were checked to have the required properties by measuring the roughness exponent directly on the generated surfaces. The measured value was found to be slightly smaller than expected $\zeta \approx 0.75 \pm 0.05$. Then the contact profile $z(x)$ was constructed by scanning through the surfaces, displaced horizontally by x , and computing the maximum overlap. The profile $z(x)$ was checked to be itself self-affine with the same exponent $\zeta \approx 0.78 \pm 0.05$ as that of the surfaces.

We studied the evolution of the standard deviation A of $z(x)$ as a function of the width of the surfaces L_y . A is shown to slightly decrease as L_y increases. This variation is however very small and tends to vanish for sufficiently large L_y . Therefore, we conclude that the size effect can be neglected.

4. Results.

4.1 CASE A : GRAVITY LOADING. — Before discussing the results, let us note that there is an intrinsic velocity scale in the problem, so that one can define a dimensionless velocity v . Let L be the profile length, and ℓ the lower cut-off distance of the self-affine description (taken as the size of a facet), and let A be the standard deviation of $z(x)$ over the system size. For realistic surfaces exhibiting a self-affine character, the lower cut-off scale ℓ could be for instance a « grain size », although other limits can be considered. For the sake of simplicity, we will call « grain size » this lower cut-off in the rest of this paper. With these parameters, the statistical properties of the profile are completely determined in our model. Using the self-affinity of the profile $z(x)$, a typical slope can be introduced at the grain size : $s \approx (A/\ell)(L/\ell)^{-\zeta}$, where ζ is the self-affine exponent. A and therefore s are proportional to the overall roughness of the contact surfaces at fixed roughness exponent, and system size.

Depending on the imposed horizontal velocity, the typical flight distance can cover any range from 0 to the system size. However it seems reasonable to expect a different behavior when the typical flight distance is smaller or larger than the only intermediate length scale that appears in our model : ℓ . This allows us to introduce a « natural » local scale for the horizontal velocity, U_ℓ , such that the typical ballistic flight distance will be comparable to the grain size. A simple computation leads to $U_\ell = \sqrt{\ell g/s}$. We can now scale the velocity with respect to U_ℓ to obtain the dimensionless reduced velocity :

$$v = V/U_\ell = V \sqrt{\frac{A}{\ell^2 g}} \left(\frac{\ell}{L} \right)^{\zeta/2} \quad (2)$$

In the numerical simulations, g was kept constant, and $\zeta = 0.8$ in order to be consistent with experimental observations on crack surfaces. The local friction coefficient μ was set to one in all cases. Similarly, the length unit was such that $\ell = 1$. The above reduced form for the velocity was checked numerically by varying L , A and V independently. In the following, we report results only in terms of this reduced velocity.

Figure 2 shows the evolution of the temporal and realisation average of the friction force as a function of the velocity. Two distinct regimes can clearly be identified : for small velocities the friction coefficient $\varphi(v) = F(v)/mg$ is constant, whereas for larger velocities, it drops abruptly, φ being proportional to v^{-2} .

Fig. 2. — Evolution of the temporal and realisation average of the friction coefficient $\varphi(v)$ (symbol $\circ$) as a function of the dimensionless velocity v , for the case of a simple gravity loading. The plain curve gives the expected friction coefficient from the measured mean contact distance (see Fig. 3).

Fig. 3. — Average ballistic distance $x^*(v)/\ell$ as a function of the reduced velocity v (symbol $\circ$), for the case of a simple gravity loading. The plain line is a power-law fit with a slope 2.

The most important effect to explain such an abrupt weakening of the friction force is the occurrence of longer and longer jumps during which the friction force is obviously zero. If we now assume that the friction force during contacts has on average the same value as the spatial average of F as given in equation (1) we can write

$$F(v) = F(0) t(v) \quad (3)$$

where $F(0)$ is the friction for zero velocity, and $t(v)$ is the fraction of time spent on contact compared to the total time $t(v) = T_{\text{contact}} / (T_{\text{contact}} + T_{\text{flight}})$. In figure 2 we have not plotted the r.h.s. of equation (3) since it cannot be distinguished from $F(v)$.

In order to progress in the understanding of the velocity weakening, we can analyze the distribution of ballistic times or distances.

We have analyzed the evolution of the mean flight distance $x^*(v)/\ell$ with the velocity, as shown in figure 3, $x^*(v)/\ell$ is clearly proportional to v^2 . This proportionality is obeyed over the entire range of velocity, including the velocities for which the apparent friction is velocity independent.

From this observation it is possible to obtain an expression for the mean friction coefficient. Indeed, the fraction of time spent on contact, $t(v)$, can be estimated to be

$$t(v) \approx \frac{\ell}{\ell + x^*(v)} \quad (4)$$

using the fact that the mean distance of continuous contact is of the order of the lower cut-off distance ℓ of the profile (i.e. the size of the grains).

A few consequences are to be noted.

i) The first regime of velocity independent friction is limited to the case where the mean distance of ballistic flights is smaller than ℓ , i.e. the lower cut-off of the self-affine regime. Below this length scale, the friction is dominated by the time spent in close contact, and thus the ballistic periods appear as negligible corrections to the static limit.

ii) The variation of $x^*(v)$ with v can be recovered using a simple heuristic argument considering the intersection of the largest parabola with the envelope of profiles starting from a given point, i.e.

$$A^*(L, \zeta) x^{*\zeta} \approx -g/2 V^2 x^{*2} + s x^*$$

where A^* is a small normalization factor. This argument leads to the conclusion that a v^2 dependence is always encountered for ζ exponents smaller than 1. This includes the case of simple rough surfaces with no size scaling. For an exponent $\zeta > 1$, a nontrivial power-law dependence occurs: $x \propto v^{2/(2-\zeta)}$. We did not push the analysis of this situation any further since the self-affine roughness exponent is generally expected to lie in the range $[0, 1]$.

iii) We note, however, that the features of the self-affine profile are still present in the macroscopic friction law, in the form of the prefactor of the velocity in equation (2).

The shape of the distribution of flight distances can also be analysed. For velocities such that $x^*(v)$ is much larger than ℓ , a typical distribution of x is shown in log-log scale in figure 4. Clearly two regimes can be distinguished: the large distance distribution consists in a peak, the position of which is roughly given by the mean value of x , i.e. x^* . This position is thus proportional to v^2 as above mentioned. For small distances, the distribution exhibits a power-law decay. This power-law reveals the self-affine nature of $z(x)$, and thus of the surfaces in contact. Indeed for small distances, the vertical displacement of the upper solid is dominated by the fluctuation of $z(x)$ in the neighborhood of the « take-off » point. Therefore, the probability that the flight distance is x is simply equal to the probability to encounter the profile

Fig. 4. — Statistical distribution $N(x)$ of ballistic flight distances x in log-log scale, for the case of a simple gravity loading. The dashed line shows the expected power-law distribution for small distances, with slope $-(2 - \zeta)$. The parameters used for this example were: $\zeta = 0.8$, $L = 2^{16}$, $v = 2.53$ and the distribution has been collected from 100 independent realizations.

at the same height as the take-off point for the first time at a distance x . In fact the latter probability distribution is one of the tools [21, 22] one can use to analyse the self-affine character of a signal. The probability varies as a power-law of x : $x^{-(2-\zeta)}$ where ζ is the roughness exponent. In figure 4 we have plotted a power-law with the expected slope.

As a consequence, despite the fact that the apparent macroscopic friction coefficient is only dependent on the self-affine nature of the surfaces through the expression of the cross-over velocity, the self-affinity could be analysed experimentally by recording the temporal fluctuations of the friction force at high velocity, possibly through acoustic measurements.

4.2 CASE B : EXTERNAL ELASTIC STIFFNESS. — This situation gives rise to a new scaling. The reduced velocity has to be constructed on the stiffness constant k rather than on the acceleration of gravity g . In order to obtain a velocity scale from k , dimensional analysis indicates that a length scale has to be invoked. The maximum normal force can be written kA , and thus kA/m plays the role of an effective gravity. This leads to the definition of a new reduced velocity

$$v = \lambda/\ell \left(\frac{\ell}{L} \right)^{\zeta/2} \quad (5)$$

with $\lambda = V/\omega$ and $\omega = \sqrt{k/m}$. It is interesting to note that the elastic stiffness introduces a natural wavelength for the jumps, λ , which does not depend on the local slope.

Figure 5 shows the evolution of the temporal and realisation averaged friction force as a function of the sliding velocity. This figure must be compared with the one in the case of a pure gravity loading (Fig. 2). The main difference is the appearance of a second velocity weakening regime with a v^{-1} dependence at higher velocities. These two velocity weakening regimes are related to the occurrence of longer and longer jumps, and the decrease of the mean contact time. Relation (3) holds for both regimes. However, in the elastic case the statistics at very high velocities is biased and under sampled. This explains why, even though relation (3) predicts correctly the change in the velocity weakening, it is slightly shifted from the computed apparent friction coefficient at very high velocities.

Fig. 5. — Same as figure 2, for the case of an external elastic loading. The computed averaged friction coefficient is shown with symbol (○). The plain curve gives the expected friction coefficient from the measured mean contact distances (see Fig. 6). The dotted line is a power law fit with slope -2 , the dashed line curves are power-law fits with slope -1 both for the computed friction and the expected friction curves.

In order to understand the transition between these two velocity weakening regimes, we have again analyzed the evolution of the mean jump length $x^*(v)/\ell$ with the velocity. Figure 6 clearly shows that above a given velocity, this jump length is no longer proportionnal to v^2 but rather to v . In particular :

i) for the first weakening regime at low enough velocities, the mean jump distance remains small compared to the natural wavelength, and we face a situation similar to the gravity case, using the above given correspondence with k , with a v^{-2} friction dependence. In this regime, as shown for the ballistic flights, the mean jump distance increases as v^2 while the natural wavelength only increases as v ;

Fig. 6. — Same as figure 3, for an external elastic loading. The average jump distances $x^*(v)/\ell$ are shown with symbol (○). The dotted line is a power-law fit with slope 2, while the dashed line is a power-law fit with slope 1.

ii) for the second weakening regime at higher velocities, the mean jump distance is of the order of the natural wavelength which acts as a natural upper cut-off. As expected from relation (3), and observed in figure 5, the friction coefficient scales now as v^{-1} since now the mean jump distance can only increase as the natural wavelength with v . The variation of $x^*(v)$ with v can be recovered using the same simple heuristic argument considered above where the parabola is replaced by

$$-g/\omega^2(\cos\{|\omega/V|x\} - 1) - s|V/\omega|\sin\{|\omega/V|x\},$$

for high V and $|V/\omega| \approx x$;

iii) the cross-over between these two regimes is governed by a characteristic velocity which must depend on the elastic stiffness and the roughness of the surface.

In figure 6, a cross-over can be observed at the beginning for small mean jump distances, before the v^2 regime. For this regime, it can be shown that the mean jump distance will be

$$\langle x \rangle \approx 2sv^2/g^* \langle 1/z_d - z_{\min} \rangle$$

where $g^* = k/m$ and z_d is the height of the « taking-off » point. This leads to a logarithmic divergence for $z \rightarrow z_{\min}$, so that the average becomes sensitive on the lower cut-off, which depends on the velocity. This small effect is suppressed as soon as one superimposes a gravity loading to the elastic force.

The shape of the distribution of jump lengths can also be analyzed. As for the gravity case, a typical distribution of x is shown in figure 7 for velocities such that $x^*(v)$ is much larger than ℓ . Again, it can be noticed that for small distances, the vertical displacement of the upper slider is dominated by the fluctuation of $z(x)$ in the neighborhood of the « take-off » point, and the probability varies as a power-law of x : $x^{-(2-\zeta)}$. The tail of the distribution is smoother than that observed for the gravity case.

Fig. 7. — Same as figure 4, for the case of an external elastic loading. The parameters used for this example were: $\zeta = 0.8$, $L = 2^{16}$, $v = 2.8$, and the distribution is collected from 100 independent realizations. The dashed line curve is a power-law fit with slope $-(2 - \zeta)$, for small jump lengths.

5. Conclusion.

One has to bear in mind that the model studied here is indeed too simple to readily apply to most realistic situations, and is only meant to provide a simple reference case where the velocity dependence of the friction can be fully analyzed in terms of the geometry of the contact surfaces. A major limitation is the fact that both solid bodies in contact are undeformable. For two deformable bodies in contact one can expect, besides local non-linear dissipations, two major changes : i) first, the contact point will become a contact *region* with a finite extent and a complex distribution of normal and tangential stress ; ii) moreover, more than one contacting region could be obtained leading at least to two additional length scales — the size of the contacting region and the distance between different contacts. Complicated comminution processes are also known to take place at the contacting regions leading to a rather complicated problem with a third body.

Among possible extensions of our model, a special attention should be paid to account for :

i) the inclusion of a finite anelastic local dissipation associated with the impact on the asperities of the surfaces. In this respect, the present model can be regarded as one with an infinite dissipation. Recent numerical simulations in this direction indicate that such local dissipation may affect the velocity weakening effect ;

ii) the case of two matching surfaces produced by fracture and with no previous sliding history. In such a case, in order for the sliding displacement to be possible those two surfaces would have to disengage themselves leading to an initial mean vertical displacement which is a power-law of the sliding distance. This give rise to a slip weakening rather than a velocity weakening ;

and iii) the driving mechanism which is another parameter that can be explored since in our model the a velocity was kept constant.

Interestingly enough, besides all these limitations and further possible extensions, the model allowed us to analyze the velocity weakening of the apparent friction coefficient of two rough solids in contact in terms of the geometry of the contacting surfaces. The apparent friction force is computed by considering explicitly the separation between the two surfaces during the sliding motion. In that sense our approach is related to that of Tolstoi [15]. Two generic cases were considered for the normal force : a pure body force and an external elastic stiffness. The mean friction coefficient has been cast in a simple expression which only depends on a characteristic velocity. The latter has been related to the statistical features of the surfaces. The velocity weakening of the apparent friction force has been shown to depend on the type of normal loading. In the case of an external elastic stiffness, a typical $1/v$ weakening has been identified for large velocities.

Finally, the statistical distribution of ballistic flight distances has been shown to reveal the self-affinity of the contact surfaces. It would be of high interest to test these predictions on laboratory test samples.

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